Vanishing of the cosmological constant in nonfactorizable geometry

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We generalize the results of Randall and Sundrum to a wider class of four-dimensional space-times including the four-dimensional Schwarzschild background and de Sitter universe. We solve the equation for graviton propagation in a general four dimensional background and find an explicit solution for a zero mass bound state of the graviton. We find that this zero mass bound state is normalizable only if the cosmological constant is *strictly* zero, thereby providing a dynamical reason for the vanishing of cosmological constant within the context of this model. We also show that the results of Randall and Sundrum can be generalized without any modification to the Schwarzschild background.

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I. INTRODUCTION

Any realistic theory of gravity should be able to reproduce the r^{-1} behavior of the gravitational potential in the Newtonian limit. Generically, the potential falls off like r^{-d+3}, where d is the number of extra dimensions with infinite extend. Thus, to obtain r^{-1} behavior, higher dimensional theories of gravity have been assuming compactness ($\approx$ Planck length) of the extra dimensions. Thus, in the conventional way of extracting an effective lower-dimensional theory from higher dimensions, one performs a Kaluza-Klein reduction in which the extra dimensions are warped up into a compact space (of the order of Planck length) such as a torus or a sphere (see Ref. [1] and references therein). Provided that the scale size of these internal dimensions is sufficiently small in relation to the energy scale of excitations in the lower dimension, then the mass gap separating the massless modes from the massive modes will be sufficient to ensure that the internal dimensions are essentially unobservable, and the world will essentially appear to be effectively lower dimensional. If an extra dimension is noncompact, there would be continuum modes with masses extending down to zero, when seen from the lower-dimensional viewpoint.

Recent developments in string theory have shown that if matter fields are localized on a 3-brane in $1+3+d$ dimensions, while gravity can propagate in the extra dimensions, then the extra dimensions can be large [2,3]. In this scenario, the Planck scale M_P is traded for the size of the extra dimensions felt by gravity. Likewise, gauge coupling unification can be preserved and remain perturbative, but it now occurs at scales as low as a TeV. One can therefore have gravity and gauge coupling unification occurring at as low a scale as a few hundred GeV to 1 TeV. This new scenario has been claimed to be experimentally testable [4] and offers a simple qualitative explanation to the fermion mass hierarchy problem [6].

In these large extra spatial dimensions, deviations from Newtonian potential will be detected at the scale of the extra dimensions. The form of the Newtonian potential can be obtained for a pointlike mass, in these models, by means of Gauss' law [2]. Denoting by r the radial distance in $4+d$ dimensions and by r_b the radial distance as measured on the 3-brane, we find for distances r much greater than the typical size of the extra dimension L a potential of the form

$$V_{(4)} = -G_N \frac{M}{r_b}, \tag{1}$$

where $G_N = m_p^{-2}$ is Newton's constant in four dimensions. On the other hand, for $r \ll L$ the potential becomes

$$V_{(4+d)} = -G_{(4+d)} \frac{M}{r^{1+d}}, \tag{2}$$

with $G_{(4+d)} = M_{(4+d)}^{-2-d} = L^d G_N$. This implies that the huge Planck mass $m_p^2 = M_{(4+d)}^{2+d} L^d$ and, for sufficiently large L and d, the bulk mass scale $M_{(4+d)}$ (eventually identified with the fundamental string scale) can be as small as 1 TeV. Since

$$L \sim [1 \ \ \mathrm{TeV}/M_{(4+d)}]^{1+2/d} 10^{31/d-16} \ \ \mathrm{mm}, \tag{3}$$

demanding that Newton's law is not violated for distances larger than 1 mm restricts $d \geqslant 2$ [2,5]. Further bounds are obtained by estimating the production of Kaluza Klein gravitons and support higher values of d [9].

On the other hand, Randall and Sundrum (RS) [7] have shown that these extra dimensions in five-dimensional space-times need not be compact. They have shown that for $d=1$, gravity can be localized on a single 3-brane (where the standard model particles are confined) even when the fifth dimension is infinite. The noncompact localization arises via the exponential warp or conformal factor in the nonfactorizable metric:

$$ds^2 = \exp(-2k|y|)[dt^2 - d\mathbf{x}^2] - dy^2. \tag{4}$$

The metric signature we adopt is $(+----)$. For $y \neq 0$, this metric satisfies the five dimensional Einstein's equation with negative five dimensional cosmological constant, $\Lambda \approx -k^2$. The brane is located at $y=0$, and the induced metric on the brane is a Minkowski metric. The bulk is a five dimensional anti–de Sitter metric, with $y=0$ as boundary, so

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that $y<0$ is identified with $y>0$, reflecting the Z_2 symmetry with the brane as fixed point, that arises in the string theory.

Perturbations of the metric (4) shows that the Newtonian potential on the brane is recovered at lowest order:

$$V(r)=\frac{GM}{r}\left(1+\frac{2}{3k^2r^2}\right). \tag{5}$$

Thus, the four dimensional gravity is recovered at high energies, with a first-order correction that is constrained by current submillimeter experiments [8]. The zero mode produces the standard $1/r$ gravitational potential along the brane, and the Kaluza-Klein modes give rise to corrections of order $1/r^3$. [The general line element of the form in Eq. (4) has been obtained earlier by Gogberashvili [10] by setting the momentum toward the large extra, fifth, dimension to be zero.]

The corrections to the Newtonian gravitational potential $V_N(r)\propto(m_1m_2/r)$ have been investigated earlier by several authors from different points of view. Duff [11] had obtained a similar result by computing the one-loop corrections to the (flat) graviton propagator. In his analysis, the single graviton exchange provided the linearized Schwarzschild line element, which in the weak field limit is the standard $1/r$ potential and the inclusion of the quantum corrections to one-loop order modifies gives rise to corrections of order $1/r^3$. Since the lowest order corrections have to be linear in $G\hbar$, it is obvious from dimensional grounds that the correction will multiply V_N by a factor of the form $[1+a(G\hbar/c^3r^2)]$ where a is numerical coefficient. [While this is the leading *quantum* correction, it may be noted that the lowest order post-Newtonian approximation will give a correction of the form $[1+b(G(m_1+m_2)/c^2r)]$, where b is a numerical factor, which has a slower fall-off with distance.]

Danoghue [12] has obtained similar results by treating gravity as an effective field theory. He argues that the leading quantum corrections, in powers of the energy or inverse powers of the distance, can be computed in quantum gravity through the knowledge of the low-energy structure of the theory (effective field theory). He shows that the one loop corrections to the graviton propagator gives the $1/r^3$ corrections to the Newtonian potential. He also emphasizes that the correction to low energy gravity, treated as an effective theory is remarkably unique and the leading quantum correction to the potential is $(1/r^3)$. (There have been other similar analysis in the literature where the classical and quantum corrections to the Newtonian potential have been calculated. See, for example, Ref. [13].)

In the case of RS, there is no background Schwarzschild metric and they merely study the graviton perturbations around the *flat* four-dimensional spacetime. Their approach is essentially to look at the corrections to the graviton propagator arising from a set of continuum states with mass $m>0$. The analysis by itself is classical and indeed, the corrections to V_N which they find does not depend on $\hbar$ directly; of course, they provide an *interpretation* which is quantum mechanical. In contrast, much of the earlier work, concentrated on quantum gravitational corrections, have used the background Schwarzschild line element.

This raises the question: Is it possible to generalize the ideas of RS to a situation in which the four-dimensional metric is nontrivial (say, a Schwarzschild metric or de Sitter universe)? Will we get the same mass spectrum for the graviton modes and the same correction term to $V_N(r)$? The fact that Duff and Danoghue obtained similar results suggests that this could be the case — though it needs to be explicitly demonstrated.

In this paper, we show that the main results of RS have a simple mathematical origin and can indeed be generalized to a wider class of models. We will provide a *general* solution to the zero mass graviton mode in *arbitrary* background and — as an illustration — will work out explicitly the case that incorporates a spherically symmetric solution in four dimensions. (This will include as special cases, the Schwarzschild and de Sitter manifolds.) It is important to show that the properties of the graviton propagation, and the effective gravitational potential do not change under such a generalization. We shall provide exact solutions which demonstrate that such is indeed the case; these solutions also provide some insight into the structure of the solution and will possibly allow us to study — for example — models for black hole evaporation in this context.

In Sec. II, we will solve the equation for graviton propagating in general four dimensional space-time and obtain an explicit solution for the zero mass bound state of the graviton. In Sec. III, we perform the analysis for the four dimensional spherically symmetric space-times and show explicitly that the four dimensional cosmological constant should vanish. Finally in Sec. IV we summarize the results and discuss the implication of the result in the compactified Randall-Sundrum model.

II. GENERALIZATION OF RANDALL-SUNDRUM MODEL

In this section, we study the plane wave gravitons, $h_{\mu\nu}$, propagating in the five dimensional space-time,

$$ds^2=g_{ab}dx^adx^b=\exp[-2a(y)][g^{(4)}_{\mu\nu}dx^\mu dx^\nu]-dy^2, \tag{6}$$

with the condition that it satisfies the full five dimensional Einstein's equation with the five dimensional cosmological constant. We use the lowercase Latin letters for the full five dimensions and the lowercase Greek letters for four dimensions. (We follow the notation of RS closely to provide easy comparison.)

Denoting the perturbed metric by $\tilde{g}_{ab}=g_{ab}+h_{ab}$ and using the gauge

$$h_{55}=h_{5\mu}=0, \quad \nabla^\mu h_{\mu\nu}=0, \quad h^\mu_\mu=0, \tag{7}$$

it is easy to see that $h_{\mu\nu}$ can be written as plane wave gravitons, i.e., $h_{\mu\nu}=e_{\mu\nu}\Phi$ where $e_{\mu\nu}$ is the polarization tensor. The equation satisfied by Φ can be separated with the ansatz $\Phi(x^\mu,y)=A(x^\mu)Z(y)$. Substituting into the wave equation, separating the variables using a constant m^2, we find that A satisfies the standard wave equation for a particle of mass m while Z satisfies the equation

$$\frac{d^2Z}{dy^2}+(-4\dot{a}^2(y)+2\ddot{a}(y)+m^2\exp[2a(y)])Z=0. \quad (8)$$

(The essential steps leading to the above equation are given in the Appendix.) This reduces to Eq. (8) of RS, when we use their solution $a(y)=k|y|$. We are interested in the allowed range of values for m and whether we can get an acceptable solution for $m=0$. By inspection, it is clear that this equation has a solution for $m=0$, given by

$$Z=\exp[-2a(y)]. \quad (9)$$

In fact, this is *precisely* the ground state wave function which RS obtain (after a series of algebraic transformations) for their special case of $a(y)=k|y|$. The physical meaning, mathematical simplicity and generality of the result is hidden by (i) their transformations and (ii) the fact that they never give $\psi(y)$ but only $\hat{\psi}(z)$ in their paper. [Note that Eq. (9) is a valid solution to Eq. (8) with $m=0$ a long as $a(y)$ is continuous even if its derivative is discontinuous at the origin.]

This is the first result of this paper and shows that the existence of a zero mass graviton is a very general result and does not require much of the extra assumptions in RS except that Z should be well behaved and *normalizable* as a function of y, in the relevant range. (Note that the ground state wave function for an arbitrary four dimensional line element is exactly the conformal or warp factor in the generalized Randall-Sundrum model.) This clearly shows that the stability of the 3-brane can be explicitly shown in the RS model by obtaining the zero mass graviton wave function which is well behaved and normalizable. The question arises as to the conditions under which we will obtain a normalizable function for $Z(y)$. Such an analysis for a general $a(y)$ is complicated and hence we will illustrate it explicitly for a special case. In the next section, we take a simple case by assuming that the four dimensional spacetime is spherically symmetric and show that for the case of nonzero four dimensional cosmological constant, the zero mass ground state wave function is non-normalizable.

We would also like to point out the following point: The other eigenvalues and eigenfunctions can be found by converting Eq. (8) into an eigenvalue equation for m^2. In general, an equation of the form

$$\frac{d^2S}{dx^2}+(EV(x)-4k^2)S=0 \quad (10)$$

[where E and k^2 are constants, $V(x)$ is a continuous function of x] can be transformed to an eigenvalue equation for E by changing the independent variable from x to z by

$$z=\int dx V(x)^{1/2} \quad (11)$$

and dependent variable from S to $\hat{S}=SV^{-1/4}$. This will give a modified Schroedinger equation of the form

$$\frac{d^2\hat{S}}{dz^2}+\left[-\frac{1}{16}\left(\frac{d(\ln[V(x)])}{dz}\right)^2+\frac{4k^2}{V(x)}-\frac{1}{4}\frac{d^2(\ln[V(x)])}{dz^2}\right]\hat{S}=-E\hat{S}, \quad (12)$$

where, x in the above expression is expressed in terms of z using Eq. (11).

III. SPECIAL CASE: SPHERICALLY SYMMETRIC SPACE-TIME

In the previous section, we have shown that the existence of the zero mass graviton is a very general result in the case of the RS model. However, the analysis of the normalization of (zero mass) ground state wavefunction for a general four dimensional space-time is complicated. Here, we take a simple case by assuming that the four dimensional spacetime is spherically symmetric and is of the form

$$ds^2=\exp(-2a(y))[A(r)dt^2-B(r)dr^2-r^2d\Omega^2]-dy^2, \quad (13)$$

where $d\Omega^2$ is the angular line element and $a(y)$, $A(r)$ and $B(r)$ need to be determined via the five-dimensional Einstein's equations. We consider the latter to be of the form

$$G_{ab}=\Lambda g_{ab} \quad (14)$$

with possible nonzero vacuum energy density Λ in five dimensions. Inserting the ansatz (13) for the metric, the only nonvanishing components of the Einstein tensor, G, are the diagonal components. The Einstein's equation, for (00) and (11) components, reduces to

$$\frac{1}{r^2}-\frac{1}{r^2B(r)}+\frac{B'(r)}{rB^2(r)}=\exp[-2a(y)]R(y) \quad (15)$$

$$-\frac{1}{B(r)}\left[\frac{1}{r^2}-\frac{B(r)}{r^2}+\frac{A'(r)}{rA(r)}\right]=\exp[-2a(y)]R(y) \quad (16)$$

where

$$R(y)=\Lambda+6\dot{a}^2(y)-3\ddot{a}(y), \quad (17)$$

and the prime denotes derivative with respect to r. Combining the two equations, we obtain $B(r)=1/A(r)$. Substituting for $B(r)$ in the above equations and to the (22) and (33) components of the Einstein's equation, we get

$$-A(r)\left(\frac{1}{r^2}-\frac{A'(r)}{rA(r)}-\frac{1}{r^2A(r)}\right)=\exp[-2a(y)]R(y) \quad (18)$$

$$-\frac{1}{2r}(2A'(r)+rA''(r))=\exp[-2a(y)]R(y) \tag{19}$$

$$-\left(\frac{A''(r)}{2}+2\frac{A'(r)}{r}+\frac{A(r)-1}{r^2}\right)=\exp[-2a(y)] \times(\Lambda+6\dot{a}^2(y)). \tag{20}$$

Solving the above equations gives $A(r)$ to be

$$A(r)=1-\frac{C}{r}-\frac{\lambda}{3}r^2, \tag{21}$$

where C and λ are the constants of integration. This four-dimensional metric is the well known Schwarzschild–de Sitter metric for the choice of $C>0$, where λ is the four-dimensional cosmological constant, in the sense that the four dimensional metric with $A(r)$ given by Eq. (21) corresponds to a four dimensional space-time with this cosmological constant. (We use the term cosmological constant in four dimensions in the above sense and it should not be confused with the other possible ways of defining the cosmological constant — for example, from the brane tension, etc. Note that the sign of λ is still undetermined.) Substituting the form of $A(r)$ in the original equations, the differential equation for $a(y)$ becomes

$$\frac{d^2a(y)}{dy^2}=\frac{\lambda}{3}\exp(2a(y)). \tag{22}$$

It is clear that the conformal factor will have *only* the λ dependence and will be independent of C. (Normally the four dimensional space-time can have a nonvanishing cosmological constant only when there is a source in the right-hand side of the four dimensional Einstein's equations. In our case, if we write the five dimensional G_{ab} in terms of four dimensional Einstein tensor $G_{\mu\nu}$ and extra terms arising from the fifth dimension, it is possible to show that the effective source for $G_{\mu\nu}$ is exactly that corresponding to a four dimensional cosmological constant λ.)

Solving Eq. (22), it is easy to obtain the form of $a(y)$ such that it reduces to the RS result of $a(y)=k|y|$ when $\lambda=0$. We get

$$\exp[-2a(y)]=\exp[2k|y|][\exp[-2k|y|]-(\lambda/12k^2)]^2 \tag{23}$$

with k being a constant related to Λ by $\Lambda=-6k^2$. This shows that $\Lambda<0$ for an acceptable solution. Equations (23), (21) with the result $A(r)=1/B(r)$ completely determine the metric. The modulus sign in $|y|$ will make the derivatives of $a(y)$ discontinuous at the origin $y=0$ which can be taken to be the location of the membrane as in the RS case.

Equation (23) allows us to draw an important conclusion which is the second key result of this paper. Note that the conformal factor $Z=\exp[-2a(y)]$ depends on λ but *not* on C. In the limit of $\lambda\to 0$ the conformal factor for the four-dimensional world line element is same as in the RS model. Thus, the original analysis of RS can be generalized *without any modifications* to the case in which the four dimensional spacetime is described by Schwarzschild line-element [$\lambda=0$, $C>0$ in Eq. (21)] as well suggesting that the zeroth order gravitational interaction, in the form of Schwarzschild line element, gets "corrected" by the conformal factor. This could possibly be the reason why the one-loop corrections to the Schwarzschild metric in the earlier analysis of Duff [11] also gives a similar result.

The above reason is strengthened by the results in Refs. [14,15]: In a recent paper, Duff [14] has shown that the propagator for the continuum graviton modes, in the RS picture, incorporates all quantum effects of matter on the brane. Using the Duff's analysis, Alvarez and Mazzitelli [15] have shown that for the conformal fields and up to quadratic order in the curvature, the nonlocal effective action is equivalent to the $d+1$ action for classical gravity in AdS_{d+1} restricted to a $d-1$ brane.

The condition on the *four-dimensional* cosmological constant λ is more interesting. The ground state wave function $Z=\exp(-2a(y))$ in Eq. (23) is not normalizable for $\lambda\neq 0$ and hence we do not get a massless [$m=0$ in Eq. (8)] graviton for $\lambda\neq 0$. An examination of the general solution to Eq. (22) confirms this conclusion. Using $Z=\exp(-2a(y))$, the first integral to Eq. (22) can be written as

$$\frac{dZ}{dy}=\pm\left(4\beta_1 Z^2+\frac{4\lambda}{3}Z\right)^{1/2}, \tag{24}$$

where β_1 is the constant of integration. For $\beta_1<0$, the solution is oscillatory with nodes and hence is not of interest. For the case $\beta_1=k^2>0$, we obtain the solution to be

$$Z=-\frac{\lambda}{6k^2}+\frac{1}{16k^2}\exp(\pm 2k(y-y_0)) +\frac{\lambda^2}{9k^2}\exp(\mp 2k(y-y_0)), \tag{25}$$

where y_0 is the constant of integration. In the case of $\lambda=0$, the wave function (Z) is normalizable and reduces to the ground state wave function obtained by RS with a suitable choice of the signs for $y>0$ and $y<0$ [we take the solution to be varying as $\exp(-2ky)$ for $y>0$ and $\exp(2ky)$ for $y<0$ with the membrane being located at $y=0$]. However, when $\lambda\neq 0$, the wave function is not bounded as $|y|\to\infty$ (for any combination of signs in the argument of the exponential) and hence is not normalizable for nonzero λ. This is because the third term on the right-hand side of Eq. (25) (which is nonzero when $\lambda\neq 0$) comes with an argument to the exponential having a different sign compared to the second term. This shows clearly that the nature of the solution for $Z(y)$ — which acts as the ground state wave function for zero mass graviton mode — is very different when $\lambda\neq 0$ compared to the case considered by RS. [The above result can be understood in a slightly different manner: The ground state wave function in Eq. (9) is the same as the conformal factor of the line element (6). If the ground state wave function blows up as $y\to\infty$ then the conformal or warp factor in the Randall-Sundrum line element will be very large for large y. Hence, the brane located at $y=0$ is unstable to the metric perturbations.]

IV. CONCLUSIONS AND DISCUSSIONS

To conclude, we have shown that the existence of a zero graviton mode is general, i.e., it exists for a wide class of four dimensional metrics in the case of the RS model. In particular, the results of RS are valid without modifications for a four dimensional Schwarzschild black hole. But the presence of nonzero cosmological constant in four dimensions modifies the RS results. The presence of a nonzero cosmological constant does not provide a normalizable ground state wave function corresponding to the zero mass graviton. Hence, we have obtained a dynamical reason for the strict vanishing of the cosmological constant *within the context of these models*. The stability of the 3-brane to different classes of matter fields in the context of the general five dimensional metric is under investigation.

We would like to point out to the reader the difference in the approach taken here and to the earlier works [16]. The earlier analysis of the Schwarzschild metric on the brane was performed by taking the case $a(y)=k|y|$. In this case, it is easy to demonstrate that the $\hat{R}_{ab}$ solves the RS equations of motion, provided the four dimensional brane is Ricci flat ($R_{\mu\nu}=0$). Hence in these analyses, replacing the Ricci flat branes with the flat branes was by forcing the conformal or warp factor to be the same as that of RS.

Our analysis in this paper is geared towards understanding the stability of the 3-brane against the metric perturbations (in the five dimensions) for a general four dimensional space-time. We have shown that the stability of the 3-brane in the RS model can be explicitly shown in the RS by obtaining the zero mass ground state graviton wave function which is well behaved and normalizable. Here we have performed this analysis for a four dimensional spherically symmetric metric and obtained the general form of the four dimensional spherically symmetric metric along with the conformal factor $a(y)$ by solving the five dimensional vacuum Einstein's equation (with nonzero Λ). The general solution we obtained shows that there the conformal or warp factor is independent of the Schwarzschild mass (see Sec. III). However, the analysis (of the four dimensional Schwarzschild metric in the 3-brane) by earlier authors is by forcing the conformal or warp factor to be same as that of RS and hence replacing the Ricci flat branes with the flat branes. The reason for the conformal factor to be independent of the Schwarzschild mass [constant C in Eq. (21)] is not clear in the earlier works.

An interesting alternative scenario would be to use the model by RS in Ref. [6]. In this scenario, we can set up two 3-branes where the 3-branes are extended in the x_μ directions and are located at some fixed points in the y axis and thus restricting the extra dimensions to be compactified. (In this model, it is assumed that the branes do not contribute to the energy momentum tensor.) By restricting the extra dimensions to be compactified, we can obtain normalizable zero mass gravitons. Such an analysis leads to two different situations depending on whether (i) $\lambda>12k^2$ or (ii) $\lambda<12k^2$. The first possibility, even if $k\approx$TeV, will give a large cosmological constant. The other case, which is more plausible, gives us the upper bound on the compactification scale (radius) of the extra dimensions. (Some of these issues have been discussed in Ref. [17].) These issues are under current investigation.

Finally, we would like to mention the following curious fact: In conventional four-dimensional general relativity, linearizing the Einstein-Hilbert action,

$$S_{gravity}=-\frac{c^3}{16\pi G}\int\sqrt{-g(x)}\,[R(x)+2\lambda]d^4x \qquad (26)$$

[where $R(x)$ is the Ricci scalar, λ is the cosmological constant and $g_{\mu\nu}$ is the general four-dimensional metric], we obtain

$$\Box^{(4)}h_{\mu\nu}=-\lambda h_{\mu\nu}. \qquad (27)$$

The cosmological constant appears as a mass term in the linearized spin-2 wave equation. Vanishing of cosmological constant is required for this equation to be interpreted as representing the massless spin-2 particles (gravitons) in general. [The graviton propagation in de Sitter background (which is a maximally symmetric space-time) has been performed (see for example Ref. [18]) and it was shown that gravitons possess only two physical propagating degrees of freedom. A detailed analysis for a *general* background has not been performed and in these cases the vanishing of the cosmological constant is required to interpret it as representing massless gravitons (corresponding to a long range interaction).] However, by making the cosmological constant very small one can obtain a long range interaction for gravity. Our analysis here shows that even *an arbitrarily small* cosmological constant will make the ground state wave function (corresponding to a massless graviton) non-normalizable, requiring the cosmological constant to strictly vanish. Whether there exists a deeper connection between the two results is not clear and is under investigation.

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APPENDIX

For the sake of completeness, we outline the essential steps leading to Eq. (8) in Sec. II. Defining $\Theta^{c}_{\mu\nu}\equiv g^{ac}h_{\mu\nu;a}$ (the semicolon on the right-hand side represents the covariant derivative), we have

$$\nabla^a\nabla_a h_{\mu\nu}=g^{ac}\nabla_c(h_{\mu\nu;a})=\Theta^c_{\mu\nu;c}$$

$$=\Theta^c_{\mu\nu,c}+\Theta^m_{\mu\nu}\Gamma^c_{mc}-\Theta^c_{m\nu}\Gamma^m_{c\mu}-\Theta^c_{m\mu}\Gamma^m_{c\nu} \tag{A1}$$

$$=(-g)^{-1/2}\partial_c(\sqrt{-g}\Theta^c_{\mu\nu})-\Theta^c_{m\nu}\Gamma^m_{c\mu}-\Theta^c_{m\mu}\Gamma^m_{c\nu}. \tag{A2}$$

Evaluating Θ's in the right-hand side (RHS) of the expression (A1), we obtain

$$\Theta^c_{\mu\nu}=g^{ca}h_{\mu\nu;a}$$

$$=g^{ca}[\partial_a h_{\mu\nu}-\Gamma^\sigma_{\mu a}h_{\sigma\nu}-\Gamma^\gamma_{\nu a}h_{\mu\sigma}] \tag{A3}$$

$$\Theta^c_{m\nu}=g^{ca}h_{m\nu;a}=g^{ca}h_{\eta\nu;a}\quad (\text{using the gauge condition } h_{5\mu}=0)$$

$$=g^{ca}[\partial_a h_{\eta\nu}-\Gamma^\sigma_{\eta a}h_{\sigma\nu}-\Gamma^\sigma_{\nu a}h_{\eta\sigma}] \tag{A4}$$

$$\Theta^c_{\mu m}=g^{ca}h_{\mu m;a}=g^{ca}h_{\mu\eta;a}\quad (\text{using the gauge condition } h_{5\mu}=0)$$

$$=g^{ca}[\partial_a h_{\mu\eta}-\Gamma^\sigma_{\mu a}h_{\eta\sigma}-\Gamma^\sigma_{\eta a}h_{\mu\sigma}]. \tag{A5}$$

We know

$$\Gamma^i_{kl}=\frac{1}{2}g^{im}\left[\frac{\partial g_{mk}}{\partial x^l}+\frac{\partial g_{ml}}{\partial x^k}-\frac{\partial g_{kl}}{\partial x^m}\right] \tag{A6}$$

$$\Gamma^\sigma_{\mu a}=\frac{1}{2}g^{\sigma m}\left[\frac{\partial g_{m\mu}}{\partial x^a}+\frac{\partial g_{ma}}{\partial x^\mu}-\frac{\partial g_{\mu a}}{\partial x^m}\right]. \tag{A7}$$

Γ can be easily evaluated for the line element (6) and is given by

$$\Gamma^\sigma_{\mu a}=\frac{1}{2}g^{\sigma\beta}\left[\frac{\partial g_{\beta\mu}}{\partial x^a}+\frac{\partial g_{\beta a}}{\partial x^\mu}-\frac{\partial g_{\mu a}}{\partial x^\beta}\right]\delta^a_\alpha-\dot{a}(\psi)\delta^\sigma_\mu\delta^a_5. \tag{A8}$$

Thus Eqs. (A3), (A4), and (A5) will get modified to the form

$$\Theta^c_{\mu\nu}=g^{ca}[\partial_a h_{\mu\nu}+2\dot{a}(\psi)\delta^a_5 h_{\mu\nu}]-\frac{g^{ca}}{2}[g^{\sigma\beta}(\partial_a g_{\beta\mu}+\partial_\mu g_{\beta a}-\partial_\beta g_{\mu a})\delta^a_\alpha h_{\sigma\nu}+g^{\sigma\beta}(\partial_a g_{\beta\nu}+\partial_\nu g_{\beta a}-\partial_\beta g_{\nu a})\delta^a_\alpha h_{\mu\sigma}] \tag{A9}$$

$$\Theta^c_{m\nu}=g^{ca}h_{\eta\nu;a}=g^{ca}[\partial_a h_{\eta\nu}+2\dot{a}(\psi)\delta^a_5 h_{\eta\nu}]-\frac{g^{ca}}{2}[g^{\sigma\beta}(\partial_a g_{\beta\eta}+\partial_\eta g_{\beta a}-\partial_\beta g_{\eta a})\delta^a_\alpha h_{\sigma\nu}$$
$$+g^{\sigma\beta}(\partial_a g_{\beta\nu}+\partial_\nu g_{\beta a}-\partial_\beta g_{\nu a})\delta^a_\alpha h_{\eta\sigma}] \tag{A10}$$

$$\Theta^c_{\mu m}=g^{ca}h_{\mu\eta;a}=g^{ca}[\partial_a h_{\eta\mu}+2\dot{a}(\psi)\delta^a_5 h_{\eta\mu}]-\frac{g^{ca}}{2}[g^{\sigma\beta}(\partial_a g_{\beta\mu}+\partial_\mu g_{\beta a}-\partial_\beta g_{\mu a})\delta^a_\alpha h_{\sigma\eta}$$
$$+g^{\sigma\beta}(\partial_a g_{\beta\eta}+\partial_\eta g_{\beta a}-\partial_\beta g_{\eta a})\delta^a_\alpha h_{\mu\sigma}]. \tag{A11}$$

We obtain the full expression of $h^{;a}_{\mu\nu;a}$ by substituting these expressions obtained for Θ in Eq. (A1). The first term in the RHS of Eq. (A1) is

$$(-g)^{-1/2}\partial_c(\sqrt{-g}\Theta^c_{\mu\nu})=\Box^{(4)}h_{\mu\nu}+\partial_5^2 h_{\mu\nu}-2\dot{a}(\psi)\partial_5 h_{\mu\nu}-8\dot{a}^2(\psi)h_{\mu\nu}+2\ddot{a}(\psi)h_{\mu\nu}-\frac{(-g)^{-1/2}}{2}\partial_\tau[\sqrt{-g}g^{\tau\alpha}g^{\sigma\beta}$$
$$\times(\partial_\alpha g_{\beta\mu}+\partial_\mu g_{\beta\alpha}-\partial_\beta g_{\mu\alpha})h_{\sigma\nu}]-\frac{(-g)^{-1/2}}{2}\partial_\tau[\sqrt{-g}g^{\tau\alpha}g^{\sigma\beta}(\partial_\alpha g_{\beta\nu}+\partial_\nu g_{\beta\alpha}-\partial_\beta g_{\nu\alpha})h_{\mu\sigma}], \tag{A12}$$

where $\Box^{(4)}$ denotes the four dimensional D'Alembertian operator. The second and third terms in the RHS of the expression (A1) are

$$\Theta^{c}_{\eta\nu}\Gamma^{\eta}_{c\mu}=\frac{1}{2}III_{a}g^{\tau\alpha}\partial_{\alpha}h_{\mu\nu}-\dot{a}(\psi)\partial_{5}h_{\mu\nu}-2\dot{a}^{2}(\psi)r\partial_{5}h_{\mu\nu} - \frac{g^{ca}}{4}[I_{a}\times III_{a}\times\delta^{a}_{\alpha}h_{\sigma\nu}+II_{a}\times III_{a}\delta^{a}_{\alpha}h_{\eta\sigma}] \tag{A13}$$

$$\Theta^{c}_{\mu\eta}\Gamma^{\eta}_{c\nu}=\frac{1}{2}III_{b}\,g^{\tau\alpha}\partial_{\alpha}h_{\mu\nu}-\dot{a}(\psi)\partial_{5}h_{\mu\nu}-2\dot{a}^{2}(\psi)\partial_{5}h_{\mu\nu} - \frac{g^{ca}}{4}[I_{b}\times III_{b}\times\delta^{a}_{\alpha}h_{\sigma\nu}+II_{b}\times III_{b}\times\delta^{a}_{\alpha}h_{\eta\sigma}] \tag{A14}$$

where $I_{a,b}$, $II_{a,b}$, $III_{a,b}$ are the terms which depend only on the four-dimensional coordinates x^{μ}. Combining the terms in the expressions (A12), (A13), (A14) and rearranging them, we get

$$h^{;a}_{\mu\nu;a}=\Box^{(4)}h_{\mu\nu}+\partial_{5}^{2}h_{\mu\nu}-4\dot{a}^{2}h_{\mu\nu}+2\ddot{a}h_{\mu\nu} + \text{terms depending on the 4D coordinates.} \tag{A15}$$

Thus, it is easy to see from the above relation that $h_{\mu\nu}$ can be written as plane wave gravitons, i.e., $h_{\mu\nu}=e_{\mu\nu}\Phi$. The equation satisfied by Φ can be separated with the ansatz $\Phi(x^{\mu},y)=A(x^{\mu})Z(y)$. Substituting into the wave equation, separating the variables using a constant m^2, we find that A satisfies the standard wave equation for a particle of mass m while Z satisfies the equation

$$\frac{d^{2}Z}{dy^{2}}+(-4\dot{a}^{2}(y)+2\ddot{a}(y)+m^{2}\exp[2a(y)])Z=0. \tag{A16}$$

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