# Minimal supersymmetric standard model prediction for $W^{\pm}H^{\mp}$ production by gluon fusion at the CERN Large Hadron Collider

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We discuss the associated  $W^{\pm}H^{\mp}$  production in pp collision for the CERN Large Hadron Collider. A complete one-loop calculation of the loop-induced subprocess  $gg \rightarrow W^{\pm}H^{\mp}$  is presented in the framework of the minimal supersymmetric standard model (MSSM), and the possible enhancement of the hadronic cross section is investigated under the constraint from the squark direct-search results and the low-energy precision data. Because of the large destructive interplay in the quark-loop contributions between triangle-type and box-type diagrams, the squark-loop contributions turn out to be comparable with the quark-loop ones. In particular, the hadronic cross section via gluon fusion can be extensively enhanced by squark-pair threshold effects in the box-type diagrams, so that it can reach the size of the hadronic cross section via the subprocess  $b\bar{b} \rightarrow W^{\pm}H^{\mp}$  which appears at the tree level.

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### I. INTRODUCTION

In the standard model (SM), a single neutral Higgs boson is predicted as a direct consequence of the SM mechanism of electroweak symmetry breaking. The detection of this particle is, therefore, one of the most essential tasks at present and future collider experiments, especially at the CERN Large Hadron Collider (LHC) [1]. On the other hand, charged Higgs bosons (as well as a *CP*-odd neutral Higgs boson) are predicted in extended versions of the SM model, including the minimal supersymmetric standard model (MSSM). Since the discovery of such an additional Higgs boson will immediately indicate physics beyond the SM, there is increasing interest in theoretical and experimental studies to provide the basis for its accurate exploration.

At hadron colliders, a mode for charged Higgs boson  $H^{\pm}$  detection may be the top-quark-top-antiquark pair production from gluon fusion and  $q\bar{q}$  annihilation and subsequent decay  $t \rightarrow bH^+ \rightarrow b\tau^+ \nu$ , if the mass of  $H^{\pm}$  is smaller than  $m_t - m_b$ . For heavier  $H^{\pm}$ , the main modes for  $H^{\pm}$  production may be those associated with heavy quarks, such as  $gb \rightarrow H^-t$  [2] and  $qb \rightarrow q'bH^-$  [3]. Although these processes give rather large production rates, they suffer from also large QCD backgrounds, especially when the  $H^{\pm}$  mass is above the threshold of  $t\bar{b}$  pair production. Pair production of  $H^{\pm}$  via the tree-level  $q\bar{q}$  annihilation subprocesses and via the loop-induced gluon-fusion mechanism has also been studied in the literature [4-7].

A further possibility is single-charged-Higgs-boson production together with a charged W gauge boson. In this paper we focus on a discussion of this process of associated  $W^{\pm}H^{\mp}$  production for the LHC. There are mainly two partonic subprocesses that contribute to the hadronic cross section  $pp \rightarrow W^-H^+$ :  $b\bar{b}$  annihilation (at the tree level) and gg fusion (at the one-loop level). The production cross section based on  $gg \rightarrow W^{\pm}H^{\mp}$  can be comparable to that via  $b\bar{b}$  $\rightarrow W^{\pm}H^{\mp}$ , because of the large number of gluons in the high energy proton beams at the LHC. The associated  $W^{\pm}H^{\mp}$ production via  $b\overline{b}$  annihilation and gg fusion was discussed at first in [8] for a supersymmetric 2-Higgs-doublet model including the loop contributions from top and bottom quarks with the approximation  $m_b = 0$ . That work has been extended in [9] including a non-zero b-quark mass, thus allowing investigation of the process for arbitrary values of  $\tan \beta$ . The rate of  $W^{\pm}H^{\mp}$  production mediated by quark loops turned out to be sizable, especially for low and high values of  $\tan \beta$ . The background, which mainly comes from  $t\bar{t}$  production, has been analyzed in [10]. These studies correspond to a MSSM scenario where the scalar quarks are sufficiently heavy to decouple from the loop contributions.

In more general scenarios the squarks need not be heavy, and taking the MSSM seriously requires the inclusion of also the squark loops in the gluon-fusion mechanism and to study their effects on the predictions for  $pp \rightarrow W^-H^+$ . There are several reasons to underline the importance of the squark contributions. In the subprocess  $gg \rightarrow W^{\pm}H^{\mp}$ , the quarkloop contributions are destructive between the triangle-type diagrams and the box graphs [8,9] in the MSSM; the effects from both types are almost of the same size, so that the cross section obtained from the summed t-b loop contributions is more than one order of magnitude smaller than that with only triangle- or box-type diagrams separately. By this mechanism of cancellation in the quark-loop terms the relative importance of the squark-loop contribution increases. On top, the cross section for  $gg \rightarrow W^{\pm}H^{\mp}$  can be sizably enhanced by threshold effects of squark pairs, such that in special cases the hadronic cross section via gluon fusion can be as large as that via  $b\bar{b} \rightarrow W^{\pm}H^{\mp}$ . Moreover, the present calculation of the  $b\bar{b}$  annihilation is expected to overestimate the cross section, as has been mentioned [8,9] by recalling the problem of double counting [11]. Owing to the presence

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of also superpartners in the virtual states of the loop diagrams, the gluon-fusion mechanism is more sensitive to the detailed structure of the model than the tree-level process of  $b\overline{b}$  annihilation.<sup>1</sup>

In this article we extend the previous calculations by including also the scalar-quark sector in the loop diagrams for gg fusion. We give analytical results and a detailed discussion of the effects in the hadronic cross section. Our results are for general parameters of the MSSM; for the numerical discussion, constraints from the direct search and from the precision data [13,14] are taken into account.<sup>2</sup>

The paper is organized as follows. In Sec. II the calculation for the partonic cross section,  $gg \rightarrow W^-H^+$ , is explained, and numerical results for the partonic and hadronic cross sections are presented in Sec. III. Our conclusion is given in Sec. IV. All relevant coupling constants and analytic formulas for the amplitudes are presented in the Appendixes for a comprehensive documentation.

## II. PARTONIC PROCESS $gg \rightarrow W^-H^+$

#### A. Cross section

In our kinematical conventions, the momenta of the initial state gluons, k and  $\overline{k}$ , are chosen as incoming and outgoing for the momenta, p and  $\overline{p}$ , of the final state particles:

$$g(k,a,\sigma) + g(\bar{k},b,\bar{\sigma}) \rightarrow W^{-}(p,\lambda) + H^{+}(\bar{p})$$

Besides by their momenta, the initial state gluons are characterized by their color indices a,b and their helicities  $\sigma, \overline{\sigma}$  $(=\pm 1)$ , and the final state *W* boson is characterized by its helicity  $\lambda$  (=0,±1). We make use of the parton kinematical invariants

$$\hat{s} = (k + \bar{k})^2$$
,  $\hat{t} = (k - p)^2$ ,  $\hat{u} = (k - \bar{p})^2$ 

obeying the relation

$$\hat{s} + \hat{t} + \hat{u} = m_{H^{\pm}}^2 + m_W^2.$$

The spin- and color-averaged cross section for the parton process

$$\frac{d\sigma}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2} \sum_{\lambda=0,\pm} \frac{1}{4} \sum_{\sigma,\bar{\sigma}=\pm 1} \frac{(\mathrm{CF})}{64} |\mathcal{M}_{\sigma\bar{\sigma}\lambda}|^2,$$
  
with  $(\mathrm{CF}) = \sum_{a,b=1}^{8} \left[ \mathrm{Tr} \left\{ \frac{\lambda^a}{2} \frac{\lambda^b}{2} \right\} \right]^2 = 2$  (1)

contains the helicity amplitudes

$$\mathcal{M}_{\sigma\bar{\sigma}\lambda} = \varepsilon^{\mu}_{\sigma}(k)\varepsilon^{\nu}_{\bar{\sigma}}(\bar{k})\varepsilon^{*\rho}_{\lambda}(p)\tilde{\mathcal{M}}_{\mu\nu\rho}, \qquad (2)$$

where  $\varepsilon_{\sigma}^{\mu}(k)$ ,  $\varepsilon_{\overline{\sigma}}^{\nu}(\overline{k})$  and  $\varepsilon_{\lambda}^{*\rho}(p)$  are the polarization vectors for incoming gluons and outgoing *W* bosons. As a general feature of the amplitude, the transversality of gluons gives useful identities

$$k^{\mu}\varepsilon^{\nu}_{\overline{\sigma}}(\overline{k})\varepsilon^{*\rho}_{\lambda}(p)\widetilde{\mathcal{M}}_{\mu\nu\rho} = \varepsilon^{\mu}_{\sigma}(k)\overline{k}^{\nu}\varepsilon^{*\rho}_{\lambda}(p)\widetilde{\mathcal{M}}_{\mu\nu\rho} = 0, \quad (3)$$

which allows cross-checks of our one-loop calculation of  $\tilde{\mathcal{M}}_{\mu\nu\rho}$ .

In the parton center-of-mass (c.m.) frame, the momenta may be expressed by

$$k^{\mu} = \left(\frac{\sqrt{\hat{s}}}{2}, 0, 0, \frac{\sqrt{\hat{s}}}{2}\right),\tag{4}$$

$$\bar{k}^{\mu} = \left(\frac{\sqrt{\hat{s}}}{2}, 0, 0, -\frac{\sqrt{\hat{s}}}{2}\right), \tag{5}$$

$$p^{\mu} = (E_W, p_W)$$
$$= (E_W, |\vec{p}_W| \sin \Theta, 0, |\vec{p}_W| \cos \Theta), \quad (6)$$

and then the polarization vectors are given by

$$\varepsilon_{\sigma}^{\mu}(k) = \frac{1}{\sqrt{2}}(0,1,i\sigma,0),$$
 (7)

$$\varepsilon_{\overline{\sigma}}^{\mu}(\overline{k}) = \frac{1}{\sqrt{2}}(0, 1, -i\overline{\sigma}, 0) \tag{8}$$

and

$$\varepsilon_{\lambda=0}^{*\mu}(p) = \left(\frac{|\vec{p}_W|}{m_W}, \frac{E_W}{m_W}\sin\Theta, 0, \frac{E_W}{m_W}\cos\Theta\right), \tag{9}$$

$$\varepsilon_{\lambda=\pm}^{*\mu}(p) = \frac{1}{\sqrt{2}}(0, i\lambda \cos\Theta, 1, -i\lambda \sin\Theta).$$
(10)

Finally, the integrated partonic cross section

$$\sigma_{gg \to W^-H^+}(\hat{s}, \alpha_S(\mu_R)) = \int_{\hat{t}_{\min}(\hat{s})}^{\hat{t}_{\max}(\hat{s})} d\hat{t} \frac{d\sigma}{d\hat{t}}$$
(11)

is evaluated by numerical integration over the kinematically allowed  $\hat{t}$  range for a given c.m. energy  $\sqrt{\hat{s}}$ . The renormalization scale  $\mu_R$  in the modified minimal subtraction scheme ( $\overline{\text{MS}}$ ) strong coupling constant  $\alpha_S(\mu_R)$  is chosen as  $\sqrt{\hat{s}}$ , the hard energy scale of the process.

<sup>&</sup>lt;sup>1</sup>For the subprocess  $b\bar{b} \rightarrow W^- H^+$ , the electroweak one-loop corrections have recently been studied in Ref. [12], which can give rise to a 10–15 % reduction of the lowest-order result.

<sup>&</sup>lt;sup>2</sup>Quite recently, the squark-loop contributions to gluon fusion have also been derived [15] with a study of their effects in a supergravity-inspired grand unified theory (GUT) scenario.

TABLE I. Choices for the the soft supersymmetry breaking parameters  $M_{\tilde{Q}}$ ,  $M_{\tilde{U}}$ ,  $M_{\tilde{D}}$ ,  $X_t$  and  $X_b$  without squark mixing (i.e.  $X_t=X_b=0$ ). For all cases,  $\mu$  is fixed to be zero. The resulting spectrum of the third generation squark masses is also displayed.

	Case A			Case B			Case C		
$M_{\tilde{O}}$ [GeV]	250			300			350		
$\tilde{M}_{\tilde{U}}$ [GeV]	250			300			350		
$M_{\tilde{D}}$ [GeV]	250			300			350		
$X_t$ [GeV]	0			0			0		
$X_b$ [GeV]	0			0			0		
$\tan \beta$	1.5	6	30	1.5	6	30	1.5	6	30
$m_{\tilde{t}_1}$ [GeV]	303	300	300	345	343	343	390	387	387
$m_{\tilde{t}_2}$ [GeV]	304	303	303	346	345	345	390	390	389
$m_{\tilde{b}_1}$ [GeV]	250	251	251	300	301	301	350	351	351
$m_{\tilde{b}_2}$ [GeV]	253	257	257	302	306	306	352	355	355

#### B. Calculation of the amplitude

In the MSSM, there is no tree-level contribution to the subprocess  $gg \rightarrow W^-H^+$ . The process is induced at the one-loop level by diagrams with quark loops (see Appendix D 1) and with squark loops (see Appendix D 2). In the loop diagrams, also the neutral Higgs bosons  $h^0$ ,  $H^0$  and  $A^0$  of the MSSM appear as well as the scalar partners of the quarks. The notation and couplings are collected in Appendix C.

The quark-loop diagrams can be subdivided into box-type diagrams and into triangle diagrams with *s*-channel exchange of a neutral Higgs boson. In each box diagram, the quarks in the loop couple directly to the outgoing charged Higgs boson  $H^+$ , while in the triangle diagrams the quarks couple to one of the neutral Higgs bosons  $h^0$ ,  $H^0$  and  $A^0$ . Since these Yukawa interactions are all proportional to the quark masses, the contributions from the loop diagrams of the third-generation quarks (t,b) are dominant.

It has been known that there is a striking feature among the quark-loop contributions to the amplitude in the MSSM. Owing to supersymmetry, the box-diagram contribution to the amplitude and the triangle-type one are almost of the same size but the relative sign is negative, so that strong destructive interference occurs. The cross section resulting from all the quark-loop diagrams is therefore one to two orders of magnitude smaller than the cross section obtained from only box-type diagrams or only triangle diagrams, separately.

By the destructive interference in the quark-loop contributions, the squark-loop effects become relatively large. The squark one-loop diagrams are subdivided into (1) diagrams with a 2-point loop and an intermediate *s*-channel neutral CP-even Higgs boson, (2) diagrams with a triangle and an intermediate neutral CP-even Higgs boson, (3) diagrams with a triangle without an intermediate Higgs boson, and (4) box-type diagrams. As has been verified explicitly, each group separately satisfies the transversality relation (3).

The cross section has been derived in two completely independent calculations and perfect agreement has been achieved. Furthermore, these results have been confirmed by

TABLE II. Choices for the the soft supersymmetry breaking parameters  $M_{\tilde{Q}}$ ,  $M_{\tilde{U}}$ ,  $M_{\tilde{D}}$ ,  $X_t$  and  $X_b$  for three values of tan  $\beta$  and the resulting spectrum of the third-generation squark masses and the mixing angle in the  $\tilde{t}$ -sector. For all cases,  $\mu$  is fixed to be zero.

		Case 1	l	Case 2			Case 3		
$M_{\tilde{Q}}$ [GeV]		250			300			350	
$M_{\tilde{U}}$ [GeV]		250			300			350	
$M_{\tilde{D}}$ [GeV]		250			300			350	
$X_b$ [GeV]		0			0			0	
$\tan \beta$	1.5	6	30	1.5	6	30	1.5	6	30
$X_t$ [GeV]	-470	-464	-463	-628	-621	-620	-813	-806	-806
$m_{\tilde{t}_1}$ [GeV]	101	100	101	101	101	101	102	102	102
$m_{\tilde{t}_2}$ [GeV]	417	414	414	479	476	476	542	540	540
$m_{\tilde{b}_1}$ [GeV]	251	251	251	300	301	301	350	351	351
$m_{\tilde{b}_2}$ [GeV]	253	257	257	302	306	306	352	355	355
$\theta_{\tilde{t}}$ [°]	44.9	44.7	44.7	44.9	44.8	44.8	44.9	44.8	44.8

utilizing FEYNARTS and FORMCALC [16]. The agreement of these three independent calculations and the transversality test establish strong confidence in our result.

#### **III. NUMERICAL RESULTS**

#### A. Parameters

We take  $m_Z$ ,  $m_W$  and  $G_F$  as the input electroweak parameters, and use the values  $m_Z=91.1882$  GeV,  $m_W=80.419$  GeV and  $G_F=1.16639 \times 10^{-5}$  GeV<sup>-2</sup> [13]. For the strong coupling constant  $\alpha_S(\mu_R)$ , we use the formula including the two-loop QCD corrections for  $n_f=5$  with  $\Lambda_{QCD}^5=170$  MeV which can be found in [13]. The masses of the top and bottom quarks are fixed here as  $m_t=174.3$  GeV and  $m_b=4.7$  GeV.

The sfermion parameter sets are chosen in accordance with direct search results [13]. In addition, the stringent experimental constraints on the new physics parameters from the electroweak precision measurements are respected. In Refs. [14,17], this kind of constraint on the new physics parameters has been studied in the framework of the MSSM by using the Z-pole data,  $m_W$  measurements, and low-energy neutral-current data. Our parameter sets are chosen to be in accordance with the region inside the 99% C.L. contour on the  $S_Z - T_Z$  plane presented in [17].

In cases without sfermion mixing, it is expected that lighter sfermions give larger one-loop contributions while heavy sfermions tend to decouple from the observables. We examine the cross section under this situation by introducing three cases, specified as A, B, C in Table I. Another interesting situation is the case with large  $\tilde{t}_L - \tilde{t}_R$  mixing. The magnitude of the mixing is determined by the off-diagonal part of the  $\tilde{t}$ -mass matrix, especially  $X_t$  (see Appendix A). To study the cross section in this situation, we select three parameter sets (cases 1, 2, 3 in Table II), in which the maximal  $\tilde{t}_L - \tilde{t}_R$  mixing occurs with the mixing angle  $\theta_{\tilde{t}} \sim 45^\circ$  and a light  $\tilde{t}_1$  with  $m_{\tilde{t}_1} \sim 100$  GeV.



FIG. 1. Partonic cross section evaluated using all Feynman graphs (solid lines), only quark loop graphs (dashed lines) and only squark loop graphs (dotted lines) for a sequence of charged-Higgs-boson masses (100,210,360,450 GeV) and  $\tan \beta = 1.5$ . The parameter set 1 (see Table II) is used in the squark sector.

#### **B.** Partonic cross section

Before proceeding to the hadronic cross section,  $pp \rightarrow W^-H^+$ , we want to illustrate the partonic cross section of the subprocess  $gg \rightarrow W^-H^+$  numerically. Although the partonic process is not accessible experimentally, it is useful for understanding the features of the hadronic cross section, in particular its dependence on the MSSM parameters. For gluon-fusion processes, the threshold region  $(\sqrt{\hat{s}} \sim m_W + m_{H^{\pm}})$ , where gluon pairs are most numerous in proton collisions, gives the dominant contribution to the hadronic cross section. Not intending completeness, we display here only one parameter set (case 1 of Table II) for the squark sector as an example.

In Fig. 1 the integrated partonic cross section is shown as a function of  $\sqrt{\hat{s}}$  for a sequence of  $m_{H^{\pm}}$  values. Each plot displays three curves: the cross section evaluated from all (solid lines), only squark-loop (dotted lines) and only quarkloop (dashed lines) diagrams. The last case corresponds to the scenario where the squarks decouple.

For  $m_{H^{\pm}} = 100$  GeV [Fig. 1(a)] the cross section is mainly dominated by the quark-loop diagrams. The  $\tilde{b}\tilde{b}$  thresholds at  $\sqrt{\hat{s}} \sim 2m_{\tilde{b}_1} \sim 2m_{\tilde{b}_2}$  from the squark-box diagrams are visible. Nevertheless, the  $\tilde{b}\tilde{b}$  thresholds turn out to be of less importance for the hadronic cross section, because they are too far off the threshold of the associated  $H^+W^-$  production. Another effect of the squark-loop diagrams is present near the production threshold, where the tail of the  $\tilde{t}_1\tilde{t}_1$  threshold still has influence and gives rise to a slightly enhanced hadronic cross section with respect to the squark decoupling case (see Fig. 4).

At a charged-Higgs-boson mass of 210 GeV [Fig. 1(b)] the cross section is largely dominated by the quark-loop diagrams. The peak corresponds to the  $t\bar{t}$ -threshold effect in the quark-box diagrams. A constructive squark-loop effect appears above the sbottom pair threshold, but this is negligible in the hadronic cross section.

For  $m_{H^{\pm}} = 360$  GeV [Fig. 1(c)] a novel enhancement of the partonic cross section occurs around the thresholds of sbottom pair production  $\sqrt{\hat{s}} \sim 2m_{\tilde{b}_1} \sim 2m_{\tilde{b}_2}$ , where the squark-loop contributions are dominant. Since the peak in the partonic cross section is near the  $W^-H^+$  threshold where



FIG. 2. Hadronic cross section for  $H^+W^-$  production via gluon fusion versus the charged-Higgs-boson mass  $m_{H^{\pm}}$  for two values of tan  $\beta$  (1.5,6). The three squark scenarios without mixing (A, B, C in Table I) are compared with the case of decoupling squarks (solid lines).

a lot of gluons are supplied, a large squark contribution appears also in the hadronic cross section, which corresponds to the local maximum of the hadronic cross section at around  $m_{H^{\pm}} = 360$  GeV in Fig. 4.

For  $m_{H^{\pm}} = 450$  GeV [Fig. 1(d)] there is neither a quarkpair nor a squark-pair threshold slightly above the  $W^-H^+$ threshold, but the enhancement due to virtual squarks is still quite relevant.

### C. Hadronic cross section

The hadronic inclusive cross section for  $W^-H^+$  production in proton-proton collisions at a total hadronic c.m. energy  $\sqrt{S}$  can be written as a convolution [18]

$$\sigma(pp \to W^-H^+ + X) = \sum_{\{n,m\}} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}_{nm}^{pp}}{d\tau} \times \sigma_{nm \to W^-H^+}(\tau S, \alpha_S(\mu_R))$$
(12)

with the parton luminosity

$$\frac{d\mathcal{L}_{nm}^{pp}}{d\tau} = \int_{\tau}^{1} \frac{dx}{x} \frac{1}{1+\delta_{nm}} \bigg[ f_{n/p}(x,\mu_F) f_{m/p}\bigg(\frac{\tau}{x},\mu_F\bigg) + f_{m/p}(x,\mu_F) f_{n/p}\bigg(\frac{\tau}{x},\mu_F\bigg) \bigg], \qquad (13)$$



FIG. 3. Hadronic cross section for  $H^+W^-$  production via gluon fusion versus tan  $\beta$  for three values of the charged-Higgs-boson mass  $m_{H^{\pm}}$  (100,300,1000 GeV). The three squark cases without mixing (A, B, C in Table I) are compared with the case of decoupling squarks (solid lines).

where  $f_{n/p}(x,\mu_F)$  denotes the density of partons of type *n* in the proton carrying a fraction *x* of the proton momentum at the scale  $\mu_F$ . In our case the sum over unordered pairs of partons  $\{n,m\}$  reduces to two terms; i.e., there are two parton subprocesses contributing to inclusive  $W^-H^+$  hadroproduction: gluon fusion and  $b\bar{b}$  annihilation. Our main concern is the gluon-fusion process and the possible enhancement of the cross section through virtual squark effects with respect to the approximation of decoupling squarks. We will also compare the cross section for gluon fusion with the one for  $b\bar{b}$  annihilation. The numerical evaluation has been carried out with the Martin-Roberts-Stirling set G [MRS(G)] gluon distribution functions [19] and with the renormalization and factorization scale  $\mu_R$ ,  $\mu_F$  chosen as equal.

The input parameters in the MSSM Higgs sector can be chosen to be  $m_{H^{\pm}}$  and tan  $\beta$ . Thus we study here the dependence of the hadronic cross section for  $W^-H^+$ production via gluon fusion on these two parameters for the different squark scenarios mentioned in Sec. III A.





FIG. 4. Hadronic cross section for  $H^+W^-$  production via gluon fusion versus the charged-Higgs-boson mass  $m_{H^{\pm}}$  for two values of tan  $\beta$  (1.5,6). The three squark scenarios with large mixing (1,2,3 in Table II) are compared with the case of decoupling squarks (solid lines).

#### 1. Unmixed sfermions

Figure 2 shows the variation of the hadronic cross section<sup>3</sup> with  $m_{H^{\pm}}$  for the squark scenarios without mixing (cases A, B, C; see Table I) and for two values of  $\tan \beta$  (1.5 and 6). The cross section is decreasing rapidly with increasing charged-Higgs boson mass, except for the region around  $m_{H^{\pm}} = 210$  GeV where a peak appears. The decrease of the cross section comes from the gluon luminosity, while the rise of the cross section in the peak area is due to the top-garkpair threshold in the quark-box diagrams, which gives rise to a sharp peak in the partonic cross section see [Fig. 1(b)] at  $\sqrt{\hat{s}} = 2m_t$  when the production threshold for the  $W^-H^+$  is near  $2m_t$ . The squark scenarios without mixing (cases A, B, C; see Table I) show generally an enhancement of the cross section with respect to the decoupling case of about 25-35 % over the depicted  $m_{H^{\pm}}$  range, except for the peak region between about 180 GeV and 250 GeV. Clearly, in this range the quark loop graphs dominate and the squark contribution corrects the quark-loop result only by a few percent.



FIG. 5. Hadronic cross section for  $H^+W^-$  production via gluon fusion versus tan  $\beta$  for three values of the charged-Higgs-boson mass  $m_{H^{\pm}}$  (100,360,1000 GeV). The squark case 1 (thick lines) is compared to the case of decoupling squarks (thin lines).

The tan  $\beta$  dependence of the hadronic cross section is shown in Fig. 3 for the non-mixing cases (cases A, B, C, Table I). The case in which all squarks decouple is also shown for comparison. One can see that the squark-loop contributions enhance the hadronic cross section for tan  $\beta$  values mainly below the point where the cross section takes its minimum, The enhancement is bigger for smaller  $m_{H^{\pm}}$ , e.g. 40% for squark case A at  $m_{H^{\pm}} = 100$  GeV and tan  $\beta = 5$ . For  $m_{H^{\pm}} = 100$  GeV and 300 GeV the enhancement is highest for the squark case A with the lowest squark mass scale, and lowest for the squark case C with the highest squark mass scale, while the situation is just the opposite at  $m_{H^{\pm}} = 1000$  GeV.

## 2. Maximal $\tilde{t}_L$ - $\tilde{t}_R$ mixing

In Fig. 4 the hadronic cross section for the squark scenarios with maximal  $\tilde{t}_L - \tilde{t}_R$  mixing (cases 1, 2, 3; see Table II) is shown as a function of  $m_{H^{\pm}}$  for tan  $\beta = 1.5$  and 6, comparing the three squark scenarios with the squark decoupling case. The general feature of all three scenarios is that there is a second peak besides the one due to quark-box diagrams. This second peak originates from the squark-pair threshold effects in the squark-loop diagrams. If the  $W^-H^+$ 

<sup>&</sup>lt;sup>3</sup>With the "hadronic cross section" we denote henceforth the cross section for  $W^-H^+$  production via gluon fusion and take care that no confusion arises when the hadronic cross section for production of  $W^-H^+$  via  $b\bar{b}$  annihilation is addressed.





FIG. 6. Hadronic cross section for  $H^+W^-$  production via gluon fusion versus tan  $\beta$  for three values of the charged-Higgs-boson mass  $m_{H^{\pm}}$  (100,410,1000 GeV). The squark case 2 (thick lines) is compared to the case of decoupling squarks (thin lines).

production threshold is somewhat below  $2m_{\tilde{b}_{1(2)}}$ , then the bottom-squark threshold of the box diagrams results in a pronounced peak in the parton cross section at  $\sqrt{\hat{s}}$  $= 2m_{\tilde{b}_{1(2)}}$ , which is even slightly higher than the peak due to quark loops [see Figs. 1(b) and 1(c)]. The magnitude of the peak can be traced back to strongly enhanced couplings of the charged Higgs to  $\tilde{t}_1\tilde{b}_1$  and  $\tilde{t}_2\tilde{b}_1$  involving the nondiagonal entries of the  $\tilde{t}$ -mass matrix (see Appendix C). It turns out that in the region of low tan  $\beta$  the contribution to the hadronic cross section from gluon fusion can be comparable and even slightly larger than the contribution from  $b\bar{b}$ annihilation in the MSSM. This important feature is also shown in Fig. 4, where the hadronic cross section for  $W^-H^+$ production via  $b\bar{b}$  annihilation for tan  $\beta = 1.5$  is also depicted for comparison.

The tan  $\beta$  dependence of the cross section in the largemixing cases is shown in Figs. 5–7. The squark cases with mixing are all rather similar in their behavior. Therefore we concentrate on case 3 in Fig. 7 as the most interesting example for a more explicit discussion. In Fig. 7, the hadronic cross section is displayed for three values of  $m_{H^{\pm}}$  (100,470, 1000 GeV) in case 3 (thick lines) and it is compared to the

FIG. 7. Hadronic cross section for  $H^+W^-$  production via gluon fusion versus tan  $\beta$  for three values of the charged-Higgs-boson mass  $m_{H^{\pm}}$  (100,470,1000 GeV). The squark case 3 (thick lines) is compared to the case of decoupling squarks (thin lines).

case of decoupling squarks (thin lines). From the logarithmic plot one can read off the fact that the enhancement due to squark effects in case 3 with respect to the squark decoupling case keeps almost the same relative size for  $\tan \beta$  values below the position of the minimum of the hadronic cross section. With further increasing  $\tan \beta$  the enhancement diminishes slowly. Taking the case of  $m_{H^{\pm}} = 470$  GeV as an example, the magnitude of enhancement ranges from a factor of about 3 in the range  $1 \le \tan \beta \le 10$  to 1.1 at  $\tan \beta = 50$ . In addition, there is a thin dot-dashed line in Fig. 7 showing the hadronic cross section which originates only from the  $b\bar{b}$ -annihilation subprocess for the most interesting  $m_{H^{\pm}}$ value of 470 GeV, where the contribution by gluon fusion to the inclusive hadron process gets as important as the one by  $b\bar{b}$  annihilation for tan  $\beta = 1.5$  (see Fig. 4). It turns out that this special feature in the large mixing cases is valid for small values of  $\tan \beta$ . The range of validity is limited from above roughly by the position of the minimum of the hadronic cross section originating only from  $b\bar{b}$  annihilation, which is at  $\tan \beta \approx 5$  in the case considered here. This is because for tan  $\beta$  greater than 5, the  $b\bar{b}$ -annihilation cross section is again rising, while the gluon fusion cross section has not approached its minimum yet.

The figures discussed in this paper are based on calculations assuming stable virtual particles; i.e., the finite widths of quarks and squarks have been neglected. This treatment is a reasonable approximation since the widths of the heavy quarks and squarks are small compared to their masses. The widths of the squarks depend on the mass spectrum of the gauginos, which determine the kinematically allowed squark-decay channels but otherwise do not enter our calculation. For example, the light bottom squark, which plays the crucial role in the threshold effects in the squark-box amplitudes, has a width of the order of 1 GeV, or much less if the decay channel  $\tilde{b}_1 \rightarrow t \chi^-$  is kinematically closed (see e.g. [22]).

For a quantitative statement, we have estimated the finitewidth effects for the squarks and the top quark in our calculation. Taking into account a finite squark width yields a slight reduction of the threshold peak in the hadronic cross section (Fig. 4) and no change outside of the peak. Specifically, we get a reduction of the peak value displayed in Fig. 4 of 1-3% in the various scenarios for squark widths of 0.1 GeV and of about 10% for 1 GeV, almost independent of tan  $\beta$ . The inclusion of the top quark width results in a reduction of 9% for tan  $\beta=1.5$  and 18% for tan  $\beta=6$  of the threshold peak located at  $m_{H^{\pm}}=210$  GeV in Fig. 4.

### **IV. CONCLUSIONS**

We have discussed the charged-Higgs-boson production process associated with a *W* boson at hadron colliders. The hadronic cross section from the subprocess  $gg \rightarrow W^-H^+$  has been calculated in the MSSM and the squark-loop contribu-

tion examined in comparison with the quark-loop effects for various MSSM parameters, which are chosen respecting bounds from the electroweak precision measurement and the squark direct search. We find that the squark (top squark and bottom squark) loop effects can be of about the same order as the *t*-*b* loop effects. In addition, for the maximum mixing between  $\tilde{t}_L$  and  $\tilde{t}_R$  with the mixing angle  $\theta_t \sim \pi/4$ , the hadronic cross section from  $gg \rightarrow W^{\pm}H^{\mp}$  is extensively enhanced by the threshold effects of  $\tilde{t}_1$  and  $\tilde{b}_{1,2}$ , where  $\tilde{t}_1$  is the lighter top squark. Therefore, the hadronic cross section via gluon fusion can reach the size of the cross section via the tree  $b\bar{b}$  annihilation subprocess for smaller tan  $\beta$ .

#### ACKNOWLEDGMENTS

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#### APPENDIX A: SQUARK MASSES AND MIXING

Squarks are introduced as the super-partners of quarks, so that there are three generations of isospin doublets and singlets corresponding to the quarks. The scalar partners of the L- and R-chiral quarks, in general, mix to form the mass eigenstates.

For the third generation of squarks, the mass-squared matrices in the L-R basis have the form

$$M_{\tilde{t}}^{2} = \begin{pmatrix} M_{\tilde{Q}}^{2} + m_{t}^{2} + m_{Z}^{2} \left(\frac{1}{2} - e_{t} s_{w}^{2}\right) \cos 2\beta & m_{t} X_{t} \\ m_{t} X_{t} & M_{\tilde{U}}^{2} + m_{t}^{2} + m_{Z}^{2} e_{t} s_{w}^{2} \cos 2\beta \end{pmatrix},$$
(A1)

and

$$M_{\tilde{b}}^{2} = \begin{pmatrix} M_{\tilde{Q}}^{2} + m_{b}^{2} + m_{Z}^{2} \left( -\frac{1}{2} - e_{b} s_{w}^{2} \right) \cos 2\beta & m_{b} X_{b} \\ m_{b} X_{b} & M_{\tilde{D}}^{2} + m_{b}^{2} + m_{Z}^{2} e_{b} s_{w}^{2} \cos 2\beta \end{pmatrix},$$
(A2)

where

$$X_t = A_t - \mu \cot \beta, \tag{A3}$$

$$X_b = A_b - \mu \tan \beta. \tag{A4}$$

For squarks of the first two generations, the mass matrices are obtained analogously. In Eqs. (A1) and (A2), the symbols  $e_t$  and  $e_b$  denote the electric charge of top and bottom quarks;  $\mu$  is the supersymmetric Higgs mass parameter,  $M_{\tilde{Q}}$  the soft-breaking mass parameter for the squark iso-doublet  $(\tilde{t}_L, \tilde{b}_L)$ , and  $M_{\tilde{U}}$  and  $M_{\tilde{D}}$  are the soft-breaking mass parameters for the iso-singlets  $\tilde{t}_R$  and  $\tilde{b}_R$ . They can be different for each generation, but for simplicity we will assume equal values for all generations in our numerical analysis.  $A_t$  and  $A_b$  are the parameters of the soft-breaking scalar three-point interactions of top and bottom squarks with the Higgs fields.

We restrict our analysis to real parameters, so that the mass matrices for sfermions ( $\tilde{f}_L$  and  $\tilde{f}_R$ ) are real and can be diagonalized by introducing the mixing angles  $\theta_{\tilde{f}}$ . The mass eigenstates  $\tilde{f}_1$  and  $\tilde{f}_2$  are obtained as

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{\tilde{f}} & -\sin \theta_{\tilde{f}} \\ \sin \theta_{\tilde{f}} & \cos \theta_{\tilde{f}} \end{pmatrix} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}.$$

The off-diagonal parts in the mass-squared matrices (A1),(A2) are proportional to the fermion masses. Therefore, the mixing effects are important mainly for third generation squarks.

### **APPENDIX B: HELICITY AMPLITUDES**

In the analytic expressions for the helicity amplitudes scalar 3- and 4-point functions appear for which the following conventions and shorthand notation are introduced:

$$C_0^{lm,ABC} = \frac{1}{i\pi^2} \int d^4k \frac{1}{[k^2 - m_A^2][(k+p_l)^2 - m_B^2][(k+p_l+p_m)^2 - m_C^2]},$$
  
$$D_0^{klm,ABCD} = \frac{1}{i\pi^2} \int d^4k \frac{1}{[k^2 - m_A^2][(k+p_k)^2 - m_B^2][(k+p_k+p_l)^2 - m_C^2][(k+p_k+p_l+p_m)^2 - m_D^2]}$$

Furthermore, in the box amplitudes tensor coefficients appear, where the upper indices denote momenta and masses in the same way as above. The lower index of the tensor coefficients gives the number in the tensor decomposition and corresponds precisely to the naming in [20] except for the fact that the Minkowski metric is used as implemented in the FORTRAN package AAFF [21], which has been used in the numerical evaluation. To match the definition of the loop integrals the external momenta are all chosen as incoming. Thus we have the following translation to the definition of Sec. II A:

$$p_1 = k, \quad p_2 = \overline{k}, \quad p_3 = -p, \quad p_4 = -\overline{p}.$$

In some *C* functions a sum of two momenta  $p_l + p_k$  appears as one of the momentum arguments, which is then denoted by (*lk*). The amplitude for  $gg \rightarrow W^-H^+$  is divided into contributions from quark and squark loops and further into triangle-type and box-type diagrams:

$$\mathcal{M}_{\sigma\bar{\sigma}\lambda} = \mathcal{M}_{\sigma\bar{\sigma}\lambda}^{q,\triangle} + \mathcal{M}_{\sigma\bar{\sigma}\lambda}^{q,\Box} + \mathcal{M}_{\sigma\bar{\sigma}\lambda}^{\bar{q},\triangle} + \mathcal{M}_{\sigma\bar{\sigma}\lambda}^{\bar{q},\Box} .$$
(B1)

Regarding the squark loops, we include the 2-point loops in the triangle contribution and the triangles without Higgs exchange in the box contribution.

#### 1. Quark contributions

Triangle contributions:

$$\mathcal{M}_{\sigma\bar{\sigma\lambda}}^{q,\Delta} = (K_{1\sigma\bar{\sigma\lambda}} + K_{2\sigma\bar{\sigma\lambda}})T_1^{q,\Delta} + (K_{10\sigma\bar{\sigma\lambda}} + K_{11\sigma\bar{\sigma\lambda}})T_2^{q,\Delta}$$
(B2)

$$\begin{split} T_{1}^{q,\triangle} &= \frac{ig_{S}^{2}}{\pi^{2}} \left( g [H^{0}H^{\pm}W^{\mp}] \left\{ \frac{m_{t}g_{s}[H^{0}tt]}{\hat{s} - m_{H}^{2}} \left[ \left( \frac{\hat{s}}{2} - 2m_{t}^{2} \right) C_{0}^{12,ttt} - 1 \right] + \frac{m_{b}g_{s}[H^{0}bb]}{\hat{s} - m_{H}^{2}} \left[ \left( \frac{\hat{s}}{2} - 2m_{b}^{2} \right) C_{0}^{12,bbb} - 1 \right] \right\} + g [h^{0}H^{\pm}W^{\mp}] \\ & \times \left\{ \frac{m_{t}g_{s}[h^{0}tt]}{\hat{s} - m_{h}^{2}} \left[ \left( \frac{\hat{s}}{2} - 2m_{t}^{2} \right) C_{0}^{12,ttt} - 1 \right] + \frac{m_{b}g_{s}[h^{0}bb]}{\hat{s} - m_{h}^{2}} \left[ \left( \frac{\hat{s}}{2} - 2m_{b}^{2} \right) C_{0}^{12,bbb} - 1 \right] \right\} \right) \end{split} \\ T_{2}^{q,\triangle} = \frac{g_{S}^{2}g [A^{0}H^{\pm}W^{\mp}]}{\pi^{2}(\hat{s} - m_{A}^{2})} (m_{t}g_{p}[A^{0}tt]C_{0}^{12,ttt} + m_{b}g_{p}[A^{0}bb]C_{0}^{12,bbb}). \end{split}$$

The universal helicity factors  $K_{i\sigma\sigma\lambda}$  are listed below in Appendix B 3. Box contributions:

$$\mathcal{M}_{\sigma\bar{\sigma}\lambda}^{q,\Box} = \sum_{i=1}^{24} K_{i\sigma\bar{\sigma}\lambda} T_i^{q,\Box}$$
(B3)

$$T_{9}^{q,\Box} = \frac{8382}{2\sqrt{2}\pi^{2}} \frac{1}{4} \left[ m_{t}g_{+}(-D_{11}^{124,tttb} + D_{11}^{214,tttb} + D_{12}^{124,tttb} - D_{12}^{214,tttb}) + m_{b}g_{-}(D_{11}^{123,bbbt} - D_{11}^{213,bbbt} - D_{13}^{123,bbbt} + D_{13}^{213,bbbt}) \right]$$

$$T_{10}^{q,\Box} = \frac{g_{S}^{2}g_{2}}{2\sqrt{2}\pi^{2}} \frac{1}{4} \left[ m_{t}g_{+}(R_{6} + D_{12}^{124,tttb} - D_{12}^{214,tttb} + D_{13}^{214,tttb} + D_{23}^{124,tttb} + D_{24}^{124,tttb} - D_{25}^{124,tttb} - D_{26}^{124,tttb}) + m_{b}g_{-}(R_{7} + R_{8}) \right]$$

$$T_{9}^{q,\Box} = \frac{g_{S}^{2}g_{2}}{2\sqrt{2}\pi^{2}} \frac{1}{4} \left[ m_{t}g_{+}(-D_{11}^{124,tttb} + D_{11}^{214,tttb} + D_{12}^{124,tttb} - D_{12}^{214,tttb}) + m_{b}g_{-}(D_{11}^{123,bbbt} - D_{11}^{213,bbbt} - D_{13}^{123,bbbt} + D_{13}^{213,bbbt}) \right]$$

$$\begin{split} R_{3}^{\pm} &= 8(D_{312}^{132,bbt} + D_{312}^{231,bbt} + D_{313}^{123,bbt} + D_{313}^{213,bbt} - D_{313}^{214,ttb} - D_{313}^{124,ttb} + D_{27}^{132,bbt}) - 2(2C_{12}^{2(13),bbt} - C_{12}^{24,ttb} + C_{0}^{24,ttb} + C_{11}^{132,bbt}) + 2(C_{0}^{23,ttb} + C_{12}^{2(14),ttb} + C_{11}^{32,btt}) \\ &+ C_{11}^{(13)2,btt}) \mp 2(C_{0}^{23,ttb} + C_{12}^{2(14),ttb} + C_{11}^{32,btt}) \\ T_{7}^{q,\Box} &= \frac{g_{S}^{2}g_{2}}{2\sqrt{2}\pi^{2}} \frac{1}{4} \Big[ m_{t}g_{+}(R_{4} + D_{13}^{132,bbtt}) + m_{b}g_{-}(R_{5} - D_{0}^{123,bbtt} + D_{0}^{213,bbt} - D_{11}^{123,bbtt} + D_{11}^{132,bbtt} + D_{11}^{213,bbtt}) \Big] \\ T_{8}^{q,\Box} &= \frac{g_{S}^{2}g_{2}}{2\sqrt{2}\pi^{2}} \frac{1}{4} \Big\{ m_{t}g_{+} \Big[ -R_{4}(p_{1} \leftrightarrow p_{2}) - D_{13}^{132,bbtt} \Big] + m_{b}g_{-} \Big[ -R_{5}(p_{1} \leftrightarrow p_{2}) - D_{11}^{23,bbtt} \Big] \Big\} \\ R_{4} &= -D_{0}^{214,ttb} - D_{11}^{214,ttb} - D_{12}^{23,bbtt} - D_{12}^{23,bbtt} - D_{13}^{123,bbtt} + D_{13}^{124,tttb} \\ R_{5} &= D_{0}^{132,bbtt} + D_{11}^{21,bbtt} - D_{12}^{23,bbtt} + D_{13}^{124,tttb} \Big] \end{split}$$

$$+D_{311}^{124,ttib} + D_{312}^{214,ttib} + D_{312}^{213,bbbt} + D_{313}^{231,bbtt} - D_{313}^{214,ttib} - D_{313}^{124,ttib} - 4(C_{11}^{(12)3,bbt} + C_{11}^{(12)4,ttb} - C_{12}^{(12)4,ttb})$$

$$T_{5}^{q,\Box} = \frac{ig_{5}^{2}g_{2}}{2\sqrt{2}\pi^{2}} \frac{1}{8} \{m_{t}g_{+}[\hat{s}(-D_{0}^{214,ttb} + D_{0}^{124,ttb} + D_{12}^{132,bbtt} + D_{13}^{123,bbtt} + D_{13}^{213,bbt}) - (2m_{W}^{2} - \hat{s} - 2\hat{t})(D_{13}^{124,ttb} + D_{13}^{214,ttb} + D_{13}^{214,ttb} + D_{13}^{214,ttb} + D_{13}^{213,bbtt} + D_{13}^{213,bbtt}) - (2m_{W}^{2} - \hat{s} - 2\hat{t})(D_{13}^{124,ttb} + D_{13}^{214,ttb} + D_{13}^{21$$

$$\begin{split} T_4^{q,\Box} &= T_3^{q,\Box}(p_1 \leftrightarrow p_2) \\ R_2 &= \hat{s}(-D_{11}^{124,tttb} - D_{11}^{123,bbbt} - D_{11}^{132,bbtt} - D_{12}^{213,bbbt} - D_{12}^{214,tttb} + D_{13}^{214,tttb} + D_{13}^{124,tttb} - D_{13}^{231,bbtt}) + 8(D_{311}^{123,bbbt} + D_{311}^{132,bbtt} + D_{311}^{132,bbtt} - D_{312}^{214,tttb} - D_{$$

$$= 3D_{23} + D_{23} - D_{23} - D_{23} - D_{23} - D_{33} - D_{33}^{214,ttb} - D_{33}^{214,ttb} + D_{36}^{123,bbt} + D_{37}^{123,bbt} + D_{37}^{231,bbt} + D_{38}^{231,bbt} + D_{39}^{213,bbt} + D_{39}^{214,ttb} - D_{33}^{214,ttb} - D_{33}^{214,ttb} + D_{36}^{123,bbt} + D_{37}^{123,bbt} + D_{38}^{231,bbt} + D_{39}^{213,bbt} + D_{39}^{214,ttb} - D_{33}^{214,ttb} - D_{33}^{214,ttb} + D_{36}^{231,bbt} + D_{37}^{231,bbt} - D_{39}^{214,ttb} + D_{27}^{231,bbt} - D_{27}^{214,ttb} - D_{0}^{123,bbt} \hat{s} + (\hat{t} - m_{H^{\pm}}^{2})(D_{0}^{132,bbt} - D_{0}^{124,ttb}) + 4(D_{27}^{132,bbt} + D_{27}^{213,bbt} - D_{27}^{214,ttb} - D_{0}^{214,ttb}) + 2(C_{0}^{34,bt} - C_{0}^{34,btb}) + R_{2}] + m_{b}g_{-}[(m_{H^{\pm}}^{2} - 2\hat{s} - \hat{t})D_{0}^{123,bbt} - D_{0}^{132,bbtt} \hat{s} - (-\hat{t} + \hat{s} + m_{H^{\pm}}^{2})D_{0}^{213,bbt} - D_{0}^{124,ttb} \hat{s} - (-\hat{u} + m_{W}^{2})D_{0}^{231,bbtt} + 4(3D_{27}^{132,bbtt} + 3D_{27}^{213,bbtt} + D_{27}^{214,ttb} + D_{27}^{231,bbtt} - D_{0}^{124,tttb} \hat{s} - (-\hat{u} + m_{W}^{2})D_{0}^{231,bbtt} + 4(3D_{27}^{132,bbtt} + 3D_{27}^{213,bbtt} + D_{27}^{214,ttb} + D_{27}^{231,bbtt} + D_$$

$$T_{1}^{q,\Box} = \frac{ig_{3}^{2}g_{2}}{4\sqrt{2}\pi^{2}} [m_{t}g_{+}(R_{1}+D_{12}^{132,bbtt}+D_{22}^{132,bbtt}-D_{22}^{231,bbtt}-D_{13}^{124,ttb}-D_{13}^{214,ttb}+D_{23}^{123,bbtt}-D_{23}^{124,ttb}+D_{23}^{213,bbtt}-D_{23}^{214,ttb}+D_{23}^{214,ttb}+D_{23}^{213,bbtt}+D_{23}^{213,bbtt}+D_{23}^{214,ttb}+D_{23}^{214,ttb}+D_{23}^{214,ttb}+D_{23}^{214,ttb}+D_{23}^{214,ttb}+D_{23}^{214,ttb}+D_{23}^{214,ttb}+D_{23}^{214,ttb}+D_{23}^{214,ttb}+D_{23}^{213,bbtt}+D_{23}^{213,bbtt}+D_{23}^{213,bbtt}+D_{23}^{213,bbtt}+D_{23}^{213,bbtt}+D_{23}^{214,ttb}+D_{23}^{214,tt$$

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$$\begin{split} T_{11}^{*} &= \frac{g_{1}^{2}g_{2}}{2\sqrt{2}\pi^{2}} \frac{1}{4} \{m_{18} - [R_{6}(p_{1} \rightarrow p_{2}) + p_{11}^{21,100} + p_{12}^{21,100} + p_{12}^{21,100} + p_{21}^{21,100} + p_{21}^{21,1$$

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$$\begin{split} T_{16}^{*} & = \frac{g_{3}^{*} g_{2}}{2\sqrt{2} \pi^{2}} \frac{1}{8} \{ m_{28}^{*} + (2m_{12}^{*} (D_{11}^{*1,1} + + b) - D_{11}^{*1,1} + b) - m_{pm} (D_{12}^{*12,0,bet} + D_{01}^{*12,0,bet} + D_{11}^{*12,0,bet} + D_{11}^{*12,0,be$$

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$$\begin{split} T^{q,\square}_{24} = & \frac{g_{S}^{2}g_{2}}{2\sqrt{2}\pi^{2}} \frac{1}{8} \{ m_{t}g_{+} [2m_{b}^{2}D_{0}^{132,bbtt} - 2m_{t}^{2}D_{0}^{214,tttb} + (m_{b}^{2} - m_{t}^{2} + m_{H^{\pm}}^{2})D_{12}^{132,bbtt} + (\hat{u} - m_{H^{\pm}}^{2})D_{12}^{231,bbtt} \\ & + (m_{b}^{2} - m_{t}^{2} - m_{H^{\pm}}^{2})D_{13}^{214,tttb} - m_{t}m_{b}D_{13}^{123,bbtt} - (C_{0}^{34,tbt} - 2C_{0}^{24,ttb} + C_{12}^{(12)4,ttb} + C_{12}^{2(13),bbt} - 6D_{27}^{214,tttb} + 6D_{27}^{132,bbtt} \\ & + R_{11})] + m_{b}g_{-} [2m_{b}^{2}D_{0}^{123,bbt} - (\hat{u} - m_{H^{\pm}}^{2})(D_{0}^{132,bbtt} + D_{12}^{132,bbtt}) + (\hat{t} - m_{H^{\pm}}^{2})(D_{0}^{231,bbtt} + D_{12}^{231,bbtt}) + (\hat{u} + m_{b}^{2})D_{13}^{123,bbtt} \\ & - (C_{11}^{(23)1,btt} - C_{0}^{34,btb} - C_{0}^{13,ttb} - C_{11}^{31,ttt} + C_{0}^{23,ttb} - C_{0}^{24,ttb} + C_{11}^{32,bbtt} - C_{0}^{24,btt} + 6D_{27}^{123,bbtt} + R_{11})] \\ & + 2m_{t}^{2}m_{b}g_{p}[H^{+} \text{ out, t in, b out}]D_{13}^{123,bbtt} \} \end{split}$$

$$R_{11} = C_0^{14,ttb} - C_{11}^{(13)2,btt} + C_{12}^{14,ttb} - C_{12}^{24,ttb}.$$

Here the shorthand

$$g_{\pm} = g_s [H^+ \text{ out}, t \text{ in}, b \text{ out}] \pm g_p [H^+ \text{ out}, t \text{ in}, b \text{ out}]$$

has been used.

## 2. Squark contributions

Triangle contributions:

$$\mathcal{M}_{\sigma\bar{\sigma}\lambda}^{\tilde{q},\Delta} = (K_{1\sigma\bar{\sigma}\lambda} + K_{2\sigma\bar{\sigma}\lambda})T_{1}^{\tilde{q},\Delta}$$
(B4)  
$$T_{1}^{\tilde{q},\Delta} = \sum_{i=1}^{2} \frac{ig_{s}^{2}}{4\pi^{2}} \bigg[ \frac{g[H^{0}H^{\pm}W^{\mp}]}{\hat{s} - m_{H}^{2}} [g[H^{0}\tilde{t}_{i}\tilde{t}_{i}](1 + 2m_{\tilde{t}_{i}}^{2}C_{0}^{12,\tilde{t}_{i}\tilde{t}_{i}\tilde{t}_{i}}) + g[H^{0}\tilde{b}_{i}\tilde{b}_{i}](1 + 2m_{\tilde{b}_{i}}^{2}C_{0}^{12,\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}})]$$
$$+ \frac{g[h^{0}H^{\pm}W^{\mp}]}{\hat{s} - m_{h}^{2}} [g[h^{0}\tilde{t}_{i}\tilde{t}_{i}](1 + 2m_{\tilde{t}_{i}}^{2}C_{0}^{12,\tilde{t}_{i}\tilde{t}_{i}\tilde{t}_{i}}) + g[h^{0}\tilde{b}_{i}\tilde{b}_{i}](1 + 2m_{\tilde{b}_{i}}^{2}C_{0}^{12,\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}})]\bigg].$$

Box contributions:

$$\mathcal{M}_{\sigma\bar{\sigma}\lambda}^{\tilde{q},\Box} = \sum_{i=1}^{6} K_{i\sigma\bar{\sigma}\lambda} T_{i}^{\tilde{q},\Box}$$

$$T_{1}^{\tilde{q},\Box} = \sum_{i,j=1}^{2} \frac{ig_{s}^{2}}{2\pi^{2}} (D_{12}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{t}_{j}\tilde{t}_{j}} - D_{12}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}\tilde{b}_{i}} + D_{22}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{t}_{j}\tilde{t}_{j}} - D_{22}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}\tilde{b}_{i}} + D_{24}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{t}_{j}\tilde{t}_{j}} - D_{24}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{t}_{j}} - D_{36}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}\tilde{b}_{i}} + D_{36}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}\tilde{b}_{i}} + D_{36}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{t}_{j}\tilde{t}_{j}} - D_{36}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}\tilde{b}_{i}} + D_{36}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} + D_{37}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} - D_{37}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} - D_{37}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} + D_{37}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} + D_{37}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} - D_{37}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} + D_{37}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} + D_{37}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} + D_{$$

$$T_{2}^{\tilde{q},\Box} = \sum_{i,j=1}^{2} \frac{ig_{S}^{2}}{2\pi^{2}} (D_{26}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{t}_{j}\tilde{t}_{j}} - D_{26}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}\tilde{b}_{i}\tilde{t}_{j}} - D_{38}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}} + D_{38}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}} - D_{38}^{123,\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}} - D_{33}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} - D_{33}^{132,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} - D_{33}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} - D_{33}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} - D_{33}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{b}_{i}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}} - D_{39}^{124,\tilde{t}_{j}\tilde{t}_{j}\tilde{t}_{j}\tilde$$

$$\begin{split} T_{3}^{\tilde{q},\Box} &= \sum_{i,j=1}^{1} -\frac{lg_{S}}{4\pi^{2}} (-2D_{27}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{i}_{j}\tilde{i}_{j}} + 2D_{27}^{132,\tilde{c}_{j}\tilde{i}_{j}\tilde{b}_{i}\tilde{b}_{i}} - 2D_{311}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{i}_{j}\tilde{i}_{j}} + 2D_{311}^{132,\tilde{c}_{j}\tilde{i}_{j}\tilde{b}_{i}\tilde{i}_{j}} + 2D_{311}^{132,\tilde{c}_{j}\tilde{i}_{j}\tilde{b}_{i}\tilde{b}_{i}} + 2D_{311}^{132,\tilde{c}_{j}\tilde{i}_{j}\tilde{b}_{i}\tilde{b}_{i}} + 2D_{311}^{132,\tilde{c}_{j}\tilde{i}_{j}\tilde{i}_{j}\tilde{b}_{i}} + 2D_{311}^{132,\tilde{c}_{j}\tilde{i}_{j}\tilde{i}_{j}\tilde{b}_{i}} + 2D_{311}^{132,\tilde{c}_{j}\tilde{i}_{j}\tilde{i}_{j}\tilde{b}_{i}} + 2D_{311}^{132,\tilde{c}_{j}\tilde{i}_{j}\tilde{i}_{j}\tilde{b}_{i}} + 2D_{311}^{132,\tilde{c}_{j}\tilde{i}_{j}\tilde{i}_{j}\tilde{b}_{i}} + 2D_{311}^{123,\tilde{c}_{j}\tilde{i}_{j}\tilde{i}_{j}\tilde{b}_{i}} - 2D_{311}^{124,\tilde{c}_{j}\tilde{i}_{j}\tilde{b}_{i}} - 2D_{311}^{124,\tilde{c}_{j}\tilde{i}_{j}\tilde{i}_{j}\tilde{b}_{i}} -$$

$$\begin{split} T_{4}^{\tilde{q},\square} &= \sum_{i,j=1}^{2} -\frac{ig_{S}^{2}}{4\pi^{2}} (-2D_{313}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{i}_{j}\tilde{j}_{j}} + 2D_{313}^{132,\tilde{c}_{i}\tilde{j}_{j}\tilde{b}_{i}\tilde{b}_{i}} + C_{0}^{34,\tilde{b}_{i}\tilde{i}_{j}\tilde{b}_{i}} - C_{0}^{34,\tilde{c}_{j}\tilde{b}_{i}\tilde{i}_{j}} + C_{11}^{(12)3,\tilde{b}_{i}\tilde{b}_{i}\tilde{i}_{j}} - C_{11}^{(12)3,\tilde{i}_{j}\tilde{i}_{j}\tilde{b}_{i}} - 2D_{27}^{123,\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}\tilde{i}_{j}} \\ &+ 2D_{27}^{123,\tilde{i}_{j}\tilde{i}_{j}\tilde{i}_{j}\tilde{b}_{i}} - 2D_{312}^{123,\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}\tilde{i}_{j}} + 2D_{312}^{123,\tilde{i}_{j}\tilde{i}_{j}\tilde{i}_{j}\tilde{b}_{i}} + 2D_{312}^{124,\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}} - 2D_{312}^{124,\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}} - 2D_{313}^{124,\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}\tilde{i}_{j}} \\ &+ 2D_{313}^{124,\tilde{i}_{j}\tilde{i}_{j}\tilde{b}_{i}})g[\tilde{t}_{j}\tilde{b}_{i}W^{\pm}]g[H^{\pm}\tilde{t}_{j}\tilde{b}_{i}] \\ T_{5}^{\tilde{q},\square} &= \sum_{i,j=1}^{2} \frac{ig_{S}^{2}}{4\pi^{2}} (C_{0}^{24,\tilde{b}_{i}\tilde{b}_{i}\tilde{i}_{j}} - C_{0}^{24,\tilde{i}_{j}\tilde{b}_{i}\tilde{b}_{i}} - C_{11}^{(13)2,\tilde{b}_{i}\tilde{b}_{i}\tilde{i}_{j}} + 2D_{313}^{124,\tilde{i}_{j}\tilde{b}_{i}\tilde{b}_{i}} + 2D_{27}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{i}_{j}\tilde{i}_{j}} - 2D_{312}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{i}_{j}} + 2D_{312}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{i}_{j}} - 2D_{312}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}} + 2D_{313}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}} - 2D_{312}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}} - 2D_{313}^{132,\tilde{b}_{i}\tilde{b}_{i}\tilde{b}_{i}} - 2D_{313}^{132,\tilde{b}_{i}\tilde{$$

## 3. Helicity factors

The factors  $K_{i\sigma\sigma\lambda}$  contain all the helicity information of the amplitude. They are obtained by contracting kinematical tensors with the helicity four-vectors defined in Sec. II A. In the following these factors are listed. The symbol  $\lambda_T$  denotes a transverse polarization of the *W* boson, i.e.  $\lambda_T = \pm 1$ :

$$\begin{split} & K_{1,\sigma\bar{\sigma}0} = K_{2,\sigma\bar{\sigma}0} = \frac{[4\hat{s}|\tilde{p}_{W}|^{2} - (\hat{t} - \hat{u})^{2}][2|\tilde{p}_{W}|^{2}\sqrt{\hat{s}} + E_{W}(\hat{u} - \hat{t})]}{32(\hat{s}|\tilde{p}_{W}|m_{W})}, \\ & K_{3,\sigma\bar{\sigma}0} = \frac{1}{8m_{W}} \left( -2|\tilde{p}_{W}|\sqrt{\hat{s}} + \frac{E_{W}(\hat{t} - \hat{u})}{|\tilde{p}_{W}|} \right) (\sigma\bar{\sigma} + 1), \\ & K_{4,\sigma\bar{\sigma}0} = K_{3,\sigma\bar{\sigma}0}(\hat{t} \leftrightarrow \hat{u}), \\ & K_{5,\sigma\bar{\sigma}0} = K_{6,\sigma\bar{\sigma}0} = \frac{E_{W}}{m_{W}} \left( \frac{1}{8} \frac{(\hat{t} - \hat{u})^{2}}{\hat{s}|\tilde{p}_{W}|} - \frac{1}{2} |\tilde{p}_{W}| \right), \\ & K_{7,\sigma\bar{\sigma}0} = -i(\hat{s}/2)\sigma K_{5,\sigma\bar{\sigma}0}, \\ & K_{8,\sigma\bar{\sigma}0} = i(\hat{s}/2)\bar{\sigma}K_{5,\sigma\bar{\sigma}0}, \\ & K_{9,\sigma\bar{\sigma}0} = 0, \\ & K_{10,\sigma\bar{\sigma}0} = \frac{i}{16m_{W}} \left( \frac{E_{W}(\hat{t} - \hat{u})}{|\tilde{p}_{W}|} - 2\sqrt{\hat{s}}|\tilde{p}_{W}| \right) \hat{s}(\sigma + \bar{\sigma}), \\ & K_{11,\sigma\bar{\sigma}0} = K_{10,\sigma\bar{\sigma}0}(\hat{t} \leftrightarrow \hat{u}), \\ & K_{12,\sigma\bar{\sigma}0} = K_{13,\sigma\bar{\sigma}0} = -K_{8,\sigma\bar{\sigma}0}, \\ & K_{14,\sigma\bar{\sigma}0} = \frac{i[2|\tilde{p}_{W}|^{2}\sqrt{\hat{s}} - E_{W}(\hat{t} - \hat{u})](2\sqrt{\hat{s}}E_{W} - \hat{t} + \hat{u})}{32|\tilde{p}_{W}|m_{W}}} (\sigma + \bar{\sigma}), \\ & K_{15,\sigma\bar{\sigma}0} = \frac{i[2|\tilde{p}_{W}|^{2}\sqrt{\hat{s}} + E_{W}(\hat{t} - \hat{u})](2\sqrt{\hat{s}}E_{W} - \hat{t} + \hat{u})}{32|\tilde{p}_{W}|m_{W}} (\sigma + \bar{\sigma}), \end{split}$$

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$$\begin{split} & K_{16,\sigma\bar{\sigma}0} = K_{7,\sigma\bar{\sigma}0} m_W^2 / (E_W \sqrt{\hat{s}}), \quad K_{17,\sigma\bar{\sigma}0} = K_{8,\sigma\bar{\sigma}0} m_W^2 / (E_W \sqrt{\hat{s}}), \\ & K_{18,\sigma\bar{\sigma}0} = (2/\hat{s}) K_{10,\sigma\bar{\sigma}0}, \quad K_{19,\sigma\bar{\sigma}0} = -K_{15,\sigma\bar{\sigma}0} (\hat{t} \leftrightarrow \hat{u}), \\ & K_{20,\sigma\bar{\sigma}0} = -K_{14,\sigma\bar{\sigma}0} (\hat{t} \leftrightarrow \hat{u}), \quad K_{21,\sigma\bar{\sigma}0} = -K_{16,\sigma\bar{\sigma}0}, \\ & K_{22,\sigma\bar{\sigma}0} = -K_{17,\sigma\bar{\sigma}0}, \quad K_{23,\sigma\bar{\sigma}0} = -K_{18,\sigma\bar{\sigma}0} (\hat{t} \leftrightarrow \hat{u}), \\ & K_{24,\sigma\bar{\sigma}0} = -\frac{im_W (\hat{t} - \hat{u})}{4 |\tilde{p}_W| \sqrt{\hat{s}}} (\sigma + \bar{\sigma}). \end{split}$$

In the following an additional shorthand is used:

$$\begin{split} p_t^2 &= \frac{\hat{t}\hat{u} - m_W^2 m_{H^{\pm}}^2}{\hat{s}}, \\ K_{1,\sigma\bar{\sigma}\lambda_T} &= -K_{2,\sigma\bar{\sigma}\lambda_T} = \frac{i}{32} \left( 4\sqrt{\hat{s}} |\vec{p}_W| - \frac{(\hat{t}-\hat{u})^2}{|\vec{p}_W|\sqrt{\hat{s}}} \right) p_t \sqrt{2}\lambda_T, \\ K_{3,\sigma\bar{\sigma}\lambda_T} &= -K_{4,\sigma\bar{\sigma}\lambda_T} = -\frac{i}{8} \frac{\sqrt{2}p_t \sqrt{\hat{s}}(\sigma\bar{\sigma}+1)\lambda_T}{|\vec{p}_W|}, \\ K_{5,\sigma\bar{\sigma}\lambda_T} &= -\frac{i}{8} \left( \frac{(\hat{t}-\hat{u})\lambda_T}{\sqrt{\hat{s}}|\vec{p}_W|} + 2\sigma \right) p_t \sqrt{2}, \\ K_{6,\sigma\bar{\sigma}\lambda_T} &= K_{5,\sigma\bar{\sigma}\lambda_T}(\sigma \rightarrow -\bar{\sigma}), \\ K_{7,\sigma\bar{\sigma}\lambda_T} &= \frac{1}{2} \left( \frac{1}{4}\hat{s}\sigma\bar{\sigma} - \frac{1}{8} \frac{\sqrt{\hat{s}}(\hat{t}-\hat{u})\sigma\lambda_T}{|\vec{p}_W|} \right) p_t \sqrt{2}, \\ K_{8,\sigma\bar{\sigma}\lambda_T} &= K_{7,\sigma\bar{\sigma}\lambda_T}(\sigma \rightarrow -\bar{\sigma}, \bar{\sigma} \rightarrow -\sigma), \\ K_{9,\sigma\bar{\sigma}\lambda_T} &= K_{7,\sigma\bar{\sigma}\lambda_T}(\sigma \rightarrow -\bar{\sigma}, \bar{\sigma} \rightarrow -\sigma), \\ K_{10,\sigma\bar{\sigma}\lambda_T} &= -K_{11,\sigma\bar{\sigma}\lambda_T} = i(\hat{s}/2)K_{3,\sigma\bar{\sigma}\lambda_T}, \\ K_{12,\sigma\bar{\sigma}\lambda_T} &= i(\hat{s}/2)\sigma K_{5,\sigma\bar{\sigma}\lambda_T}, \\ K_{13,\sigma\bar{\sigma}\lambda_T} &= -K_{15,\sigma\bar{\sigma}\lambda_T}(\sigma \rightarrow -\bar{\sigma}), \\ K_{14,\sigma\bar{\sigma}\lambda_T} &= -K_{15,\sigma\bar{\sigma}\lambda_T} = -\frac{1}{32} \frac{(2\sqrt{\hat{s}}E_W - \hat{t} + \hat{u})\sqrt{\hat{s}}p_t \sqrt{2}(\sigma + \bar{\sigma})\lambda_T}{|\vec{p}_W|}, \\ K_{16,\sigma\bar{\sigma}\lambda_T} &= \frac{1}{16} \bigg[ \bigg( -\frac{E_W(\hat{t}-\hat{u})}{|\vec{p}_W|} + 2|\vec{p}_W|\sqrt{\hat{s}} \bigg) \sigma\lambda_T + (\hat{t}-\hat{u}) - 2\sqrt{\hat{s}}E_W \bigg] p_t \sqrt{2}, \\ K_{17,\sigma\bar{\sigma}\lambda_T} &= K_{16,\sigma\bar{\sigma}\lambda_T}(\sigma \rightarrow -\bar{\sigma}), \end{split}$$

$$\begin{split} K_{18,\sigma\bar{\sigma}\lambda_{T}} &= K_{23,\sigma\bar{\sigma}\lambda_{T}} = \frac{1}{8} \frac{p_{t}\sqrt{\hat{s}\sqrt{2}(\sigma+\bar{\sigma})\lambda_{T}}}{|\tilde{p}_{W}|}, \\ K_{19,\sigma\bar{\sigma}\lambda_{T}} &= -K_{20,\sigma\bar{\sigma}\lambda_{T}} = K_{15,\sigma\bar{\sigma}\lambda_{T}}(\hat{t}\leftrightarrow\hat{u}), \\ K_{21,\sigma\bar{\sigma}\lambda_{T}} &= \frac{1}{16} \bigg[ \left( \frac{E_{W}(\hat{t}-\hat{u})}{|\tilde{p}_{W}|} + 2|\tilde{p}_{W}|\sqrt{\hat{s}} \right) \sigma\lambda_{T} + (\hat{t}-\hat{u}) + 2\sqrt{\hat{s}}E_{W} \bigg] p_{t}\sqrt{2}, \\ K_{22,\sigma\bar{\sigma}\lambda_{T}} &= K_{21,\sigma\bar{\sigma}\lambda_{T}}(\sigma\to-\bar{\sigma}), \\ K_{24,\sigma\bar{\sigma}\lambda_{T}} &= -\sqrt{2}(E_{W}/\sqrt{\hat{s}})K_{18,\sigma\bar{\sigma}\lambda_{T}}. \end{split}$$

### APPENDIX C: MSSM COUPLINGS

In the following all couplings of the third generation quarks and squarks to MSSM Higgs particles, which are relevant to the process, are collected. The factor  $g_2 = e/s_w$  denotes the SU(2) coupling constant of the weak interaction,  $s_w = \sin \theta_w$ ,  $c_w = \cos \theta_w$  and  $t_w = \tan \theta_w$  with the weak mixing angle  $\theta_w$ . The scalar and pseudoscalar couplings in the Higgs-boson–fermion interactions are distinguished by adding subscripts *s* and *p* to the coupling symbols. Analogously, the subscripts *v* and *a* distinguish the vector and axial vector coupling of quarks to the *W* boson.

#### 1. Quark couplings to Higgs bosons

Neutral Higgs bosons:

$$g_s[H^0tt] = -g_2 \frac{m_t}{2m_W} \frac{\sin \alpha}{\sin \beta}, \quad g_s[H^0bb] = -g_2 \frac{m_b}{2m_W} \frac{\cos \alpha}{\cos \beta}, \tag{C1}$$

$$g_s[h^0 tt] = -g_2 \frac{m_t}{2m_W} \frac{\cos \alpha}{\sin \beta}, \quad g_s[h^0 bb] = +g_2 \frac{m_b}{2m_W} \frac{\sin \alpha}{\cos \beta}, \tag{C2}$$

$$g_p[H^0tt] = g_p[h^0tt] = 0, \quad g_p[H^0bb] = g_p[h^0bb] = 0,$$
 (C3)

$$g_p[A^0tt] = ig_2 \quad \frac{m_t}{2m_W} \cot\beta, \quad g_p[A^0bb] = ig_2 \quad \frac{m_b}{2m_W} \tan\beta, \tag{C4}$$

$$g_s[A^0tt] = g_s[A^0bb] = 0.$$
(C5)

Charged Higgs bosons:

$$g_{s}[H^{+} \text{ out}, t \text{ in}, b \text{ out}] = g_{2} \frac{m_{b} \tan \beta + m_{t} \cot \beta}{2\sqrt{2}m_{W}} = g_{s}[H^{-} \text{ out}, b \text{ in}, t \text{ out}],$$
(C6)

$$g_p[H^+ \text{ out}, t \text{ in}, b \text{ out}] = -g_2 \frac{m_b \tan \beta - m_t \cot \beta}{2\sqrt{2}m_W} = -g_p[H^- \text{ out}, b \text{ in}, t \text{ out}].$$
(C7)

### 2. Squark couplings to Higgs bosons

Neutral Higgs bosons:

$$g[H^{0}\tilde{t}_{1}\tilde{t}_{1}] = g_{2} \left[ \frac{m_{t}}{2m_{w}\sin\beta} \left[ (A_{t}\sin\alpha - \mu\cos\alpha)\sin2\theta_{\tilde{t}} - 2m_{t}\sin\alpha \right] + \frac{m_{Z}\cos(\alpha + \beta)}{6c_{w}} \left[ (5 - 8c_{w}^{2})\cos^{2}\theta_{\tilde{t}} - 4s_{w}^{2} \right] \right], \quad (C8)$$

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$$g[H^{0}\tilde{t}_{2}\tilde{t}_{2}] = g_{2} \left[ \frac{m_{t}}{2m_{W}\sin\beta} \left[ -(A_{t}\sin\alpha - \mu\cos\alpha)\sin 2\theta_{\tilde{t}} - 2m_{t}\sin\alpha \right] + \frac{m_{Z}\cos(\alpha + \beta)}{6c_{w}} \left[ -(5 - 8c_{w}^{2})\cos^{2}\theta_{\tilde{t}} + (1 - 4s_{w}^{2}) \right] \right],$$
(C9)

$$g[H^0\tilde{b}_1\tilde{b}_1] = g_2 \bigg[ \frac{m_b}{2m_w \cos\beta} [(A_b\cos\alpha - \mu\sin\alpha)\sin2\theta_{\tilde{b}} - 2m_b\cos\alpha] + \frac{m_Z\cos(\alpha + \beta)}{6c_w} [(4c_w^2 - 1)\cos^2\theta_{\tilde{b}} + 2s_w^2] \bigg],$$
(C10)

$$g[H^{0}\tilde{b}_{2}\tilde{b}_{2}] = g_{2} \left[ \frac{m_{b}}{2m_{W}\cos\beta} [-(A_{b}\cos\alpha - \mu\sin\alpha)\sin 2\theta_{\tilde{b}} - 2m_{b}\cos\alpha] + \frac{m_{Z}\cos(\alpha + \beta)}{6c_{W}} [-(4c_{W}^{2} - 1)\cos^{2}\theta_{\tilde{b}} + (1 + 2c_{W}^{2})] \right],$$
(C11)

$$g[h^{0}\tilde{q}_{i}\tilde{q}_{i}] = g[H^{0},\tilde{q}_{i},\tilde{q}_{i}] \quad (\sin\alpha \to \cos\alpha, \ \cos\alpha \to -\sin\alpha). \tag{C12}$$

Charged Higgs bosons:

$$g[H^{\pm}\tilde{t}_{1}\tilde{b}_{1}] = \frac{g_{2}}{2\sqrt{2}m_{W}} \{+2m_{t}m_{b}(\tan\beta + \cot\beta) \quad \sin\theta_{\tilde{b}}\sin\theta_{\tilde{t}}\sin2\beta[m_{b}^{2}(1+\tan^{2}\beta) + m_{t}^{2}(1+\cot^{2}\beta) - 2m_{W}^{2}] \\ \times \cos\theta_{\tilde{b}}\cos\theta_{\tilde{t}} - 2m_{b}(\mu + A_{b}\tan\beta) \quad \sin\theta_{\tilde{b}}\cos\theta_{\tilde{t}} - 2m_{t}(\mu + A_{t}\cot\beta) \quad \cos\theta_{\tilde{b}}\sin\theta_{\tilde{t}}\},$$
(C13)

$$g[H^{\pm}\tilde{t}_{1}\tilde{b}_{2}] = \frac{g_{2}}{2\sqrt{2}m_{W}} \{-2m_{t}m_{b}(\tan\beta + \cot\beta) \cos\theta_{\tilde{b}}\sin\theta_{\tilde{t}} + \sin 2\beta[m_{b}^{2}(1 + \tan^{2}\beta) + m_{t}^{2}(1 + \cot^{2}\beta) - 2m_{W}^{2}] \sin\theta_{\tilde{b}}\cos\theta_{\tilde{t}} + 2m_{b}(\mu + A_{b}\tan\beta) \cos\theta_{\tilde{b}}\cos\theta_{\tilde{t}} - 2m_{t}(\mu + A_{t}\cot\beta) \sin\theta_{\tilde{b}}\sin\theta_{\tilde{t}}\},$$
(C14)

$$g[H^{\pm}\tilde{t}_{2}\tilde{b}_{1}] = \frac{g_{2}}{2\sqrt{2}m_{W}} \{-2m_{t}m_{b}(\tan\beta + \cot\beta) \quad \sin\theta_{\tilde{b}}\cos\theta_{\tilde{t}} + \sin 2\beta[m_{b}^{2}(1 + \tan^{2}\beta) + m_{t}^{2}(1 + \cot^{2}\beta) - 2m_{W}^{2}] \cos\theta_{\tilde{b}}\sin\theta_{\tilde{t}} + 2m_{b}(\mu + A_{b}\tan\beta) \quad \sin\theta_{\tilde{b}}\sin\theta_{\tilde{t}} 2m_{t}(\mu + A_{t}\cot\beta) \quad \cos\theta_{\tilde{b}}\cos\theta_{\tilde{t}}\},$$
(C15)

$$g[H^{\pm}\tilde{t}_{2}\tilde{b}_{2}] = \frac{g_{2}}{2\sqrt{2}m_{W}} \{2m_{t}m_{b}(\tan\beta + \cot\beta) \cos\theta_{\tilde{b}}\cos\theta_{\tilde{t}} + \sin 2\beta[m_{b}^{2}(1 + \tan^{2}\beta) + m_{t}^{2}(1 + \cot^{2}\beta) - 2m_{W}^{2}] \\ \times \sin\theta_{\tilde{b}}\sin\theta_{\tilde{t}} + 2m_{b}(\mu + A_{b}\tan\beta) \cos\theta_{\tilde{b}}\sin\theta_{\tilde{t}} + 2m_{t}(\mu + A_{t}\cot\beta) \sin\theta_{\tilde{b}}\cos\theta_{\tilde{t}}\}.$$
(C16)

## 3. Couplings to gauge bosons

*Quarks and squarks:* The indices  $\alpha, \beta, a, b$  are color indices; *i* and *j* denote squark mass eigenstates:

$$g_v[q^{\alpha}q^{\beta}g^a] = g_s \frac{\lambda_{\alpha\beta}^a}{2}, \quad g_a[qqg] = 0, \tag{C17}$$

$$g[\tilde{q}^{\alpha}\tilde{q}^{\beta}g^{a}] = -g_{S}\frac{\lambda^{a}_{\alpha\beta}}{2},$$
(C18)

$$g[\tilde{q}^{\alpha}\tilde{q}^{\beta}g^{a}g^{b}] = g_{S}^{2} \left\{ \frac{\lambda^{a}}{2}, \frac{\lambda^{b}}{2} \right\}_{\alpha\beta}, \tag{C19}$$

$$g_{v}[t^{\beta}b^{\alpha}W^{\pm}] = \frac{g_{2}}{2\sqrt{2}}\delta_{\alpha\beta}, \quad g_{a}[t^{\beta}b^{\alpha}W^{\pm}] = \frac{g_{2}}{2\sqrt{2}}\delta_{\alpha\beta}, \tag{C20}$$

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$$g[\tilde{t}_i \tilde{b}_j W^{\pm}] = -\frac{g_2}{\sqrt{2}} R_{ij}, \qquad (C21)$$

$$g[\tilde{t}_i^{\beta}\tilde{b}_j^{\alpha}g^aW^{\pm}] = \sqrt{2}g_2g_SR_{ij}\frac{\lambda_{\alpha\beta}^a}{2}.$$
(C22)

The  $R_{ij}$  are the following functions of the squark mixing angles:

$$\begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} = \begin{pmatrix} \cos \theta_{\tilde{t}} \cos \theta_{\tilde{b}} & \sin \theta_{\tilde{t}} \cos \theta_{\tilde{b}} \\ \cos \theta_{\tilde{t}} \sin \theta_{\tilde{b}} & \sin \theta_{\tilde{t}} \sin \theta_{\tilde{b}} \end{pmatrix}.$$
 (C23)

Higgs bosons:

$$g[h^{0}H^{\pm}W^{\mp}] = \frac{g_{2}}{2}\sin(\beta - \alpha),$$
(C24)

$$g[H^0 H^{\pm} W^{\mp}] = -\frac{g_2}{2} \cos(\beta - \alpha),$$
 (C25)

$$g[A^0H^{\pm}W^{\mp}] = -i\frac{g_2}{2}.$$
 (C26)

### **APPENDIX D: FEYNMAN GRAPHS**

Feynman graphs with opposite direction of charge flow are not depicted.







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