Quantization of electric charge, the neutrino, and generation universality

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It is shown that electric charge quantization is unconnected to the Majorana neutrino in the nonuniversal generation leptoquark-bilepton flavor dynamics which includes the right-handed neutrino and an explicit U(1) factor in the gauge semisimple group.

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Two fundamental questions left unsolved within the standard model of nuclear and electromagnetic interactions [1] are the flavor question and the fermion mass hierarchy, which are addressed [2–6] in the leptoquark-bilepton flavor dynamics [2,3,7] having the local gauge symmetries $G_{3m1} \equiv \mathrm{SU}(3)_c \otimes \mathrm{SU}(m)_L \otimes \mathrm{U}(1)_{L+R}$ with $m=3,4$. There is also a cubic seesaw relation $m(\nu_l) \sim m^3(l)/M_W^2$ constraining the neutrino mass to the charged lepton masses [8] implicating the 10^{-5} eV value for the lightest neutrino mass, and also the interesting possibility of double beta decay depending less on the neutrino mass [9] than in many extensions of the standard model, as well as the associated Majoron emission process [10]. The nontrivial issue that in the $m=3$ model there is a Peccei-Quinn symmetry [11] with an invisible axion was also shown, solving the strong-CP problem [12]. Although in the minimal G_{331} leptoquark-bilepton model the massive neutrinos are Majorana fermions [13,14], as a result of the nonuniversal generation structure, the electric charge quantization and Majorana neutrino connection is lost. Such facts agree with a pioneering scrutiny [15] involving a large class of gauge models containing a U(1) factor in the gauge group, the right-handed neutrino, and generation universality. Using the lightest leptons as the particles which determine the approximate symmetry with the generation nonuniversality and if lepton charges are only 0, ± 1, SU(4) is the highest symmetry group to be included in the electroweak sector. There is no room for $m>4$. A model with $\mathrm{SU}(4) \otimes \mathrm{U}(1)$ symmetry was proposed more than one decade ago by Voloshin [16] but quarks were not included there.

Let us consider the largest leptoquark-bilepton gauge semisimple G_{341} group extension. The electric charge operatorial formula [17]

$$\frac{\mathcal{Q}}{|e|} = \Lambda_3 + \xi\Lambda_8 + \zeta\Lambda_{15} + \varsigma N \Lambda_0 \tag{1}$$

is embedded in the traceless neutral generators Λ_i, $i=3,8,15$, of the SU(4) gauge group,

$$\Lambda_3 = \frac{1}{2}\,\mathrm{diag}(1,-1,0,0), \tag{2a}$$

$$\Lambda_8 = \frac{1}{2\sqrt{3}}\,\mathrm{diag}(1,1,-2,0), \tag{2b}$$

$$\Lambda_{15} = \frac{1}{2\sqrt{6}}\,\mathrm{diag}(1,1,1,-3), \tag{2c}$$

and $\Lambda_0 = \mathrm{diag}(1,1,1,1)$, with the specific embedding parameters $\xi = -1/\sqrt{3}$, $\zeta = -2\sqrt{6}/3$, $\varsigma = 1$, where N is the new charge associated with the symmetric nonchiral Abelian semisimple factor of the G_{341} local symmetry. In the $\overline{\mathbf{4}}$ representation the neutral generators are [18]

$$\overline{\Lambda}_3 = \frac{1}{2}\,\mathrm{diag}(0,0,1,-1), \tag{3a}$$

$$\overline{\Lambda}_8 = \frac{1}{2\sqrt{3}}\,\mathrm{diag}(0,2,-1,-1), \tag{3b}$$

$$\overline{\Lambda}_{15} = \frac{1}{2\sqrt{6}}\,\mathrm{diag}(3,-1,-1,-1). \tag{3c}$$

The SU(4) maximal subalgebras are $\mathrm{SU}(3)\otimes\mathrm{U}(1)$, $\mathrm{SU}(2)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)$, Sp(2), and $\mathrm{SU}(2)\otimes\mathrm{SU}(2)$. The isomorphism is $\mathrm{SU}(4)\sim\mathrm{SO}(6)$. Three families of independent leptonic chiral flavor gauge symmetry eigenstates transform collectively,

$$f_{lL} = \begin{pmatrix} \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix} \\ \begin{pmatrix} \nu_{lR} \\ l_R \end{pmatrix}^c \end{pmatrix} \sim (\mathbf{1}_c, \mathbf{4}_L, N=0), \tag{4}$$

with the label $l = e, \mu, \tau$ and the charge-conjugated fields $l^c = C\overline{l}^{\mathrm{T}}$, ν_l^c, where $C \equiv i\gamma^2\gamma^0$. One quark family has the attributions

$$Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ u' \\ J \end{pmatrix}_L \sim (\mathbf{3}_c, \mathbf{4}_L, +2/3), \tag{5}$$

with the singlets

$$u_{1R} \sim (\mathbf{3}_c, \mathbf{1}_R, +2/3), \tag{6a}$$

$$d_{1R} \sim (\mathbf{3}_c, \mathbf{1}_R, -1/3), \tag{6b}$$

$$u'_R \sim (\mathbf{3}_c, \mathbf{1}_R, +2/3), \tag{6c}$$

$$J_R \sim (\mathbf{3}_c, \mathbf{1}_R, +5/3), \tag{6d}$$

and the other two families have the following common transformation properties:

$$Q_{\alpha L} = \begin{pmatrix} j_\alpha \\ d'_\alpha \\ u_\alpha \\ d_\alpha \end{pmatrix}_L \sim (\mathbf{3}_c, \overline{\mathbf{4}}_L, -1/3), \tag{7}$$

$\alpha = 2,3$ for the chiral left-handed fields, and

$$j_{\alpha R} \sim (\mathbf{3}_c, \mathbf{1}_R, -4/3), \tag{8a}$$

$$d'_{\alpha R} \sim (\mathbf{3}_c, \mathbf{1}_R, -1/3), \tag{8b}$$

$$u_{\alpha R} \sim (\mathbf{3}_c, \mathbf{1}_R, +2/3), \tag{8c}$$

$$d_{\alpha R} \sim (\mathbf{3}_c, \mathbf{1}_R, -1/3), \tag{8d}$$

where u', J, j_α, and d'_α are new quark flavors with electric charges $+\frac{2}{3}|e|$, $+\frac{5}{3}|e|$, $-\frac{4}{3}|e|$, and $-\frac{1}{3}|e|$, respectively, where the downlike quark flavors transport the quantum of electric charge, being

$$\begin{aligned} |e| &= \frac{gt}{(1+4t^2)^{1/2}} = \frac{g'}{(1+4t^2)^{1/2}} \\ &= 1.602\,176\,462(63) \times 10^{-19}\ \mathrm{C} \\ &= 4.803\,204\,20(19) \times 10^{-10}\ \mathrm{esu}, \end{aligned} \tag{9}$$

the proton charge [19,20], and $t \equiv g'/g$, where g and g' are the SU(4) and U(1) gauge coupling constants. The electric-charged and neutral leptons acquire mass through the symmetric decuplet $(\mathbf{1}_c, \overline{\mathbf{10}}_S, 0)$ of scalar fields,

$$\overline{H} = \begin{pmatrix} H_1^0 & H_1^+ & H_2^0 & H_2^- \\ H_1^+ & H_1^{++} & H_3^+ & H_3^0 \\ H_2^0 & H_3^+ & H_4^0 & H_4^- \\ H_2^- & H_3^0 & H_4^- & H_2^{--} \end{pmatrix}, \tag{10}$$

in the Yukawa interactions

$$\mathcal{L}_{\mathrm{Y}}^f = -\frac{1}{2} G_{ll'} \overline{(f_{lL})^c} \overline{H} f_{l'L}, \tag{11}$$

having the general form of a Majorana mass term after spontaneous symmetry breaking. Since the lepton mass term transforms as $\overline{(f_{lL})^c} f_{l'R} \sim (\mathbf{1}_c, \mathbf{4} \otimes \mathbf{4}, 0) = (\mathbf{1}_c, \mathbf{6}_A \oplus \mathbf{10}_S, 0)$ and the sextet will leave one lepton massless and two others degenerate for three families, it is necessary to introduce the $N=0$ symmetric decuplet which plays no role in generating quark masses. The explicit fermion bilinears and Higgs bosons Yukawa couplings are

$$\begin{aligned} \overline{(f_L)^c}\overline{H} f_L = {} & \overline{(\nu_l^c)}_R \nu_{lL} H_1^0 + \overline{(l^c)}_R \nu_{lL} H_1^+ + \overline{\nu}_{lR} \nu_{lL} H_2^0 + \overline{l}_R \nu_{lL} H_2^- \\ & + \overline{(\nu_l^c)}_R l_L H_1^+ + \overline{(l^c)}_R l_L H_1^{++} + \overline{\nu}_{lR} l_L H_3^+ + \overline{l}_R l_L H_3^0 \\ & + \overline{(\nu_l^c)}_R \nu_{lL}^c H_2^0 + \overline{(l^c)}_R \nu_{lL}^c H_3^+ + \overline{\nu}_{lR} \nu_{lL}^c H_4^0 \\ & + \overline{l}_R \nu_{lL}^c H_4^- + \overline{(\nu_l^c)}_R l_L^c H_2^- + \overline{(l^c)}_R l_L^c H_3^0 \\ & + \overline{\nu}_{lR} l_L^c H_4^- + \overline{l}_R l_L^c H_2^{--} \end{aligned} \tag{12}$$

and after the spontaneous symmetry breaking steps

$$G_{341} \to G_{331} \to G_{321} \to G_{31} \tag{13}$$

remain the mass terms

$$\begin{aligned} -\mathcal{L}_{\mathrm{Y}}^{\mathrm{mass}} = \frac{1}{2} \big(& G_{\nu_l \nu_{l'}} \langle H_1^0 \rangle_0 \overline{(\nu_l^c)}_R \nu_{l'L} + G_{\nu_l \nu_{l'}} \langle H_2^0 \rangle_0 \overline{\nu}_{lR} \nu_{l'L} \\ & + G_{ll'} \langle H_3^0 \rangle_0 \overline{l}_R l'_L + G_{\nu_l^c \nu_{l'}^c} \langle H_2^0 \rangle_0 \overline{(\nu_l^c)}_R \nu_{l'L}^c \\ & + G_{\nu_l \nu_{l'}} \langle H_1^0 \rangle_0 \overline{\nu}_{lR} \nu_{l'L}^c + G_{ll'} \langle H_3^0 \rangle_0 \overline{(l^c)}_R l_L^c \big) \end{aligned} \tag{14}$$

and the neutrinos could be Dirac-Majorana particles [21].

The vanishing anomalies conditions containing the $\mathrm{U}(1)_N$ fermionic attributions imply the following constraints between the N's:

$$\mathrm{Tr}([\mathrm{SU}(3)_c]^2[\mathrm{U}(1)_N]) = 0:$$

$$\begin{aligned} & 3(N_{Q_{1L}} + 2N_{Q_{\alpha L}}) - 3(N_{U_{1,\alpha R}} + N_{D_{1,\alpha R}}) \\ & \quad - N_{J_R} - 2N_{j_{\alpha R}} - N_{u'_R} - 2N_{d'_{\alpha R}}) = 0; \end{aligned} \tag{15a}$$

$$\mathrm{Tr}([\mathrm{SU}(4)_L]^2[\mathrm{U}(1)_N]) = 0:$$

$$3(N_{Q_{1L}} + 2N_{Q_{\alpha L}}) + \sum_l N_l^3 = 0; \tag{15b}$$

$$\mathrm{Tr}([\mathrm{U}(1)_N]^3) = 0:$$

$$\begin{aligned} & 3(N_{Q_{1L}}^3 + 2N_{Q_{\alpha L}}^3) - 3(N_{U_{1,\alpha R}}^3 + N_{D_{1,\alpha R}}^3) \\ & \quad + N_{J_R}^3 - 2N_{j_{\alpha R}}^3 - N_{u'_R}^3 - 2N_{d'_{\alpha R}}^3 + \sum_l N_l^3 = 0, \end{aligned} \tag{15c}$$

$$\mathrm{Tr}([\text{gravitational anomaly term}]^2[\mathrm{U}(1)_N]) = 0:$$

$$\begin{aligned} & 3(N_{Q_{1L}} + 2N_{Q_{\alpha L}}) - 3(N_{U_{1,\alpha R}} + N_{D_{1,\alpha R}}) \\ & \quad + N_{J_R} - 2N_{j_{\alpha R}} - N_{u'_R} - 2N_{d'_{\alpha R}} + \sum_l N_l = 0, \end{aligned} \tag{15d}$$

where $N_{U_{1,\alpha}}$ and $N_{D_{1,\alpha}}$ are the $\mathrm{U}(1)_N$ quantum numbers of the standard quark flavors. The Witten global anomaly [22] does not involve the $\mathrm{U}(1)_N$ quantum numbers. The three leptonic classical constraints are

$$N_l = 0, \tag{16}$$

coming from the Abelian gauge invariance of $\mathcal{L}_{\rm Y}$ where the electric charge quantization of the leptonic sector is already contained. From the quantum and classical gauge invariance constraints the electric charges of fundamental leptons and quarks, in the $|e|$ unit, are [23]

$$\mathcal{Q}_{\nu_l}=0, \quad \mathcal{Q}_l=-1, \quad \mathcal{Q}_D=-\frac{1}{3},$$
$$\mathcal{Q}_U=-2\mathcal{Q}_D=-2\mathcal{Q}_{d'_\alpha}=\mathcal{Q}_{u_1}, \tag{17}$$
$$\mathcal{Q}_J=-5\mathcal{Q}_D, \quad \mathcal{Q}_{j_\alpha}=4\mathcal{Q}_D,$$

even for massless neutrinos. There are no new constraints coming from the cancellation of mixed gauge and gravitational anomalies. If we consider the global symmetry associated with the conservation of leptobaryon quantum number,

$$\mathcal{F}=L+B=\sum_l L_l+B, \tag{18}$$

which prevents neutrinos from getting a mass to be explicitly broken, then Majorana mass terms arise if $\langle H^0_{1,4}\rangle \neq 0$ turning off the vacuum expectation values (VEVs) of the H^0_2 and H^0_3 nondiagonal fields. However, turning on the VEVs of all neutral scalar fields the neutrinos are Dirac-Majorana fermions [21].

Now let us add right-handed neutrinos as gauge flavor singlets. If $\langle H^0_1\rangle_0=\langle H^0_2\rangle_0=\langle H^0_4\rangle_0=0$, but $\langle H^0_3\rangle_0\neq 0$ for the charged lepton masses, the Dirac mass terms for the neutral fermions,

$$-\mathcal{L}_{\rm Y}^{\nu_l,\langle\eta\rangle_0}=G_{f_l\nu_l}\overline{f}_{lL}\langle\eta\rangle_0\nu_{l'R}+\text{H.c.}, \tag{19}$$

arise in the Yukawa couplings through the multiplet of scalar fields,

$$\eta=\begin{pmatrix}\eta_1^0\\ \eta_1^-\\ \eta_2^0\\ \eta_2^+\end{pmatrix}\sim(\mathbf{1}_c,\mathbf{4},0), \tag{20}$$

also necessary in the quark sector:

$$-\mathcal{L}_{\rm Y}^{Q,\eta}=F_{1k}\overline{Q}_{1L}U_{kR}\eta+F'_{\alpha k}\overline{Q}_{\alpha L}D_{kR}\overline{\eta}+\text{H.c.}, \quad k=1,2,3, \tag{21}$$

where

$$\overline{\eta}=\begin{pmatrix}\eta_2^+\\ \eta_2^0\\ \eta_1^-\\ \eta_1^0\end{pmatrix}\sim(\mathbf{1}_c,\overline{\mathbf{4}},0). \tag{22}$$

The classical gauge invariance of the $\mathcal{L}_{\rm Y}^{\nu_l,\langle\eta\rangle_0}$ leptonic terms implies $N_{\nu_l}=0$. The L and B attributions of the leptoquark fermions are $L_{j_\alpha}=-L_J=+2$, $B_J=B_{j_\alpha}=+\frac{1}{3}$, and the bilepton gauge bosons have lepton number $L=\pm 2$. The bileptons [26] are contained also in the stable-proton SU(15) grand unified theory [27] with nonchiral fermions and anomaly cancellation through mirror fermions.

The quantization of electric charge is inevitable in G_{3m1} models of leptoquark fermions and bilepton bosons [24,25] with three nonrepetitive fermion generations breaking generation universality and does not depend on the character of the neutral fermions. Each generation is anomalous and is not a replica of one another so that the quantum anomalies cancel when the number of generations is a multiple of the number of color charges. There is no connection between electric charge quantization and the massless Weyl and the massive Dirac, Majorana, or Dirac-Majorana fundamental neutral fermions. If $\mathcal{F}$ symmetry is explicitly broken, the neutrinos are Majorana fermions when $\langle H^0_{1,4}\rangle_0\neq 0$ and Dirac-Majorana fermions when all neutral components of the symmetric decuplet acquire their vacuum expectation values. This is another clue about the promising perspectives that neutrinos address in the direction of new physics. The G_{3m1} leptoquark-bilepton models have the leptonic representation content structure of grand unified theories in which the electric charge operator contains only the $\mathrm{SU}(m)_L$ diagonal generators whose number is the rank of the group.

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