Improved approach to the heavy-to-light form factors in the light-cone QCD sum rules

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(Received 9 November 2000; published 19 March 2001)

A systematic analysis shows that the main uncertainties in the form factors are due to the twist-3 wave functions of the light mesons in the light-cone QCD sum rules. We propose an improved approach, in which the twist-3 wave functions do not make any contribution and therefore the possible pollution by them can be avoided, to reexamine $B\to\pi$ semileptonic form factors. Also, a comparison between previous results and our results from the light-cone QCD sum rules is made. Our method will be beneficial to the precise extraction of $|V_{ub}|$ from the experimental data on the processes $B\to\pi l\tilde{\nu}_l$.

DOI: 10.1103/PhysRevD.63.094001 PACS number(s): 13.20.He, 11.55.Hx

I. INTRODUCTION

Heavy-to-light exclusive decays are an important basis for understanding and testing the standard model (SM), since they can provide a signal of CP-violation phenomena and, perhaps, a window into new physics beyond the SM; therefore, it is of crucial interest to make a reliable prediction of these exclusive processes. We have to confront calculations of the hadronic matrix elements, in which all the long-distance QCD dynamics are included. At present, an exact estimate of them is impossible, to the present knowledge of QCD, from first principles, and one must resort to phenomenological approaches. Usually, some of the methods used widely are QCD sum rules, chiral perturbation theory (CHPT), heavy quark effective theory (HQET), and the quark model. Each of them has advantages and disadvantages. For example, CHPT and HQET, as two effective theories at low energy, can describe light-to-light and heavy-to-heavy exclusive transitions, respectively, but they are not suitable for a study of heavy-to-light processes. It is more complicated to calculate the heavy-to-light decays. In this case, the QCD sum rule method was adopted extensively. However, some questions still remain. The most striking problem is that the resulting sum rules for form factors behave very badly in the heavy quark limit $m_q\to\infty$. The reason is that in the operator product expansion (OPE) at the small distance $x\approx 0$, one omits the effect of the finite correlation length between the quarks in the physical vacuum. In order to overcome the defect, light-cone QCD sum-rule approach is developed in Ref. [1] and is regarded as an advanced tool to deal with heavy-to-light exclusive processes. Especially, the results consistent with the physical picture can be driven in this framework. Compared with the traditional QCD sum rules, the light-cone QCD sum-rule approach is of the following different points: the OPE is carried out near the light cone $x^2\approx 0$, instead of at the short distance $x\approx 0$ and the nonperturbative dynamics are parametrized as so-called light-cone wave functions, instead of the vacuum condensates. There are a lot of applications of light-cone QCD sum rules in literature. For a detailed description of this method, see Ref. [2].

At first sight, the heavy-to-light decays can be calculated by perturbative QCD (PQCD) due to the hard gluon exchange (the large Q^2 transfer). A detailed analysis [3] shows that the reliable PQCD calculation depends on whether the singularities can be eliminated or suppressed by the distribution amplitude. The singularities include on-shell gluon, on-shell light quark, and on-shell heavy quark. Carlson and Milana [4] argued that the on-shell heavy quark in the hard scattering travels only a short distance and the factorization of the formalism still holds. Even that, one can find that the reliable PQCD contribution may dominate only as m_b takes some special values and $\phi_\pi=\phi_\pi^{as}$ [3]. In order to make PQCD applicable, Ref. [5] adapts the modified hard-scattering approach to the case of the heavy-light form factor by a resummation of Sudakov logarithms, which may suppress the soft contribution beyond naive power counting. However, this approach still somehow depends on the end-point behavior of the light-meson's distribution amplitude.

Recently, a QCD factorization formula [6] was proposed for $B\to\pi\pi$, πK, and πD. It makes great progress in dealing with nonleptonic decays of B meson. In this approach, the amplitudes for these decays are expressed in terms of the semileptonic form factors, hadronic light-cone distribution amplitudes, and hard-scattering functions that are calculable in PQCD, and the semileptonic form factors and the distribution amplitudes are taken as inputs since the form factors can be measured experimentally and the distribution ampli-

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tudes are a universal function of the single meson state. Theoretically, the precise calculations of heavy-to-light form factors are of a great interest. Especially, it will be helpful for a clear understanding of $B\to\pi l\tilde{\nu}_l$ $(l=e,\mu)$ which provides us with a good chance to extract the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$ from the available data.

The fact that a considerable long-distance contribution may dominate the heavy-light form factor has been a motivation for applying the light-cone QCD sum rules to the $B\to\pi$ weak form factor [7]. In this approach, the nonperturbative dynamics is parametrized as so-called light-cone wave functions classified by their twist. Remarkably, the main uncertainties in the sum-rule results arise from light-cone wave functions. Now only the twist-2 wave functions, which dominate the contributions to the sum rules, have systematically been investigated. This is not the case, however, for the twist-3 and the twist-4 wave functions, which are understood poorly. On the other hand, although QCD radiative corrections to the twist-2 term are considered in Ref. [8], for improving the predictions, and their impact on the sum rule is found out to be negligibly small, numerical results are less convincing, because we have no reason to believe that $O(\alpha_s)$ corrections to the twist-3 terms can safely be neglected. From the above analyses, we can conclude that the great uncertainty, if possible, would be due to the uncertainties in the twist-3 wave functions and the lack of the corresponding $O(\alpha_s)$ corrections, in the existing calculations of $B\to\pi$ form factors in the framework of the light-cone sum rules.

In the present paper, we suggest an improved approach to calculating heavy-to-light weak form factors, and then apply it to reanalyze $B\to\pi l\tilde{\nu}_l$. The striking advantage of the method is, as will be shown in the following, that contributions of the twist-3 wave functions vanish at all from the light-cone sum rule in question, such that the possible pollution by them is effectively avoided. It will be beneficial to enhance the reliability of the light-cone sum-rule calculations.

II. CORRELATOR

Let us start with the following definition of $B\to\pi$ weak form factors $f(q^2)$ and $\tilde{f}(q^2)$:

$$\langle\pi(p)|\overline{u}\gamma_\mu b|B(p+q)\rangle=2f(q^2)p_\mu+\tilde{f}(q^2)q_\mu, \quad (1)$$

with q being the momentum transfer. Following Refs. [9], we choose to use a chiral current

$$\Pi_\mu(p,q)=i\int d^4x e^{iqx}\langle\pi(p)|T\{\overline{u(x)}\gamma_\mu(1+\gamma_5)b(x),\overline{b(0)}i(1+\gamma_5)d(0)\}|0\rangle$$
$$=\Pi(q^2,(p+q)^2)p_\mu+\tilde{\Pi}(q^2,(p+q)^2)q_\mu, \quad (2)$$

which is different from that in Refs. [3] and [4] to calculate $f(q^2)$ and $\tilde{f}(q^2)$. Here the T product of the chiral current operator is inserted between the vacuum and the on-shell π meson state.

First, we discuss the hadronic representation for the correlator. This can be done by inserting the complete intermediate states with the same quantum numbers as the current operator $\overline{b}i(1+\gamma_5)d$ in the correlator. By isolating the pole term of the lowest pseudoscalar B meson, we have the hadronic representation in the following:

$$\Pi_\mu^H(p,q)=\Pi^H[q^2,(p+q)^2]p_\mu+\tilde{\Pi}^H[q^2,(p+q)^2]q_\mu$$
$$=\frac{\langle\pi|\overline{u}\gamma_\mu b|B\rangle\langle B|\overline{b}\gamma_5 d|0\rangle}{m_B^2-(p+q)^2}$$
$$+\sum_H\frac{\langle\pi|\overline{u}\gamma_\mu(1+\gamma_5)|B^H\rangle\langle B^H|\overline{b}i(1+\gamma_5)d|0\rangle}{m_{B^H}^2-(p+q)^2}. \quad (3)$$

Note that the intermediate states B^H contain not only pseudoscalar resonances of the masses greater than m_B, but also scalar resonances with $J^p=0^+$, corresponding to the operator $\overline{b}d$. With Eq. (1) and the definition $\langle B|\overline{b}i\gamma_5 d|0\rangle=m_B^2f_B/m_b$, the invariant amplitudes Π^H and $\tilde{\Pi}^H$ read off

$$\Pi^H[q^2,(p+q)^2]=\frac{2f(q^2)m_B^2f_B}{m_b[m_B^2-(p+q)^2]}+\int_{s_0}^\infty\frac{\rho^H(s)}{s-(p+q)^2}ds$$
$$+\text{subtractions}, \quad (4)$$

and

$$\tilde{\Pi}^H[q^2,(p+q)^2]=\frac{\tilde{f}(q^2)m_B^2f_B}{m_b[m_B^2-(p+q)^2]}+\int_{s_0}^\infty\frac{\tilde{\rho}^H(s)}{s-(p+q)^2}ds$$
$$+\text{subtractions}, \quad (5)$$

where we have replaced the contributions of higher resonances and continuum states with dispersion integrations, in which the threshold parameter s_0 should be set near the squared mass of the lowest scalar B meson, and the spectral densities $\rho^H(s)$ and $\tilde{\rho}^H(s)$ can be approximated by invoking the quark-hadron duality ansatz

$$\rho^H(s)(\tilde{\rho}^H(s))=\rho^{QCD}(s)(\tilde{\rho}^{QCD}(s))\theta(s-s_0). \quad (6)$$

If we confine ourselves to discussing the semileptonic decays $B\to\pi l\tilde{\nu}_l$ $(l=e,\mu)$, the contributions of $\tilde{f}(q^2)$ to the decay amplitudes are small enough to be negligible, due to the smallness of the final-state lepton masses, and therefore only the form factor $f(q^2)$ needs considering.

On the other hand, we have to calculate the corrector in QCD theory, to obtain the desired sum rule for $f(q^2)$. It is possible by using the light-cone OPE method. To this end, we work in the large spacelike momentum regions $(p+q)^2-m_b^2\ll 0$ for the $b\overline{d}$ channel, and $q^2\ll m_b^2-O(1\ \mathrm{GeV}^2)$ for the momentum transfer, which correspond to the small light-cone distance $x^2\approx 0$ and are required by the validity of the OPE. In addition, the chiral limit $p^2=m_\pi^2=0$ is taken throughout this discussion, for simplicity. The leading contribution to the OPE is easy to drive by contracting the b-quark operators to a free propagator. After further considering the effect of the background gluon field, we can write down a full b-quark propagator

$$\begin{aligned}\langle 0|Tb(x)\overline{b}(0)|0\rangle &= iS_b^{(0)}(x)-ig_s\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\\ &\times\int_0^1 dv\left[\frac{1}{2}\frac{\hat{k}+m}{(m_b^2-k^2)^2}G^{\mu\nu}(vx)\sigma_{\mu\nu}\right.\\ &\left.+\frac{1}{m_b^2-k^2}vx_\mu G^{\mu\nu}(vx)\gamma_\nu\right].\end{aligned} \tag{7}$$

Here $G_{\mu\nu}$ is the gluonic field strength, g_s denotes the strong-coupling constant, and $S_b^0(x)$ expresses a free b-quark propagator

$$S_b^{(0)}(x)=\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\frac{\hat{k}+m}{k^2-m_b^2}. \tag{8}$$

Consider first the leading contribution from the free b-quark propagator. Carrying out the OPE for the corrector and making use of the Eq. (8), we have

$$\begin{aligned}\Pi^{(\overline{q}q)} &= -2m_b i\int\frac{d^4x d^4k}{(2\pi)^4}e^{i(q-k)x}\\ &\times\frac{1}{k^2-m_b^2}\langle\pi(p)|T\overline{u}(x)\gamma_\mu\gamma_5 d(0)|0\rangle,\end{aligned} \tag{9}$$

for the two-particle contribution $\Pi^{(\overline{q}q)}$. An important observation, as has been emphasized, is that only the leading nonlocal matrix element $\langle\pi(p)|T\overline{u}(x)\gamma_\mu\gamma_5 d(0)|0\rangle$ contributions to the corrector, while the nonlocal matrix elements $\langle\pi(p)|\overline{u}(x)i\gamma_5 d(0)|0\rangle$ and $\langle\pi(p)|\overline{u}(x)\sigma_{\mu\nu}\gamma_5 d(0)|0\rangle$ whose leading terms are of twist-3, disappear in our approach. Proceeding to Eq. (9), we discuss the light-cone expansion of $\langle\pi(p)|T\overline{u}(x)\gamma_\mu\gamma_5 d(0)|0\rangle$. In general, for a nonlocal quark-antiquark operator, we expand it around $x=0$, and then parametrize the operator matrix elements of any definitive twist by the so-called light-cone wave functions. In the present case, the nonlocal matrix element $\langle\pi(p)|T\overline{u}(x)\gamma_\mu\gamma_5 d(0)|0\rangle$ can be expanded as

$$\begin{aligned}&\langle\pi(p)|T\overline{u}(x)\gamma_\mu\gamma_5 d(0)|0\rangle\\ &\quad=-ip_\mu f_\pi\int_0^1 du e^{iupx}[\varphi_\pi(u)+x^2 g_1(u)]\\ &\quad+f_\pi\left(x_\mu-\frac{x^2p_\mu}{px}\right)\int_0^1 du e^{iupx}g_2(u),\end{aligned} \tag{10}$$

to the twist-4 accuracy, where $\varphi_\pi(u)$ is the twist-2 wave function, while both $g_1(u)$ and $g_2(u)$ have twist-4. Substituting Eq. (10) into Eq. (9) and integrating over x and k yields

$$\begin{aligned}\Pi^{(\overline{q}q)}[q^2,(p+q)^2] &= 2f_\pi m_b\left[\int_0^1\frac{du}{u}\varphi_\pi(u)\frac{1}{s-(p+q)^2}\right.\\ &-8m_b^2\int_0^1\frac{du}{u^3}g_1(u)\frac{1}{[s-(p+q)^2]^3}\\ &+2\int_0^1\frac{du}{u^2}G_2(u)\frac{1}{[s-(p+q)^2]^2}\\ &\left.+4\int_0^1\frac{du}{u^3}G_2(u)\frac{q^2+m_b^2}{[s-(p+q)^2]^3}\right],\end{aligned} \tag{11}$$

with $G_2(u)=\int_0^u g_2(v)dv$. In deriving Eq. (11) the relation $u=(m_b^2-q^2)/s-q^2$ has been used, and thus it should be understood that s is the function of argument u. A further discussion involves the evaluations of higher Fock-state effects. This can be done by taking into account the second term in Eq. (7) in the OPE of the correlator. A straightforward calculation gives, for the three-particle contribution $\Pi_\mu^{(\overline{q}qg)}$,

$$\begin{aligned}&\Pi_\mu^{(\overline{q}qg)}[q^2,(p+q)^2]\\ &\quad=ig_s m_b\int\frac{d^4kd^4xdv}{(2\pi)^4(m_b^2-k^2)}\\ &\quad\times e^{i(q-k)x}[\langle\pi(p)|\overline{d}(x)\gamma_\mu G^{\alpha\beta}(vx)\sigma_{\alpha\beta}u(0)|0\rangle\\ &\quad+\langle\pi(p)|\overline{d}(x)\gamma_\mu\gamma_5 G^{\alpha\beta}(vx)\sigma_{\alpha\beta}u(0)|0\rangle].\end{aligned} \tag{12}$$

Considering $\langle\pi(p)|\overline{d}(x)\gamma_\mu G^{\alpha\beta}(vx)\sigma_{\alpha\beta}u(0)|0\rangle=0$, as required by the parity conservation in strong interaction, and using the identity

$$\gamma_\mu\sigma_{\alpha\beta}=i(g_{\mu\alpha}\gamma_\beta-g_{\mu\beta}\gamma_\alpha)+\epsilon_{\mu\alpha\beta\nu}\gamma^\nu\gamma_5, \tag{13}$$

we further have

$$\begin{aligned}&\Pi_\mu^{(\overline{q}qg)}[q^2,(p+q)^2]\\ &\quad=im_b\int\frac{d^4kd^4xdv}{(2\pi)^4(m_b^2-k^2)}\\ &\quad\times e^{i(q-k)x}[ig_{\mu\alpha}\langle\pi(p)|\overline{u}(x)\gamma_\beta\gamma_5 g_s G^{\alpha\beta}(vx)d(0)|0\rangle\\ &\quad+\langle\pi(p)|\overline{u}(x)\gamma^\nu g_s\tilde{G}_{\mu\nu}(vx)d(0)|0\rangle],\end{aligned} \tag{14}$$

with $\tilde{G}_{\mu\nu}(vx)=\frac{1}{2}\epsilon^{\mu\nu\sigma\tau}G^{\sigma\tau}(vx)$. It should be noted that the situation here is the same as that in Eq. (11); the nonlocal matrix element $\langle\pi|\overline{u}(x)\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta}(vx)d(0)|0\rangle$, which has the twist-3 in leading order in the light-cone expansion, vanishes from the OPE. As a result, a self-consistency is kept in our approach. The matrix elements in Eq. (14) can be parametrized in terms of the three-particle wave functions of twist-4 $\varphi_\perp$, $\varphi_\parallel$, $\tilde{\varphi}_\perp$, and $\tilde{\varphi}_\parallel$ defined by

$$\begin{aligned}\langle\pi(p)|\overline{d}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)u(0)|0\rangle &= f_\pi\left[q_\beta\left(g_{\alpha\mu}-\frac{x_\alpha q_\mu}{qx}\right)-q_\alpha\left(g_{\beta\mu}-\frac{x_\beta g_\mu}{qx}\right)\right]\\ &\quad\times\int D\alpha_i\varphi_\perp(\alpha_i)\exp iqx(\alpha_1+v\alpha_3)\\ &\quad+f_\pi\frac{q_\mu}{qx}(q_\alpha x_\beta-q_\beta x_\alpha)\\ &\quad\times\int D\alpha_i\varphi_\parallel(\alpha_i)\exp iqx(\alpha_1+v\alpha_3),\end{aligned} \tag{15}$$

$$\begin{aligned}\langle\pi(p)|\overline{d}(x)\gamma_\mu g_s\tilde{G}_{\alpha\beta}(vx)u(0)|0\rangle &= if_\pi\left[q_\beta\left(g_{\alpha\mu}-\frac{x_\alpha q_\mu}{qx}\right)-q_\alpha\left(g_{\beta\mu}-\frac{x_\beta q_\mu}{qx}\right)\right]\\ &\quad\times\int D\alpha_i\tilde{\varphi}_\perp(\alpha_i)\exp iqx(\alpha_1+v\alpha_3)\\ &\quad+if_\pi\frac{q_\mu}{qx}(q_\alpha x_\beta-q_\beta x_\alpha)\\ &\quad\times\int D\alpha_i\tilde{\varphi}_\parallel(\alpha_i)\exp iqx(\alpha_1+v\alpha_3),\end{aligned} \tag{16}$$

with $D\alpha_i=d\alpha_1 d\alpha_2 d\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)$. Completing the integrations over x and k, we have

$$\begin{aligned}\Pi^{(\overline{q}qg)}[q^2,(p+q)^2] &= 2m_b f_\pi\int_0^1 dv\int D\alpha_i\\ &\quad\times\frac{2\varphi_\perp(\alpha_i)+2\tilde{\varphi}_\perp(\alpha_i)-\varphi_\parallel(\alpha_i)-\tilde{\varphi}_\parallel(\alpha_i)}{[s-(p+q)^2]^2(\alpha_1+v\alpha_3)^2},\end{aligned} \tag{17}$$

with parameter s defined by the relation $\alpha_1+v\alpha_3=(m_b^2-q^2)/s-q^2$. The final light-cone QCD expansion of the correlator can be written down as

$$\Pi^{QCD}[q,(p+q)]=\Pi^{(\overline{q}q)}[q,(p+q)]+\Pi^{(\overline{q}qg)}[q,(p+q)] \tag{18}$$

III. SUM RULE FOR $f(q^2)$

Further, to carry out the subtraction procedure of the continuum spectrum we need to convert the QCD representation (18) into a dispersion integration. In Π^{QCD}, the term proportional to $1/s-(p+q)^2$ in integrand is already a dispersion integration with respect to $(p+q)^2$ so that subtraction of the continuum can be made by simply changing the lower limit of integration from 0 to $\Delta=(m_b^2-q^2)/s_0-q^2$, while those with higher power of $1/s-(p+q)^2$, after the partial integration, become the following form

$$I=\int_{m_b^2}^{\infty}\frac{F(s)}{s-(p+q)^2}ds, \tag{19}$$

which is a dispersion integration with the perturbative spectrum densities $F(s)$. For instance, we have $F(s)=[d^2f(s)]/ds^2$, with $f(s)=8f_\pi m_b^3 g_1[u(s)](q^2-s)/(m_b^2-q^2)^2$, for the contribution of the twist-4 wave function $g_1(u)$ in Eq. (11). In this case, the subtraction of the continuum corresponds to a simple replacement $\infty\to s_0$.

Now, the light-cone QCD sum rule for $f(q^2)$ can be obtained, by making the Borel transformations with respect to $(p+q)^2$ in the hadronic and the QCD expressions and equating them. The result is

$$\begin{aligned}f(q^2)=\frac{m_b^2 f_\pi}{m_B^2 f_B}e^{m_B^2/M^2}\Bigg\{&\int_\triangle^1\frac{du}{u}\exp-\frac{m_b^2-q^2(1-u)}{uM^2}\left[\varphi_\pi(u)-\frac{4m_b^2}{u^2M^4}g_1(u)+\frac{2}{uM^2}\int_0^u g_2(v)dv\left(1+\frac{m_b^2+q^2}{uM^2}\right)\right]\\ &+\int_0^1 dv\int D\alpha_i\frac{\theta(\alpha_1+v\alpha_3-\Delta)}{(\alpha_1+v\alpha_3)^2M^2}\exp-\frac{m_b^2-(1-\alpha_1-v\alpha_3)q^2}{M^2(\alpha_1+v\alpha_3)}[2\varphi_\perp(\alpha_i)+2\tilde{\varphi}_i\perp(\alpha_i)-\varphi_\parallel(\alpha_i)-\tilde{\varphi}_\parallel(\alpha_i)]\\ &-4m_b^2e^{-s_0/M^2}\left[\frac{1}{(m_b^2-q^2)^2}\left(1+\frac{s_0-q^2}{M^2}\right)g_1(\Delta)-\frac{1}{(s_0-q^2)(m_b^2-q^2)}\frac{dg_1(\Delta)}{du}\right]\\ &-2e^{-s_0/M^2}\left[\frac{m_b^2+q^2}{(s_0-q^2)(m_b^2-q^2)}g_2(\Delta)-\frac{1}{(m_b^2-q^2)}\left(1+\frac{m_b^2+q^2}{m_b^2-q^2}\right)\left(1+\frac{s_0-q^2}{M^2}\right)\int_0^\Delta g_2(v)dv\right]\Bigg\}.\end{aligned} \tag{20}$$

We would like to stress that the terms proportional to exponential factor e^{-s_0/M^2} arise from the substructions of the continuum, and may not be neglected for our present purposes.

Before proceeding further we need to make a choice of input parameters entering the sum rule for $f(q^2)$. To begin with, let us specify the set of pion wave functions. For the leading twist-2 wave function $\varphi_\pi(u)$, the asymptotic form is exactly given by PQCD [10] $\varphi_\pi(u,\mu\to\infty)=6u(1-u)$, nonperturbative corrections can be included in a systematic way in terms of the approximate conformal invariance of QCD

$$\varphi_\pi(u,\mu)=6u(1-u)[1+a_2(\mu)C_2^{3/2}(2u-1)+a_4(\mu)C_4^{3/2}(2u-1)+\cdots], \tag{21}$$

with the Gegenbaer polynomials

$$C_2^{3/2}(2u-1)=\frac{3}{2}[5(2u-1)^2-1], \tag{22}$$

$$C_4^{3/2}(2u-1)=\frac{15}{8}[21(2u-1)^4-14(2u-1)^2+1]. \tag{23}$$

The coefficients in the expansion $a_n(\mu)$ can be determined by a certain nonperturbative approach. As we know, there are many models for the twist-2 wave function [11]. In order to make a comparison with the previous result, we follow Ref. [12] and use

$$a_2(\mu_0=0.5\ \text{GeV})=\frac{2}{3},\quad a_4(\mu_0=0.5\ \text{GeV})=0.43, \tag{24}$$

which result from an analysis of light-cone sum rules for the πNN and the $\omega\rho\pi$ couplings. Furthermore, the use of the renormalization-group equation gets

$$a_2(\mu_b)=0.35,\quad a_4(\mu_b)=0.18, \tag{25}$$

at the scale $\mu_b=\sqrt{m_B^2-m_b^2}\approx 2.5$ GeV, which characterizes the mean virtuality of the b quark. For the twist-4 wave functions, we use the results for the three-particle wave functions [12]

$$\varphi_\perp(\alpha_i)=30\delta^2(\alpha_1-\alpha_2)\alpha_3^2\left[\frac{1}{3}+2\epsilon(1-2\alpha_3)\right],$$

$$\tilde{\varphi}_\perp(\alpha_i)=30\delta^2\alpha_3^2(1-\alpha_3)\left[\frac{1}{3}+2\epsilon(1-2\alpha_3)\right],$$

$$\varphi_\parallel(\alpha_i)=120\delta^2\epsilon(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3,$$

$$\tilde{\varphi}_\parallel(\alpha_i)=-120\delta^2\alpha_1\alpha_2\alpha_3\left[\frac{1}{3}+\epsilon(1-3\alpha_3)\right], \tag{26}$$

with $\delta^2(\mu_b)=0.17\ \text{GeV}^2$ and $\varepsilon(\mu_b)=0.36$. Further, a relation can be obtained between the two-particle twist-4 wave functions and the above by equation of motion such that we have [12]

$$g_1(u)=\frac{5}{2}\varepsilon^2u^2\overline{u^2}+\frac{1}{2}\varepsilon\delta^2\Big[u\overline{u}(2+13u\overline{u})+10u^3\ln u\left(2-3u+\frac{6}{5}u^2\right)+10\overline{u}^3\ln\overline{u}\left(2-3\overline{u}+\frac{6}{5}\overline{u^2}\right)\Big],$$

$$g_2(u)=\frac{10}{3}\delta^2u\overline{u}(u-\overline{u}). \tag{27}$$

Unlike the case of the twist-2 wave functions, these twist-4 wave functions seem to be very difficult to test by experiment, for they usually are of negligible contributions in the sum rules.

Another important input is the decay constant of B meson f_B. The QCD sum rule for f_B has been discussed many times. However, all these estimates are not applicable in our sum rule for $f(q^2)$. The reason is that in the present case a chiral current correlator is adopted to avoid pollution by the twist-3 wave functions, so that a similar correlator has to be used, for consistency, in the sum-rule calculation of f_B. To this end, we consider the following two-point correlator:

$$K(q^2)=i\int d^4xe^{iqx}\langle 0|\overline{q}(x)(1+\gamma_5)b(x),\ \overline{b}(0)(1-\gamma_5)q(0)|0\rangle. \tag{28}$$

The calculation should be limited to leading order in QCD, since the QCD radiative corrections to the sum rule for $f(q^2)$ are neglected as well. A standard manipulation yields three self-consistent sets of results [9]: (1) $f_B=165$ MeV for $m_b=4.7$ GeV, and $s_0=33\ \text{GeV}^2$, (2) $f_B=120$ MeV for $m_b=4.8$ GeV and $s_0=32\ \text{GeV}^2$, and (3) $f_B=85$ MeV for $m_b=4.9$ GeV and $s_0=30\ \text{GeV}^2$. The above results correspond to the best fit in s_0 and will be used as inputs in numerical analyses of the sum rule for $f(q^2)$. At this point, a few comments are in order: (1) some vacuum condensate parameters vanish from the sum rule for f_B, and thus some inherent uncertainties in the sum rule are reduced, and (2) the threshold parameters s_0 turn out to be of values less than those in the conventional sum rule for f_B. This is consistent with the case in the sum rule for $f(q^2)$. As for the B meson mass m_B and the pion decay constant f_π, we take the present world average value $m_B=5.279$ GeV, and $f_\pi=0.132$ GeV.

IV. NUMERICAL RESULT

With these inputs, we can carry out the numerical analysis. The first step is, according to the standard procedure, to look for a range of the Borel parameter M^2, in which the numerical results are quite stable for a given threshold s_0.

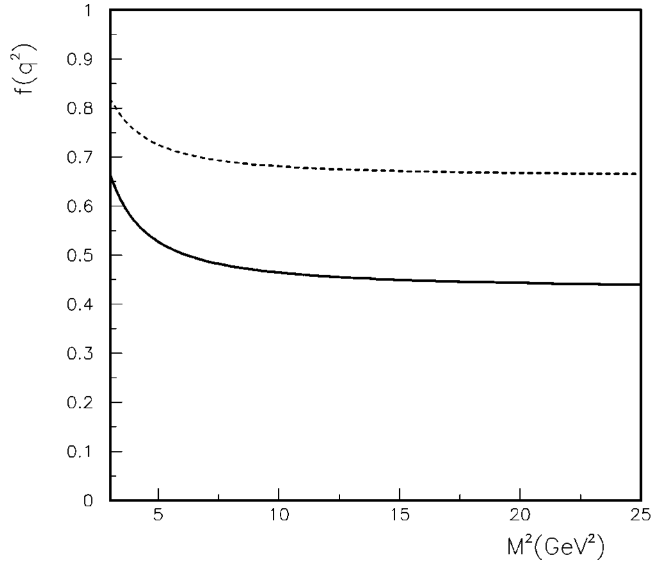

FIG. 1. Sensitivity of the form factor $f(q^2)$ to the Borel parameter M^2. Considered are the two typical cases of $q^2=10\ \mathrm{GeV}^2$ (solid) and $q^2=16\ \mathrm{GeV}^2$ (dashed), with $S_0=32\ \mathrm{GeV}^2$ and $m_b=4.8$ GeV.

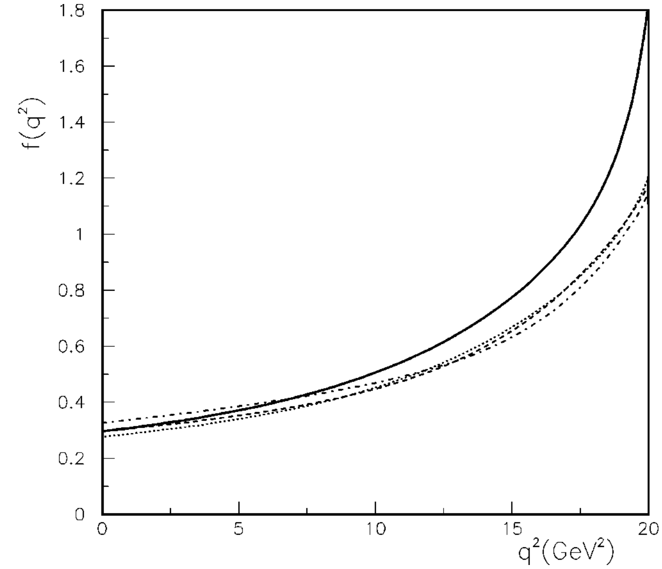

FIG. 2. The light-cone QCD sum rules for form factor $f(q^2)$ of $B\to\pi$ semileptonic transitions at $M^2=12\ \mathrm{GeV}^2$. The solid curve expresses the results in Ref. [7], while the dotted, the dashed, and the dashed-dot curves correspond to our predictions, with (i) $m_b=4.7$ GeV, $s_0=33\ \mathrm{GeV}^2$, (ii) $m_b=4.8$ GeV, $s_0=32\ \mathrm{GeV}^2$, and (iii) $m_b=4.9$ GeV, $s_0=30\ \mathrm{GeV}^2$, respectively.

Then, what remains to be done is to determine the fiducial interval of M^2, from which the desired sum-rule results can be read off, by the requirement that the contributions of the twist-4 wave functions do not exceed 10%, while those of the continuum states are not more than 30%.

In the present case, the reasonable range of M^2, for the threshold s_0 given above, is found to be $8\ \mathrm{GeV}^2\leqslant M^2\leqslant 17\ \mathrm{GeV}^2$ with the different central values as q^2 changes. In such a ''window,'' $f(q^2)$ depends very weakly on M^2, up to $q^2=18\ \mathrm{GeV}^2$. This is shown, for example, in Fig. 1, where the two typical cases, corresponding to $q^2=10\ \mathrm{GeV}^2$ and 16 GeV^2, are considered for an illustrative purpose. This allows us to estimate safely the variation of $f(q^2)$ with q^2, at a certain specific value of M^2. The numerical results at $M^2=12\ \mathrm{GeV}^2$, together with the previous light-cone sum-rule prediction [7] are plotted in Fig. 2, for a comparison. We find $f(0)=0.27$, 0.29, and 0.33 [corresponding to set(3), set(2), and set(1), respectively], which are in basic agreement with the result in Ref. [7] $f(0)=0.29$. As a mater of fact, numerical agreement between the two different approaches exists up to $q^2=10\ \mathrm{GeV}^2$, the differences being within 20%. The obvious numerical derivation, however, begins to appear beyond 10 GeV^2 and our results turn out to be less than those of Ref. [7] by about $(35-40)\%$, near $q^2=18\ \mathrm{GeV}^2$. Apparently, the fact that $f(q^2)$ is less sensitive to M^2 cannot account for the disagreement. To clarify this issue, both approaches have to undergo a more systematic investigation, including a complete evaluation of $O(\alpha_s)$ corrections and a detailed analysis of the uncertainties in the twist-3 wave functions. Indeed, the radiative corrections, as it has been shown in Ref. [8], are negligibly small for the twist-2 term. It perhaps is not the case in our approach and for the twist-3 terms in the sum rules of [7]. In the region $q^2\geqslant 18\ \mathrm{GeV}^2$, applicability of the light-cone sum rules is questionable, as has been mentioned, such that a comparison between the different approaches is meaningless.

A systematic discussion on the sources of uncertainties for $f(q^2)$ is needed. All the above calculations correspond to taking the central values of threshold parameters, which are determined in the two point sum rule for f_B. To look at the numerical impact of the uncertainties in threshold parameter on the sum rule for $f(q^2)$, we make use of the analytic form, instead of the numerical results, for the two-point sum rule for f_B in the numerical calculations. It is shown that the resulting $f(q^2)$ varies by $(10-15)\%$ relative to the central values, depending on m_b and q^2. Also, we investigate the sensitivity of $f(q^2)$ to the simultaneous variations of s_0 and m_b in the regions $30\ \mathrm{GeV}^2\leqslant s_0\leqslant 33\ \mathrm{GeV}^2$ and $4.7\ \mathrm{GeV}\leqslant m_b\leqslant 4.9$ GeV, finding that the induced change in $f(q^2)$ in the case is less than 5% in the total range of q^2, for the most stable values of f_B, and therefore is negligible.

In addition, there also are the uncertainties related to the light-cone wave functions of π meson. For example, the wave function, which is closed to the asymptotic form, will give a smaller value of $f(0)$. However, the twist-2 wave function is universal for the different processes. The uncertainties due to it can be controlled well as soon as one can obtain more reliable twist-2 wave functions to fit them. For the twist-4 wave functions, considering that they have only the effect of about $(4-6)\%$ on $f(q^2)$, as shown, we can imagine that the contributions of wave functions beyond the twist-4 are anyway negligibly small. In fact, this signals that we need not be careful about the sensitivity of $f(q^2)$ to wave functions of twist-4 and beyond twist-4. As the twist-3 wave functions go, the numerical calculations show that their contributions are comparable with those of the twist-2, amounting to about 50% in Ref. [7]. Remarkably, the reliability of these wave functions has to be subject to a test in that case. Nevertheless, this causes no problem in the present case, for all the twist-3 wave functions make a vanishing contribution to the sum rule in question, up to all orders in PQCD.

III. SUMMARY

To summarize, we have re-examined that weak form factor $f(q^2)$ for B decays into light pseudoscalar mesons, taking $B\to\pi$ semileptonic transitions as an illustrative example, in the light-cone QCD sum rule framework. The aim is to control the nonperturbative dynamics in the sum rules, to the best of our ability, and further to enhance the predictivity and reliability of numerical results. To this end, a chiral-current correlator is worked out. It is explicitly shown that the twist-3 light-cone wave functions, which have not been understood very well, can be effectively eliminated from the sum rule for $f(q^2)$. Consequently, the possible pollution by them is avoided in the final expression. The results presented here will be beneficial to the precision extracting of the CKM matrix element $|V_{ub}|$ from the exclusive processes $B\to\pi l\tilde{\nu}_l$ ($l=e,\mu$), by confronting the theoretical predictions with the experimentally available data.

In comparison to a previous estimate based on the light-cone sum rules, we find that the numerical agreement exists between the two different sum rules for $f(q^2)$ in the region of momentum transfer $0\leqslant q^2\leqslant 10$ GeV^2; beyond this region, a remarkable numerical deviation begins to appear; in particular, near $q^2=18$ GeV^2 (maximum value required by the light-cone OPE) our numerical results are less than that in Ref. [7] by about $(35-40)\%$. Also, the possible uncertainties in the sum rule $f(q^2)$ due to the parameter m_b are discussed. At present we have not included the PQCD radiative corrections. It is expected that our result does not change much after including the PQCD radiative corrections since the twist-3 light-cone wave functions are eliminated in our approach.

ACKNOWLEDGMENTS

One of the authors X.Y.W. thanks L. Z. Hai, Y. D. Shan, and G. H. Zhu for useful comments. This work is supported in part by the National Science Foundation of China.

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