Texture specific mass matrices and CP violating asymmetry in $B_d^0(\overline{B_d^0}) \to \psi K_S$

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(Received 2 December 2000; revised manuscript received 10 January 2001; published 29 March 2001)

In the context of texture 4-zero and texture 5-zero hierarchical quark mass matrices, the CP violating asymmetry in $B_d^0(\overline{B_d^0}) \to \psi K_S(\sin 2\beta)$ has been evaluated by considering quark masses at the m_Z scale. For a particular viable texture 4-zero-mass matrix the range of $\sin 2\beta$ is 0.27–0.60 and for the corresponding texture 5-zero case it is 0.31–0.59. Furthermore, our calculations reveal a crucial dependence of $\sin 2\beta$ on light quark masses as well as the phase in this sector.

DOI: 10.1103/PhysRevD.63.097301 PACS number(s): 12.15.Ff, 11.30.Er, 13.25.Hw, 14.40.−n

The recent first measurements of time-dependent CP asymmetry $a_{\psi K_S}$ in $B_d^0(\overline{B_d^0}) \to \psi K_S$ decay by the BABAR and BELLE collaborations suggest that these values could be smaller than the expectations from the standard model analysis of the Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle. For example, the reported asymmetry by BABAR and BELLE are

$$a_{\psi K_S} = 0.12 \pm 0.37 \pm 0.09 \quad \text{BABAR} \quad [1], \tag{1}$$

$$a_{\psi K_S} = 0.45^{+0.43}_{-0.44}{}^{+0.07}_{-0.09} \quad \text{BELLE} \quad [2], \tag{2}$$

whereas the earlier collider detector at Fermilab (CDF) measurements gave [3]

$$a_{\psi K_S}^{\mathrm{CDF}} = 0.79^{+0.41}_{-0.44}, \tag{3}$$

and a recent global analysis of the CKM unitarity triangle [4] gives the value

$$a_{\psi K_S}^{\mathrm{SM}} = 0.75 \pm 0.06. \tag{4}$$

Recently, several authors [5–9] have studied the implications of the possibility of low value of $a_{\psi K_S}$ in comparison to the CDF measurements as well as the standard model expectations. In particular, Silva and Wolfenstein [6] have examined the possibilities of physics beyond the standard model in the case $a_{\psi K_S} \leqslant 0.2$.

In the context of texture specific mass matrices, $a_{\psi K_S}$ has been evaluated in the leading approximations [10,11], however without going into the detailed implications of $a_{\psi K_S}$ on the texture as well as the mass scale at which the quark masses are evaluated. Recently, it has been demonstrated [12–15] that texture 4-zero quark mass matrices not only accommodate the CKM phenomenology but are also able to reproduce a neutrino mixing matrix that can accommodate the solar neutrino problem, atmospheric neutrino problem, and the oscillations observed at the Liquid Scintillation Neutrino Detector. In particular, Randhawa *et al.* [12] have shown that there is a unique set of viable texture 4-zero-mass matrices in the quark sector as well as in the lepton sector. The purpose of the present Brief Report is to investigate in detail and beyond the leading order the implications of $a_{\psi K_S}$ measurements for the particular viable case of texture 4-zero-mass matrices as well as for texture 5-zero-mass matrices. It would also be interesting to examine the implications of low values of $a_{\psi K_S}$, in particular of $a_{\psi K_S} \leqslant 0.2$, a benchmark for physics beyond the standard model as advocated by Silva and Wolfenstein [6].

We begin with the unique set of texture 4-zero quark mass matrices considered by Randhawa *et al.* [12], for example,

$$M_u = \begin{pmatrix} 0 & A_u & 0 \\ A_u^* & D_u & B_u \\ 0 & B_u^* & C_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & A_d & 0 \\ A_d^* & D_d & B_d \\ 0 & B_d^* & C_d \end{pmatrix}, \tag{5}$$

where $A_u = |A_u| e^{i\alpha_u}$, $A_d = |A_d| e^{i\alpha_d}$, $B_u = |B_u| e^{i\beta_u}$, and $B_d = |B_d| e^{i\beta_d}$. The elements of the mass matrices follow the following mass hierarchy [12,16,17]

$$A_i \ll D_i \sim B_i \ll C_i, \quad i = u, d. \tag{6}$$

Within the standard model, $a_{\psi K_S}$ is related to the angle β of the unitarity triangle, expressed as

$$a_{\psi K_S}^{\mathrm{SM}} = \sin 2\beta, \quad \beta \equiv \arg\left[\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right], \tag{7}$$

$\sin 2\beta$ can be calculated by evaluating the elements V_{cd}, V_{cb}, V_{td}, and V_{tb} from the above mass matrices.

The above matrices can be diagonalized exactly and the corresponding CKM matrix elements can easily be found; for details we refer the reader to Ref. [16]. However for the sake of readability of manuscript as well as for facilitating the discussion to evaluate $\sin 2\beta$, we reproduce below the exact expressions of V_{cd}, V_{cb}, V_{td}, and V_{tb}:

$$V_{cd} = -a e^{-i\phi_1} + c\sqrt{(1-R_u)(1-R_d)} + c\sqrt{(b^2+R_u)(d^2+R_d)}\, e^{i\phi_2}, \tag{8}$$

$$V_{cb} = -acd^2\sqrt{\frac{d^2+R_d}{1-R_d}}\, e^{-i\phi_1} + \sqrt{(1-R_u)(d^2+R_d)} - \sqrt{(b^2+R_u)(1-R_d)}\, e^{i\phi_2}, \tag{9}$$

$$V_{td}=ab^2\sqrt{\frac{b^2+R_u}{1-R_u}}\,e^{-i\phi_1}+c\sqrt{(b^2+R_u)(1-R_d)}\\ -c\sqrt{(1-R_u)(d^2+R_d)}\,e^{i\phi_2}, \tag{10}$$

$$V_{tb}=acb^2d^2\sqrt{\frac{(b^2+R_u)(d^2+R_d)}{(1-R_u)(1-R_d)}}\,e^{-i\phi_1}\\ +\sqrt{(b^2+R_u)(d^2+R_d)}+\sqrt{(1-R_u)(1-R_d)}e^{i\phi_2}, \tag{11}$$

where $a=\sqrt{m_u/m_c}$, $b=\sqrt{m_c/m_t}$, $c=\sqrt{m_d/m_s}$, $d=\sqrt{m_s/m_b}$, $\phi_1=\alpha_u-\alpha_d$, $\phi_2=\beta_u-\beta_d$, $R_u=D_u/m_t$, and $R_d=D_d/m_b$. In principle, $\sin 2\beta$ can be calculated using Eqs. (7)–(11), however before doing that we first ensure that by varying the various input parameters, the CKM matrix elements are within their respective range given by the Particle Data Group (PDG) [18]. In carrying out these calculations, we have taken the quark masses at m_Z scale as recently advocated by Fusaoka and Koide [14] as well as by Fritzsch and Xing [15]. For the sake of completion, however, we have also repeated the whole analysis with masses at 1 GeV scale, the scale conventionally used.

To facilitate the analysis, without loss of generality, we first consider $\phi_2=0$ as advocated by several authors [12,15]. As mentioned earlier we have carried out our calculations at two different mass scales, i.e., at m_Z scale and at 1 GeV, the corresponding input masses are summarized in Table I. For calculating the limits on $\sin 2\beta$, we scanned the full ranges of all the input masses at different confidence levels as well as at both the scales, varying ϕ_1 from 0° to 180°. It may be of interest to point out that while carrying out the variations in R_u and R_d, we have restricted their variation upto 0.2 only as the values higher than that are not able to reproduce the CKM elements within their range given by PDG. This is in accordance with our earlier calculations [12] as well as the hierarchical structure of mass matrices described by Eq. (6). Having taken care of the CKM matrix elements being within the limits mentioned by PDG, we proceed to find a range for $\sin 2\beta$ using expressions (7)–(10). A similar exercise is carried out for (i) $D_u=0$, $D_d\neq 0$, (ii) $D_u\neq 0$, and $D_d=0$, the two cases corresponding to texture 5-zero matrices.

TABLE I. Running quark masses $m_q(\mu)$ (in units of GeV) [14].

	At $\mu=1$ GeV	At $\mu=m_Z$
m_u	$0.004\,88\pm 0.000\,57$	$0.002\,33^{+0.000\,42}_{-0.000\,45}$
m_d	$0.009\,81\pm 0.000\,65$	$0.004\,69^{+0.000\,60}_{-0.000\,66}$
m_s	0.1954 ± 0.0125	$0.0934^{+0.0118}_{-0.0130}$
m_c	$1.506^{+0.048}_{-0.037}$	$0.677^{+0.056}_{-0.061}$
m_b	$7.18^{+0.59}_{-0.44}$	3.0 ± 0.11
m_t	475^{+86}_{-71}	181 ± 13

In Table II, we have summarized the results of our calculations at the different mass scales for texture 4-zero and texture 5-zero-mass matrices. From the table one can immediately find that the range of $\sin 2\beta$ in the case of texture 4-zero-mass matrices, with input masses at m_Z scale and at 1σ C.L., is given by

$$\sin 2\beta=0.27-0.60. \tag{12}$$

This range looks to be narrow in comparison with the BELLE and BABAR results and is ruled out by the standard model (SM) analysis. The corresponding range for $\sin 2\beta$ narrows further when quark masses are considered at the 1 GeV scale, for example,

$$\sin 2\beta=0.39-0.54. \tag{13}$$

This can be easily understood from the fact that the light quark masses at the m_Z scale show much more scatter compared to masses at the 1 GeV scale. In view of the sensitive dependence of $\sin 2\beta$ on the quark masses, in Figs. 1–4, we have plotted the variation of $\sin 2\beta$ with mass ratios m_u/m_c, m_d/m_s, m_c/m_t, and m_s/m_b. From these figures, it is easy to conclude that $\sin 2\beta$ is very sensitively dependent on the

TABLE II. The range for $\sin 2\beta$ at different confidence levels of quark masses.

	Masses at $\mu=m_Z$		Masses at $\mu=1$ GeV	
	Texture 4 zeros	Texture 5 zeros ($D_d=0$, $D_u\neq 0$)	Texture 4 zeros	Texture 5 zeros ($D_d=0$, $D_u\neq 0$)
$\sin 2\beta$ (with quark masses at 1σ confidence level)	0.27–0.60	0.31–0.59	0.39–0.54	0.45– 0.54
$\sin 2\beta$ (with quark masses at 2σ confidence level)	0.057–0.68	0.08–0.65	0.27–0.57	0.38–0.56
$\sin 2\beta$ (with quark masses at 3σ confidence level)	0.04–0.75	0.05–0.73	0.06–0.61	0.07–0.58

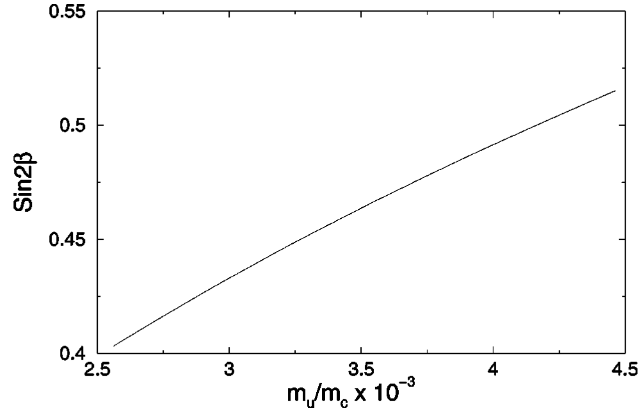

FIG. 1. Variation of $\sin 2\beta$ with m_u/m_c at the m_Z scale. All other masses are at their mean values, whereas $\phi_1 = 90°$, $\phi_2 = 0$, and $R_u = R_d = 0.1$.

ratios of the light quark masses, m_u/m_c and m_d/m_s, while variations in m_c/m_t and m_s/m_b do not affect $\sin 2\beta$ much. This gets further emphasized when one closely examines the figures, for example, $\sin 2\beta$ varies from 0.40 to 0.52 when m_u/m_c varies from 0.0026 to 0.0045, while it varies only from 0.464 to 0.457 when m_c/m_t varies from 0.0032 to 0.0044. Similarly $\sin 2\beta$ varies from 0.52 to 0.41 when m_d/m_s varies from 0.0383 to 0.0658, while it varies only from 0.458 to 0.461 when m_s/m_b varies from 0.0258 to 0.0364. It is perhaps desirable to mention that while considering the above-mentioned variation of $\sin 2\beta$ on a given mass ratio all other masses have been kept at their mean values at the m_Z scale, whereas $\phi_1 = 90°$ and $R_u = R_d = 0.1$.

In view of the scale sensitivity of $\sin 2\beta$, it is perhaps desirable to study the affect of quark masses on $\sin 2\beta$ at higher confidence levels of quark masses in comparison to the 1σ C.L. corresponding to Eqs. (12)–(15). In the Table II, we have also listed the results for $\sin 2\beta$ with the input quark masses being at their 2σ and 3σ confidence levels. A look at the table reveals that when the quark masses are considered at 2σ C.L., we obtain the following ranges for $\sin 2\beta$ for the set of texture 4-zero matrices given in Eq. (5): 0.057–0.68. These ranges get further broadened when masses are considered at their 3σ C.L., for example, 0.04–0.75. Thus we see that with input masses at their 2σ and 3σ C.L., the entire range of BABAR and BELLE is covered, once again emphasizing the sensitivity of $\sin 2\beta$ on the quark masses. This brings into focus the better evaluation of light quark masses.

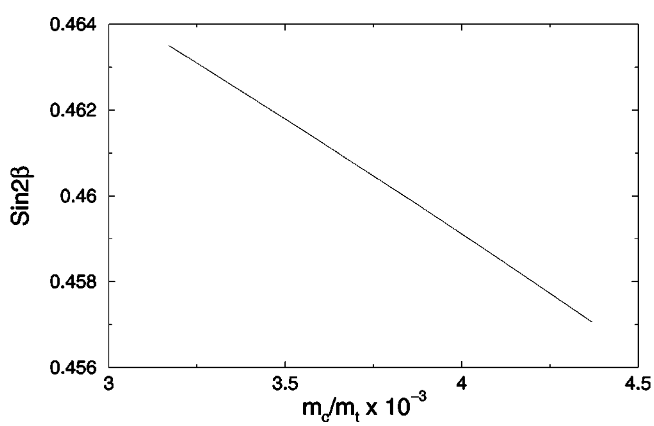

FIG. 2. Variation of $\sin 2\beta$ with m_c/m_t at the m_Z scale. All other masses are at their mean values, whereas $\phi_1 = 90°$, $\phi_2 = 0$, and $R_u = R_d = 0.1$.

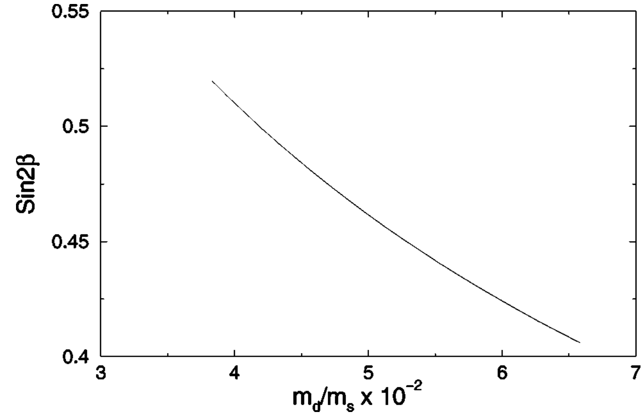

FIG. 3. Variation of $\sin 2\beta$ with m_d/m_s at the m_Z scale. All other masses are at their mean values, whereas $\phi_1 = 90°$, $\phi_2 = 0$, and $R_u = R_d = 0.1$.

Further scrutiny of the Table II reveals interesting results for the texture 5-zero case. For example, we obtain the following range for $\sin 2\beta$ with $D_u \neq 0$, $D_d = 0$, and with quark masses at the m_Z scale and at 1σ C.L.:

$$\sin 2\beta = 0.31 - 0.59, \tag{14}$$

and correspondingly with the masses at 1 GeV we get

$$\sin 2\beta = 0.45 - 0.54. \tag{15}$$

Thus, in comparison to the corresponding ranges for texture 4-zero matrices, the lower bound on $\sin 2\beta$ goes up somewhat while there is not much change in the upper bound. This can be understood by an examination of Eqs. (7) and

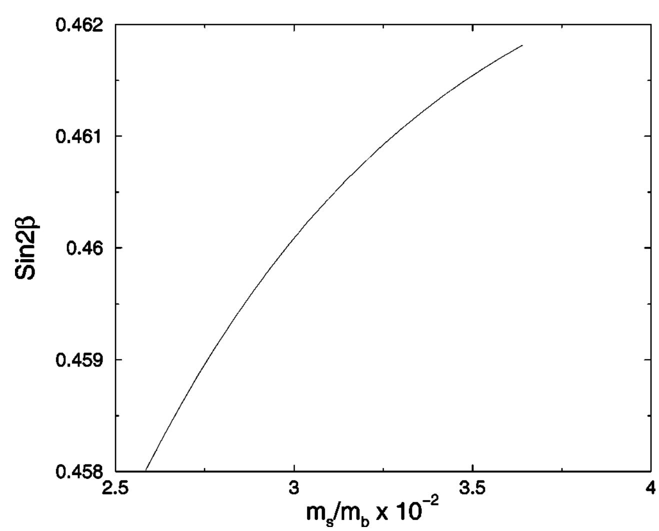

FIG. 4. Variation of $\sin 2\beta$ with m_s/m_b at the m_Z scale. All other masses are at their mean values, whereas $\phi_1 = 90°$, $\phi_2 = 0$, and $R_u = R_d = 0.1$.

(10), where $D_d=0$ results in lowering the upper bound on $|V_{td}|$, thus pushing up the lower bound on $\sin 2\beta$. In the other texture 5-zero case, for example, $D_d\neq 0$ and $D_u=0$, we find that it is not meaningful to talk of the range of $\sin 2\beta$ as in this case the CKM matrix elements do not show overlap with the PDG CKM matrix even after the full variation of all the parameters.

A few comments are in order. In view of the dependence of $\sin 2\beta$ on ϕ_1 and ϕ_2 through V_{cd}, V_{cb}, V_{td}, and V_{tb}, we have also studied the case when both ϕ_1 and ϕ_2 are taken nonzero. The results in this case do not show much deviation from the case when $\phi_2=0$ and ϕ_1 is varied. However when $\phi_1=0$ and ϕ_2 is given full variation, interestingly we find that we are not able to reproduce the CKM matrix elements and hence finding a range for $\sin 2\beta$ in this case is meaningless. Thus, it seems that the CP violating phase resides only in the light quark sector, in agreement with the conclusions of Fritzsch and Xing [10] based on leading-order calculations only.

Interestingly, from Eqs. (12)–(15), we find that a value of $\sin 2\beta$ lower than 0.2 would rule out, with good deal of confidence, texture 4-zero and texture 5-zero matrices. This would force one to consider texture 3-zero and texture 2-zero-mass matrices, which will be discussed elsewhere. Therefore it seems that a sharper measurement of $\sin 2\beta$ will have strong bearing on the specific textures of mass matrices.

To conclude, we have found a range for $\sin 2\beta$ using texture 4-zero and texture 5-zero hierarchical quark mass matrices with input quark masses at m_Z scale. In the texture 4-zero case with masses at 1σ C.L., we get $\sin 2\beta=0.27$–0.60 and in the texture 5-zero case we get $\sin 2\beta=0.31$–0.59. Both texture 5-zero and texture 4-zero matrices are ruled out if $\sin 2\beta$ is found to be $\leqslant 0.2$ and one may have to go to texture 3-zero matrices. Our analysis indicates a sensitive dependence of $\sin 2\beta$ on the light quark masses as well as the phase in this sector.

M.G. would like to thank S.D. Sharma for useful discussions. M.R. would like to thank CSIR, Government of India, for financial support and also the Chairman of the Department of Physics, for providing facilities to work in the department.

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