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Interference of spin-2 self-dual modes

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We study the effects of interference between the self-dual and anti-self-dual massive modes of linearized Einstein-Chern-Simons topological gravity. The dual models to be used in the interference process are carefully analyzed with special emphasis on their propagating spectrum. We identify the opposite dual aspects necessary for the application of the interference formalism on this model. The soldered theory so obtained displays explicitly massive modes of the Proca type. It may also be written in a form of the Polyakov-Weigman identity for a better appreciation of its physical contents.

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I. INTRODUCTION

This paper is devoted to the analysis and exploration of the effects of interference between the self- and anti-self-dual gravitational modes of the linearized Einstein-Chern-Simons theory put forward by Aragone and Khoudeir [1] as a spin-2 extension of the self-dual models proposed by Townsend, Pilch, and van Nieuwenhuizen [2] many years ago. This study is done in the context of the soldering technique [3,4] that is dimensionally independent and designed to work with distinct manifestations of the dual symmetry [5,6].

Duality symmetry is currently the focus of intense study both in physics and mathematics [7]. The physical meaning of this well-appreciated concept is being gradually clarified [8–10]. In particular, the study of electromagnetic duality has been revived [8,11,12] and a natural self-dual structure identified [13–16]. Although initially explored in the context of the 4D Maxwell theory to provide an explanation to charge quantization [17], its scope has been considerably enlarged and extended to other dimensions. The idea of self-duality has been extended outside the electromagnetic context and to all space-time dimensions, both even and odd. In the context of the latter, self-dual models in three space-time dimensions have been studied and their mathematical structure closely related to global aspects of anomalies have been highlighted. The practical connection of self-dual models as well as topologically massive models with the investigation of planar physics like quantum Hall effect and high T_c superconductivity is well understood. More important to our studies, the extension of these theories to gravity has also been formulated [18]. Much effort has been made in the analysis of several technical aspects of self-dual actions and analogies among such actions in different dimensions have been suggested [19,20].

On the other hand, the role of the soldering formalism as a quantitative technique is being progressively unveiled and its consequences in diverse dimensions explored. In two space-time dimensions a new interpretation for the phenomenon of dynamical mass generation, known as the Schwinger mechanism [21], has been proposed that explores the ability of the soldering formalism to embrace interference effects [22,23]. The effects of the interference have also been computed in a study of the chiral diffeomorphism algebra for the $\mathcal{W}_2$ [24] and $\mathcal{W}_3$ [25] gravities, in the separation of the no-mover mode of the Siegel chiral boson theory [26] and in some structures of the chiral Wess-Zumino-Witten (WZW) theory [27]. Extensions of this mechanism to three [19] and four [20] space-time dimensions have been examined recently [19]. In particular, interference in three and four space-time dimensions in the electromagnetic context was also the object of a recent investigation [5].

The object of this work is to investigate certain structures of dualities that, as far as we are aware, have not been explored before. We clearly show the possibility to fuse or solder the self- and anti-self-dual massive degrees of freedom of the associated self-dual gravity into an effective action that naturally contains the two modes in an explicitly massive form. Through the soldering operation, the self- and anti-self-dual field operators are then shown to correspond to the square root of the massive operator.

The soldering technique is developed in the next section in the context of the spin-1 three-dimensional self-dual theory. Section III contains our main proposal. There the soldering formalism is applied to the case of spin-2 self-dual gravity generating a new and interesting result. We discuss the outcome of our studies in the last section.

II. SOLDERING OF THE SPIN-1 SELF-DUAL MODES

This section is devoted to the analysis of the soldering process in the spin-1 self-dual theories. This is done to introduce the method and our notation. The three dimensional self-dual model, first discussed by Townsend, Pilch, and van Nieuwenhuizen [2], is given by the following action:

$$S_\chi[f]=\int d^3x\left(\frac{\chi}{2m}\epsilon_{\mu\nu\lambda}f^\mu\partial^\nu f^\lambda+\frac{1}{2}f_\mu f^\mu\right), \qquad (1)$$

where the signature of the topological terms is dictated by the sign of χ. Here the mass parameter m is inserted for dimensional reasons and $\epsilon_{012}=1$.

A. Physical spectrum

We will discuss now the propagating degrees of freedom of this model. To this end we use the Hamiltonian reduction technique put forward in [28] and [29]. A first insight is given by the equations of motion which, in the absence of sources, is given by

$$f_\mu=\frac{\chi}{m}\epsilon_{\mu\nu\lambda}\partial^\nu f^\lambda. \tag{2}$$

From there the following relations may be easily verified:

$$\partial_\mu f^\mu=0,$$
$$(\Box+m^2)f_\mu=0, \tag{3}$$

showing that only the transverse sector of f_μ is a propagating mode. The counting of degrees of freedom can however be put in a more formal presentation. Let us rewrite Eq. (1) in a 2+1 decomposition that reads, after a global change of sign,

$$S_\chi[f]=\int d^3x[a_b^f\dot{f}^b-V(f,\partial f)], \tag{4}$$

where $(a,b=1,2)$ and the overdot means time derivative as usual. Our goal is to construct the symplectic matrix in an iterative fashion. By inspection the symplectic variables are identified as

$$\xi_{\{I\}}=(f_0,f_1,f_2), \tag{5}$$

and the canonical one-form reads

$$a_0^f=0,$$
$$a_b^f=\frac{\chi}{2m}\epsilon_{ab}f^a. \tag{6}$$

The symplectic potential, playing the role of the Hamiltonian density is

$$V(f,\partial f)=\frac{1}{2}f_\mu f^\mu+\frac{\chi}{m}\epsilon_{ab}f_0\partial^a f^b. \tag{7}$$

The symplectic matrix is defined by [28]

$$\mathcal{F}^{(0)}_{\{I\},\{J\}}(x,y)=\frac{\delta a_{\{J\}}(x)}{\delta\xi_{\{J\}}(y)}-\frac{\delta a_{\{J\}}(y)}{\delta\xi_{\{I\}}(x)}, \tag{8}$$

and its value is given by

$$\mathcal{F}^{(0)}_{\{I\},\{J\}}(x,y)=\begin{array}{c} \\ f_0\\ f_1\\ f_2\end{array}\begin{array}{c}\begin{array}{ccc} f_0 & f_1 & f_2\end{array}\\ \begin{pmatrix}\mathcal{F}^{f_0f_0} & \mathcal{F}^{f_0f_1} & \mathcal{F}^{f_0f_2}\\ \mathcal{F}^{f_1f_0} & \mathcal{F}^{f_1f_1} & \mathcal{F}^{f_1f_2}\\ \mathcal{F}^{f_2f_0} & \mathcal{F}^{f_2f_1} & \mathcal{F}^{f_2f_2}\end{pmatrix}\end{array},$$

where

$$\mathcal{F}^{f_0f_0}(x,y)=\mathcal{F}^{f_0f_j}(x,y)=\mathcal{F}^{f_jf_0}(x,y)=0,$$
$$\mathcal{F}^{f_if_j}(x,y)=\frac{\chi}{m}\epsilon^{ij}\delta(x-y). \tag{9}$$

This operator has an obvious zeromode

$$\int d^3y\,\mathcal{F}^{(0)}_{\{I\},\{J\}}(x,y)\mathcal{V}_{\{J\}}(y)=0, \tag{10}$$

with

$$\mathcal{V}_{\{J\}}(y)=\begin{pmatrix}u(y)\\0\\0\end{pmatrix}$$

and $u(y)$ being an arbitrary function. This zeromode selects a true symplectic constraint [29] as

$$\Omega=\int d^3y[\partial_{\{J\}}H_0]^T\mathcal{V}_{\{J\}}(y), \tag{11}$$

where T stands for matrix transposition. A simple algebra shows that

$$\Omega=f_0+\frac{\chi}{2m}\epsilon_{ab}\partial^a f^b. \tag{12}$$

Due to the iterative nature of this procedure, one may interpret this constraint as a secondary symplectic constraint. The first-iterated action now reads

$$S_\chi^{(1)}[f]=\int d^3x[a_i^f\dot{f}^i+a^\lambda(f)\dot{\lambda}-V(f,\partial f)], \tag{13}$$

where λ is the symplectic multiplier and $a^\lambda(f)=\Omega$. Notice that we have now an enlarged set of symplectic variables $\xi\to\overline{\xi}=(\xi,\lambda)$ and canonical one-form $a_I\to\overline{a_I}=(a_I,\Omega)$. The first-iterated symplectic matrix defined as

$$\mathcal{F}^{(1)}_{\{I\},\{J\}}(x,y)=\frac{\delta\overline{a}_{\{I\}}(x)}{\delta\overline{\xi}_{\{J\}}(y)}-\frac{\delta\overline{a}_{\{J\}}(y)}{\delta\overline{\xi}_{\{I\}}(x)} \tag{14}$$

now reads

$$\mathcal{F}^{(1)}_{\{I\},\{J\}}(x,y)=\begin{array}{c} \\ f_0\\ f_1\\ f_2\\ \lambda\end{array}\begin{array}{c}\begin{array}{cccc} f_0 & f_1 & f_2 & \lambda\end{array}\\ \begin{pmatrix}\mathcal{F}^{f_0f_0} & \mathcal{F}^{f_0f_1} & \mathcal{F}^{f_0f_2} & \mathcal{F}^{f_0\lambda}\\ \mathcal{F}^{f_1f_0} & \mathcal{F}^{f_1f_1} & \mathcal{F}^{f_1f_2} & \mathcal{F}^{f_1\lambda}\\ \mathcal{F}^{f_2f_0} & \mathcal{F}^{f_2f_1} & \mathcal{F}^{f_2f_2} & \mathcal{F}^{f_2\lambda}\\ \mathcal{F}^{\lambda f_0} & \mathcal{F}^{\lambda f_1} & \mathcal{F}^{\lambda f_2} & \mathcal{F}^{\lambda\lambda}\end{pmatrix}\end{array},$$

where the new elements are given by

$$\mathcal{F}^{f_0\lambda}(x,y)=-\mathcal{F}^{\lambda f_0}(x,y)=-\delta(x-y),$$
$$\mathcal{F}^{f_1\lambda}(x,y)=-\mathcal{F}^{\lambda f_1}(x,y)=\frac{\chi}{2m}\partial_y\delta(x-y),$$
$$\mathcal{F}^{f_2\lambda}(x,y)=-\mathcal{F}^{\lambda f_2}(x,y)=-\frac{\chi}{2m}\partial_x\delta(x-y),$$
$$\mathcal{F}^{\lambda\lambda}(x,y)=0. \tag{15}$$

Since this matrix is now invertible, the associated zeromode is a trivial one so that the model has no more constraints [28]. The associate Dirac brackets are immediately obtained taking the inverse elements of Eq. (14). We are now in a position to realize the counting of degrees of freedom. We have three symplectic variables (f_0, f_1, and f_2; recall that λ is a symplectic multiplier) and one constraint (Ω) resulting in two phase-space degrees of freedom or one configuration space degree of freedom. This result confirms our previous Lagrangian analysis [Eqs. (3)].

B. Effects of interference

It is useful to clarify the meaning of the self-duality inherent in this action. A field dual to f_μ is defined as

$$*f_\mu = \frac{1}{m}\epsilon_{\mu\nu\lambda}\partial^\nu f^\lambda. \tag{16}$$

Repeating the dual operation, we find that

$$*(*f_\mu) = \frac{1}{m}\epsilon_{\mu\nu\lambda}\partial^\nu * f^\lambda = f_\mu \tag{17}$$

obtained by exploiting Eq. (3), thereby validating the definition of the dual field. Combining these results with Eq. (2), we conclude that

$$f_\mu = -\chi * f_\mu . \tag{18}$$

Hence, depending on the signature of χ, the theory will correspond to a self-dual or an anti-self-dual model. After this brief digression on the definition and meaning of self-dual components, we start the discussion regarding the effects of their interference.

The technique of soldering [3] constitutes essentially in lifting simultaneously the gauging of the dual global symmetry of each component into a local version for the combined system and in this way defining the effective action. It must be stressed that the fusing process always needs two opposite aspects of a symmetry to be present and this is indifferent of the space-time dimension. The crucial point is that the components are considered as functions of distinct variables. A naive addition of these (anti-) self-dual actions, if considered as functions of the same variables, leads to a trivial result. In the same manner a direct sum of the actions also would not lead to anything new. It is exactly the soldering process that leads to a nontrivial effective action.

Let us consider the self-dual and the anti-self-dual models as

$$S_+[f_\mu] = \int d^3x \left(\frac{1}{2m}\epsilon_{\mu\nu\lambda} f^\mu \partial^\nu f^\lambda + \frac{1}{2} f_\mu f^\mu \right),$$

$$S_-[g_\mu] = \int d^3x \left(-\frac{1}{2m}\epsilon_{\mu\nu\lambda} g^\mu \partial^\nu g^\lambda + \frac{1}{2} g_\mu g^\mu \right), \tag{19}$$

where f_μ and g_μ are the distinct bosonic vector fields. To effect the soldering we have to consider the gauging of the following symmetry:

$$\delta f_\mu = \delta g_\mu = \epsilon_{\mu\rho\sigma}\partial^\rho \alpha^\sigma, \tag{20}$$

which will be referred to as soldering symmetry. Under such transformations, the Lagrangians change as

$$\delta\mathcal{L}_\pm = J_\pm^{\rho\sigma}(h_\mu)\partial_\rho \alpha_\sigma, \quad h_\mu = f_\mu,\ g_\mu, \tag{21}$$

where the corresponding antisymmetric Neither currents are

$$J_\pm^{\rho\sigma}(h_\mu) = \epsilon^{\mu\rho\sigma} h_\mu \pm \frac{1}{m}\epsilon^{\gamma\rho\sigma}\epsilon_{\mu\nu\gamma}\partial^\mu h^\nu. \tag{22}$$

Next we introduce the soldering field coupled with the antisymmetric currents. In the two-dimensional case this was a vector. Its natural extension now is the antisymmetric second-rank Kalb-Ramond tensor field $B_{\rho\sigma}$ transforming in the usual way,

$$\delta B_{\rho\sigma} = \partial_\rho \alpha_\sigma - \partial_\sigma \alpha_\rho . \tag{23}$$

Then it is easy to see that the modified actions

$$S_\pm^{(1)}[h_\mu] = S_\pm[h_\mu] - \frac{1}{2}\int d^3x J_\pm^{\rho\sigma}(h_\mu) B_{\rho\sigma} \tag{24}$$

transform as

$$\delta S_\pm^{(1)} = -\frac{1}{2}\int d^3x\, \delta J_\pm^{\rho\sigma} B_{\rho\sigma} \tag{25}$$

under Eqs. (20) and (23). The final modification consists in adding a term to ensure gauge invariance of the soldered Lagrangian. This is achieved by

$$S_\pm^{(2)} = S_\pm^{(1)} + \frac{1}{4}\int d^3x B^{\rho\sigma} B_{\rho\sigma} . \tag{26}$$

A straightforward algebra shows that the following combination:

$$\begin{aligned} S_S(f,g,B) &= S_+^{(2)}(f) + S_-^{(2)}(g) \\ &= S_+(f) + S_-(g) - \frac{1}{2}\int d^3x \\ &\quad \times [B^{\rho\sigma}\{J_{\rho\sigma}^+(f) + J_{\rho\sigma}^-(g)\} + B^{\rho\sigma}B_{\rho\sigma}], \end{aligned} \tag{27}$$

is invariant under the gauge transformations (20) and (23). The gauging of the soldering symmetry is therefore complete. To return to a description in terms of the original variables, the ancillary soldering field is eliminated from Eq. (27) by using the equations of motion

$$B_{\rho\sigma} = \frac{1}{2}[J_{\rho\sigma}^+(f) + J_{\rho\sigma}^-(g)]. \tag{28}$$

Inserting this solution in Eq. (27), the final soldered Lagrangian is expressed solely in terms of the currents involving the original fields

$$\begin{aligned} S_{eff}(f,g) &= S_+(f) + S_-(g) - \frac{1}{8}\int d^3x [J_{\rho\sigma}^+(f) + J_{\rho\sigma}^-(g)] \\ &\quad \times [J_+^{\rho\sigma}(f) + J_-^{\rho\sigma}(g)]. \end{aligned} \tag{29}$$

It is now crucial to note that by using the explicit structures for the currents, the above Lagrangian is no longer a function of f_μ and g_μ separately, but depends only on the invariant combination

$$A_\mu=\frac{1}{\sqrt{2}m}(g_\mu-f_\mu) \tag{30}$$

with

$$S_{eff}(A_\mu)=\int d^3x\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{m^2}{2}A_\mu A^\mu\right], \tag{31}$$

where

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu \tag{32}$$

is the usual field tensor expressed in terms of the basic entity A_μ. Notice that the effective variable is an invariant combination (under the soldering transformations) of the original variables. The soldering mechanism has precisely fused the self- and anti-self-dual symmetries to yield a massive Maxwell theory that accommodates naturally the two degrees of freedom corresponding to these symmetries, thus preserving the degrees of freedom counting throughout its formalism. It is also interesting to observe that the noninvariant nature of the basic dual components under the ordinary gauge transformations has been preserved. Were the original systems pure gauge invariant systems (like Maxwell-Chern-Simons), the resulting soldered action would correspond to the Stueckelberg-Proca action [5].

III. SOLDERING OF THE GRAVITATIONAL SELF-DUAL MODELS

Now we pass to consider the higher spin case. Let us begin by examining the following first-order Lagrangian, which describes a spin-2 self-dual model in $\mathcal{D}=(2+1)$ space-time dimensions [1,30],

$$S_\chi=\int d^3x\left[\frac{\chi}{2m}\epsilon^{\mu\alpha\beta}\eta^{\nu\lambda}h_{\mu\nu}\partial_\alpha h_{\beta\lambda}-\frac{1}{2}h_{\mu\nu}h^{\nu\mu}+\frac{1}{2}h^2\right], \tag{33}$$

where $h_{\mu\nu}$ is a nonsymmetric second-order tensor, $h\equiv\eta^{\mu\nu}h_{\mu\nu}$, and the mass parameter m is introduced on dimensional basis. Our convention is $\eta^{\mu\nu}=\mathrm{diag}(-1,+1,+1)$. The first term is the usual Chern-Simons term, whereas the last two form the Fierz-Pauli mass term. This Lagrangian is linearized about the dreinbein field $e_{\mu\nu}$ as $e_{\mu\nu}=\eta_{\mu\nu}+\kappa h_{\mu\nu}$. It can be shown [30] that the signature of χ determines the field's helicity. So we can think of $S_\pm$ as describing theories of opposite helicities. The equivalence between this self-dual model (33) and the so-called (linearized) topologically massive gravity [18] is shown by means of an associated master action [1].

A. Physical spectrum

In this subsection we discuss the physical content of this theory. To this end we use the Hamiltonian reduction technique put forward in [28] and [29] and briefly discussed for the spin-1 case in Sec. II A. Some insight may be obtained already at the Lagrangian level allowing us to discuss the (propagating) spectrum of this theory. Independent variations of $h_{\mu\nu}$ gives the field equations for the model,

$$h^{\nu\mu}-\eta^{\mu\nu}h=\frac{1}{m}\epsilon^{\mu\alpha\beta}\eta^{\nu\lambda}\partial_\alpha h_{\beta\lambda}. \tag{34}$$

Taking the divergence and rotational of Eq. (34) leads to the following expression:

$$\begin{aligned}4m^2(h^{\nu\mu}-\eta^{\mu\nu}h)=&-2\Box(h^{\mu\nu}+h^{\nu\mu})-2\partial^\mu\partial^\nu h\\&+\partial^\mu[\partial_\lambda(2h^{\nu\lambda}+h^{\lambda\nu})]\\&+\partial^\nu[\partial_\lambda(2h^{\mu\lambda}+h^{\lambda\mu})]\end{aligned} \tag{35}$$

that only reduces to a Klein-Gordon equation for the symmetric, transverse, and traceless sector of $h_{\mu\nu}$. This gives a clear indication of the nonpropagating nature of the antisymmetric sector of $h_{\mu\nu}$. In this case we have a (massive) propagation mode (this result was also shown by evaluating the vacuum amplitude in the presence of an external source [1]) where the harmonic gauge condition

$$\partial_\mu(h^{\mu\nu}-\eta^{\mu\nu}h)=0 \tag{36}$$

is naturally satisfied.

To confirm the prediction above we develop next a canonical analysis of this theory using the symplectic approach [28,29]. This will permit a proper counting of the propagating degrees of freedom. Let us start writing Eq. (33) in a 2+1 decomposition,

$$\begin{aligned}S_\chi=\int d^3x\frac{1}{2m}[&-2h_{00}(\chi\epsilon_{ij}\partial_i h_{j0}+mh_{ii})\\&+2h_{0k}(\chi\epsilon_{ij}\partial_i h_{jk}+mh_{k0})-\chi h_{ik}\epsilon_{ij}\dot{h}_{jk}+\chi h_{i0}\epsilon_{ij}\dot{h}_{j0}\\&+m(h_{ii}h_{jj}-h_{ij}h_{ji})],\end{aligned} \tag{37}$$

where (i,j and $k=1,2$) and the dot means time derivative. Next we introduce the following redefinition [30]:

$$n=h_{00},$$

$$N_i=h_{i0},$$

$$M_i=h_{0i},$$

$$H_{ij}=\frac{1}{2}(h_{ij}+h_{ji}),$$

$$V=\frac{1}{2}\epsilon_{ij}h_{ij}, \tag{38}$$

where we have separated the symmetric (H_{ij}) and antisymmetric (V) parts of h_{ij}. After this, the action (37) assumes the form

$$S_\chi = \int d^3x \frac{1}{2m}\bigg[-2\chi \dot{H}_{ij}\bigg\{ \delta_{ij}V - \frac{1}{4}(\epsilon_{ik}H_{kj}+\epsilon_{jk}H_{ki})\bigg\} - \chi \dot{N}_i \epsilon_{ij}N_j - 2n(\chi\epsilon_{ij}\partial_i N_j + H_{ii}) + 2M_k(\chi\epsilon_{ij}\partial_i H_{jk} + mN_k - \chi\partial_k V) - m(H_{ij}H_{ij} - H_{ii}H_{jj}) - 2mVV\bigg]. \tag{39}$$

Notice that n and M_k are not true dynamical variables but just Lagrangian multipliers enforcing the constraints

$$\psi \equiv \chi\epsilon_{ij}\partial_i N_j + mH_{ii},$$
$$\psi_k \equiv \chi\epsilon_{ij}\partial_i H_{jk} + mN_k - \chi\partial_k V. \tag{40}$$

As before we construct the symplectic matrix in an iterative fashion. Following the prescription of [29] we perform a further redefinition

$$n \to -\dot{\eta},$$
$$M_i \to \dot{\mu}_i, \tag{41}$$

to obtain the (zeroth-iterated) action

$$S_\chi^{(0)}[\xi] = \int d^3x[a^\eta(\xi)\dot{\eta} + a_k^\mu(\xi)\dot{\mu}_k + a_{ij}^H(\xi)\dot{H}_{ij} + a_i^N(\xi)\dot{N}_i - H_0]. \tag{42}$$

Here $\xi = H_{ij}, N_i, V$ are the symplectic variables and $a_{\{I\}}(\xi)$ are the canonical one-form defined by

$$a^\eta(\xi) = \frac{1}{m}\psi,$$

$$a_k^\mu(\xi) = \frac{1}{m}\psi_k,$$

$$a_{ij}^H(\xi) = \frac{-\chi}{m}\bigg[\delta_{ij}V - \frac{1}{4}(\epsilon_{ik}H_{kj} + \epsilon_{jk}H_{ki})\bigg],$$

$$a_i^N(\xi) = \frac{-\chi}{2m}\epsilon_{ij}N_j,$$

$$a^V(\xi) = 0, \tag{43}$$

and

$$H_0 = 2m(H_{ij}H_{ij} - H_{ii}H_{jj}) + 2m^2V^2. \tag{44}$$

The (zeroth-order) symplectic matrix defined as [28,29]

$$\mathcal{F}^{(0)}_{\{I\},\{J\}}(x,y) = \frac{\delta a_{\{J\}}(x)}{\delta \xi_{\{I\}}(y)} - \frac{\delta a_{\{J\}}(y)}{\delta \xi_{\{I\}}(x)} \tag{45}$$

gives

$$\mathcal{F}^{(0)}_{\{I\},\{J\}}(x,y) = \begin{array}{c} \\ \eta \\ \mu_i \\ H_{ij} \\ N_i \\ V \end{array} \begin{array}{c} \begin{matrix} \eta & \mu_l & H_{ln} & N_l & V \end{matrix} \\ \begin{pmatrix} \mathcal{F}^{\eta\eta} & \mathcal{F}_l^{\eta\mu} & \mathcal{F}_{ln}^{\eta H} & \mathcal{F}_l^{\eta N} & \mathcal{F}_l^{\eta V} \\ \mathcal{F}_i^{\mu\eta} & \mathcal{F}_{i,l}^{\mu\mu} & \mathcal{F}_{i,ln}^{\mu H} & \mathcal{F}_{i,l}^{\mu N} & \mathcal{F}_{i,l}^{\mu V} \\ \mathcal{F}_{ij}^{H\eta} & \mathcal{F}_{ij,l}^{H\mu} & \mathcal{F}_{ij,ln}^{HH} & \mathcal{F}_{ij,l}^{HN} & \mathcal{F}_{ij}^{HV} \\ \mathcal{F}_i^{N\eta} & \mathcal{F}_{i,l}^{N\mu} & \mathcal{F}_{i,ln}^{NH} & \mathcal{F}_{i,l}^{NN} & \mathcal{F}_{i,l}^{NV} \\ \mathcal{F}^{V\eta} & \mathcal{F}_l^{V\mu} & \mathcal{F}_{ln}^{VH} & \mathcal{F}_l^{VN} & \mathcal{F}_l^{VV} \end{pmatrix} \end{array},$$

where the nonvanishing matrix elements read

$$\mathcal{F}_{ln}^{\eta H}(x,y) = \frac{1}{m}\delta_{ln}\delta(x-y),$$

$$\mathcal{F}_l^{\eta N}(x,y) = -\frac{\chi}{m}\epsilon_{lp}\partial_p^x\delta(x-y),$$

$$\mathcal{F}_{i,ln}^{\mu H}(x,y) = -\frac{\chi}{m}\delta_{in}\epsilon_{lp}\partial_p^x\delta(x-y),$$

$$\mathcal{F}_{i,l}^{\mu N}(x,y) = \delta_{il}\delta(x-y),$$

$$\mathcal{F}_i^{\mu V}(x,y) = -\frac{1}{m}\partial_i^x\delta(x-y),$$

$$\mathcal{F}_{ij}^{H\eta}(x,y) = -\frac{1}{m}\delta_{ij}\delta(x-y),$$

$$\mathcal{F}_{ij,l}^{H\mu}(x,y) = \frac{\chi}{m}\delta_{jl}\epsilon_{ip}\partial_p^y\delta(x-y),$$

$$\mathcal{F}_{ij,ln}^{HH}(x,y) = \frac{1}{4m}[2\epsilon_{il}\delta_{jn} + \epsilon_{jl}\delta_{in} - \epsilon_{ni}\delta_{lj}]\delta(x-y),$$

$$\mathcal{F}_{ij}^{HV}(x,y) = -\frac{1}{m}\delta_{ij}\delta(x-y),$$

$$\mathcal{F}_i^{N\eta}(x,y) = \frac{\chi}{m}\epsilon_{ip}\partial_p^y\delta(x-y),$$

$$\mathcal{F}_{i,l}^{NN}(x,y) = -\frac{1}{m}\epsilon_{il}\delta(x-y),$$

$$\mathcal{F}_l^{V\mu}(x,y) = \frac{\chi}{m}\partial_l^y\delta(x-y),$$

$$\mathcal{F}_{ln}^{VH}(x,y) = \frac{1}{m}\delta_{ln}\delta(x-y), \tag{46}$$

and

$$\mathcal{F}^{\eta\eta}(x,y)=\mathcal{F}_l^{\eta\mu}(x,y)=\mathcal{F}^{\eta V}(x,y)=\mathcal{F}_i^{\mu\eta}(x,y)=\mathcal{F}_{i,l}^{\mu\mu}(x,y)=\mathcal{F}_{ij,l}^{HN}(x,y)=0,$$

$$\mathcal{F}_{i,l}^{N\mu}(x,y)=\mathcal{F}_{i,ln}^{NH}(x,y)=\mathcal{F}_i^{NV}(x,y)=\mathcal{F}^{V\eta}(x,y)=\mathcal{F}_l^{VN}(x,y)=\mathcal{F}^{VV}(x,y)=0. \quad (47)$$

This is a singular matrix. The easiest way of seeing this is by noticing the presence of a zero mode

$$\int d^3y\,\mathcal{F}^{(0)}_{\{I\},\{J\}}(x,y),\mathcal{V}_{\{J\}}(y)=0 \quad (48)$$

with

$$\mathcal{V}_{\{J\}}(y)=\begin{pmatrix}\mathcal{V}^{\eta}(y)\\ \mathcal{V}_l^{\mu}(y)\\ \mathcal{V}_{ln}^{H}(y)\\ \mathcal{V}_l^{N}(y)\\ \mathcal{V}^{V}(y)\end{pmatrix},$$

whose explicit elements read

$$\mathcal{V}^{\eta}(y)=\mathcal{V}_l^{\mu}(y)=0,$$

$$\mathcal{V}_{ln}^{H}(y)=\frac{2}{m}\epsilon_{ln}u(y),$$

$$\mathcal{V}_l^{N}(y)=\frac{1}{m^2}\partial_l u(y),$$

$$\mathcal{V}^{V}(y)=-\frac{1}{m}u(y). \quad (49)$$

Here $u(y)$ is an arbitrary function satisfying proper boundary conditions. This zero mode signals the presence of another constraint given by

$$\Omega=\int d^3y[\partial_{\{J\}}H_0]^T\mathcal{V}_{\{J\}}(y), \quad (50)$$

where T stands for matrix transposition. A simple algebra shows that

$$\Omega=V. \quad (51)$$

Due to the iterative nature of this procedure, one may interpret this constraint as a secondary symplectic constraint in complete analogy with Dirac's procedure [30]. The first-iterated action now reads

$$S_{\chi}^{(1)}[\xi]=\int d^3x[a^{\eta}(\xi)\dot{\eta}+a_k^{\mu}(\xi)\dot{\mu}_k+a_{ij}^{H}(\xi)\dot{H}_{ij}+a_i^{N}(\xi)\dot{N}_i+a^{\lambda}(\xi)\dot{\lambda}-H_0], \quad (52)$$

where λ is the symplectic multiplier and $a^{\lambda}(\xi)=\Omega$. Notice that we have now an enlarged set of symplectic variables $\xi\to\overline{\xi}=(\xi,\lambda)$ and momenta $a_I\to\overline{a_I}=(a_I,\Omega)$. The first-iterated symplectic matrix defined as

$$\mathcal{F}^{(1)}_{\{I\},\{J\}}(x,y)=\frac{\delta\overline{a}_{\{I\}}(x)}{\delta\overline{\xi}_{\{J\}}(y)}-\frac{\delta\overline{a}_{\{J\}}(y)}{\delta\overline{\xi}_{\{I\}}(x)} \quad (53)$$

now reads

$$\mathcal{F}^{(1)}_{\{I\},\{J\}}(x,y)=\begin{array}{c} \\ \eta\\ \mu_i\\ H_{ij}\\ N_i\\ V\\ \lambda\end{array}\begin{array}{c}\begin{array}{cccccc}\eta & \mu_l & H_{ln} & N_l & V & \lambda\end{array}\\ \begin{pmatrix}\mathcal{F}^{\eta\eta} & \mathcal{F}_l^{\eta\mu} & \mathcal{F}_{ln}^{\eta H} & \mathcal{F}_l^{\eta N} & \mathcal{F}^{\eta V} & \mathcal{F}^{\eta\lambda}\\ \mathcal{F}_i^{\mu\eta} & \mathcal{F}_{i,l}^{\mu\mu} & \mathcal{F}_{i,ln}^{\mu H} & \mathcal{F}_{i,l}^{\mu N} & \mathcal{F}_i^{\mu V} & \mathcal{F}_i^{\mu\lambda}\\ \mathcal{F}_{ij}^{H\eta} & \mathcal{F}_{ij,l}^{H\mu} & \mathcal{F}_{ij,ln}^{HH} & \mathcal{F}_{ij,l}^{HN} & \mathcal{F}_{ij}^{HV} & \mathcal{F}_{ij}^{H\lambda}\\ \mathcal{F}_i^{N\eta} & \mathcal{F}_{i,l}^{N\mu} & \mathcal{F}_{i,ln}^{NH} & \mathcal{F}_{i,l}^{NN} & \mathcal{F}_i^{NV} & \mathcal{F}_i^{N\lambda}\\ \mathcal{F}^{V\eta} & \mathcal{F}_l^{V\mu} & \mathcal{F}_{ln}^{VH} & \mathcal{F}_l^{VN} & \mathcal{F}^{VV} & \mathcal{F}^{V\lambda}\\ \mathcal{F}^{\lambda\eta} & \mathcal{F}_l^{\lambda\mu} & \mathcal{F}_{ln}^{\lambda H} & \mathcal{F}_l^{\lambda N} & \mathcal{F}^{\lambda V} & \mathcal{F}^{\lambda\lambda}\end{pmatrix}\end{array},$$

where the new λ elements are given by

$$\mathcal{F}^{V\lambda}(x,y)=-\frac{1}{m}\delta(x-y),$$

$$\mathcal{F}^{\lambda V}(x,y)=\frac{1}{m}\delta(x-y), \quad (54)$$

and

$$\mathcal{F}^{\eta\lambda}(x,y)=\mathcal{F}_i^{\mu\lambda}(x,y)=\mathcal{F}_{ij}^{H\lambda}(x,y)=\mathcal{F}_i^{N\lambda}(x,y)=\mathcal{F}^{\lambda\lambda}(x,y)=0,$$

$$\mathcal{F}^{\lambda\eta}(x,y)=\mathcal{F}_l^{\lambda\mu}(x,y)=\mathcal{F}_{ln}^{\lambda H}(x,y)=\mathcal{F}_l^{\lambda N}(x,y)=0. \quad (55)$$

By following the steps above, it can be shown that the corresponding zero mode is trivial, so that there are no more constraints. It is now a simple task to perform the counting of degrees of freedom in this system. There are six true symplectic variables (H_{ij}, N_i, and V; we recall that η, μ_k, and λ are just multipliers) and four constraints (ψ,ψ_k, and Ω) totaling two independent phase-space variables or one degree of freedom as discussed above.

B. Effects of interference: Spin 2

Next we discuss the meaning of self- and anti-self-duality in this model. We define the duality transformation as

$$^{\star}h^{\nu\mu}\equiv\frac{1}{m}\epsilon^{\mu\alpha\beta}\eta^{\nu\lambda}\partial_{\alpha}h_{\beta\lambda}. \quad (56)$$

In order to give a sensible definition for self- and anti-self-duality, this operation must be idempotent. Indeed we can show that

$$^\star(^\star h^{\nu\mu}) = h^{\nu\mu} \tag{57}$$

by using the equations of motion, guaranteeing the existence of self- and anti-self-dual solutions. Observe that this duality construction does not depend on considering $h_{\mu\nu}$ as a symmetric, transverse, and traceless field. It is valid also for a nonsymmetrical field. Let us write explicitly the separated actions leading to these dual solutions in terms of two distinct and independent variables,

$$S_+(f) = \int d^3x\left[\frac{1}{2m}\epsilon^{\mu\alpha\beta}\eta^{\nu\lambda}f_{\mu\nu}\partial_\alpha f_{\beta\lambda} - \frac{1}{2}f_{\mu\nu}f^{\nu\mu} + \frac{1}{2}f^2\right], \tag{58}$$

$$S_-(g) = \int d^3x\left[-\frac{1}{2m}\epsilon^{\mu\alpha\beta}\eta^{\nu\lambda}g_{\mu\nu}\partial_\alpha g_{\beta\lambda} - \frac{1}{2}g_{\mu\nu}g^{\nu\mu} + \frac{1}{2}g^2\right]. \tag{59}$$

Here $S_\pm$ represents the self-dual and anti-self-dual theories, $f_{\mu\nu}$ and $g_{\mu\nu}$ being their fields, respectively. This separation will be crucial below, when performing the soldering of these theories. Note that, since we are interested in propagating modes, we can safely put both $f \equiv \eta^{\mu\nu}f_{\mu\nu}$ and $g \equiv \eta^{\mu\nu}g_{\mu\nu}$ equal to zero.

Let us discuss next the soldering of the above actions. Consider the following local transformation:

$$\delta h^\pm_{\mu\nu} = \partial_\mu \xi_\nu \tag{60}$$

with ξ being an infinitesimal parameter. As noted earlier $h^+_{\mu\nu} \equiv f_{\mu\nu}$ and $h^-_{\mu\nu} \equiv g_{\mu\nu}$.

Under the field transformation (60), the self- (S_+) and anti-self-dual (S_-) actions transform as

$$\delta S^\pm = \int d^3x\, \partial_\mu \xi_\nu J^{\nu\mu}_\pm, \tag{61}$$

where the associated Neither currents are given by

$$J^{\nu\mu}_\pm = \pm\frac{1}{m}\epsilon^{\mu\alpha\beta}\eta^{\nu\lambda}\partial_\alpha h_{\beta\lambda} - h_{\nu\mu}. \tag{62}$$

Although Eq. (60) is not a symmetry transformation for both S_+ and S_-, the soldering formalism will enable us to find a nontrivial composite theory, which is invariant by Eq. (60). To proceed, we again make use of an iterative Neither procedure. Introducing an auxiliary field $B_{\mu\nu}$ (the soldering field), which is coupled with the currents $J^\pm_{\mu\nu}$ so as to act as a counterterm to establish the invariance, we get the following iterated Lagrangians,

$$S_\pm \to S^{(1)}_\pm = S_\pm - \int d^3x B_{\mu\nu} J^{\nu\mu}_\pm. \tag{63}$$

If we impose the following transformation for $B_{\mu\nu}$:

$$\delta B_{\mu\nu} = \partial_\mu \xi_\nu, \tag{64}$$

then it is possible to find an effective theory invariant by both transformations (60) and (64),

$$S_{eff} = S^{(1)}_+ + S^{(1)}_- + \int d^3x B_{\mu\nu}B^{\nu\mu}. \tag{65}$$

This action is written solely in terms of the original fields after the auxiliary field $B_{\mu\nu}$ is eliminated by its equations of motion. In fact, by using the explicit structures for the currents (62), the effective Lagrangian (65) is no longer a function of the individual dual components $h_{\mu\nu}$ and $f_{\mu\nu}$, but only a combination, invariant under the soldering transformations (60),

$$A_{\mu\nu} = \frac{1}{m}(f_{\mu\nu} - g_{\mu\nu}). \tag{66}$$

Indeed after some algebra we find

$$S_{eff} = \int d^3x\left[-\frac{1}{4}F_{[\mu\nu]\lambda}F^{[\mu\nu]\lambda} + \frac{m^2}{2}A_{\mu\nu}A^{\mu\nu}\right], \tag{67}$$

where

$$F_{[\sigma\rho]\tau} = \partial_\sigma A_{\tau\rho} - \partial_\rho A_{\tau\sigma} \tag{68}$$

is the associated field tensor for the basic entity $A_{\mu\nu}$. We have succeeded in producing the fusion of self- and anti-self-dual massive degrees of freedom into a massive Maxwell-like theory for a new entity $A_{\mu\nu}$ that naturally contains both massive propagations.

Let us next rewrite our result into two different forms that will help to further clarify the physical meaning of the soldered action. Firstly we observe that the effective Lagrangian (67) can be written in the following factorized form:

$$S_{eff} = \int d^3x[\Omega^+_{\mu\nu}(A)\Omega^{\mu\nu}_-(A)] \tag{69}$$

with

$$\Omega^\pm_{\mu\nu}(A) = A_{\mu\nu} \mp \frac{1}{2m}(\eta_{\nu\lambda}\epsilon_{\mu\alpha\beta} + \eta_{\mu\lambda}\epsilon_{\nu\alpha\beta})\partial^\alpha A^{\lambda\beta}. \tag{70}$$

In this form it becomes clear that the soldered effective action indeed contains both the self- and anti-self-dual solutions, but in terms of the gauge invariant field $A_{\mu\nu}$. By solving the equations of motion for Eq. (69), we get

$$\left[\eta_{\mu\lambda}\eta_{\nu\beta} \mp \frac{1}{2m}(\eta_{\nu\lambda}\epsilon_{\mu\alpha\beta} + \eta_{\mu\lambda}\epsilon_{\nu\alpha\beta})\partial^\alpha\right]\left[\eta^{\mu\sigma}\eta^{\lambda\rho} \pm \frac{1}{2m}(\eta^{\lambda\sigma}\epsilon^{\mu\gamma\rho} + \eta^{\mu\sigma}\epsilon^{\lambda\gamma\rho})\partial_\gamma\right]A_{\rho\sigma} = 0.$$

It can be appreciated from the above expression that the self- and anti-self-dual operators may be interpreted as the square-root operators of the massive Maxwell equations very much like the Dirac operator is interpreted as the square root of the massive Klein-Gordon operator.

Finally, let us display the result in terms of a relation that includes individual components through a Polyakov-Weigman-like relation. Indeed, a simple algebra shows that

$$S_{eff}(h-f)=S_{eff}(h)+S_{eff}(f)-2\int d^3x\Omega^{+}_{\mu\nu}(h)\Omega^{\mu\nu}_{-}(f). \tag{71}$$

This identity states that the gauge invariant action on the left-hand side can be written in terms of the gauge variant components on the right-hand side, but a contact term is necessary to restore the symmetry. This is the basic content of the (2D) Polyakov-Weigman identity. As our analysis shows, such identities will always occur whenever dual aspects of a symmetry are being soldered to yield an enlarged effective action. In that case it was the chiral symmetry, while here it is 3D self-duality.

IV. CONCLUSIONS

In this work we studied the effects of interference between the self-dual modes of both the spin-1 vector model and the linearized Einstein-Chern-Simons topological gravity. We reviewed the physical spectrum of these models, first in a heuristic Lagrangian way and finally at a more formal presentation using the symplectic Hamiltonian reduction. The constraints associated with these models were found and their propagating degrees of freedom were shown to be a massive transverse field for the spin-1 model and a massive, symmetric, transverse, and traceless spin-2 mode for the self-dual gravity.

The appropriate duality transformations have been disclosed for both models and have been shown to lead to a self-dual structure. The ideas and notions of the soldering formalism, were elaborated by considering the self- and anti-self-dual formulations of the models. In particular the constraint nature of the theory is not modified. Here the soldering of second-class self-dual models led to a second-class Proca-like theory but we had the opportunity to observe that the soldering of first-class systems leads to first-class systems as well. The important point of departure being that the new group of symmetry is not a mere direct product of the individual components [24]. The interference between these opposite duality aspects has led to a nontrivial theory encompassing and extending the symmetries of both aspects in a single effective theory.

Moreover, the effective soldered theory is naturally provided with a discrete set of transformations that swaps the self- and anti-self-dual components. This theory could be recast in a variety of different forms illuminating the physical nature of the interference effects.

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