Lepton flavor violation in the two Higgs doublet model type III

Rodolfo A. Diaz,* R. Martinez,† and J.-Alexis Rodriguez‡
Departamento de Fisica, Universidad Nacional de Colombia, Bogota, Colombia
(Received 17 October 2000; revised manuscript received 30 January 2001; published 6 April 2001)

We consider the two Higgs doublet model (2HDM) of type III which leads to flavor changing neutral currents (FCNC) at the tree level. In the framework of this model we can have, in principle, two situations: the case (a) when both doublets acquire a vacuum expectation value different from zero and the case (b) when one of them is zero. In addition, we show that we can make two types of rotations for the flavor mixing matrices which generates four types of different Lagrangians. Two of the four possible Lagrangians correspond to the 2HDM type I and type II plus flavor changing (FC) interactions. The analytical expressions of the partial lepton number violating widths $\Gamma(\mu\rightarrow eee)$ and $\Gamma(\mu\rightarrow e\gamma)$ are derived for the cases (a) and (b) and both types of rotations. In all cases these widths go asymptotically to zero in the limit when all Higgs boson masses go to infinity. We present from our analysis upper bounds for the flavor changing transition $\mu\rightarrow e$, and we show that such bounds are sensitive to the VEV structure and the type of rotation utilized.

DOI: 10.1103/PhysRevD.63.095007 PACS number(s): 14.80.Cp, 13.10.+q, 13.35.Bv

I. INTRODUCTION

Flavor changing neutral currents (FCNC) are forbidden at the tree level in the standard model (SM). However, they could be present at one loop level as in the case of $b\rightarrow s\gamma$ [1], $K^0\rightarrow\mu^+\mu^-$ [2], $K^0-\overline{K^0}$ [3], $t\rightarrow c\gamma$ [4], etc. In general, many extensions of the SM permit FCNC at the tree level. The introduction of new representations of fermions different from doublets produce them by means of the Z coupling [5]. In addition, they are generated at the tree level in the Yukawa sector by adding a second doublet to the SM [6]. Such couplings also appear in supersymmetry (SUSY) theories without R parity [7]. Theories with FCNC were previously considered unattractive because they were strongly constrained experimentally, especially due to the small K_L-K_S mass difference. Nevertheless, nowadays it is hoped to observe such physical processes in laboratory, as a result many theories were proposed (see above).

Owing to the continuous improvements in experimental accuracies, lepton flavor violation (LFV) has become a very important possible source of new physics. Experiments to search directly for LFV have been performed for many years, all with null results so far. Experimental limits have resulted from searches for $K_L^0\rightarrow\mu^+e^-$ [8], $K_L^0\rightarrow\pi^0\mu^+e^-$ [9], $K^+\rightarrow\pi^+\mu^+e^-$ [10], $\mu^+\rightarrow e^+\gamma$ [11], $\mu^+\rightarrow e^+e^+e^-$ [12] and $\mu^-N\rightarrow e^-N$ [13].

There are several mechanisms to avoid FCNC at the tree level. Glashow and Weinberg [14] proposed a discrete symmetry in the two Higgs doublet model (2HDM) which forbids the couplings that generate such rare decays, hence they do not appear at the tree level. Another possibility is to consider heavy exchange of scalar or pseudoscalar Higgs fields [15] or by cancellation of large contributions with opposite sign. Another mechanism was proposed by Cheng and Sher arguing that a natural value for the FC couplings from different families should be of the order of the geometric average of their Yukawa couplings [16].

Taking this *natural* assumption and since Yukawa couplings in the SM vary with mass, it is plausible that the same occurs for FC couplings. Hence it is expected that FCNC involving the third generation can be larger, while the ones involving the first generation are hoped to be small [15,17]. Another clue that suggests large mixing between the second and third generation in the charged leptonic sector, is the large mixing between second and third generation of the neutral leptonic sector. This is predicted by experiments with atmospheric neutrinos [18].

The increasing interest in LFV processes is due to the strong restrictions that experiments have imposed on them. This consequently determines small regions of parameters for new physics of any theory beyond the SM. Some specific decays have been widely studied within the framework of supersymmetric extensions, because in supersymmetric theories the presence of FCNC induced by R-parity violation generates massive neutrinos and neutrino oscillations [19]. In recent papers the decays $\mu\rightarrow e\gamma$ and $\mu\rightarrow 3e$ with polarized muons have been examined in the context of supersymmetric grand unified theories to get bounds in the $m_{\tilde{e}_R}-|A_0|$ plane [20].

On the other hand, a muon collider could provide very interesting new constraints on FCNC, for example $\mu\mu\rightarrow\mu\tau(e\tau)$ mediated by Higgs exchange [23] which test the mixing between the second and third generations. Additionally, the muon collider could be a Higgs factory and it is well known that the Higgs sector is crucial for FCNC [24]. Finally, effects on the coupling of muon and tau in the 2HDM framework owing to anomalous magnetic moment of the muon could be significantly improved by E821 experiment at Brookhaven National Laboratory [23].

Additionally, in the quark sector bounds on FCNC come from $\Delta F=2$ processes, rare B decays, $Z\rightarrow\overline{b}b$ and the

*Electronic address: radiaz@ciencias.ciencias.unal.edu.co
†Electronic address: romart@ciencias.ciencias.unal.edu.co
‡Electronic address: alexro@ciencias.ciencias.unal.edu.co

ρ-parameter [21]. Reference [21] also explored the implications of FCNC at the tree level for $e^+e^-(\mu^+\mu^-)\to t\overline{c}+\overline{t}c$, $t\to c\gamma(Z,g)$, $D^0-\overline{D}^0$, and $B_s^0-\overline{B}_s^0$. Moreover, there are other important processes involving FCNC. For instance, the decay $B^-(D^-)\to K^-\mu^+\tau^-$ which depends on $\mu-\tau$ mixing and vanishes in the SM. Hence it is very sensitive to new physics. Another one is $B^-(D^-)\to K^-\mu^+e^-$ whose form factors have been calculated in [15,22].

The simplest model which exhibits FCNC at the tree level is the model with one extra Higgs doublet, known as the two Higgs doublet model (2HDM). There are several kinds of such models. In the model type I, one Higgs doublet provides masses to the up and down quarks, simultaneously. In the model type II, one Higgs doublet gives masses to the up quarks and the other one to the down quarks. These former two models have the discrete symmetry mentioned above to avoid FCNC at the tree level [14]. However, the discrete symmetry is not necessary in whose case both doublets generate the masses of the quarks of up-type and down-type, simultaneously. In the literature, the latter is known as the model type III [25–27]. It has been used to look for physics beyond the SM and specifically for FCNC at the tree level [21,15]. In general, both doublets could acquire a vacuum expectation value (VEV), but we can absorb one of them rotating the Higgs fields properly. Nevertheless, we shall show that a substantial difference arises from the case in which both doublets get the vacuum expectation value (VEV), this is because a rotation of the Higgs fields implies to fix one parameter of the model. Therefore we will study the model type III considering two cases. In the first case, the two Higgs doublets acquire VEV [case (a)]. In the second one, only one Higgs doublet acquire VEV [case (b)]. In the latter case the free parameter $\tan\beta$ is removed from the theory making the analysis simpler.

In Sec. II, we describe the model and define the notation we shall use throughout the document. In Sec. III, we show that we can make two kinds of rotations for the flavor mixing matrices which generates four types of different Lagrangians, and that in the framework of the first rotation we arrive to the case (b) from the case (a) in the limit $\tan\beta\to\infty$, while with the second rotation we obtain (b) from (a) in the limit $\tan\beta\to 0$. Furthermore, we find that two of the four possible Lagrangians correspond to the models of types I and II plus flavor changing (FC) interactions.

In Sec. IV we get bounds on LFV in the 2HDM type III based on the decays $\mu\to e\gamma$ and $\mu\to eee$. Such decays are examined in the context of both cases (a) and (b) according to the classification made above, and with both types of rotations. We find that such constraints depend on whether we use cases (a) or (b) and on what kind of rotation is utilized.

II. THE MODEL

The 2HDM type III is an extension of the SM plus a new Higgs doublet and three new Yukawa couplings in the quark and leptonic sectors. The mass terms for the up-type or down-type sectors depend on two matrices or two Yukawa couplings. The rotation of the quarks and leptons allows us to diagonalize one of the matrices but not both simultaneously, so one of the Yukawa couplings remains nondiagonal, generating the FCNC at the tree level.

The Yukawa Lagrangian is as follows:

$$\begin{aligned}-\pounds_Y=&\ \eta_{ij}^{U,0}\overline{Q}_{iL}^0\widetilde{\Phi}_1U_{jR}^0+\eta_{ij}^{D,0}\overline{Q}_{iL}^0\Phi_1D_{jR}^0+\xi_{ij}^{U,0}\overline{Q}_{iL}^0\widetilde{\Phi}_2U_{jR}^0\\&+\xi_{ij}^{D,0}\overline{Q}_{iL}^0\Phi_2D_{jR}^0+\eta_{ij}^{E,0}\overline{l}_{iL}^0\Phi_1E_{jR}^0+\xi_{ij}^{E,0}\overline{l}_{iL}^0\Phi_2E_{jR}^0\\&+\text{H.c.},\end{aligned}\tag{2.1}$$

where $\Phi_{1,2}$ are the Higgs doublets, η_{ij}^0 and ξ_{ij}^0 are nondiagonal 3×3 matrices and i,j are family indices. D refers to the three down quarks $D\equiv(d,s,b)^T$, U refers to the three up quarks $U\equiv(u,c,t)^T$, and E to the three charged leptons. The superscript 0 indicates that the fields are not mass eigenstates yet. In the so-called model type I, the discrete symmetry forbids the terms proportional to ξ_{ij}^0, meanwhile in the model type II the discrete symmetry forbids terms proportional to $\xi_{ij}^{U,0},\eta_{ij}^{D,0},\eta_{ij}^{E,0}$.

In this kind of model (type III), we consider two cases. In the case (a) we assume the VEV as

$$\langle\Phi_1\rangle_0=\begin{pmatrix}0\\ v_1/\sqrt{2}\end{pmatrix},\quad \langle\Phi_2\rangle_0=\begin{pmatrix}0\\ v_2/\sqrt{2}\end{pmatrix}\tag{2.2}$$

and we take the complex phase of v_2 equal to zero since we are not interested in CP violation. The mass eigenstates of the scalar fields are given by [28]

$$\begin{pmatrix}G_W^\pm\\ H^\pm\end{pmatrix}=\begin{pmatrix}\cos\beta & \sin\beta\\ -\sin\beta & \cos\beta\end{pmatrix}\begin{pmatrix}\phi_1^\pm\\ \phi_2^\pm\end{pmatrix},$$

$$\begin{pmatrix}G_Z^0\\ A^0\end{pmatrix}=\begin{pmatrix}\cos\beta & \sin\beta\\ -\sin\beta & \cos\beta\end{pmatrix}\begin{pmatrix}\sqrt{2}\,\mathrm{Im}\,\phi_1^0\\ \sqrt{2}\,\mathrm{Im}\,\phi_2^0\end{pmatrix},$$

$$\begin{pmatrix}H^0\\ h^0\end{pmatrix}=\begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\sqrt{2}\,\mathrm{Re}\,\phi_1^0-v_1\\ \sqrt{2}\,\mathrm{Re}\,\phi_2^0-v_2\end{pmatrix},\tag{2.3}$$

where $\tan\beta=v_2/v_1$ and α is the mixing angle of the CP-even neutral Higgs sector. $G_{Z(W)}$ are the would-be Goldstone bosons for $Z(W)$, respectively. And A^0 is the CP-odd neutral Higgs boson. $H^\pm$ are the charged physical Higgs bosons.

The case (b) corresponds to the case in which the VEV are taken as

$$\langle\Phi_1\rangle_0=\begin{pmatrix}0\\ v_1/\sqrt{2}\end{pmatrix},\quad \langle\Phi_2\rangle_0=\begin{pmatrix}0\\ 0\end{pmatrix}.\tag{2.4}$$

The mass eigenstates scalar fields in this case are [29,30]

$$G_W^\pm=\phi_1^\pm,\quad H^\pm=\phi_2^\pm,$$

$$G_Z^0=\sqrt{2}\,\mathrm{Im}\,\phi_1^0,\quad A^0=\sqrt{2}\,\mathrm{Im}\,\phi_2^0,\tag{2.5}$$

and the neutral CP-even fields are the same as in the former model just replacing $v_2=0$. A very important difference between both models is that $G_{Z(W)}$ is a linear combination of components of Φ_1 and Φ_2 in the model (a), meanwhile in the model (b) is a component of the doublet Φ_1.

III. GENERATION OF MODELS TYPE I AND II FROM TYPE III

To convert the Lagrangian (2.1) into mass eigenstates we make the unitary transformations

$$D_{L,R}=(V_{L,R})D^0_{L,R}$$

$$U_{L,R}=(T_{L,R})U^0_{L,R} \tag{3.1}$$

from which we obtain the mass matrices. In the context of case (a)

$$M_D^{diag}=V_L\left[\frac{v_1}{\sqrt{2}}\eta^{D,0}+\frac{v_2}{\sqrt{2}}\xi^{D,0}\right]V_R^\dagger,$$

$$M_U^{diag}=T_L\left[\frac{v_1}{\sqrt{2}}\eta^{U,0}+\frac{v_2}{\sqrt{2}}\xi^{U,0}\right]T_R^\dagger. \tag{3.2}$$

We can solve for $\xi^{D,0},\xi^{U,0}$ obtaining

$$\xi^{D,0}=\frac{\sqrt{2}}{v_2}V_L^\dagger M_D^{diag}V_R-\frac{v_1}{v_2}\eta^{D,0},$$

$$\xi^{U,0}=\frac{\sqrt{2}}{v_2}T_L^\dagger M_D^{diag}T_R-\frac{v_1}{v_2}\eta^{U,0}. \tag{3.3}$$

Let us call Eqs. (3.3) rotations of type I. Replacing them into Eq. (2.1) the expanded Lagrangian for up and down sectors are

$$\begin{aligned}-\pounds^{(a,I)}_{Y(U)}=&-\frac{g}{\sqrt{2}M_W}\bar{U}M_U^{diag}KP_LDG_W^+-\frac{ig}{2M_W}\bar{U}M_U^{diag}\gamma_5UG_Z^0+\frac{g}{2M_W\sin\beta}\bar{U}M_U^{diag}U(\sin\alpha H^0+\cos\alpha h^0)\\&-\frac{ig\cot\beta}{2M_W}\bar{U}M_U^{diag}\gamma_5UA^0-\frac{g\cot\beta}{\sqrt{2}M_W}\bar{U}M_U^{diag}KP_LDH^++\frac{i}{\sqrt{2}\sin\beta}\bar{U}\eta^U\gamma_5UA^0-\frac{1}{\sqrt{2}\sin\beta}\bar{U}\eta^UU\\&\times[\sin(\alpha-\beta)H^0+\cos(\alpha-\beta)h^0]+\frac{1}{\sin\beta}\bar{U}\eta^UKP_LDH^++\text{H.c.},\end{aligned} \tag{3.4}$$

$$\begin{aligned}-\pounds^{(a,I)}_{Y(D)}=&\frac{g}{\sqrt{2}M_W}\bar{U}KM_D^{diag}P_RDG_W^++\frac{ig}{2M_W}\bar{D}M_D^{diag}\gamma_5DG_Z^0+\frac{g}{2M_W\sin\beta}\bar{D}M_D^{diag}D(\sin\alpha H^0+\cos\alpha h^0)\\&+\frac{g\cot\beta}{\sqrt{2}M_W}\bar{U}KM_D^{diag}P_RDH^++\frac{ig\cot\beta}{2M_W}\bar{D}M_D^{diag}\gamma_5DA^0-\frac{i}{\sqrt{2}\sin\beta}\bar{D}\eta^D\gamma_5DA^0-\frac{1}{\sqrt{2}\sin\beta}\bar{D}\eta^DD\\&\times[\sin(\alpha-\beta)H^0+\cos(\alpha-\beta)h^0]-\frac{1}{\sin\beta}\bar{U}K\eta^DP_RDH^++\text{leptonic sector}+\text{H.c.},\end{aligned} \tag{3.5}$$

where K is the CKM matrix and $\eta^{U(D)}=T_L(V_L)\eta^{U(D),0}T_R^\dagger(V_R)^\dagger$ and similarly for $\xi^{U(D)}$. The superindex (a,I) refers to the case (a) and rotation type I. The leptonic sector is obtained from Eq. (3.5) replacing the down (up) quarks by the charged leptons (neutrinos).

It is easy to check that if we add Eqs. (3.4) and (3.5) we obtain a Lagrangian consisting of the one in the 2HDM type I [28], plus the FC interactions, i.e., $\pounds^{a,I}_{Y(U)}+\pounds^{a,I}_{Y(D)}$. Therefore, we obtain the Lagrangian of type I from Eqs. (3.4) and (3.5) by setting $\eta^D=\eta^U=0$. In addition, it is observed that the case (b) in both up and down sectors can be calculated just taking the limit $\tan\beta\to\infty$.

On the other hand, from Eq. (3.2), we can also solve for $\eta^{D,0},\eta^{U,0}$ instead of $\xi^{D,0},\xi^{U,0}$, to get

$$\eta^{D,0}=\frac{\sqrt{2}}{v_1}V_L^\dagger M_D^{diag}V_R-\frac{v_2}{v_1}\xi^{D,0},$$

$$\eta^{U,0}=\frac{\sqrt{2}}{v_1}T_L^\dagger M_U^{diag}T_R-\frac{v_2}{v_1}\xi^{U,0}, \tag{3.6}$$

which we call rotations of type II. Replacing them into Eq. (2.1) the expanded Lagrangian for up and down sectors become

$$-\pounds_{Y(U)}^{(a,II)}=-\frac{g}{\sqrt{2}M_W}\bar{U}M_U^{diag}KP_LDG_W^+-\frac{ig}{2M_W}\bar{U}M_U^{diag}\gamma_5UG_Z^0+\frac{g}{2M_W\cos\beta}\bar{U}M_U^{diag}U(\cos\alpha H^0-\sin\alpha h^0)$$
$$+\frac{ig\tan\beta}{2M_W}\bar{U}M_U^{diag}\gamma_5UA^0+\frac{g}{\sqrt{2}M_W}\tan\beta\bar{U}M_U^{diag}KP_LDH^+-\frac{1}{\cos\beta}\bar{U}\xi^UKP_LDH^++\frac{1}{\sqrt{2}\cos\beta}\bar{U}\xi^UU$$
$$\times[\sin(\alpha-\beta)H^0+\cos(\alpha-\beta)h^0]-\frac{i}{\sqrt{2}\cos\beta}\bar{U}\xi^U\gamma_5UA^0+\text{H.c,} \tag{3.7}$$

$$-\pounds_{Y(D)}^{(a,II)}=\frac{ig}{2M_W}\bar{D}M_D^{diag}\gamma_5DG_Z^0+\frac{g}{\sqrt{2}M_W}\bar{U}KM_D^{diag}P_RDG_W^++\frac{g}{2M_W\cos\beta}\bar{D}M_D^{diag}D(\cos\alpha H^0-\sin\alpha h^0)$$
$$-\frac{ig\tan\beta}{2M_W}\bar{D}M_D^{diag}\gamma_5DA^0-\frac{g\tan\beta}{\sqrt{2}M_W}\bar{U}KM_D^{diag}P_RDH^++\frac{1}{\cos\beta}\bar{U}K\xi^DP_RDH^++\frac{1}{\sqrt{2}\cos\beta}\bar{D}\xi^DD$$
$$\times[\sin(\alpha-\beta)H^0+\cos(\alpha-\beta)h^0]+\frac{i}{\sqrt{2}\cos\beta}\bar{D}\xi^D\gamma_5DA^0+\text{leptonic sector}+\text{H.c.} \tag{3.8}$$

The superindex (a,II) refers to the case (a) and rotation type II. In this situation the case (b) is obtained in the limit $\tan\beta\to 0$, for up and down sectors. Moreover, if we add the Lagrangians (3.4) and (3.8) we find the Lagrangian of the 2HDM type II [28] plus the FC interactions, $\pounds_{Y(U)}^{a,I}+\pounds_{Y(D)}^{a,II}$. Similarly like before, Lagrangian type II is obtained setting $\xi^D=\eta^U=0$. Therefore, Lagrangian type II is generated by making a rotation of type I in the up sector and a rotation of type II in the down sector, it is valid since ξ^U and ξ^D are independent each other and same to $\eta^{U,D}$.

In addition, we can build two additional Lagrangians by adding $\pounds_{Y(U)}^{(a,II)}+\pounds_{Y(D)}^{(a,II)}$ and $\pounds_{Y(U)}^{(a,II)}+\pounds_{Y(D)}^{(a,I)}$. So four different Lagrangians are generated from the case (a). On the other hand, terms involving would-be Goldstone bosons are the same in all the Lagrangians in the R gauge, while in the unitary gauge they vanish [28].

Finally, we can realize that in both cases (a) and (b) with both types of rotations, FCNC processes vanishes when all Higgs boson masses go to infinity. We shall show it by using the rare processes $\mu\to eee$ and $\mu\to e\gamma$.

IV. LFV PROCESSES

In the present work, we study the processes $\mu\to e\gamma$ and $\mu\to eee$ in the 2HDM type III. The decay width of $\mu\to e\gamma$ in both cases (a) and (b) comes from one loop corrections, where we have used a muon running into the loop. The first interaction vertex is proportional to the muon mass and the final vertex is proportional to the flavor changing transition $\mu\to e$. The decay widths in the two types of rotations are given by

$$\Gamma^{(a,I)}(\mu\to e\gamma)=\frac{G_F\alpha_{em}m_\mu^7\eta_{\mu e}^2}{4\pi^4\sqrt{2}\sin^4\beta}|\sin\alpha\sin(\alpha-\beta)F_1(m_{H^0})+\cos\alpha\cos(\alpha-\beta)F_1(m_{h^0})-\cos\beta F_2(m_{A^0})|^2,$$

$$\Gamma^{(a,II)}(\mu\to e\gamma)=\frac{G_F\alpha_{em}m_\mu^7\xi_{\mu e}^2}{4\pi^4\sqrt{2}\cos^4\beta}|\cos\alpha\sin(\alpha-\beta)F_1(m_{H^0})-\sin\alpha\cos(\alpha-\beta)F_1(m_{h^0})+\sin\beta F_2(m_{A^0})|^2 \tag{4.1}$$

where

$$F_1(x)=-2F_2(x)=\frac{\log[x^2/m_\mu^2]}{x^2}. \tag{4.2}$$

The decay widths for the process $\mu\to eee$ in the two cases read

$$\Gamma^{(a,I)}(\mu\to eee)=\frac{G_Fm_\mu^5m_e^2\eta_{\mu e}^2}{\sqrt{2}512\pi^3\sin^4\beta}\left|\frac{\sin\alpha\sin(\alpha-\beta)}{m_{H^0}^2}+\frac{\cos\alpha\cos(\alpha-\beta)}{m_{h^0}^2}-\frac{\cos\beta}{m_{A^0}^2}\right|^2,$$

$$\Gamma^{(a,II)}(\mu\to eee)=\frac{G_Fm_\mu^5m_e^2\xi_{\mu e}^2}{\sqrt{2}512\pi^3\cos^4\beta}\left|\frac{\cos\alpha\sin(\alpha-\beta)}{m_{H^0}^2}-\frac{\sin\alpha\cos(\alpha-\beta)}{m_{h^0}^2}+\frac{\sin\beta}{m_{A^0}^2}\right|^2. \tag{4.3}$$

The corresponding expressions for the case (b) are obtained taking the appropriate limits, in rotation type I $\tan\beta\to\infty$ and in rotation type II $\tan\beta\to 0$. These FC processes vanish when all Higgs boson masses go to infinity.

Now, by using the experimental upper bounds for LFV processes [11,12]

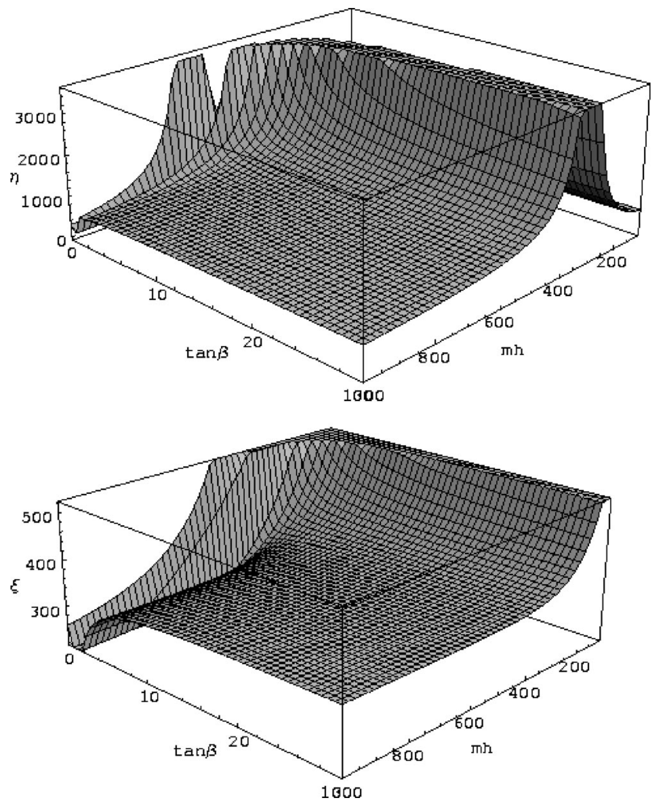

FIG. 1. Figure 1 corresponds to 3D plots of the fraction of FC couplings coming from the ratio of the muon contribution and tau contribution in the radiative corrections for the process $\mu\to e\gamma$. We set $\alpha=\pi/16$, $m_{H^0}=300$ GeV and m_{A^0} is going to infinity. The figure on the top corresponds to (a,I) and the other one to (a,II).

$$\Gamma(\mu\to e\gamma)\leqslant 3.59\times 10^{-30}\ \text{GeV},$$

$$\Gamma(\mu\to eee)\leqslant 3.0\times 10^{-31}\ \text{GeV}, \tag{4.4}$$

we get restrictions to $\eta(\xi)$ parameters which generate FC at the tree level. We see that the upper bounds imposed by $\mu\to e\gamma$ are much more restrictive.

We use a muon running into the loop for the calculation of $\mu\to e\gamma$ instead of a tau as customary. We take the quotient $\Gamma^{(a,\tau)}/\Gamma^{(a,\mu)}$ where $\Gamma^{(a,\mu)}$ represents the width of $\mu\to e\gamma$ with a muon into the loop for the case (a), and similarly for $\Gamma^{(a,\tau)}$ with tau into the loop. Supposing that $\Gamma^{(a,\mu)}/\Gamma^{(a,\tau)}\approx 1$ and setting $m_{H^0}=300$ GeV, $\alpha=\pi/16$ and $m_A\to\infty$, we plot the quotients

$$\frac{\eta_{\mu e}}{\eta_{\mu\tau}\eta_{\tau e}},\quad \frac{\xi_{\mu e}}{\xi_{\mu\tau}\xi_{\tau e}} \tag{4.5}$$

vs m_h^0 and $\tan\beta$. We notice from Fig. 1 that the values obtained for the fraction cover a wide range. Consequently, it is not necessary that the tau contribution is more important than the muon one.

We turn now to derive constraints for the parameters of the Higgs sector. Let us consider the process $\mu\to e\gamma$ in both cases and both types of rotations for different values of the Higgs boson masses and mixing angles. In Fig. 2 we take

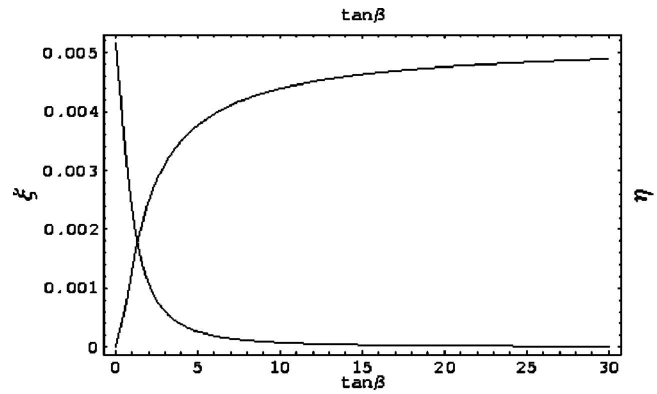

FIG. 2. Figure 2 illustrates the differences between the models (a,I) and (a,II) with respect to the parameter $\tan\beta$. We have taken the Higgs boson mass m_{A^0} to infinity and $\alpha=\pi/16$ and $m_{h^0}=m_{H^0}=300$ GeV. The curve that increases with $\tan\beta$ corresponds to the model (a,I).

m_{A^0} going to infinity. We plot $\eta(\xi)_{\mu e}$ vs $\tan\beta$, for $\alpha=\pi/16$ and $m_{H^0}=m_{h^0}=300$ GeV for the models (a,I),(a,II), respectively. We can observe that the behavior of the models are quite different in a long range of $\tan\beta$. Additionally, near to the critical points of $\tan\beta(=0,\infty)$, the models take complementary values.

The 3D plots $(\eta(\xi)_{\mu e}, m_h^0, m_A^0)$ are shown in Fig. 3 for $m_H^0=500$ GeV, $\alpha=\pi/16$ and $\tan\beta=1$. They represent the models (a,I) and (a,II), similar to Fig. 2. Once again, we realize that the behavior of both models is quite different.

Figure 4 corresponds to the models (a,II) and (b,II) in which $m_{H^0}=300$ GeV and $\alpha=\pi/16$. For the model (a,II)

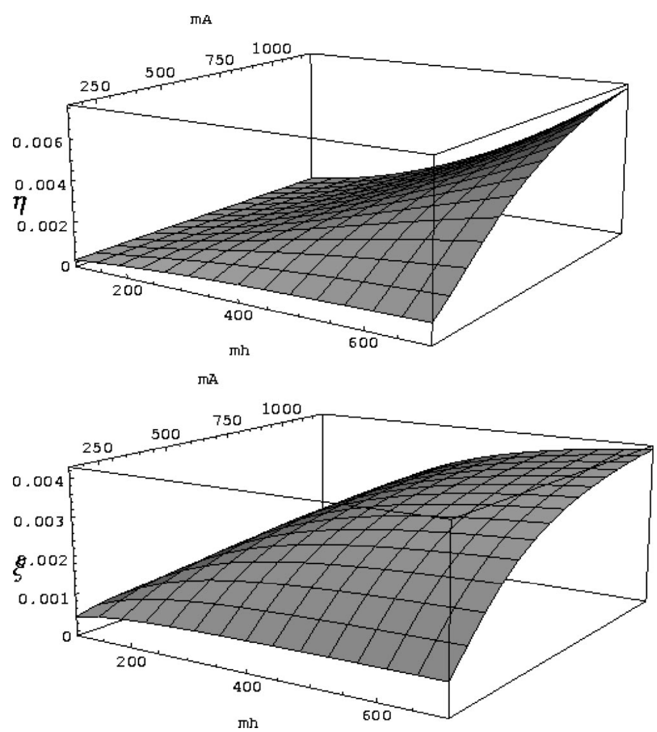

FIG. 3. Figure 3 is for the parameter space $[\eta(\xi)_{\mu e}, m_h^0, m_A^0]$ for the models (a,I) and (a,II), respectively. We set $\tan\beta=1$, $m_{H^0}=500$ GeV, and $\alpha=\pi/16$.

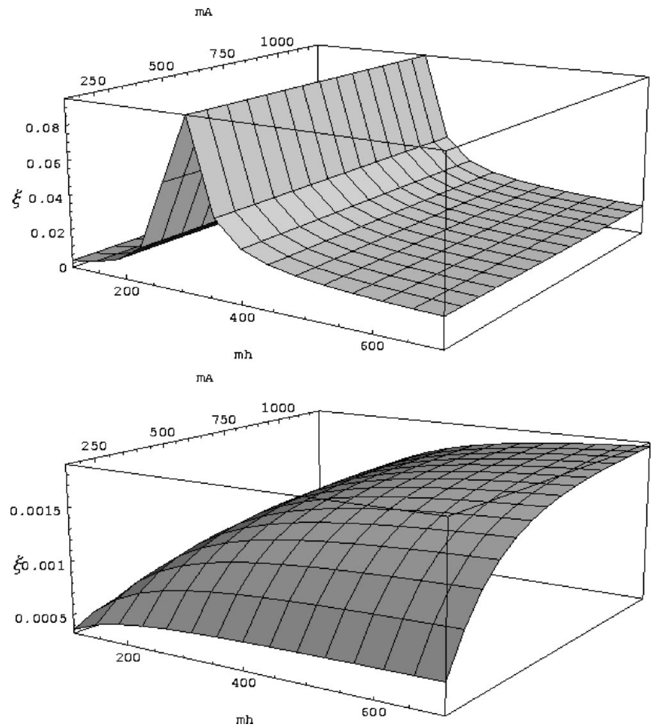

FIG. 4. Figure 4 is for the parameter space ($\xi_{\mu e}, m_h^0, m_A^0$) for the models (a,II) and (b,II), respectively. We set $m_{H^0}=300$ GeV$\alpha=\pi/16$. We use $\tan\beta=1$ for the model (a).

we use $\tan\beta=1$. These graphics illustrate that the cases (a) and (b) are substantially different.

V. CONCLUSIONS

In the present work we examine a 2HDM type III which produces FCNC at the tree level. We classified the model type III according to the VEV taken by the Higgs bosons and to the method used to rotate the mixing matrices. All that, in order to write down the Lagrangian in the mass eigenstates. When both doublets acquire a VEV we have called it case (a) and when only one doublet acquire a VEV we have called it case (b). On the other hand, when we write $\xi^{D,0},\xi^{U,0}$ in terms of $\eta^{D,0},\eta^{U,0}$ plus the mass matrices, it is called here a rotation of type I. And, when we solve for $\eta^{D,0},\eta^{U,0}$ in terms of $\xi^{D,0},\xi^{U,0}$ and the mass matrices we call it a rotation of type II.

In addition, we observe that the 2HDM of type I plus FC interactions is generated by adding the Lagrangian of type (a,I) in the up sector and the Lagrangian of type (a,I) in the down sector, $\pounds^{a,I}_{Y(U)}+\pounds^{a,I}_{Y(D)}$. Meanwhile, the Lagrangian of type II plus FC interactions is generated by adding the Lagrangian of type (a,I) in the up sector and the Lagrangian of type (a,II) in the down sector, $\pounds^{a,I}_{Y(U)}+\pounds^{a,II}_{Y(D)}$. Other two different combinations are possible, i.e., $\pounds^{(a,II)}_{Y(U)}+\pounds^{(a,I)}_{Y(D)}$ and $\pounds^{(a,II)}_{Y(U)}+\pounds^{(a,II)}_{Y(D)}$. Moreover, if we began with a Lagrangian of type (a,I) we would obtain the Lagrangian (b,I) taking the limit $\tan\beta\to\infty$, while if we started with a Lagrangian of type (a,II) we would obtain the Lagrangian (b,II) in the limit $\tan\beta\to 0$.

In order to emphasize the difference between the cases (a) and (b) we can notice that case (b) could be obtained from the case (a) by rotating the Higgs fields properly, in order to set one of the VEV equal to zero. However, making a rotation implies to fix one parameter of the model, in this case $\tan\beta=v_2/v_1\to 0, \quad \infty$. Nevertheless, it is well known that physical observables are in general sensitive to $\tan\beta$, consequently the case (b) is a particular occurrence of case (a).

Furthermore, to illustrate the importance of the classification made in Secs. II and III, we show graphics to find bounds on the FC coupling $\eta(\xi)_{\mu e}$ coming from the process $\mu\to e\gamma$ and we realize that such bounds are sensitive to the type of rotation and also to the structure of the VEV. We also calculate the process $\mu\to 3e$ for both kind of rotations but the constraints obtained were less restrictive than the ones obtained with the process $\mu\to e\gamma$.

Finally, to evaluate such bounds we have used a muon running into the loop for the process $\mu\to e\gamma$ instead of a tau as usual. Consequently, we plot the quotient (18) in terms of m_{h^0} and $\tan\beta$, getting a wide range of allowed values for that quotient. So, the tau contribution is not necessarily more important than the muon one.

ACKNOWLEDGMENTS

We acknowledge to M. Nowakowski for his suggestions and for the careful reading of the manuscript. This work was supported by COLCIENCIAS.

[1] M. Ciuchini *et al.*, Phys. Lett. B **316**, 127 (1993); Nucl. Phys. **B421**, 41 (1994); S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, *ibid.* **B353**, 591 (1991).
[2] H. Stern and M. K. Gaillard, Ann. Phys. (N.Y.) **76**, 580 (1973); C. S. Kim, J. L. Rosner, and C. P. Yuan, Phys. Rev. D **42**, 96 (1990).
[3] T. Imami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981).
[4] J. L. Diaz-Cruz *et al.*, Phys. Rev. D **41**, 891 (1990); G. Eilam, J. Hewett, and A. Soni, *ibid.* **44**, 1473 (1991); G. Couture, C. Hamzanoi, and H. Konig, *ibid.* **52**, 1713 (1995).
[5] J. L. Hewett and T. Rizzo, Phys. Rep. **183**, 193 (1989); G. Baremboim *et al.*, Phys. Lett. B **422**, 277 (1998); V. Barger, M. Berger, and R. Phillips, Phys. Rev. D **52**, 1663 (1995); R. Martinez, J.-Alexis Rodriguez, and M. Vargas, *ibid.* **60**, 077504 (1999); F. del Aguila, J. Aguilar Saavedra, and R. Miquel, Phys. Rev. Lett. **82**, 1628 (1999).
[6] J. Liu and L. Wolfenstein, Nucl. Phys. **B289**, 1 (1987).
[7] M. Nowakowski and A. Pilaftsis, Nucl. Phys. **B461**, 19 (1996); A. Joshipura and M. Nowakowski, Phys. Rev. D **51**, 5271 (1995); G. Ross and J. W. F. Valle, Phys. Lett. **151B**, 375 (1985).
[8] K. Arisaka *et al.*, Phys. Rev. Lett. **70**, 1049 (1993).
[9] K. Arisaka *et al.*, Phys. Lett. B **432**, 230 (1993).
[10] A. M. Lee *et al.*, Phys. Rev. Lett. **64**, 165 (1990).

[11] R. D. Bolton *et al.*, Phys. Rev. D **38**, 2077 (1983); M. L. Brooks *et al.*, Phys. Rev. Lett. **83**, 1521 (1999).
[12] U. Bellgardt *et al.*, Nucl. Phys. **B299**, 1 (1998).
[13] C. Dohmen *et al.*, Phys. Lett. B **317**, 631 (1993).
[14] S. Glashow and S. Weinberg, Phys. Rev. D **15**, 1958 (1977).
[15] Marc Sher and Yao Yuan, Phys. Rev. D **44**, 1461 (1991).
[16] T. P. Cheng and M. Sher, Phys. Rev. D **35**, 3484 (1987).
[17] D. Atwood, L. Reina, and A. Soni, Phys. Rev. D **55**, 3156 (1997); G. Cvetic, S. Hwang, and C. S. Kim, *ibid.* **58**, 116003 (1998).
[18] Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998).
[19] Kaustubh Agashe and M. Graessner, Phys. Rev. D **61**, 075008 (2000).
[20] Y. Okada, K. Okumura, and Y. Shimizu, Phys. Rev. D **61**, 094001 (2000).
[21] D. Atwood, L. Reina, and A. Soni, Phys. Rev. D **53**, 1199 (1996); **54**, 3296 (1996); Phys. Rev. Lett. **75**, 3800 (1993).
[22] G. López Castro, R. Martinez, and G. Munoz, Phys. Rev. D **58**, 033003 (1998).
[23] Marc Sher, Phys. Lett. B **487**, 151 (2000).
[24] *Workshop on Physics at the First Muon Collider and at the Front End of the Muon Collider*, edited by S. Geer and R. Raja (AIP Publishing, Batavia, Il, 1997).
[25] W. S. Hou, Phys. Lett. B **296**, 179 (1992); D. Chang, W. S. Hou, and W. Y. Keung, Phys. Rev. D **48**, 217 (1993).
[26] M. Sher and Y. Yuan, Phys. Rev. D **44**, 1461 (1991).
[27] S. Nie and M. Sher, Phys. Rev. D **58**, 097701 (1998).
[28] For a review, see J. Gunion, H. Haber, G. Kane, and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, New York, 1990).
[29] D. Atwood, L. Reina, and A. Soni, Phys. Rev. D **55**, 3156 (1997); Phys. Rev. Lett. **75**, 3800 (1995).
[30] Shuquan Nie and Marc Sher, Phys. Rev. D **58**, 097701 (1998).