Cosmological evolution of a brane universe in a type 0 string background

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We study the cosmological evolution of a D3-brane universe in a type 0 string background. We follow the brane universe along the radial coordinate of the background and we calculate the energy density which is induced on the brane because of its motion in the bulk. We find that, for some typical values of the parameters and for a particular range of values of the scale factor of the brane universe, the effective energy density is dominated by a term proportional to $1/(\log a)^4$ indicating a slowly varying inflationary phase. For larger values of the scale factor the effective energy density takes a constant value and the brane universe enters its usual inflationary period.

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I. INTRODUCTION

There has been much recent interest in the idea that our Universe may be a brane embedded in some higher-dimensional space [1]. It has been shown that the hierarchy problem can be solved if the higher-dimensional Planck scale is low and the extra dimensions large [2]. Randall and Sundrum [3] proposed a solution of the hierarchy problem without the need for large extra dimensions but instead through curved five-dimensional spacetime AdS_5 that generates an exponential suppression of scales.

This idea of a brane universe can naturally be applied to string theory. In this context, the standard model gauge bosons as well as charged matter arise as fluctuations of the *D*-branes. The universe is living on a collection of coincident branes, while gravity and other universal interactions are living in the bulk space [4]. For example, the strongly coupled $E_8\times E_8$ heterotic string theory is believed to be an eleven-dimensional theory, the field theory limit of which was described by Horava and Witten [5]. The spacetime manifold includes a compact dimension with an orbifold structure. Matter is confined on the two ten-dimensional hypersurfaces (nine-branes) which can be seen as forming the boundaries of this spacetime.

This new concept of a brane universe naturally leads to a new approach to cosmology. Any cosmological evolution such as inflation has to take place on the brane while gravity acts globally on the whole space. In the literature there are a lot of cosmological models which study the cosmological evolution of our Universe. In most of these models the spacetime is five dimensional, where the fifth dimension is the radial dimension of an AdS_5 space. The effective Einstein equations on the brane are then solved taking under consideration the matter on the brane [6–10].

Another approach to cosmological evolution of our brane-universe is to consider the motion of the brane in higher-dimensional spacetimes. In Ref. [8] the motion of a domain wall (brane) in such a space was studied. The Israel matching conditions were used to relate the bulk to the domain wall (brane) metric, and some interesting cosmological solutions were found. In Ref. [10] a universe three-brane is considered in motion in ten-dimensional space in the presence of a gravitational field of other branes. It was shown that this motion in ambient space induces cosmological expansion (or contraction) on our universe, simulating various kinds of matter.

In this direction we have studied [11] the motion of a three-brane in the background of type 0 string theory. It was shown that the motion of the brane on this specific background, with constant values of dilaton and tachyon fields, induces a cosmological evolution which for some range of the parameters has an inflationary phase. In Ref. [12], using similar technics the cosmological evolution of the three-brane in the background of type IIB string theory was considered.

Type 0 string theories [14] are interesting because of their connection [16] to four-dimensional SU(N) gauge theory. The type 0 string does not have spacetime supersymmetry and because of that contains in its spectrum a nonvanishing tachyon field. In Ref. [14] it was argued that one could construct the dual of an SU(N) gauge theory with six real adjoint scalars by stacking N electric three-dimensional branes of the type 0 model on top of each other. The tachyon field couples to the five form field strength, which drives the tachyon to a nonzero expectation value.

Asymptotic solutions of the dual gravity background were constructed in Refs. [14,22]. At large radial coordinates the tachyon is constant and one finds a metric of the form $\mathrm{AdS}_5\times\mathrm{S}^5$ with vanishing coupling which was interpreted as a UV fixed point. The solution exhibits a logarithmic running in qualitative agreement with the asymptotic freedom property of the field theory. At small radial coordinates the tachyon vanishes and one finds again a solution of the form $\mathrm{AdS}_5\times\mathrm{S}^5$ with infinite coupling, which was interpreted as a strong coupling IR fixed point. A gravity solution which describes the flow from the UV fixed point to the IR fixed point is given in Ref. [13].

We calculate the effective energy density which is induced on the brane because of its motion in the particular background of a type 0 string. Using the approximate solutions of Refs. [14,22,13], we find that for large values of the radial coordinate r, in the UV region, the effective energy

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density takes a constant value, which means that the universe has an inflationary period. For smaller values of r, or of the scale factor α, the energy density is dominated by a term proportional to $1/(\log a)^4$, where α is the scale factor of the brane universe. This value of the energy density indicates that the universe is in a slow inflationary phase, in a ''logarithmic inflationary'' phase as we can call it, in contrast to ''constant inflationary'' phase which characterizes the usual exponential behavior. For even smaller values of r, the approximation breaks down and we cannot trust the solutions anymore. If we go to the IR region the energy density is dominated by the term $1/\alpha^4$ and again we find the ''logarithmic inflation'' for larger values of r. The approximation breaks down again for some larger values of r. It is well known that it is very difficult to connect the IR to the UV solutions. Therefore our failure to present a full cosmological evolution, relies exactly on this fact [15].

We note here that what we find is somewhat peculiar, in the sense that one does not expect the effective energy density to be dominated, for a range of values of the scale factor, by terms proportional to $1/(\log a)^4$. We understand this behavior, as due entirely to mirage matter which is induced on the brane, from this particular background.

Our work is organized as follows. In Sec. II, we develop the formalism for a brane moving in a string background with a dilaton and a Ramond-Ramond (RR) field. In Sec. III, we discuss type 0 string and except the exact solution in an $AdS_5\times S^5$ background we discuss the asymptotic UV and IR solutions of type 0 strings. In Sec. IV, we discuss the cosmological evolution of a brane in the background of type 0 string. Finally in the last section we discuss our results.

II. BRANE MOVING IN TEN-DIMENSIONAL BACKGROUND

We will consider a probe brane moving in a generic static, spherically symmetric background [10]. The brane will move in a geodesic. We assume the brane to be light compared to the background so that we will neglect the back reaction. As the brane moves the induced world-volume metric becomes a function of time, so there is a cosmological evolution from the brane point of view. The metric of a D3-brane is parametrized as

$$ds_{10}^2 = g_{00}(r)dt^2 + g(r)(d\vec{x})^2 + g_{rr}(r)dr^2 + g_S(r)d\Omega_5^2 \tag{2.1}$$

and there is also a dilation field Φ as well as a RR background $C(r)=C_{0\ldots3}(r)$ with a self-dual field strength. The dynamics on the brane will be governed by the Dirac-Born-Infeld action given by

$$S = T_3\int d^4\xi e^{-\Phi}\sqrt{-\det(\hat{G}_{\alpha\beta}+(2\pi\alpha')F_{\alpha\beta}-B_{\alpha\beta})} + T_3\int d^4\xi \hat{C}_4 + \text{anomaly terms.} \tag{2.2}$$

The induced metric on the brane is

$$\hat{G}_{\alpha\beta} = G_{\mu\nu}\frac{\partial x^\mu \partial x^\nu}{\partial\xi^\alpha\partial\xi^\beta} \tag{2.3}$$

with similar expressions for $F_{\alpha\beta}$ and $B_{\alpha\beta}$. In the static gauge, $x^\alpha=\xi^\alpha$, $\alpha=0,1,2,3$ using Eq. (2.3) we can calculate the bosonic part of the brane Lagrangian which reads

$$L = \sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j} - C(r), \tag{2.4}$$

where $h_{ij}d\varphi^i d\varphi^j$ is the line element of the unit five-sphere and

$$A(r) = g^3(r)|g_{00}(r)|e^{-2\Phi}, \quad B(r) = g^3(r)g_{rr}(r)e^{-2\Phi},$$
$$D(r) = g^3(r)g_S(r)e^{-2\Phi}, \tag{2.5}$$

and $C(r)$ is the RR background. Demanding conservation of energy E and of total angular momentum l^2 on the brane, we find

$$\dot{r}^2 = \frac{A}{B}\left(1 - \frac{A}{(C+E)^2}\frac{D+l^2}{D}\right), \quad h_{ij}\dot{\varphi}^i\dot{\varphi}^j = \frac{A^2 l^2}{D^2(C+E)^2}. \tag{2.6}$$

We can see that the above relation gives the following constraint:

$$\left(1 - \frac{A}{(C+E)^2}\frac{D+l^2}{D}\right) \geqslant 0. \tag{2.7}$$

The induced four-dimensional metric on the brane is

$$d\hat{s}^2 = (g_{00} + g_{rr}\dot{r}^2 + g_S h_{ij}\dot{\varphi}^i\dot{\varphi}^j)dt^2 + g(d\vec{x})^2. \tag{2.8}$$

In the above relation we substitute $\dot{r}^2$ and $h_{ij}\dot{\varphi}^i\dot{\varphi}^j$ from Eq. (2.6), and we get

$$d\hat{s}^2 = -d\eta^2 + g[r(n)](d\vec{x})^2 \tag{2.9}$$

with η the cosmic time which is defined by

$$d\eta = \frac{|g_{00}|g^{3/2}e^{-\Phi}}{|C+E|}dt. \tag{2.10}$$

The induced metric (2.9) on the brane is the standard form of a flat expanding universe. For this metric we can write the effective Einstein equations on the brane

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(T_{\mu\nu})_{\text{eff}}, \tag{2.11}$$

where $(T_{\mu\nu})_{\text{eff}}$ is the effective energy momentum tensor which is induced on the brane. We have assumed that our brane is light and there is no back-reaction with the bulk. We expect $(T_{\mu\nu})_{\text{eff}}$ to be a function of the quantities of the bulk.

Before we proceed, it is interesting to discuss the general case where there is matter on the brane with an energy momentum tensor $\tau_{\mu\nu}$ and a cosmological constant Λ. The Einstein equations on the brane are [17,18]

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-\Lambda q_{\mu\nu}+8\pi G\tau_{\mu\nu}+\kappa^4\pi_{\mu\nu}-E_{\mu\nu}, \tag{2.12}$$

where $q_{\mu\nu}$ is the induced metric on the brane, $\pi_{\mu\nu}$ is a function of the matter content of the brane, having the form

$$\pi_{\mu\nu}=-\frac{1}{4}\tau_{\mu\nu}\tau_{\nu}^{\alpha}+\frac{1}{12}\tau\tau_{\mu\nu}+\frac{1}{8}q_{\mu\nu}\tau_{\alpha\beta}\tau^{\alpha\beta}-\frac{1}{24}q_{\mu\nu}\tau^2, \tag{2.13}$$

and $E_{\mu\nu}$ is given by

$$E_{\mu\nu}=C^{\alpha}_{\beta\rho\sigma}n_{\alpha}n^{\rho}q^{\beta}_{\mu}q^{\sigma}_{\nu}, \tag{2.14}$$

where $C^{\alpha}_{\beta\rho\sigma}$ is the Weyl tensor [17]. As we can see from the above relation the term $E_{\mu\nu}$ is a geometrical object depending on the bulk geometry. If we now compare Eq. (2.11) with Eq. (2.12), we can see that because in our case we do not have matter on the brane, consistency requires that $(T_{\mu\nu})_{\rm eff}$ of Eq. (2.11) should be proportional to $E_{\mu\nu}$ of Eq. (2.12). One can check that using relation (2.14) one can get similar results as ours, an approach which is followed in Ref. [9].

If we now assume the usual form of a perfect fluid for the effective energy momentum tensor, we get from Eq. (2.11)

$$8\pi G\rho+\Lambda=\frac{3}{4}g^{-2}\dot{g}^2. \tag{2.15}$$

We can now define an $\rho_{\rm eff}$ from the relation

$$8\pi G\rho+\Lambda=8\pi G_N\rho_{\rm eff}. \tag{2.16}$$

Using Eq. (2.10) we get

$$\dot{g}=g'\left[\frac{|g_{00}|}{g_{rr}}-\frac{g_{00}^2}{g_s g_{rr}}\left(\frac{g^3 g_s e^{-2\Phi}+l^2}{(C+E)^2}\right)\right]^{1/2}\frac{|C+E|}{|g_{00}|g^{3/2}e^{-\Phi}}, \tag{2.17}$$

where a prime denotes differentiation with respect to r. To derive an analogue of the four-dimensional Friedman equations for the expanding four-dimensional universe on the probe D3-brane, we define the scale factor as $g=\alpha^2$ and then Eq. (2.16) with the use of Eqs. (2.15) and (2.17) becomes

$$\frac{8\pi}{3}G_N\rho_{\rm eff}=\left(\frac{\dot{\alpha}}{\alpha}\right)^2$$
$$=\frac{(C+E)^2 g_S e^{2\Phi}-|g_{00}|(g_S g^3+l^2 e^{2\Phi})}{4|g_{00}|g_{rr}g_S g^3}\left(\frac{g'}{g}\right)^2. \tag{2.18}$$

Therefore the motion of a D3-brane on a general spherically symmetric background had induced on the brane a matter density. As it is obvious from the above relation, the specific form of the background will determine the cosmological evolution on the brane.

We had defined $\rho_{\rm eff}$ through the relation (2.16). In this relation the four-dimensional Newton's constant G_N and the four-dimensional cosmological constant appears. To make our approach clear, we will discuss their physical meaning in our scheme.

We have assumed that in our brane universe there is no gravity by itself; therefore Newton's law is defined on the whole 10-dimensional space. As the brane moves, we can write on the brane the effective Einstein equations (2.11). The Newton's constant which appears in this equation has the meaning of an effective parameter determined by the background. To see its value let us go to a particular background. Let us consider the near-horizon geometry of a macroscopic D3-brane which is $AdS_5\times S^5$, with metric [10,9],

$$ds^2=\frac{r^2}{L^2}\left\{-\left[1-\left(\frac{r_0}{r}\right)^4\right]dt^2+(d\vec{x})^2\right\}+\frac{L^2}{r^2}\frac{dr^2}{\left[1-\left(\frac{r_0}{r}\right)^4\right]}$$
$$+L^2 d\Omega_5^2, \tag{2.19}$$

where $L^4=4\pi gN\alpha'^2$ with g the string coupling in 10 dimensions and N the number of D3-branes. By using Eq. (2.18) the effective energy density on the brane in this background is

$$\frac{8\pi}{3}G_N\rho_{\rm eff}=\frac{1}{L^2}\left\{\left(1+\frac{E}{\alpha^4}\right)^2-\left[1-\left(\frac{r_0}{L}\right)^4\frac{1}{\alpha^4}\right]\left(1+\frac{l^2}{L^2}\frac{1}{\alpha^6}\right)\right\}. \tag{2.20}$$

As we can see, the only scale which enters in this relation is α'. Then as it is obvious from the above relation, we can on purely dimensional grounds write $G_N=L^2$ or $G_N=2\sqrt{\pi gN}\alpha'$. We can express the G_N in terms of the 5-dimensional Newton's constant using the relation $T_3=3/4\pi GL$ [9]

$$G_N=\frac{4\pi}{3}GT_3(2\sqrt{\pi gN}\alpha')^{3/2}, \tag{2.21}$$

where T_3 is the brane tension.

The induced cosmological constant on the brane Λ can be expressed in terms of the background fields using the equation

$$-g^{-2}\dot{g}^2+g^{-1}\ddot{g}+\frac{3}{4}\gamma g^{-2}\dot{g}^2-\gamma\Lambda=0. \tag{2.22}$$

In deriving the above equation we have used the conservation of energy momentum tensor $(T_{\mu\nu})_{\rm eff}$ and an equation of state in the form $p=(\gamma-1)\rho$. Nevertheless we believe that in this approach we cannot really distinguish between ρ and Λ and a definition of the form (2.16) is more meaningful. As it is discussed in Ref. [17] in the case where we cannot distinguish between vacuum energy and matter energy we cannot truly specify G_N. Then as we discussed above, G_N is determined in a "phenomenological" way depending on the background.

III. TYPE 0 STRING BACKGROUND

Type 0 string theory is interesting because of its connection to gauge theories. This enables us to study SU(N) gauge theory by merely gravitational quantities. Another advantage of this theory is the presence of a tachyon field. Tachyonic fields in ordinary field theory create instabilities. In cosmology on the contrary, the time evolution of a tachyon field plays an important role. In two dimensions because the tachyon field is a matter field has important consequences in cosmology [19], and it can give a solution to the ''gracefull exit'' problem [20]. In four dimensions its effect to cosmology has been examined by various authors [21].

As we have shown in Ref. [11] in type 0 string the tachyon field can induce inflation on the brane. We had used an exact solution of type 0 string with constant tachyon and dilation fields. If these fields are coordinate dependent then, there is not an exact solution of the theory, but there are approximate solutions which we will discuss in the following. The action of the type 0 string is given by [14]

$$S_{10}=\int d^{10}x\sqrt{-g}\left[e^{-2\Phi}\left(R+4(\partial_\mu\Phi)^2-\frac{1}{4}(\partial_\mu T)^2-\frac{1}{4}m^2T^2-\frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}\right)-\frac{1}{4}\left(1+T+\frac{T^2}{2}\right)|F_5|^2\right]. \tag{3.1}$$

The equations of motion which result from this action, with the antisymmetric field put to zero, are

$$2\nabla^2\Phi-4(\nabla_n\Phi)^2-\frac{1}{4}m^2T^2=0, \tag{3.2}$$

$$R_{mn}+2\nabla_m\nabla_n\Phi-\frac{1}{4}\nabla_mT\nabla_nT-\frac{1}{4\times 4!}e^{2\Phi}f(T)\times\left(F_{mklpq}F_n{}^{klpq}-\frac{1}{10}G_{mn}F_{sklpq}F^{sklpq}\right)=0, \tag{3.3}$$

$$(-\nabla^2+2\nabla^n\Phi\nabla_n+m^2)T+\frac{1}{2\times 5!}e^{2\Phi}f'(T)F_{sklpq}F^{sklpq}=0, \tag{3.4}$$

$$\nabla_m[f(T)F^{mnkpq}]=0. \tag{3.5}$$

The tachyon is coupled to the RR field through the function

$$f(T)=1+T+\frac{1}{2}T^2. \tag{3.6}$$

In the background where the tachyon field acquires vacuum expectation value $T_{\rm vac}=T_0=-1$, the tachyon function (3.6) takes the value $f(T_0)=\frac{1}{2}$ which guarantee the stability of the theory [14].

Equations (3.2)–(3.5) can be solved using the following ansatz for the metric:

$$ds_{10}^2=g_{00}(r)dt^2+g(r)(d\vec{x})^2+g_{rr}(r)dr^2+g_S(r)d\Omega_5. \tag{3.7}$$

Technically it is easier to solve the above equations if we go to new variables. One can then introduce the new parameter ρ through the relation

$$\rho=\frac{e^{2\Phi_0}}{4r^4} \tag{3.8}$$

and the fields ξ and η from the relations

$$g=e^{(\Phi-\xi)/2}, \tag{3.9}$$

$$g_s=e^{(\Phi+\xi)/2-\eta}. \tag{3.10}$$

Then Eq. (3.7) takes the form

$$ds^2=-e^{(\Phi-\xi)/2}dt^2+e^{(\Phi-\xi)/2}d\vec{x}^2+e^{(\Phi+\xi)/2-5\eta}d\rho^2+e^{(\Phi+\xi)/2-\eta}d\Omega_5^2. \tag{3.11}$$

With this form of the metric the action (3.1) can be described by the following Toda-like mechanical system (an overdot denotes ρ derivative)

$$S=\int d\rho\left[\frac{1}{2}\dot{\Phi}^2+\frac{1}{2}\dot{\xi}^2+\frac{\dot{T}^2}{4}-5\dot{\eta}^2-V(\Phi,\xi,T,\eta)\right] \tag{3.12}$$

with the potential $V(\Phi,\xi,T,\eta)$ given by

$$V(\Phi,\xi,T,\eta)=g(T)e^{(1/2)\Phi+(1/2)\xi-5\eta}+20e^{-4\eta}-Q^2f^{-1}(T)e^{-2\xi}. \tag{3.13}$$

If the tachyon field takes its vacuum value and the dilation field a constant value $\Phi=\Phi_0$ one can find the electrically charged three-brane

$$g_{00}=-H^{-1/2},\quad g(r)=H^{-1/2},\quad g_S(r)=H^{1/2}r^2,\quad g_{rr}(r)=H^{1/2},\quad H=1+\frac{e^{\Phi_0}Q}{2r^4} \tag{3.14}$$

if the following ansatz for the RR field:

$$C_{0123}=A(r),\quad F_{0123r}=A'(r) \tag{3.15}$$

is used.

If T and Φ are functions of the coordinate r, then approximate solutions exist [14,22]. These solutions are valid for large (UV) and small (IR) values of the radial coordinate. The approximations of Refs. [14,22] agree in the UV region but in the IR the approximation of Ref. [14] leads to an IR fixed point, while the approximation of Ref. [22] to a confining point.

From the action (3.12) we can derive the following equations of motion [22]:

$$\ddot{\xi}+\frac{1}{2}g(T)e^{(1/2)\Phi+(1/2)\xi-5\eta}+2\frac{Q^2}{f(T)}e^{-2\xi}=0, \tag{3.16}$$

$$\ddot{\eta}+\frac{1}{2}g(T)e^{(1/2)\Phi+(1/2)\xi-5\eta}+8e^{-4\eta}=0, \tag{3.17}$$

$$\ddot{\Phi}+\frac{1}{2}g(T)e^{(1/2)\Phi+(1/2)\xi-5\eta}=0, \tag{3.18}$$

$$\ddot{T}+2g'(T)e^{(1/2)\Phi+(1/2)\xi-5\eta}+2\frac{Q^2f'(T)}{f^2(T)}e^{-2\xi}=0, \tag{3.19}$$

where $g(T)$ is the bare tachyon potential

$$g(T)=\frac{1}{2}T^2-\lambda T^4 \tag{3.20}$$

and λ is a parameter. Defining a new variable $\rho=u^{-4}$, in the UV for $u\to\infty$, or $\rho\to 0$, we can solve the equations of motion (3.16)–(3.19) to the next to leading order and find [22,14]

$$T=T_0-4\frac{g'(T_0)}{g(T_0)}\frac{1}{\log\rho}+O\left(\frac{\log(-\log\rho)}{\log^2\rho}\right), \tag{3.21}$$

$$\Phi=-2\log(C_0\log\rho)-\left[7+8\left(\frac{g'(T_0)}{g(T_0)}\right)^2\right]\left(\frac{\log(-\log\rho)}{\log\rho}\right)+\frac{B}{\log\rho}+O\left(\frac{\log^2(-\log\rho)}{\log^2\rho}\right), \tag{3.22}$$

$$\xi=\log[\sqrt{2f^{-1}(T_0)}Q\rho]-\frac{1}{\log\rho}+O\left(\frac{\log(-\log\rho)}{\log^2\rho}\right), \tag{3.23}$$

$$\eta=\frac{1}{2}\log(4\rho)-\frac{1}{\log\rho}+O\left(\frac{\log(-\log\rho)}{\log^2\rho}\right), \tag{3.24}$$

where $C_0=-4C_2^5/g(T_0)\sqrt{C_1}$ and

$$C_1=\frac{2Q}{\sqrt{2f(T_0)}}\left(1+\frac{1}{4\log\frac{u}{u_0}}\right), \quad C_2=2\left(1+\frac{1}{4\log\frac{u}{u_0}}\right). \tag{3.25}$$

The above solutions show that at the UV point, the tachyon takes a constant value and if we calculate the next to leading order effective coupling $e^{(1/2)\Phi}$,

$$e^{(1/2)\Phi}\sim\frac{1}{\log u-\left(\frac{7}{8}+\frac{g'(T_0)^2}{g(T_0)^2}\right)\log\log u} \tag{3.26}$$

we see that goes to zero for large u.

For $u\to 0$ or for large ρ there are two approximate solutions in the literature, leading to infrared fixed point [14] or to a confining fixed point [22,13]. For the first approximation we have

$$T=-\frac{16}{\log\rho}-\frac{8}{\log^2\rho}(9\log\log\rho-3)+O\left(\frac{\log^2\log\rho}{\log^3\rho}\right), \tag{3.27}$$

$$\Phi=-\frac{1}{2}\log(2Q^2)+2\log\log\rho-\frac{1}{\log\rho}9\log\log\rho+O\left(\frac{\log\log\rho}{\log^2\rho}\right), \tag{3.28}$$

$$\xi=\frac{1}{2}\log(2Q^2)+\log\rho+\frac{9}{\log\rho}+\frac{9}{2\log^2\rho}\left(9\log\log\rho-\frac{20}{9}\right)+O\left(\frac{\log^2\log\rho}{\log^3\rho}\right), \tag{3.29}$$

$$\eta=\log 2+\frac{1}{2}\log\rho+\frac{1}{\log\rho}+\frac{1}{2\log^2\rho}(9\log\log\rho-2)+O\left(\frac{\log^2\log\rho}{\log^3\rho}\right). \tag{3.30}$$

While for the second solution [22,13] we have

$$\Phi=\Phi_0+\rho-\frac{1}{16(\sqrt{5}+3)^2}e^{\Phi_0/2}e^{-(\sqrt{5}+3)\rho}, \tag{3.31}$$

$$\eta=\frac{1}{\sqrt{5}}\rho-\frac{5}{2}e^{-(4/\sqrt{5})\rho}, \tag{3.32}$$

$$\xi=\rho-\frac{1}{2}e^{-2\rho}, \tag{3.33}$$

$$T=-\frac{1}{2}e^{-2\rho}. \tag{3.34}$$

In both approximations the tachyon field in the IR point goes to zero while the effective coupling gets infinite. It is important to observe that the approximate solutions in the UV (3.21)–(3.24) and in the IR (3.28)–(3.30) of Ref. [14] are related by $y\to -y$, suggesting that they can be smoothly connected into a full interpolating solution. An attempt to connect these solutions was presented in Ref. [13]. As we mention above, the tachyon field starts at $T=-1$ at $\rho=0$ in the UV, and grows according to Eq. (3.21), then enters an oscillating regime and finally relaxes to zero according to Eq. (3.27), when $\rho=\infty$ in IR. This guarantees that $T^2e^{(1/2)\Phi}$ becomes small which leads the metric in the $AdS_5\times S^5$ form.

There is a question if we can trust the asymptotic solutions in the infrared. The problem is that when the coupling becomes strong, string corrections become important. The situation is different in the UV where we can trust our solutions because the coupling is small. The role of the α' corrections in the IR has been discussed in Refs. [22,13]. It was claimed that the α' corrections are small.

IV. COSMOLOGICAL EVOLUTION OF THE BRANE-UNIVERSE

We consider a D3-brane moving along a geodesic in the background of a type 0 string. Having all the solutions in the ultraviolet and the infrared, we can follow the cosmological evolution of our universe as it moves along the radial coordinate r. In the presence of a nontrivial tachyon field the coupling $e^{-\Phi}$ which appears in the Dirac-Born-Infeld action in Eq. (2.2), is modified by a tachyonic function $\kappa(T)=1+\frac{1}{4}T+O(T^2)$. Then we can define an effective coupling [16]

$$e_{\rm eff}^{-\Phi}=\kappa(T)e^{-\Phi}. \tag{4.1}$$

The bulk fields are also coordinate dependent and the induced metric on the brane will depend on a nontrivial way on the dilaton field. Therefore the metric in the string frame will be connected to the metric in the Einstein frame through $g_{St}=e_{\rm eff}^{\Phi/2}g_E$. All the quantities used so far were defined in the string frame. We will follow our cosmological evolution in the Einstein frame. Then the relation (2.18) becomes

$$\frac{8\pi}{3}\rho_{\rm eff}=\left(\frac{\dot{\alpha}}{\alpha}\right)^2=\frac{(C+E)^2g_S-|g_{00}|(g_Sg^3+l^2)}{4|g_{00}|g_{rr}g_Sg^3}\left(\frac{g'}{g}\right)^2. \tag{4.2}$$

Having the approximate solution in the UV given by Eqs. (3.21)–(3.24) we can calculate the metric components of the metric (3.7) and find

$$g_{yy}=\frac{1}{16}\sqrt{\frac{Q}{2}}\left(1-\frac{9}{2y}\right), \tag{4.3}$$

$$g=\frac{1}{\sqrt{2Q}}e^{y/2}\left(1-\frac{1}{2y}\right), \tag{4.4}$$

$$g_s=\sqrt{\frac{Q}{2}}\left(1-\frac{1}{2y}\right). \tag{4.5}$$

The variable y is defined by

$$\rho=e^{-y}. \tag{4.6}$$

Then we can identify g of Eq. (4.4) with the scale factor α^2 and solve for y. We get two solutions

$$y_1=-\frac{1}{4\log\alpha+\log 2Q}, \tag{4.7}$$

$$y_2=\log 2Q+4\log\alpha+\frac{1}{\log 2Q+4\log\alpha}. \tag{4.8}$$

From the solution (4.8) which has the right behavior for large α, we keep the $\log 2Q+4\log\alpha$ term. Then the RR field C becomes

$$C=\frac{e^y}{Q}-\frac{2}{Q}E_i[y]. \tag{4.9}$$

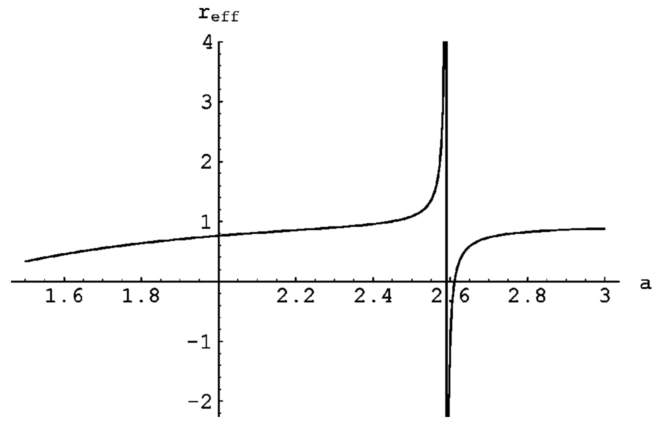

FIG. 1. The induced energy density on the brane as a function of the brane scale factor.

Then, we can calculate the effective energy density from Eq. (4.2) setting $l^2=0$ and we get

$$\begin{aligned}\frac{8\pi}{3}\rho_{\rm eff}=&\Bigg[\left(1-\frac{1}{Q\alpha^4}Ei[\log 2Q+4\log\alpha]+\frac{E}{2\alpha^4}\right)^2\\&-\frac{1}{4}\left(1-\frac{1}{2(\log 2Q+4\log\alpha)}\right)^4\Bigg]\\&\times\left(1-\frac{1}{2(\log 2Q+4\log\alpha)}\right)^{-4}\\&\times\left(1-\frac{9}{2(\log 2Q+4\log\alpha)}\right)^{-1}\\&\times\left(1+\frac{1}{(\log 2Q+4\log\alpha)^2}\right.\\&\left.\times\frac{1}{\left(1-\frac{1}{2(\log 2Q+4\log\alpha)}\right)}\right)^2.\end{aligned} \tag{4.10}$$

For some typical value of the parameters $Q=1$ and $E=1$, and for large values of α, it is obvious that $\rho_{\rm eff}$ has a constant value. Therefore an observer on the brane will see an expanding inflating universe. It is interesting to see what happens for small values of α. As α gets smaller, a term proportional to $1/(\log\alpha)^4$ starts to contribute to $\rho_{\rm eff}$. Therefore the universe for small values of scale factor has a slow expanding inflationary phase which we call the ''logarithmic inflationary'' phase. For smaller values of α we cannot trust the solution which is reflected in the fact that $\rho_{\rm eff}$ gets infinite. The behavior of the effective energy density as a function of the scale factor is shown in Fig. 1.

To have an idea how the slow inflationary phase proceeds, we can assume for the moment that the effective energy density scales as

$$\frac{8\pi}{3}\rho_{\rm eff}=\left(\frac{\dot{\alpha}}{\alpha}\right)^2=\frac{1}{(\log\alpha)^p}. \tag{4.11}$$

The solution of the above equation is

$$\alpha=e^{t^{[2/p+2]}} \tag{4.12}$$

Therefore we remain in an exponentially growing universe, but various values of p have the effect of making the universe to slow down its expansion. We note here that in order to estimate the behavior and the duration of this ''logarithmic inflationary'' phase, we have to resolve the problem of the singularity.

Going now to IR using Eqs. (3.27)–(3.30) we get for the metric components

$$g_{yy}=\frac{\sqrt{Q}}{16}2^{-3/4}\left(1-\frac{1}{2y}\right), \tag{4.13}$$

$$g=\frac{2^{-1/4}}{\sqrt{Q}}e^{-y/2}\left(1-\frac{9}{2y}\right), \tag{4.14}$$

$$g_s=2^{-3/4}\sqrt{Q}\left(1+\frac{7}{2y}\right), \tag{4.15}$$

where now y is defined by

$$\rho=e^{y}. \tag{4.16}$$

Then the identification $g=\alpha^2$ using Eq. (4.14) gives again two solutions

$$y_1=-\frac{9}{4\log\alpha+\log\sqrt{2}Q}, \tag{4.17}$$

$$y_2=-\log\sqrt{2}Q-4\log\alpha+\frac{9}{\log\sqrt{2}Q+4\log\alpha}. \tag{4.18}$$

For small α we keep from the solution y_2 of Eq. (4.18) the term $-\log\sqrt{2}Q-4\log\alpha$. Using this solution we can calculate the RR field

$$C=-\frac{e^{-y}}{Q}-\frac{2}{Q}E_i[-y]. \tag{4.19}$$

Then $\rho_{\rm eff}$ becomes

$$\frac{8\pi}{3}\rho_{\rm eff}=\left[\left(-1-2\frac{1}{\sqrt{2}Q\alpha^4}E_i[\log\sqrt{2}Q+4\log\alpha]+\frac{E}{\sqrt{2}\alpha^4}\right)^2-\frac{1}{2}\left(1+\frac{9}{2(\log\sqrt{2}Q+4\log\alpha)}\right)^4\right]$$
$$\times\left(1+\frac{9}{2(\log\sqrt{2}Q+4\log\alpha)}\right)^{-4}\left(1+\frac{1}{2(\log\sqrt{2}Q+4\log\alpha)}\right)^{-1}$$
$$\times\left(1-\frac{9}{2(4\log\alpha+\log\sqrt{2}Q)^2}\frac{1}{\left(1-\frac{9}{2(\log\sqrt{2}Q+4\log\alpha)}\right)}\right)^2. \tag{4.20}$$

As we can see, the above relation is the same as the energy density in the UV [relation (4.10)] up to some numerical factors, as expected. The difference is, that now it is valid for small α. For small α first the term $1/\alpha^8$ dominates and then the term $1/\alpha^4$. As α increases the term $1/(\log\alpha)^4$ takes over and drives the universe to a slow inflationary expansion.

We will also discuss the cosmological behavior of the IR solutions of Refs. [22] and [13]. Using Eqs. (3.31)–(3.34) we have for the metric elements

$$g_{\rho\rho}=e^{(1/2-\sqrt{5})\rho}e^{-(1/4)e^{-2\rho+(25/2)}e^{-4\rho/\sqrt{5}}}\sqrt{Q}, \tag{4.21}$$

$$g=e^{-\rho/2}e^{(1/4)e^{-2\rho}}\frac{1}{\sqrt{Q}}, \tag{4.22}$$

$$g_s=e^{(1/2-1/\sqrt{5})\rho}e^{-(1/4)e^{-2\rho}+(5/2)e^{-4\rho/\sqrt{5}}}\sqrt{Q}. \tag{4.23}$$

Then the equation $g=\alpha^2$ gives

$$\rho=-4\log\alpha-\log Q \tag{4.24}$$

and the RR field becomes

$$C=(-2e^{-2\rho}-e^{-4\rho})\frac{1}{Q}. \tag{4.25}$$

Finally $\rho_{\rm eff}$ becomes

$$\frac{8\pi}{3}\rho_{\rm eff}=(Q\alpha^4)^{-\sqrt{5}+1/2}[(-2Q\alpha^4-Q^3\alpha^{12}+E\alpha^{-4})^2$$
$$-(1+Q^2\alpha^8)]\left(1-\frac{1}{4}Q^2\alpha^8+\frac{25}{2}(Q\alpha^4)^{4/\sqrt{5}}\right)^{-1}$$
$$\times(1+Q^2\alpha^8). \tag{4.26}$$

The above calculated effective energy density, in spite of its different form, has a similar behavior as Eq. (4.20). As α increases, various negative powers of α take over until the singularity is reached where positive powers of α dominate.

V. DISCUSSION

We had followed a probe brane along a geodesic in the background of type 0 string. Assuming that the universe is described by a three-dimensional brane, we calculate the effective energy density which is induced on the brane because of this motion. We study this mirage matter as the brane-universe moves along the radial coordinate.

In our previous work [11] we found that the motion of the brane-universe in this particular background induces an inflationary phase on the brane. We made the analysis in the limited case where the dilaton and tachyon fields were constants. This assumption simplified the calculation because there is an exact solution of the equations of motion.

In this work we extend our study to a background where all the fields are functions of the radial coordinate. Then the problem becomes more complicated because there is no more an exact solution to the equations of motion. Nevertheless there are approximate solutions for large values of the radial coordinate, in the UV region and solutions for small values of the radial coordinate in the IR. In the UV the coupling of the theory is small, so we can trust the approximate solutions. In the IR, the coupling becomes strong but it was shown in the literature [22,13] that all string corrections are small.

Using these solutions, we calculate the energy densities that are induced on the brane. What we find is that for large values of the scale factor as it is measured on the brane (large values of the radial coordinate) the universe enters a slow inflationary phase, in which the energy density is proportional to an inverse power of the logarithm of the scale factor. As the scale factor grows the induced energy density takes a constant value and the universe enters a normal exponential expansion. For small values of the scale factor the induced energy density scales as the inverse powers of the scale factor and then the logarithmic terms take over and the universe enters a slow exponential expansion.

The energy densities we calculated break down for some specific values of the scale factor. This is a reflection of the fact that the approximate solutions in the IR cannot be continued to the UV. To answer the question if there is a true phase of ''logarithmic inflation'' in which the universe inflates but with a slow rate, we must resolve the problem of singularities, where our theory breaks down. We are studying the problem numerically trying to solve the equations of motion numerically [13] and see if we can smoth out the singularities. Then we can apply our technics for calculating the effective energy density.

Note added. While this work was written up to its final form, Ref. [23] appeared where a similar problem was studied, and it was found that the tachyonic background is less divergent than the one without a tachyon.

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[1] T. Regge and C. Teitelboim, *Marcel Grossman Meeting on General Relativity*, Trieste, 1975 (North Holland, Amsterdam, 1975); V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **125B**, 136 (1983).

[2] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); Phys. Rev. D **59**, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998).

[3] R. Sundrum, Phys. Rev. D **59**, 085009 (1999); **59**, 085010 (1999); L. Randall and R. Sundrum, Nucl. Phys. **B557**, 79 (1999); Phys. Rev. Lett. **83**, 3370 (1999).

[4] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995).

[5] P. Horava and E. Witten, Nucl. Phys. **B460**, 506 (1996).

[6] N. Kaloper and A. Linde, Phys. Rev. D **59**, 101303 (1999); N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and J. March-Russell, Nucl. Phys. **B567**, 189 (2000); N. Arkani-Hamed, S. Dimopoulos, G. Dvali, and N. Kaloper, Phys. Rev. Lett. **84**, 586 (2000); N. Kaloper, Phys. Rev. D **60**, 123506 (1999); P. Kanti, I. I. Kogan, K. A. Olive, and M. Pospelov, Phys. Lett. B **468**, 31 (1999).

[7] P. Binetruy, C. Deffayet, and D. Langlois, Nucl. Phys. **B565**, 269 (2000); G. Dvali and S. H. Tye, Phys. Lett. B **450**, 72 (1999); E. E. Flanagan, S. H. H. Tye, and I. Wasserman, Phys. Rev. D **62**, 024011 (2000); H. B. Kim and H. D. Kim, *ibid.* **61**, 064003 (2000); C. Csaki, M. Graesser, C. Kolda, and J. Terning, Phys. Lett. B **462**, 34 (1999); C. Csaki, M. Graesser, L. Randall, and J. Terning, Phys. Rev. D **62**, 045015 (2000); L. Anchordoqui, C. Numez, and K. Kasper, J. High Energy Phys. **10**, 050 (2000).

[8] H. A. Chamblin and H. S. Reall, Nucl. Phys. **B562**, 133 (1999); A. Chamblin, M. J. Perry, and H. S. Reall, J. High Energy Phys. **09**, 014 (1999).

[9] P. Kraus, J. High Energy Phys. **12**, 011 (1999).

[10] A. Kehagias and E. Kiritsis, J. High Energy Phys. **11**, 022 (1999).

[11] E. Papantonopoulos and I. Pappa, ''Type 0 Brane Inflation from Mirage Cosmology,'' hep-th/0001183.

[12] J. Y. Kim, Phys. Rev. D **62**, 065002 (2000).

[13] R. Grena, S. Lelli, M. Maggiore, and A. Rissone, J. High Energy Phys. **07**, 005 (2000).

[14] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. **B546**, 155 (1999).

[15] E. Papantonopoulos, *Talk given at IX Marcel Grossman meeting*, Rome, 2000.

[16] I. R. Klebanov and A. Tseytlin, Nucl. Phys. **B547**, 143 (1999).

[17] T. Shiromizu, K. Maeda, and M. Sasaki, Phys. Rev. D **62**, 024012 (2000).

[18] K. Maeda and D. Wands, Phys. Rev. D **62**, 124009 (2000).

[19] G. A. Diamandis, B. C. Georgalas, and E. Papantonopoulos,

Mod. Phys. Lett. A **10**, 1277 (1995); C. Chiou-Lahanas, G. A. Diamandis, B. C. Georgalas, X. N. Maintas, and E. Papantonopoulos, Phys. Rev. D **52**, 5877 (1995).
[20] G. A. Diamandis, B. C. Georgalas, N. E. Mavromatos, and E. Papantonopoulos, Phys. Lett. B **461**, 57 (1999).
[21] V. A. Kostelecky and M. J. Perry, Nucl. Phys. **B414**, 174 (1994); E. Raiten, *ibid.* **B416**, 881 (1994); K. Behrndt, Phys. Lett. B **301**, 29 (1993).
[22] J. A. Minahan, J. High Energy Phys. **04**, 007 (1999).
[23] Jin Young Kim, Phys. Rev. D **63**, 045014 (2001).