Dissipative fluid in Brans-Dicke theory and late time acceleration

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(Received 27 November 2000; published 3 April 2001)

We investigate the possibility of having a late time accelerated expansion phase for the universe. We use a dissipative fluid in Brans-Dicke (BD) theory for this purpose. The model does not involve any potential for the BD scalar field. We obtain the best fit values for the different parameters in our model by comparing our model predictions with SNIa data and also with the data from the ultracompact radio sources.

DOI: 10.1103/PhysRevD.63.107501 PACS number(s): 04.20.Jb, 98.80.Hw

A number of recent observations [1] suggest that Ω_m, the ratio of the matter density (baryonic+dark) to the critical density, is significantly less than unity suggesting that either the universe is open or that there are some other sources of this missing energy which makes $\Omega_{total}\sim 1$. The recent findings of BOOMERANG experiments [2] strongly suggest the second possibility of a flat universe. At the same time, the measurements of the luminosity-redshift relations observed for the 50 newly discovered type Ia supernova with redshift $z>0.35$ [3] predict that at present the universe is expanding in an accelerated fashion, suggesting the existence of a total negative pressure for the universe.

One of the possibilities is the ΛCDM model consisting of a mixture of vacuum energy or cosmological constant Λ and cold dark matter (CDM). But as the vacuum energy remains constant and the matter energy density decreases, it is necessary that their ratio must be set to a specific infinitesimally small value (10^{-120}) in the early universe so as to nearly coincide today. This is the so-called ''cosmic coincidence'' problem. Another possibility is ''quintessence'' [4], a dynamical, slowly evolving, spatially inhomogeneous component of energy density with negative pressure. An example is a time dependent scalar field slowly rolling down its potential [5]. Recently a new form of the quintessence called ''tracker field'' has been proposed to solve the cosmic coincidence problem. It has an equation of motion with an attractorlike solution in a sense that for a wide range of initial conditions the equation of motion converges to the same solution [6]. There are a number of quintessence models which have been put forward, most of which involve a minimally coupled scalar field with potentials either exponential [7] or power law [8]. There are also treatments with the nonminimally coupled scalar fields with different type of potentials [9]. It has been shown by Di Pietro and Demaret [10] that for the constant scalar field equation of state, which is a good approximation for a tracker field solution, the field equations and the conservation equations strongly constrain the scalar potential; the widely used potentials for quintessence such as the inverse power law, exponential, and the cosine form, are incompatible with these constraints. Hence it may be worthwhile to search for a model which will not involve any potential arising from a particle physics scale.

Negative pressure can also occur if the CDM fluid is not a perfect fluid but a dissipative one. Recently, it has been proposed that the CDM must self-interact in order to explain the detailed structure of the galactic halos [11]. This self-interaction will naturally create a viscous pressure whose magnitude will depend on the mean free path of the CDM particles. In a very recent work Chimento *et al.* have shown that a mixture of minimally coupled self-interacting scalar field and a perfect fluid is unable to drive the accelerated expansion and solve the cosmic coincidence problem at the same time [12], while a mixture of a dissipative CDM with bulk viscosity and a minimally coupled self-interacting scalar field can successfully drive an accelerated expansion and solve the cosmic coincidence problem simultaneously. An effective negative pressure in CDM can also be created from cosmic antifriction, which is closely related with the particle production out of the gravitational field, and can have similar dynamics like the ΛCDM model as a special case of this cosmic antifriction [13].

The present work investigates the possibility of obtaining an accelerated universe in Brans-Dicke (BD) theory with a dissipative fluid. Previously, Bartolami and Martins and Sen and Seshadri [9] have investigated such a possibility in BD theory with a perfect fluid. But both considered the potential for the BD scalar field itself that was not so in the original BD theory [14]. But in this work, we have not considered any potential for the BD scalar field. We have compared our solutions with the experimental data [3] to constrain the different parameters in our model. This simple enough model can be useful if one has to explain the quintessence model without scalar field potential.

For a flat Friedmann-Robertson-Walker (FRW) universe, with a scale factor $R(t)$, assuming the matter content is a dissipative fluid with only bulk viscosity, the BD field equations are

$$3\frac{\dot{R}^2}{R^2}=\frac{\rho_m}{\phi}+\frac{\rho_\phi}{\phi}, \tag{1}$$

$$2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}=-\frac{(p_m+\pi)}{\phi}-\frac{p_\phi}{\phi}, \tag{2}$$

$$\ddot{\phi}+3\frac{\dot{R}}{R}\dot{\phi}=\frac{\rho_m-3p_m-3\pi}{2\omega+3}, \tag{3}$$

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where ρ_ϕ and p_ϕ are the energy density and pressure corresponding to the BD scalar field and are given by

$$\rho_\phi=\left[\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi}-3\frac{\dot{R}}{R}\dot{\phi}\right], \tag{4}$$

$$p_\phi=\left[\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi}+\ddot{\phi}+2\frac{\dot{R}}{R}\dot{\phi}\right]. \tag{5}$$

The energy conservation equation for the matter field, which is not an independent equation but can be obtained using Eqs. (1)–(3) is given by

$$\dot{\rho}+3(\dot{R}/R)(\rho_m+p_m+\pi)=0. \tag{6}$$

We are considering a late time, matter-dominated universe, hence $p_m=0$ in our case.

Dissipative effects in FRW cosmology, i.e., negative π can be modeled in two ways: First the conventional bulk viscous effect in a FRW universe can be modeled within the framework of nonequilibrium thermodynamics proposed by Israel and Stewart [15]. In this theory, the transport equation for the bulk viscous pressure π takes the form

$$\pi+\tau\dot{\pi}=-3\zeta H-\frac{\tau\pi}{2}\left[3H+\frac{\dot{\tau}}{\tau}-\frac{\dot{T}}{T}-\frac{\dot{\zeta}}{\zeta}\right], \tag{7}$$

where the positive definite quantity ζ stands for the coefficient for the bulk viscosity, T is the temperature of the fluid, and τ is the relaxation time associated with the dissipative effect, i.e., the time taken by the system to reach the equilibrium state if the dissipative effect is suddenly switched off. Provided the factor in the square bracket is small, one can approximate Eq. (7) as the simple form

$$\pi+\tau\dot{\pi}=-3\zeta H, \tag{8}$$

which is widely used in the literature. One can also assume that the viscous effects are not so large as observations seem to rule out huge entropy productions in large scales [16]. The relation $\tau=\zeta/\rho_m$ can be assumed so as to ensure the viscous signal does not exceed the speed of light [17] and also $(\tau H)^{-1}=\nu$ where $\nu>1$ for a consistent hydrodynamical description for the fluid [18]. With these assumptions, Eq. (8) becomes

$$\nu H+\dot{\pi}/\pi=-3\rho_m H/\pi. \tag{9}$$

Also as demonstrated in a recent paper by Zimdahl *et al.* [13], one can also have a negative π if there exists a particle number nonconserving interaction inside the matter. This may be due to the particle production out of gravitational field. In this case, the CDM is not a conventional disspative fluid, but a perfect fluid with a varying particle number. Substantial particle production is an event that occurs in the early universe. But Zimdahl *et al.* have shown that an extremely small particle production rate can also cause the sufficiently negative π to violate the strong energy condition.

In our case, we are not *a priori* assuming any specific model for this negative π, rather only assuming the existence of a negative π; we have investigated the possibility of having the accelerated phase of the universe in BD theory, which is comparable with the observational estimates [20]. Using Eqs. (1)–(3) one can write

$$6\frac{\ddot{R}}{R}+6\frac{\dot{R}^2}{R^2}=2\omega\frac{\ddot{\phi}}{\phi}+6\omega\frac{\dot{R}\dot{\phi}}{R\phi}-\omega\frac{\dot{\phi}^2}{\phi^2}. \tag{10}$$

To solve the system of equations we have assumed the following relation between the scale factor $R(t)$ and the BD scalar field ϕ:

$$\phi=AR^\alpha, \tag{11}$$

where A and α are constants. With Eq. (11), Eq. (10) becomes

$$\dot{H}+\beta H^2=0, \tag{12}$$

where $\beta=(12-\omega\alpha^2-6\omega\alpha)/(6-2\omega\alpha)$. Equation (12) on integration yields $R=R_0(t/t_0)^{1/\beta}$, where R_0 and t_0 are positive constants. One can identify t_0 as the present epoch, i.e., the age of the universe. Now from Eq. (11) one writes $\phi=\phi_0(t/t_0)^{\alpha/\beta}$, where $\phi_0=AR_0^\alpha$. We will assume $\phi_0=1$ without any loss of generality in our subsequent calculations. The solutions for other physical quantities now become

$$\rho_m=\frac{1}{\beta^2t_0^2}\left(\frac{t}{t_0}\right)^{\alpha/\beta-2}\left[3-\frac{\omega}{2}\alpha^2+3\alpha\right], \tag{13}$$

$$\rho_\phi=\frac{1}{\beta^2t_0^2}\left(\frac{t}{t_0}\right)^{\alpha/\beta-2}\left[\frac{\omega}{2}\alpha^2-3\alpha\right], \tag{14}$$

$$p_\phi=\frac{1}{\beta^2t_0^2}\left(\frac{t}{t_0}\right)^{\alpha/\beta-2}\left[\left(\frac{\omega}{2}+1\right)\alpha^2+2\alpha-\alpha\beta\right], \tag{15}$$

$$\pi=-\frac{1}{\beta^2t_0^2}\left(\frac{t}{t_0}\right)^{\alpha/\beta-2}\left[3-2\beta+\left(\frac{\omega}{2}+1\right)\alpha^2+2\alpha-\alpha\beta\right]. \tag{16}$$

In these solutions we have essentially three parameters ω, β, and α, which are related [see just after Eq. (12)]. One has to ensure that the universe is accelerating, i.e., $0<\beta<1$ and also the density parameters for the matter and the scalar field are of the same order at present time, i.e., $\Omega_{0m}\sim\Omega_{0\phi}$. These will constrain the different parameters.

We obtain the best-fit value of β by comparing our model predictions with the SNIa data. We use the high-z data of the Supernova Cosmology Project (SCP; Perlmutter *et al.*, 1998) [3] and the low-z data from the Calan-Tololo survey (Hamuy *et al.*, 1996) [19] for our study. Of the 60 data points, we use 54 data points for our analysis (Fit C–D of the SCP data; for details of the excluded data points see Perlmutter *et al.*, 1998) [3]. The SNIa data exist up to $z\simeq0.9$. At larger red-

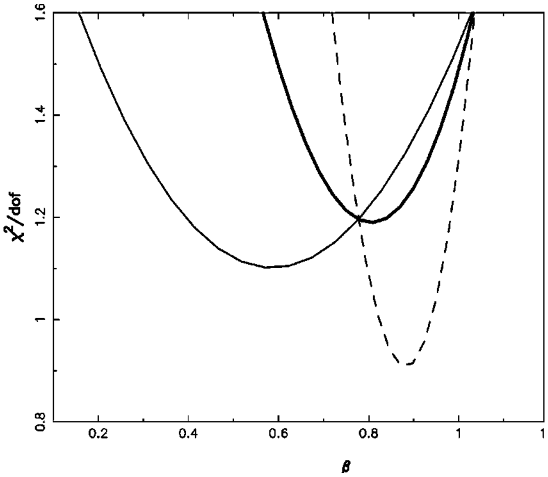

FIG. 1. χ^2/DOF is shown for SNIa (thin solid line) and ultra-compact radio sources (dashed line). The thick solid line shows the result of the joint analysis of the two data sets.

shifts, we use the data of ultracompact radio sources $(0.55 \leqslant z \leqslant 3.32$; 16 measurements) to constrain the value of β [20].

The χ^2/degree of freedom (DOF) of the comparison of our model with the two data sets is shown in Fig. 1. The joint analysis of the two data sets gives a best-fit value $\beta = 0.8$ with $\chi^2/\mathrm{DOF} = 1.18$. The good-of-fit probability for the fit $Q = 0.12$, and the 1σ error on β is $\Delta\beta = 0.05$.

In a very recent paper, using the data for the angular power spectrum of the cosmic microwave background obtained by MAXIMA-1, together with the measurements of high redshift supernova, Balbi *et al.* [21] have constrained density parameters for matter to be $0.25 < \Omega_{m0} < 0.5$. We have used this range of Ω_{m0} together with the value of β obtained above by fitting our model with a different observation, to constrain ω and α.

We have plotted in Fig. 2 the parameters ω and α for $\beta = 0.5$ and $\beta = 0.8$ and also for $\Omega_{m0} = 0.3$ and $\Omega_{m0} = 0.5$. For this we have used the relation between β, ω, and α and also Eq. (13). The two values of β correspond to the current age of the universe, 28 Gyr and 18 Gyr, respectively, where we have assumed that the present Hubble constant is $H_0 \sim 0.67 \times 10^{-10}$ per year. One can see the ranges of the two parameters α and ω, for which these values of β and Ω_{m0} are consistent, are approximately $-1.2 \leqslant \alpha \leqslant -0.8$ and $-2.5 \leqslant \omega \leqslant -1.5$. For these ranges of α, the present day variation of G, $|\dot{G}/G|_0 = |\alpha| H_0 < 10^{-10}$ per year [22]. One can also check from Eqs. (15) and (16), using these range of parameters, that both p_ϕ and π remain negative in these ranges. Also one can write from Eq. (9)

$$\nu = \frac{3[-5 - 5\alpha + \alpha^2(1 + 3\omega + \omega^2)]}{(\omega\alpha - 3)(\alpha\omega + \alpha - 1)}.$$

In order to have $\nu > 1$, which is essential for hydrodynamical description if the CDM is assumed to be a conventional viscous fluid, one cannot have a particular range for α and ω consistent with the ranges given above. Instead, for a particular value for α within the range given above, one can have a range for ω. As an example, for $\alpha = -1.2$, the range for ω to have $\nu > 1$ is $-2.25 < \omega < -1.8$. For $\alpha = -1$, it is $-2.5 < \omega < -2$ and for $\alpha = -0.8$, it is $-2.5 < \omega < -2.25$. One can see that these ranges of ω are consistent with the ranges shown in Fig. 2.

In conclusion, in recent years it has been shown that a mixture of perfect fluid and quintessence may be an interesting candidate to explain a spatially flat universe currently expanding in an accelerated manner. In these models, a minimally coupled scalar field rolling down its potential has been used to drive the accelerated expansion and also to account for the missing energy of the universe. But all these models necessarily require several fine tunings [23] of different parameters. Also, in a very recent work, Chimento *et al.* [12] have shown that one cannot simultaneously solve the cosmic coincidence problem and have a late time acceleration in FRW cosmology with a mixture of perfect fluid CDM and Q matter. On the other hand, recent investigations have predicted that CDM should be self-interacting rather than collisionless, in order to successfully explain the less dense galactic halos. Hence it is not unreasonable to think that this self-interaction may give rise to dissipative pressure π at cosmological scales. In their work, Chimento *et al.* have shown that a mixture of dissipative CDM and Q matter can indeed explain the late time acceleration and can solve the cosmic coincidence problem simultaneously. But all of these quintessence models, whether mixed with perfect fluid, or dissipative fluid suffer the problem of unwanted long range forces and the quintessence cannot be as homogeneous as it should [24].

In this work, we have investigated the possibility of having a late time acceleration without any quintessence fields. We have used a dissipative CDM model in BD theory for this purpose. The viscous pressure, together with negative pressure due to the BD scalar field, drive the late time acceleration. The BD scalar field has been used to account for the missing energy of the universe. We also have not used any potential for the BD scalar field, unlike the other nonminimally coupled scalar field models in literatures [9]. The model is simple enough and does not require much fine tuning. We have three arbitrary parameters in our model which are related through an equation. We have constrained one of the parameters β by fitting our model with the experimental

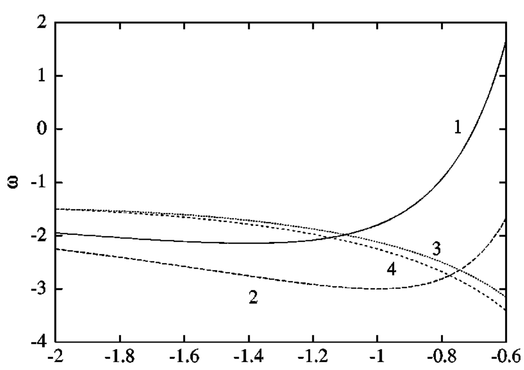

FIG. 2. The parameter space ω-α for different values of β and Ω_{m0}; (1) $\Omega_{m0} = 0.5$, (2) $\Omega_{m0} = 0.3$, (3) $\beta = 0.8$, (4) $\beta = 0.5$.

data from supernova and also from the ultracompact radio sources. The other two parameters α and ω have been constrained using this value of β and assuming also that the density parameters due to matter and the BD scalar field are of the same order today. These constraints give negative values for ω. However, the standard limit on ω is $\omega > 500$ to account for the solar system tests which sets tight constraints on post Newtonian deviations from general relativity [22]: $|\gamma_0 - 1| < 2\times 10^{-3}$ and $|\beta_0 - 1| < 2\times 10^{-3}$, where β_0 and γ_0 denote the usual post Newtonian parameters. However, these constraints come from the weak field limit of the theory. One should also keep in mind that in extended inflation, the model of La and Steinhardt [25] worked provided that ω takes a value close to 20, which is also not compatible with the solar system tests. It should also be mentioned that in order to explain the structure formation successfully in this scalar tensor theory the constraint on ω is not at all compatible with the solar system tests [26]. A negative ω is also predicted by the effective models coming from the Kaluza-Klein and superstring theories [27]. Hence it always remains a problem finding a compatibility between the astronomical observations and cosmological requirements.

The problem is to apply a theory in different scales (astronomical and cosmological) whereas experiments so far have been made only for astronomical scales. We have applied the theory to cosmological scales where there is still now no experimental tests for these scalar tensor theories, future data from supernova at higher redshift may confirm or rule out existence of the scalar partner for the graviton.

It is also important to note that as β is constant in our calculation, the universe is always in the accelerating phase, which seriously contradicts the primeval nucleosynthesis and the structure formation scenario. One way to avoid this problem is to consider ω not as a constant but as a function of the scalar field ϕ. In a recent interesting paper [28], Banerjee and Pavon have shown that with ω as a polynomial function of ϕ, one can get both radiation dominated era in the early time and accelerating phase in the late time. But in that case also asymptotically ω acquires a small negative value to have a late time accelerating phase.

Allowing ω to be a function of ϕ or redshift z to have both decelerating and accelerating phases at different times while local inhomogeneities giving rise to large value of ω consistent with solar system test, should be the complete investigation. But this will involve a detailed computational effort which is beyond the scope of this paper. What we want to stress is, if we can explain the quintessence in Brans-Dicke theory without any potential (future observations will predict whether we can or cannot) then even if ω is scale dependent, in some scale it has to satisfy the constraints given in our paper in order to explain the late time acceleration and the cosmic coincidence.

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