Symmetry nonrestoration in a Gross-Neveu model with a random chemical potential

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We study the symmetry behavior of the Gross-Neveu model in three and two dimensions with a random chemical potential. This is equivalent to a four-fermion model with charge conjugation symmetry as well as Z_2 chiral symmetry. At high temperature the Z_2 chiral symmetry is always restored. In three dimensions the initially spontaneously broken charge conjugation symmetry is not restored at high temperature, irrespective of the value of the disorder strength. In two dimensions and at zero temperature the charge conjugation symmetry undergoes a quantum phase transition from a symmetric state (for weak disorder) to a broken state (for strong disorder) as the disorder strength is varied. For any given value of disorder strength, the high-temperature behavior of the charge conjugation symmetry is the same as its zero-temperature behavior. Therefore, in two dimensions and for strong disorder strength the charge conjugation symmetry is not restored at high temperature.

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I. INTRODUCTION

Intuitively, when heated, a system with initially broken symmetry will recover its symmetry because thermal fluctuations are able to overcome potential barriers. But a counterexample was noticed by Weinberg [1]: For the four-dimensional $O(N)\times O(N)$ scalar ϕ^4 model, he showed that the system can remain in the broken phase even at arbitrarily high temperature. This phenomenon is called inverse symmetry breaking or symmetry nonrestoration (SNR), depending on whether the system was in a symmetric or a broken phase at zero temperature.

Since Weinberg's observation, SNR has been a subject of academic curiosity or a candidate for the resolution of cosmological problems caused by topological defects such as monopoles and domain walls (see Ref. [2] for a review). According to Bajc's classification [2], there are three classes of SNR mechanisms in field theory: (i) a prototype case such as the two-scalar model [1,3,4], (ii) flat directions in supersymmetric theories [5], and (iii) large charge density (or chemical potential) [6,7]. Here we restrict ourselves to class (iii).

If a large enough charge cannot be stored in thermally excited modes at high temperature, it must reside in the vacuum, and this is a sign of SNR. In field theory, a scalar field (order-parameter field) gets a positive mass term by thermal effects, but a negative one by the effects due to the chemical potential. For a fixed charge (i.e., in the canonical formalism), the chemical potential is temperature dependent. In this case, if the effect of the chemical potential on the mass exceeds the thermal effects at sufficiently high temperature, the scalar field acquires a nonzero vacuum expectation value (i.e., SNR) [6].

However, in an open system of the model which does not belong to class (i) or (ii), the symmetry may always be restored at high temperature. In the grand canonical formalism, the chemical potential and the temperature are independent parameters and so the thermal effect on the mass always surpasses the effect of the chemical potential at sufficiently high temperature, for a fixed chemical potential. For example, consider the Gross-Neveu (GN) model [8–11] with chiral symmetry. At finite chemical potential the initially spontaneously broken chiral symmetry is always restored at high temperature [12,13].

In order to find a new kind of SNR in four-fermion models, we will extend the GN model at finite chemical potential to a disordered model with random chemical potential. Recently, disordered nonrelativistic Dirac fermions in two spatial dimensions have been studied in relation to the integer quantum Hall transition [14]. Pure fermions exhibit such a transition as the value of the mass is tuned through zero, but its universality class is different from the one observed in actual experiments. Usually three types of (static) disorder are considered for a more realistic model: the random gauge potential, random chemical potential, and random mass.

Motivated by the SNR mechanism (iii), we introduce the (relativistic) GN model with random chemical potential in Sec. II. If the chemical potential has a Gaussian distribution at each site, our model is equivalent to the four-fermion model with two kinds of four-fermion interaction, $(\bar{\Psi}\Psi)^2$ and $(\bar{\Psi}\gamma_0\Psi)^2$ [see Eq. (3)], and has charge conjugation symmetry in addition to Z_2 chiral symmetry. We discuss the physical implications of a new four-fermion interaction $(\bar{\Psi}\gamma_0\Psi)^2$ at the classical level. In Sec. III we examine the behavior of these symmetries as the temperature or disorder strength is varied using the $1/N$ expansion in three and two dimensions. While Z_2 chiral symmetry is always restored at high temperature, the charge conjugation symmetry exhibits SNR. In addition, we check the validity of the mean field approximation (the leading approximation in the $1/N$ expansion) in two dimensions. In Sec. IV the fundamental origin

of SNR for charge conjugation symmetry is discussed conceptually at the quantum level. Our conclusions are presented in Sec. V.

II. GROSS-NEVEU MODEL WITH RANDOM CHEMICAL POTENTIAL

The Euclidean Lagrangian of the GN model at finite chemical potential μ is given by

$$\mathcal{L}=\bar{\Psi}(\not\partial+\mu\gamma_0)\Psi-\frac{g^2}{2N}(\bar{\Psi}\Psi)^2, \tag{1}$$

where $g^2(>0)$ is the coupling constant of the four-fermion interaction $(\bar{\Psi}\Psi)^2$ and N is the number of flavors of the Dirac fermion Ψ. The γ matrices are 4×4 and Hermitian. Let us consider the system under the influence of a random chemical potential $\rho(x)$ with the Gaussian distribution $\exp[-\int d^dx(N/2R^2)\rho^2]$ at each site where R^2 (>0) is the strength of disorder and d the dimension of the Euclidean space. The Gaussian noise [15] is characterized by correlation functions

$$\langle\rho(x)\rangle=0, \quad \langle\rho(x)\rho(x')\rangle=\frac{R^2}{N}\delta^d(x-x'), \tag{2}$$

where averages are taken at fixed Ψ.

After integrating out the random chemical potential, our model is equivalent to the four-fermion model

$$\mathcal{L}=\bar{\Psi}\not\partial\Psi-\frac{1}{2N}[g^2(\bar{\Psi}\Psi)^2+R^2(\bar{\Psi}\gamma_0\Psi)^2], \tag{3}$$

with Z_2 chiral symmetry $\{\Psi\rightarrow\gamma_5\Psi,\ \bar{\Psi}\rightarrow-\bar{\Psi}\gamma_5\}$ and charge conjugation symmetry $\{\Psi\rightarrow C\bar{\Psi}^T,\ \bar{\Psi}\rightarrow-\Psi^TC^\dagger\}$. Here the matrix C satisfies $C^\dagger C=1,\ C^\dagger\gamma_\mu C=-\gamma_\mu^T$. Under charge conjugation, $\bar{\Psi}\Psi$ and $\bar{\Psi}\gamma_0\Psi$ transform to $\bar{\Psi}\Psi$ and $-\bar{\Psi}\gamma_0\Psi$, respectively. Hence, the Lagrangian, Eq. (1), with definite chemical potential μ does not possess charge conjugation symmetry (i.e., fermion-antifermion symmetry). Note that in Eq. (3) the chemical potential term does not appear explicitly.

We will study the GN model with random chemical potential by the leading approximation of the $1/N$ expansion in three and two dimensions. To easily incorporate the $1/N$ expansion, let us rewrite it by introducing a scalar auxiliary field $\sigma(x)$ that can be interpreted as a random mass with a Gaussian distribution:

$$\mathcal{L}=\bar{\Psi}(\not\partial+\sigma+\rho\gamma_0)\Psi+\frac{N}{2g^2}\sigma^2+\frac{N}{2R^2}\rho^2. \tag{4}$$

If we derive this Lagrangian from Eq. (3), the random chemical potential $\rho(x)$ plays the role of another scalar auxiliary field. The Z_2 chiral symmetry and the charge conjugation symmetry are now expressed as $\{\Psi\rightarrow\gamma_5\Psi,\bar{\Psi}\rightarrow-\bar{\Psi}\gamma_5,\sigma\rightarrow-\sigma\}$ and $\{\Psi\rightarrow C\bar{\Psi}^T,\bar{\Psi}\rightarrow-\Psi^TC^\dagger,\rho\rightarrow-\rho\}$, respectively.

To grasp the characteristic features of the GN model with random chemical potential, let us consider the physical implications of the last term proportional to $-R^2(\text{charge density})^2$ in the equivalent four-fermion model, Eq. (3), at the classical level. Contrary to the usual electrostatics (in the limit of the large photon mass), this term implies an attractive interaction between the same kinds of charges and a repulsive one between different kinds of charges. Since the classical Lagrangian, Eq. (3), has a charge conjugation symmetry, fermions and antifermions can clump in the system with equal probability. By diffusions of the charges and repulsive forces between different kinds of charges, however, only one kind of charge (i.e., only fermions or only antifermions) remains in the system and the opposite kind of charge is expelled to the outside of the system (i.e., reservior) because our system is an open system. Charge separation occurs between our system and the reservior, which does not necessarily cause a breakdown of translational invariance of the system. It will be shown in Sec. III that this qualitative result from the classical Lagrangian, Eq. (3), is the case at zero and high temperature in three and two dimensions except for the two-dimensional system with R^2 small. The accumulation of only one kind of charge in the system leads to the spontaneous breaking of the charge conjugation symmetry. In order to obtain more reliable results the model, Eq. (3), should be studied by a nonperturbative method such as the $1/N$ expansion at the quantum level. Such an analysis by the Lagrangian, Eq. (4), will reveal in Sec. III that the two-dimensional theory with R^2 small is in a symmetric phase of the charge conjugation symmetry and so the same numbers of fermions and antifermions exist in the system.

On the other hand, the GN model is a relativistic model of superconductivity. Moreover, the attractive interaction between the same kinds of charges in the last term of Eq. (3) is similar to the BCS theory of superconductivity where the interaction between charges is mediated by phonons. In the Lagrangian, Eq. (4), two scalar auxiliary fields may be interpreted as phonon fields corresponding to lattice vibrations. The contact four-fermion interactions imply the limit of the large phonon masses which is guaranteed in the large N limit. This is the reason why the kinetic terms for phonon fields σ and ρ are suppressed in Eq. (4).

III. BEHAVIOR OF Z_2 CHIRAL SYMMETRY AND CHARGE CONJUGATION SYMMETRY AT ZERO AND HIGH TEMPERATURE

For finite-temperature field theory we adopt the imaginary-time formalism. At inverse temperature $\beta(=T^{-1})$, the fermion fields are antiperiodic on $R^{d-1}\times[0,\beta]$, while the scalar auxiliary fields are periodic. Let us introduce the notation

$$\int_p^{(T)} \equiv T \sum_{n=-\infty}^{\infty} \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}},$$

$$\int_p^{(0)} \equiv \int \frac{d^d p}{(2\pi)^d}.$$

Integrating out the fermion fields in the partition function for Eq. (4) we obtain the effective action for the auxiliary fields σ and ρ. In order to investigate the vacuum structure we need to find the finite-temperature effective potential $V_T(\sigma,\rho)$ by taking σ and ρ as constant fields: To leading order in the $1/N$ expansion,

$$\frac{V_T(\sigma,\rho)}{N} = \frac{\sigma^2}{2g^2} + \frac{\rho^2}{2R^2} - 2\int_p^{(T)} \ln[(p_0 - i\rho)^2 + \mathbf{p}^2 + \sigma^2], \tag{5}$$

where $p_0 = (2n+1)\pi/\beta \equiv \omega_n$ (n = integer) at nonzero temperature. Note that the effect of the chemical potential ρ is to shift the energy by $-i\rho$.

To evaluate the integration in Eq. (5), we need some mathematical formulas. By the standard method of contour integration [16],

$$T\sum_{n=-\infty}^{\infty} \frac{1}{(\omega_n - i\rho)^2 + \sigma^2} = \frac{1}{2|\sigma|}\left[1 - \frac{1}{1+e^{\beta(|\sigma|+|\rho|)}} - \frac{1}{1+e^{\beta(|\sigma|-|\rho|)}}\right], \tag{6}$$

$$T\sum_{n=-\infty}^{\infty} \frac{\omega_n - i\rho}{(\omega_n - i\rho)^2 + \sigma^2} = i\rho T \sum_{n=-\infty}^{\infty} \frac{\omega_n^2 - (\sigma^2 - \rho^2)}{(\omega_n^2 + \sigma^2 - \rho^2)^2 + (2\rho\omega_n)^2} = \frac{i}{2}\left[\frac{\sinh(\beta\rho)}{\cosh(\beta\sigma) + \cosh(\beta\rho)}\right]. \tag{7}$$

When the GN model is studied in the canonical formalism (i.e., with a fixed charge) [17], similar calculations appear with imaginary chemical potential. In this case a regulating factor of the form $e^{i\omega_n \tau}$ is needed in evaluating the summation in Eq. (7) and ensures a finite result in the limit $\tau \to 0$ after the Matsubara sum has been performed. By using Eqs. (6) and (7), we obtain

$$T\sum_{n=-\infty}^{\infty} \ln[(\omega_n - i\rho)^2 + \sigma^2] = T\{\ln 2 + \ln[\cosh(\beta\sigma) + \cosh(\beta\rho)]\}, \tag{8}$$

where the ζ-function regularization was used to determine the field-independent constant. At zero temperature, these formulas reduce to

$$\int \frac{dp_0}{2\pi} \frac{1}{(p_0 - i\rho)^2 + \sigma^2} = \frac{\theta(|\sigma| - |\rho|)}{2|\sigma|}, \tag{9}$$

$$\int \frac{dp_0}{2\pi} \frac{p_0 - i\rho}{(p_0 - i\rho)^2 + \sigma^2} = \frac{i}{2}\,\mathrm{sgn}(\rho)\,\theta(|\rho| - |\sigma|), \tag{10}$$

$$\int \frac{dp_0}{2\pi} \ln[(p_0 - i\rho)^2 + \sigma^2] = \max(|\sigma|, |\rho|). \tag{11}$$

Using Eqs. (6), (7) and $E_\sigma \equiv \sqrt{\mathbf{p}^2 + \sigma^2}$, we have

$$\frac{\partial}{\partial\sigma}\left(\frac{V_T}{N}\right) = \frac{\sigma}{g^2} - 2\sigma \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \frac{1}{E_\sigma} \times \left[1 - \frac{1}{1+e^{\beta(E_\sigma+\rho)}} - \frac{1}{1+e^{\beta(E_\sigma-\rho)}}\right], \tag{12}$$

$$\frac{\partial}{\partial\rho}\left(\frac{V_T}{N}\right) = \frac{\rho}{R^2} - 2\sinh(\beta\rho) \times \int \frac{d^{d-1}\mathbf{p}}{(2\pi)^{d-1}} \frac{1}{\cosh(\beta E_\sigma) + \cosh(\beta\rho)}. \tag{13}$$

To renormalize the effective potential $V_T(\sigma,\rho)$ to leading order in the $1/N$ expansion let us consider the GN model at zero temperature and in the absence of the random chemical potential because the effects of temperature and constant chemical potential do not change the ultraviolet behavior [18]. Define $1/G^2 \equiv 1/g^2 - 1/g_c^2$, with

$$\frac{1}{g_c^2} \equiv 4\int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2}.$$

In $2 < d < 4$, the GN model is in the broken phase of the Z_2 chiral symmetry for negative G^2, corresponding to strong coupling ($g^2 > g_c^2$) in the cutoff regularization, while it is in the symmetric phase for $G^2 \geqslant 0$, corresponding to weak coupling ($0 < g^2 \leqslant g_c^2$). In particular, in two dimensions, the Z_2 chiral symmetry of the GN model must be broken no matter how we choose the coupling g^2 [9]. In the broken phase,

$$\frac{1}{g^2} = 4\int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 + M^2}, \tag{14}$$

where $M = |\langle\sigma\rangle|$ (>0) is the dynamically generated fermion mass at zero temperature.

From now on, we will adopt dimensional regularization, where G^2 is equal to the regularized g^2.

A. Three dimensions

In this case, renormalization is not needed to leading order of the $1/N$ expansion (in dimensional regularization). By

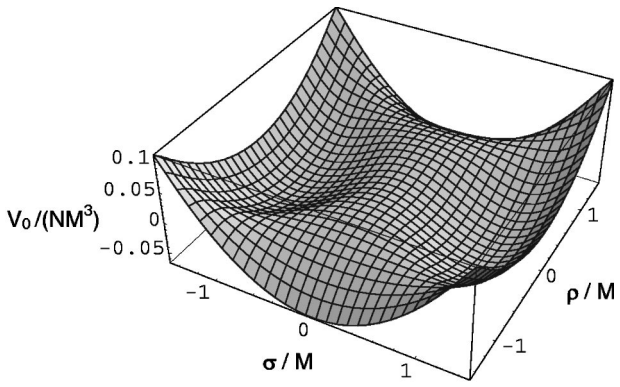

FIG. 1. The zero-temperature effective potential $V_0/(NM^3)$ in three dimensions as a function of σ/M and ρ/M for $R^2M=10$, in the case of the Z_2 chiral symmetry breaking ($G^2M=-\pi$).

making use of Eq. (11), we can find the zero-temperature effective potential $V_0(\sigma,\rho)$ directly:

$$\frac{V_0(\sigma,\rho)}{N}=\frac{\sigma^2}{2G^2}+\frac{\rho^2}{2R^2}-\frac{1}{6\pi}\times[\max^3(|\sigma|,|\rho|)-3\sigma^2\max(|\sigma|,|\rho|)], \tag{15}$$

where $1/G^2=-M/\pi$ for broken Z_2 chiral symmetry. The gap equations have four kinds of solution ($|\sigma|,|\rho|$): (i) (0,0), (ii) (M,0), (iii) ($0,2\pi/R^2$), and (iv) ($\sqrt{M(M-2\pi/R^2)},M$). The solution (iv) exists only for $M>2\pi/R^2$ and corresponds to saddle points. Figure 1 shows the zero-temperature effective potential as a function of σ/M and ρ/M, for broken Z_2 chiral symmetry. $\langle\rho\rangle=0$ is metastable, irrespective of the values of G^2 and R^2. For $|\rho|>|\sigma|$, however, $V_0(\sigma,\rho)$ is unbounded from below due to the $-|\rho|^3/(6\pi)$ term, which indicates breaking of the charge conjugation symmetry. This result stems from the fact that the term $-|\rho|^3/(6\pi)$ arising from quantum effects surpasses the effect of the probability distribution $[\rho^2/(2R^2)]$ for large $|\rho|$.

At finite temperature, using Eqs. (6), (7) and dimensional regularization, we obtain

$$\int_p^{(T)}\frac{1}{(\omega_n-i\rho)^2+E_\sigma^2}=-\frac{1}{4\pi\beta}\{\beta|\sigma|+\ln[1+2e^{-\beta|\sigma|}\cosh(\beta\rho)+e^{-2\beta|\sigma|}]\}, \tag{16}$$

$$\int_p^{(T)}\frac{\omega_n-i\rho}{(\omega_n-i\rho)^2+E_\sigma^2}=-\frac{i\,\mathrm{sgn}(\rho)}{4\pi\beta^2}\left[\beta|\sigma|\ln\left(\frac{1+e^{\beta(|\sigma|+|\rho|)}}{1+e^{\beta(|\sigma|-|\rho|)}}\right)+\mathrm{Li}_2(-e^{\beta(|\sigma|+|\rho|)})-\mathrm{Li}_2(-e^{\beta(|\sigma|-|\rho|)})\right]. \tag{17}$$

Here the polylogarithm $\mathrm{Li}_\nu(z)$ is defined (for $\nu>0$) as $\mathrm{Li}_\nu(z)=\Sigma_{k=1}^{\infty}z^k/k^\nu$ (see Ref. [19] for useful properties). From these formulas, the finite-temperature effective potential $V_T(\sigma,\rho)$ is given by

$$\frac{V_T(\sigma,\rho)}{N}=\frac{\sigma^2}{2G^2}+\frac{\rho^2}{2R^2}-\frac{\sigma^3}{3\pi}+\frac{1}{\pi\beta^3}[\mathrm{Li}_3(-e^{\beta(\sigma+\rho)})+\mathrm{Li}_3(-e^{\beta(\sigma-\rho)})-\beta\sigma\{\mathrm{Li}_2(-e^{\beta(\sigma+\rho)})+\mathrm{Li}_2(-e^{\beta(\sigma-\rho)})\}], \tag{18}$$

up to a field-independent constant. At sufficiently high temperature,

$$\frac{V_T(\sigma,\rho)}{N}\approx\left(\frac{\ln 2}{\pi}\right)T(\sigma^2-\rho^2). \tag{19}$$

While the initially spontaneously broken Z_2 chiral symmetry is restored at high temperature, charge conjugation symmetry is not. Hence our model exhibits nonrestoration of charge conjugation symmetry irrespective of the values of G^2 and R^2. We may interpret this phenomenon as an inverse symmetry breaking because $\langle\rho\rangle=0$ is metastable at zero temperature. Intuitively, SNR is related to the tachyonlike behavior of the random chemical potential [see Eqs. (19) and (24)]. In the quantum correction term of Eq. (5) the chemical potential acts as a negative mass term ($-\rho^2$) contrary to the usual positive mass term (σ^2). From a different point of view, we will discuss the origin of SNR conceptually in Sec. IV.

B. Two dimensions

For dimensional regularization, we work in $2+\epsilon$ dimensions. In terms of the fermion mass M, the zero-temperature effective potential is given by

$$\frac{V_0(\sigma,\rho)}{N}=\frac{\sigma^2}{2\pi}\left[-1+2\ln\left(\frac{\max(|\sigma|,|\rho|)+\sqrt{\max^2(|\sigma|,|\rho|)-\sigma^2}}{M}\right)\right]+\frac{\rho^2}{2R^2}-\frac{|\rho|}{\pi}\sqrt{\max^2(|\sigma|,|\rho|)-\sigma^2}, \tag{20}$$

where Eqs. (11) and (14) were used. Unlike in three dimensions, for large $|\rho|$ the effect of the probability distribution $[\rho^2/(2R^2)]$ is comparable to the last term ($\approx -\rho^2/\pi$) in Eq. (20) arising from quantum effects. The charge conjugation symmetry can be controlled by the strength of disorder R^2. The system is in the symmetric state for $0<R^2<\pi/2$, while in the broken state for $R^2>\pi/2$. Fermions and antifermions are equally probable in the symmetric state ($\langle\rho\rangle=0$), but only fermions (or antifermions) are allowed in the broken state ($\langle\rho\rangle=\pm\infty$). Our system suffers from a quantum phase transition at $R^2=\pi/2$ ($\equiv R_c^2$). The gap equations have solutions ($|\sigma|,|\rho|$): (i) (0,0), (ii) (M,0), and (iii) $[\sqrt{(2R^2-\pi)/(2R^2+\pi)}M, 2R^2M/(2R^2+\pi)]$ for $R^2\neq R_c^2$, and (i) (0,$\forall|\rho|$) and (ii) (M,0) for $R^2=R_c^2$. The solution (iii) exists only for $R^2>R_c^2$ and corresponds to saddle points. Figure 2 shows the zero-temperature effective potential as a function of σ/M and ρ/M in (a) the symmetric and (b) the broken phase for the charge conjugation symmetry.

To examine the high-temperature ($\beta\to 0$) behavior, let us introduce the dimensionless quantities $\tilde{V}_T=\beta^2 V_T$, $\tilde{\sigma}=\beta\sigma$, $\tilde{\rho}=\beta\rho$, $\tilde{M}=\beta M$. We want to expand the finite-temperature effective potential in $\tilde{\sigma}$ and $\tilde{\rho}$ at high temperature (i.e., for small $\tilde{\sigma}$ and $\tilde{\rho}$). In terms of the dimensionless quantities, Eqs. (12) and (13) are reduced to

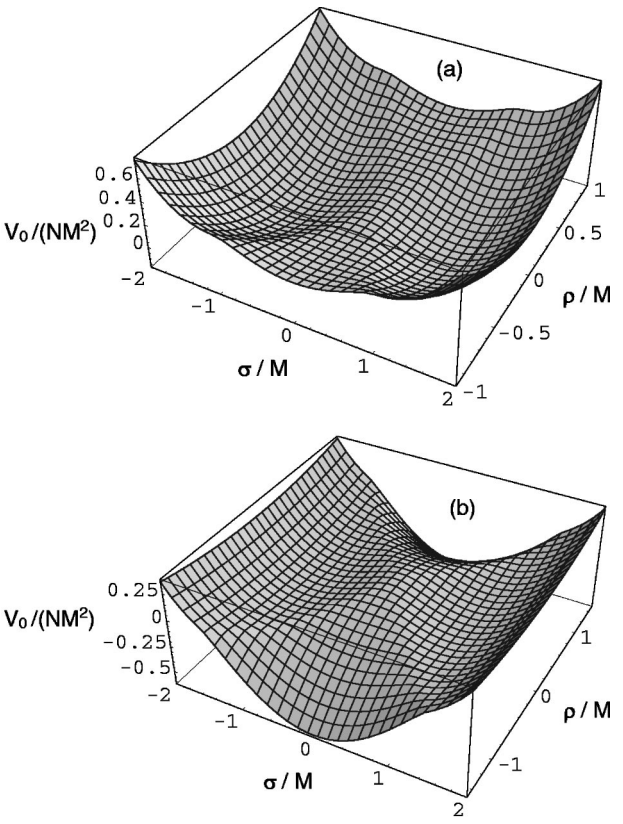

FIG. 2. The zero-temperature effective potential $V_0/(NM^2)$ in two dimensions as a function of σ/M and ρ/M, for (a) $R^2=1$ and (b) $R^2=10$.

$$\frac{\partial}{\partial\tilde{\sigma}}\left(\frac{\tilde{V}_T}{N}\right)=\frac{2\tilde{\sigma}}{\pi}\left[\ln\left(\frac{|\tilde{\sigma}|}{\tilde{M}}\right)+\int_0^\infty dx\,\frac{1}{\sqrt{x^2+\tilde{\sigma}^2}}\times\left(\frac{1}{1+e^{\sqrt{x^2+\tilde{\sigma}^2}+\tilde{\rho}}}+\frac{1}{1+e^{\sqrt{x^2+\tilde{\sigma}^2}-\tilde{\rho}}}\right)\right]$$
$$=\frac{2\tilde{\sigma}}{\pi}\left[\left\{\ln\left(\frac{\pi}{\tilde{M}}\right)-\gamma+O(\tilde{\sigma}^2)\right\}+\left\{\frac{7\zeta(3)}{4\pi^2}+O(\tilde{\sigma}^2)\right\}\tilde{\rho}^2+O(\tilde{\rho}^4)\right], \tag{21}$$

$$\frac{\partial}{\partial\tilde{\rho}}\left(\frac{\tilde{V}_T}{N}\right)=\tilde{\rho}\left[\frac{1}{R^2}-\frac{2\sinh(\tilde{\rho})}{\pi\tilde{\rho}}\times\int_0^\infty dx\,\frac{1}{\cosh(\sqrt{x^2+\tilde{\sigma}^2})+\cosh(\tilde{\rho})}\right]$$
$$=\tilde{\rho}\left[\frac{1}{R^2}-\frac{2}{\pi}+\left\{\frac{7\zeta(3)}{2\pi^3}+O(\tilde{\rho}^2)\right\}\tilde{\sigma}^2+O(\tilde{\sigma}^4)\right]. \tag{22}$$

To obtain the $O(\tilde{\rho}^0)$ term in the brackets of Eq. (21) we used the integration formula [20] for small $\tilde{\sigma}^2$:

$$\int_0^\infty dx\,\frac{1}{\sqrt{x^2+\tilde{\sigma}^2}(1+e^{\sqrt{x^2+\tilde{\sigma}^2}})}=-\frac{1}{2}\left[\ln\left(\frac{|\tilde{\sigma}|}{\pi}\right)+\gamma+O(\tilde{\sigma}^2)\right]. \tag{23}$$

Integrating Eqs. (21) and (22), at sufficiently high temperature, we obtain

$$\frac{V_T(\sigma,\rho)}{N}=\frac{1}{\pi}\left[\ln\left(\frac{\pi T}{M}\right)-\gamma\right]\sigma^2+\frac{1}{2}\left(\frac{1}{R^2}-\frac{2}{\pi}\right)\rho^2+\frac{7\zeta(3)}{4\pi^3T^2}\sigma^2\rho^2+\cdots, \tag{24}$$

up to a field-independent constant. Since the $O(\tilde{\sigma}^0)$ term in the brackets of Eq. (22) is exact, $V_T(\sigma,\rho)$ has no term higher than the ρ^2 term that consists of ρ fields only. At high temperature, the Z_2 chiral symmetry is always restored. However, the behavior of the charge conjugation symmetry is the same as that at zero temperature. Hence, charge conjugation symmetry is not restored at high temperature, for $R^2>R_c^2$.

Until now in order to find the effective potential $V_T(\sigma,\rho)$ we have used the mean field approximation (MFA) by taking σ and ρ as constant fields. This corresponds to the leading approximation in the $1/N$ expansion where the σ and ρ loops

(i.e., the fluctuations of the σ and ρ fields) are not included. Let us check the validity of our calculations. In the large N limit the MFA is good, while for large but finite N it may fail due to the contribution from kinks in two dimensions [21,22].

At first consider the case of the usual two-dimensional GN model (without the random chemical potential) where the MFA predicts a wrong critical temperature T_0 $(\neq 0)$ [21]. For $0 \leqslant T < T_0$ the MFA effective potential is the double well with two degenerate minima at $M(T)$ and $-M(T)$, the solutions of the gap equation. As a result of quantum tunneling between two degenerate minima, the system has kink solutions alternating between $M(T)$ and $-M(T)$. They have higher energies than the constant solutions $M(T)$ and $-M(T)$. The Helmholtz free energy F is related to the internal energy U and the entropy S by $F=U-TS$. Since $F=U$ at zero temperature, the constant solution is favored and the MFA is expected to be valid. If we consider the contribution from kinks explicitly, we find the average (or blocking) potential [23] which has a plateau between $M[\equiv M(0)]$ and $-M$ [21]. Even in the presence of an arbitrarily small external field this potential is tilted to favor M or $-M$. Hence the contribution from kinks does not change the physics materially and the MFA is qualitatively valid at zero temperature. For $0<T<T_0$ the number of kink configurations is sufficiently large to gain enough entropy and so their probability is overwhelming. Since the region of $\sigma=M(T)$ will, on the average, have the same weight as those of $\sigma=-M(T)$, we have $\langle\sigma\rangle=0$, which indicates the breakdown of the MFA. For $T \geqslant T_0$ the system is in the symmetric phase $[M(T)=0]$ in the MFA and thus has no kink solutions. Consequently, for the two-dimensional GN model the MFA is good only at zero and high temperature ($T \geqslant T_0$) and, by the formation of kinks, the true critical temperature turns out to be zero.

Now let us examine the validity of the MFA for the two-dimensional GN model with the random chemical potential, Eq. (3) or (4). For the purpose of the present paper we consider only the cases of zero and sufficiently high temperature. At zero temperature the MFA effective potential $V_0(\sigma,\rho)$ has degenerate minima at $(|\sigma|,|\rho|)$: $(M,0)$ for $0<R^2 \leqslant R_c^2$ and $(0,\infty)$ for $R^2>R_c^2$ [see Figs. 2(a) and 2(b)]. Hence for $0<R^2 \leqslant R_c^2$ our system may have kink solutions for σ alternating between M and $-M$, but no kinks for ρ. In this case the situation is similar to that of the usual GN model in the previous paragraph. Thus it is expected that the MFA is good at zero temperature and for $0<R^2 \leqslant R_c^2$. For $R^2>R_c^2$ the MFA effective potential is unbounded from below and there is no tunneling between two degenerate ground states because of an infinitely high barrier. So σ and ρ do not have kink solutions. At sufficiently high temperature the σ and ρ fields are decoupled from each other in $V_T(\sigma,\rho)$ and can be treated separately. By restoration of the Z_2 chiral symmetry in the MFA, σ has no kink solutions, irrespective of the value of the disorder strength. For $0<R^2<R_c^2$ the charge conjugation symmetry is preserved in the MFA and so there are no kinks for ρ. For $R^2>R_c^2$ the situation is the same as that at zero temperature. For $R^2=R_c^2$ the MFA effective potential for ρ vanishes. As R^2 is tuned through R_c^2, the charge conjugation symmetry undergoes a first-order phase transition from the symmetric phase $(\langle\rho\rangle=0)$ to the broken phase $(\langle\rho\rangle=\infty$ or $-\infty)$, following positive or negative values of $\langle\rho\rangle$ according to the value of $\langle\rho\rangle$ in the broken phase. So it is reasonable to assume that ρ has no kink solution at $R^2=R_c^2$. As a result, the MFA is reliable at zero and high temperature for all values of the disorder strength.

IV. ORIGIN OF CHARGE CONJUGATION SYMMETRY NONRESTORATION

In this section we discuss the mechanism of SNR for the charge conjugation symmetry conceptually at the quantum level. For convenience set $\sigma=0$ in Eqs. (4) and (5) because SNR is the effect of the random chemical potential. That is, we neglect the four-fermion interaction $(\bar{\Psi}\Psi)^2$ and consider the system of free massless Dirac fermions in the presence of the random chemical potential. To leading order in the $1/N$ expansion the finite-temperature effective potential $V_T(0,\rho)/N$ consists of two parts: (i) a term from probability distribution $[\rho^2/(2R^2)\equiv P_R(\rho)]$ and (ii) the grand free energy (or grand potential) for free massless Dirac fermions at constant chemical potential ρ $[\equiv\Omega_T(\rho)]$.

So it is essential to conceptually determine the value of the chemical potential that $\Omega_T(\rho)$ favors. By the symmetry of $\Omega_T(\rho)$ we can restrict ourselves to the positive chemical potential without loss of generality. At first, consider the case of zero temperature. According to Fermi-Dirac statistics, fermions fill all the energy levels to the Fermi energy (=chemical potential) and antifermions are suppressed. Hence, the larger chemical potential, the larger charge (number) density (= fermion number density − antifermion number density). This result is retained at nonzero temperature. Since the grand free energy density is minus the pressure [24], it is a decreasing function of the (positive) charge density. Consequently, the large charge density (i.e., large chemical potential) is preferred and, for all temperatures, $\Omega_T(\rho)$ is minimized at large chemical potential $(|\rho|\to\infty)$. This implies SNR for the charge conjugation symmetry in the open system of free massless Dirac fermions. Moreover, we can guess the functional form of $\Omega_T(\rho)$ by dimensional analysis: At zero temperature, $\Omega_0(\rho)\propto -|\rho|^d$ in d dimensions. At sufficiently high temperature, $\Omega_T(\rho)\propto -T\rho^2+\mathrm{O}(\rho^4/T)$ in three dimensions and $\Omega_T(\rho)\propto -\rho^2+\mathrm{O}(\rho^4/T^2)$ in two dimensions, up to a field-independent constant. These qualitative results can be checked explicitly from Eqs. (15), (19), (20), and (24).

Now let us consider the contribution $P_R(\rho)$ from the probability distribution of the random chemical potential. In three dimensions, for large $|\rho|$, $\Omega_T(\rho)$ exceeds $P_R(\rho)$ at zero and sufficiently high temperature, irrespective of the value of the disorder strength. Therefore, the initially spontaneously broken charge conjugation symmetry is not restored at high temperature. In two dimensions $P_R(\rho)$ is comparable to $\Omega_T(\rho)$. The probability distribution of the random chemical potential for weak disorder (small R^2) is dominated

at $\rho=0$ and charge conjugation symmetry is preserved at zero and high temperature. However, for strong disorder (large R^2) all values of the chemical potential have small probability density and so the fate of the charge conjugation symmetry is determined by $\Omega_T(\rho)$. Thus, in this case, the initially spontaneously broken charge conjugation symmetry is not restored at high temperature.

V. CONCLUSIONS

In the present paper we examined the symmetry behavior of the Gross-Neveu model with random chemical potential which is equivalent to the four-fermion model, Eq. (3). We used the leading approximation in the $1/N$ expansion (i.e., the mean field approximation). Our model has charge conjugation symmetry as well as Z_2 chiral symmetry. The initially spontaneously broken Z_2 chiral symmetry is always restored at high temperature. In three dimensions, the charge conjugation symmetry that is broken spontaneously at zero temperature is not restored at high temperature, irrespective of the value of the disorder strength R^2. In two dimensions, at zero temperature, the charge conjugation symmetry is not broken spontaneously for weak disorder $[0<R^2<R_c^2 (=\pi/2)]$, but broken for strong disorder $(R^2>R_c^2)$. Therefore, our system exhibits a quantum phase transition at $R^2=R_c^2$ as the value of R^2 is varied. For any given value of R^2 the high-temperature behavior of the charge conjugation symmetry is the same as its zero-temperature behavior. Hence charge conjugation symmetry remains broken at high temperature (i.e., symmetry nonrestoration) for $R^2>R_c^2$. By examining the existence of kink solutions we checked that the mean field approximation is reliable even in two dimensions at zero and high temperature.

In addition, we discussed our results on charge conjugation symmetry nonrestoration conceptually, after neglecting the four-fermion interaction $(\bar{\Psi}\Psi)^2$ for convenience because symmetry nonrestoration is the effect of the random chemical potential. The behavior of charge conjugation symmetry is determined by competition between two terms in the finite-temperature effective potential: (i) the term $\rho^2/(2R^2)$ from the Gaussian distribution for the chemical potential ρ that favors the symmetric phase $(\langle\rho\rangle=0)$ and (ii) the grand free energy for free massless Dirac fermions which favors the broken phase $(|\langle\rho\rangle|\to\infty)$.

As further work, it would be worthwhile to perform next-to-leading order calculations and consider a non-Gaussian distribution for the random chemical potential.

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