# D-brane solitons in supersymmetric sigma models

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(Received 3 November 2000; published 27 February 2001)

Massive $D=4$, $N=2$ supersymmetric sigma models typically admit domain wall ($Q$-kink) solutions and string ($Q$-lump) solutions, both preserving 1/2 supersymmetry. We exhibit a new static 1/4 supersymmetric "kink-lump" solution in which a string ends on a wall, and show that it has an effective realization as a bion of the $D=4$ super DBI action. It is also shown to have a time-dependent $Q$-kink-lump generalization which reduces to the $Q$ lump in a limit corresponding to infinite BI magnetic field. All these 1/4 supersymmetric sigma-model solitons are shown to be realized in M theory as calibrated, or "$Q$ calibrated," M5-branes in an M-monopole background.

DOI: 10.1103/PhysRevD.63.085002 PACS number(s): 11.27.+d

## I. INTRODUCTION

Although D-branes are normally defined within perturbative string theory in terms of Dirichlet boundary conditions at the end points of open strings, they may be defined more generally as branes on which strings can end. As such, D-branes may occur in field theories. An example is provided by the domain walls (alias 2-branes) of M-theory QCD (MQCD), which were shown in [1] to be surfaces on which MQCD strings may end. However, the physics of strings and walls in MQCD is quite different from that of the D2-branes of the IIA superstring theory because the end points of MQCD strings are not electric sources for a gauge field on the wall. Other examples of (non-supersymmetric) field theory D-branes have been discussed in [2], although the physics is again rather different from that of string theory D-branes.

A field theory domain wall that is a much closer analogue of the D2-brane of type IIA superstring theory is provided by the kink domain wall of massive hyper-Kähler (HK) sigma models [3]. As pointed out in [3], the effective action for the kink domain wall is the $S^1$ reduction of the $D=5$ supermembrane, and hence dual to a gauge theory. This is similar to the relation between the $D=11$ supermembrane and the $D=10$ D2-brane action [4], and the same arguments used in that case imply that the gauge theory in question is a supersymmetric one of Dirac-Born-Infeld (DBI) type. As in the $D=10$ case [5], this $D=4$ action admits 1/2 supersymmetric bion solutions that can be interpreted as strings ending on a membrane. But what are these sigma-model strings? This is one of several questions that we aim to answer in this paper. Another is whether there is a 1/4 supersymmetric sigma-model configuration representing a string ending on a domain wall, as the analogy with superstring D-branes suggests. Indeed there is, and for simple models it can be found explicitly and its properties studied in detail.

Specifically, we shall consider $D=4$ supersymmetric sigma models with a "multi-center" HK target space 4-metric of the form

$$ds^2=U d\mathbf{X}\cdot d\mathbf{X}+U^{-1}(d\varphi+d\mathbf{X}\cdot\mathbf{A})^2 \tag{1}$$

where $\boldsymbol{\nabla}\times A=\boldsymbol{\nabla}U$ and $U$ is a "multi-center" harmonic function. The only potential term consistent with maximal supersymmetry is proportional to the norm of the tri-holomorphic Killing vector field $\zeta=\partial/\partial\varphi$, and so takes the form

$$V=\frac{1}{2}\mu^2U^{-1} \tag{2}$$

where $\mu$ is a mass parameter. Introducing a coupling constant $g$ with dimensions of inverse mass, we have the sigma-model Lagrangian density

$$\mathcal{L}=-\frac{1}{2g^2}\{\eta^{\mu\nu}[U\partial_\mu\mathbf{X}\cdot\partial_\nu\mathbf{X}+U^{-1}\mathcal{D}_\mu\varphi\mathcal{D}_\nu\varphi]+\mu^2U^{-1}\} \tag{3}$$

where $\eta$ is the $D=4$ Minkowski metric (of "mostly plus" signature) and $\mathcal{D}\varphi=d\varphi+d\mathbf{X}\cdot\mathbf{A}$. When $\mu\neq0$ we have a "massive" sigma model; otherwise it is massless.

The massless sigma models typically admit 1/2 supersymmetric "lump" solitons [6] supported by a topological "lump" charge $L$. These are of course string-like solitons in $D=4$. Lump-string configurations also exist in the massive model, with a string tension that is bounded from below by the lump charge $L$, but Derrick's theorem implies that the bound is saturated only in the limit in which the string core has shrunk to zero size, yielding a singular field configuration. In other words, the massive sigma model admits Bogomol'nyi-Prasad-Sommerfield (BPS) strings that are "fundamental" in the sense that the core size vanishes. One might be tempted to ignore these strings on the grounds that

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they are singular, but there are various circumstances in which the singularity is resolved. For example, the singularity can be removed, and Derrick's theorem evaded, by incorporating time dependence. Indeed, there exists a time-dependent non-singular charged lump-string solution; its cross section is the $D=3$ $Q$-lump solution found by Abraham [7]. These solutions saturate an energy bound of the form

$$E \geqslant |L| + |Q| \tag{4}$$

where $Q$ is the Noether charge associated with the symmetry generated by $\zeta$. Although the solution is not static, it is *stationary* in the sense that the energy density is time independent, a fact that allows it to preserve some fraction of supersymmetry. This fraction was not previously determined but we shall show here that HK $Q$ lumps are 1/4 supersymmetric. Massive HK sigma models also admit kinks (static solitons that interpolate between the minima of the potential) and $Q$ kinks. The $Q$ kinks are stationary charged kinks that saturate an energy bound of the form

$$E \geqslant \sqrt{|\mathbf{K}|^2 + Q^2} \tag{5}$$

where $\mathbf{K}$ is a triplet of topological kink charges. The $Q$ kink with $Q=0$ is the static kink. Both kinks and $Q$ kinks preserve 1/2 supersymmetry.

The main result of this paper is a new non-singular *static* 1/4 supersymmetric soliton which we call the kink lump. It has a natural interpretation as a string ending on a domain wall. To see why such configurations might be anticipated, we begin by recalling that the $D=3$ $Q$ lump can be viewed as a closed loop of $D=3$ $Q$-kink string [8], so the $D=4$ $Q$-lump string can be viewed as a cylindrical tube of the $Q$-kink domain wall. If this tube is splayed out at one end, we have a (non-static) configuration representing a string ending on a wall. If we now remove the charge, we might expect to end up with a static solution of similar type but with the string core supported against collapse by its attachment to the wall. The kink lump is just such a solution. The size of the string core decreases with distance from the wall, so its shape is more accurately described as a ''spike'' than as a ''tube.'' Nevertheless, the spike has a constant energy per unit length and can therefore be interpreted as a string of fixed tension. This tension turns out to equal the tension of the singular infinite lump string, but the kink lump is completely non-singular because the ''spike'' shrinks to zero size only at infinite distance from the wall.

These results are reminiscent of the bion solution on a D2-brane [5]. For example, the end point of the bion string on a D2-brane is essentially a global vortex with a logarithmically infinite energy, which leads to a logarithmic bending of the D2-brane. We shall show that the kink lump incorporates the same logarithmic bending of the kink domain wall. Moreover, the way in which the singular lump string is ''blown up'' into a cylindrical kink domain wall is reminiscent of the way that a IIA type superstring can be ''blown up'' into a cylindrical D2-brane [9]. However, there is an important difference. The bion is a solution of the field theory governing the fluctuations of the wall, so the wall itself is not part of the solution. The bion spike remains hollow no matter how much it shrinks because the width of the wall itself is assumed to vanish. In contrast, the kink domain wall is part of the kink-lump solution and it has a definite thickness; as the spike shrinks to a size comparable to the thickness of the wall it must ''fill in'' to form a ''solid spike.'' We shall see explicitly how this happens in the kink-lump solution. The bion analogy is really more appropriate to an effective description of the kink lump as a 1/2 supersymmetric soliton of the effective theory governing fluctuations of a kink domain wall because, as mentioned above, the kink effective action is just a $D=4$ version of the $D=10$ super D2-brane, and the kink lump can indeed be identified as a bion of this theory.

Another result of this paper is a non-static but stationary generalization of the kink lump which we call a $Q$-kink lump. It can be viewed as a kink lump boosted in the ''hidden'' fifth dimension. In the limit of infinite boost, to the speed of light, the $Q$-kink lump reduces to the $Q$ lump, so the $Q$-kink lump is the generic 1/4 supersymmetric soliton of the massive HK sigma models under consideration. A boost of the $D=5$ supermembrane in the fifth dimension corresponds to the inclusion of a constant background magnetic field in the effective $D=4$ DBI action describing the kink domain wall. Using the methods of [10], we find the bion solution in this background and confirm its status as the effective description of the $Q$-kink lump. An interesting feature of this result is that the limit of infinite boost, in which the $Q$-kink lump becomes the $Q$ lump, corresponds to a limit of infinite magnetic field in the DBI theory.

Although we are concerned here with field theory solitons, most supersymmetric field theories arise as effective theories in some superstring or M-theory context, and their soliton solutions thereby acquire a superstring or M-theory interpretation. The 1/2 supersymmetric kinks and $Q$ kinks of the models discussed here were provided with several such interpretations in [11,12]. Here we shall show that the 1/4 supersymmetric kink lump extends to a solution of the M5-brane equations of motion, in a multi-M-monopole background. As such, it provides an example of a calibrated M5-brane preserving 1/16 of the supersymmetry of the M-theory vacuum. A similar result holds for the $Q$-kink lump (and hence the $Q$ lump) with the difference that the solution is time dependent. It is thus a generalization of a calibration, of a type first discussed in [13], that could be called a ''$Q$ calibration.''

We shall begin with a discussion of the sigma model field theories and their solitons, including the kink lump and the $Q$-kink lump, and their properties. We then discuss the effective description of the kink lump in terms of a $D=4$ DBI action for a sigma-model D2-brane, and show that the $Q$-kink lump can then be found by considering the DBI action in a constant background magnetic field. We then show how all these 1/4 supersymmetric solitons determine supersymmetric minimal energy configurations of the M5-brane in a multi M-monopole background. We conclude with a discussion of some other issues.

## II. KINKY LUMPS

The sigma models of relevance here have as their target space a HK 4-manifold of the type described above. The simplest choice of the harmonic function $U$ that serves our purposes is

$$U=a+\frac{1}{2}\left[\frac{1}{|\mathbf{X}-\mathbf{n}|}+\frac{1}{|\mathbf{X}+\mathbf{n}|}\right], \tag{6}$$

where $\mathbf{n}$ is a unit 3-vector and $a$ a constant. The function $U$ is singular at the two ''centers'' $\mathbf{X}=\pm\mathbf{n}$, but this is a coordinate singularity of the metric if $\varphi$ is periodically identified with period $2\pi$. When $a=0$ we have the Eguchi-Hanson metric. For $a=1$ we have the asymptotically flat metric transverse to two M monopoles. In either case, the metric is HK with the triplet of Kähler 2-forms

$$\mathbf{\Omega}=(d\varphi+d\mathbf{X}\cdot\mathbf{A})d\mathbf{X}-\frac{1}{2}U\,d\mathbf{X}\times d\mathbf{X}, \tag{7}$$

the wedge product of forms being implicit.

The 2-center metric (and, more generally, any multi-center metric with colinear centers) has an additional Killing vector field generating rotations about the $\mathbf{n}$ axis. This Killing vector field is holomorphic with respect to the complex structure $I$ associated with the Kähler 2-form $\Omega=\mathbf{n}\cdot\mathbf{\Omega}$. The 3-vector $\mathbf{X}$ of $SO(3)$ can be decomposed into the singlet $X=\mathbf{n}\cdot\mathbf{X}$ and a doublet under the $SO(2)$ subgroup that fixes $\mathbf{n}$. The HK sigma model can then be consistently truncated to a Kähler sigma model by keeping only the singlet fields $(\varphi,X)$. Because the truncation is consistent, any solution of the reduced Kähler sigma-model equations will solve the full HK sigma-model equations. The metric on this 2-dimensional Kähler subspace of the target space is

$$ds^2(K_2)=UdX^2+U^{-1}d\varphi^2 \tag{8}$$

where, for $|X|\leqslant 1$,

$$U=a+\frac{1}{1-X^2}. \tag{9}$$

The Kähler 2-form is $\Omega=\mathbf{n}\cdot\mathbf{\Omega}$ and, since one can choose $\mathbf{A}$ such that $\mathbf{n}\cdot\mathbf{A}=0$ [14], we have

$$\Omega=d\varphi\wedge dX, \tag{10}$$

which is the volume form on the 2-sphere. The lump charge $L$ is the integral of the pullback of $\Omega$, so its minimum value is $4\pi$, the area of the two-sphere. This is the tension of the singular lump string.

Although $D=5$ is the maximal dimension in which we may have a massive supersymmetric sigma model (a point that we return to in the concluding discussion), it will be sufficient for our purposes to consider a $D=4$, $N=2$ model with Lagrangian density (3). For simplicity we shall set $\mu=1$ and $g=1$. After the truncation to the $N=1$ supersymmetric Kähler sigma model described above, this yields the energy density

$$\mathcal{E}=\frac{1}{2}[U(\dot{X}^2+|\nabla X|^2)+U^{-1}(\dot{\varphi}^2+|\nabla\varphi|^2+1)], \tag{11}$$

which can be rewritten as

$$\begin{aligned}\mathcal{E}=&\frac{1}{2}[U\dot{X}^2+U^{-1}(\nabla_1\varphi)^2]+\frac{1}{2}[U^{-1}(\dot{\varphi}-v)^2\\&+U(\nabla_1X\mp\sqrt{1-v^2}\,U^{-1})^2]+\frac{1}{2}[U^{-1}\\&\times(\nabla_2\varphi-\sigma U\nabla_3X)^2+U^{-1}(\nabla_3\varphi+\sigma U\nabla_2X)^2]\\&+vU^{-1}\dot{\varphi}\pm\sqrt{1-v^2}\,\nabla_1X+\sigma(\nabla\varphi\times\nabla X)_1\end{aligned} \tag{12}$$

for constant $v$, with $|v|\leqslant 1$, and $\sigma=\pm1$. Noting that

$$Q=\int d^3x\,U^{-1}\dot{\varphi} \tag{13}$$

is a Noether charge (associated with the triholomorphic isometry of the original HK target space metric) we see that the above expression for the energy density implies (by appropriate choice of $v$) the following (formal) bound on the total energy $E$,

$$E\geqslant\sqrt{Q^2+K^2}+|L|, \tag{14}$$

where $K$ and $L$ are the topological kink and lump charges:

$$K=\int d^3x\,(\nabla X)_1,\quad L=\int d^3x\,(\nabla\varphi\times\nabla X)_1. \tag{15}$$

Note that $L$ is the pullback to the 23-plane of the Kähler 2-form $\Omega$. The bound is saturated when

$$\dot{X}=0,\quad \dot{\varphi}=v=\frac{Q}{\sqrt{Q^2+K^2}} \tag{16}$$

and

$$\nabla_1\varphi=0,\quad \nabla_1X=\pm(\sqrt{1-v^2})\,U^{-1} \tag{17}$$

and

$$\nabla_2\varphi=\sigma U\nabla_3X,\quad \nabla_3\varphi=-\sigma U\nabla_2X. \tag{18}$$

To solve these equations it will prove convenient to set

$$X=\pm\tanh u,\quad \varphi=vt+\psi, \tag{19}$$

for time independent $u,\psi$. The equation for $X$ in Eq. (17) then becomes

$$\partial_1(u+a\tanh u)=\sqrt{1-v^2}. \tag{20}$$

Let us also set

$$x^1=x,\quad x^2\pm ix^3=z. \tag{21}$$

The function $u(x,z)$ is then given implicitly by

$$u+a\tanh u=\sqrt{1-v^2}x+\sigma\log w(z,\bar{z}), \tag{22}$$

and the two real equations (18) are now equivalent to the single complex equation

$$\bar{\partial}(\psi+i\log w)=0, \tag{23}$$

where $\bar{\partial}$ indicates a partial derivative with respect to $\bar{z}$. Equivalently,

$$we^{-i\psi}=Z(z) \tag{24}$$

for arbitrary holomorphic function $Z$.

We have now found an implicit, but general, solution of Eqs. (17) and (18). For $a=0$ the solution can be given explicitly. Choosing the upper sign and $\sigma=1$ we have

$$X=\tanh[\sqrt{1-v^2}\,x+\log w] \tag{25}$$

with $\psi=-\arg Z$. For constant $Z$ both $\psi$ and $w$ are constant and we recover the $Q$-kink solution of [3]. Other choices of $Z(z)$ yield new solutions. For example, we could have $Z=\lambda/z$ for arbitrary complex constant $\lambda$. Consider, for simplicity,

$$Z(z)=\frac{1}{z}. \tag{26}$$

In this case $\psi=\arg z$ and

$$X=\tanh[\sqrt{1-v^2}\,x-\log|z|]. \tag{27}$$

For fixed $z$ we have a kink solution but for fixed $x$ we have a sigma-model lump solution. This can be seen, for example, by noting that $X\to1$ as $z\to0$ and $X\to-1$ as $z\to\infty$. For fixed $x$ the sigma model lump has scale size $\exp(\sqrt{1-v^2}x)$. This is the simplest $Q$-kink-lump solution. The static kink lump is found by setting $v=0$ while the $Q$ lump is obtained by setting $v=1$.

We now turn to a determination of the fraction of supersymmetry preserved by the kink lump, $Q$-kink lump and $Q$ lump. A formula for supersymmetric configurations of $D=6$ sigma models with $4k$-dimensional toric HK target spaces was obtained in [15]. Specializing to the $k=1$ case we conclude that supersymmetric configurations of $D=6$ sigma models are those for which the equation

$$[\Gamma^m\boldsymbol{\tau}\cdot\partial_m\mathbf{X}+iU^{-1}\Gamma^m\mathcal{D}_m\varphi]\epsilon=0 \tag{28}$$

admits solutions for non-zero $D=6$ $Sp_1$-Majorana-Weyl spinor $\epsilon$, where $\boldsymbol{\tau}$ are the Pauli matrices and $\Gamma^m$ ($m=0,1,2,3,4,5$) are the $D=6$ Dirac matrices. To apply this formula we note that the massive $D$=4 sigma model discussed here is obtained from the $D=6$ model by setting

$$\partial_4\mathbf{X}=\partial_5\mathbf{X}=0,\quad \partial_4\varphi=1,\quad \partial_5\varphi=0. \tag{29}$$

Given also that

$$\mathbf{X}=X\,\mathbf{n},\quad \dot{X}=0, \tag{30}$$

the supersymmetry condition becomes

$$[U(\boldsymbol{\tau}\cdot\mathbf{n})(\boldsymbol{\Gamma}\cdot\nabla X)+i(\boldsymbol{\Gamma}\cdot\nabla\varphi)+i\Gamma^0\dot{\varphi}+i\Gamma^4]\epsilon=0 \tag{31}$$

where $\boldsymbol{\Gamma}=(\Gamma^1,\Gamma^2,\Gamma^3)$. For the $Q$-kink lump this yields

$$\{1-[v\Gamma^{04}\mp i\sqrt{1-v^2}\,(\boldsymbol{\tau}\cdot\mathbf{n})\Gamma^{14}]\}\epsilon + \Gamma^4(\boldsymbol{\Gamma}\cdot\nabla\varphi)[1-i\sigma(\boldsymbol{\tau}\cdot\mathbf{n})\Gamma^{23}]\epsilon=0. \tag{32}$$

When $\nabla\varphi$ vanishes we have

$$(v\Gamma^{04}\mp i\sqrt{1-v^2}\Gamma^{14}\boldsymbol{\tau}\cdot\mathbf{n})\epsilon=\epsilon, \tag{33}$$

which confirms the 1/2 supersymmetry of the kink and $Q$ kink. When $\nabla\varphi$ is non-zero we have, in addition, that

$$i\Gamma^{23}(\boldsymbol{\tau}\cdot\mathbf{n})\epsilon=\epsilon. \tag{34}$$

The combined conditions imply 1/4 supersymmetry, for any $v$. We conclude that the kink lump, $Q$ lump and $Q$-kink lump are all 1/4 supersymmetric.

## III. ENERGETICS

We shall now set $a=0$ for simplicity, and again choose $\sigma=1$. Then, when the $Q$-kink-lump solution is used in the expression for the energy density, one finds that

$$\mathcal{E}=\frac{4e^{2y}}{(1+e^{2y}|Z|^2)^2}[|Z|^2+|Z'|^2] \tag{35}$$

where we have set

$$y=\sqrt{1-v^2}\,x \tag{36}$$

for convenience. If we integrate the energy density over $x$, we find the energy density on the domain wall to be

$$\mathcal{E}_{wall}=2\gamma(1+|Z'|^2/|Z|^2), \tag{37}$$

where

$$\gamma=\frac{1}{\sqrt{1-v^2}}. \tag{38}$$

By taking $Z$ to be constant we see that the wall's surface tension is $2\gamma$.

For the moment we postpone the analysis of $\mathcal{E}_{wall}$ for non-constant $Z$ and return to the unintegrated formula (35). For a general $Q$-kink-lump solution with

$$Z(z)=\sum_i\frac{\lambda_i}{z-z_i} \tag{39}$$

we have, in the limit of large $r=|z|$,

$$\mathcal{E}\sim\frac{4e^{2y}|Z|^2}{(1+e^{2y}|Z|^2)^2}. \tag{40}$$

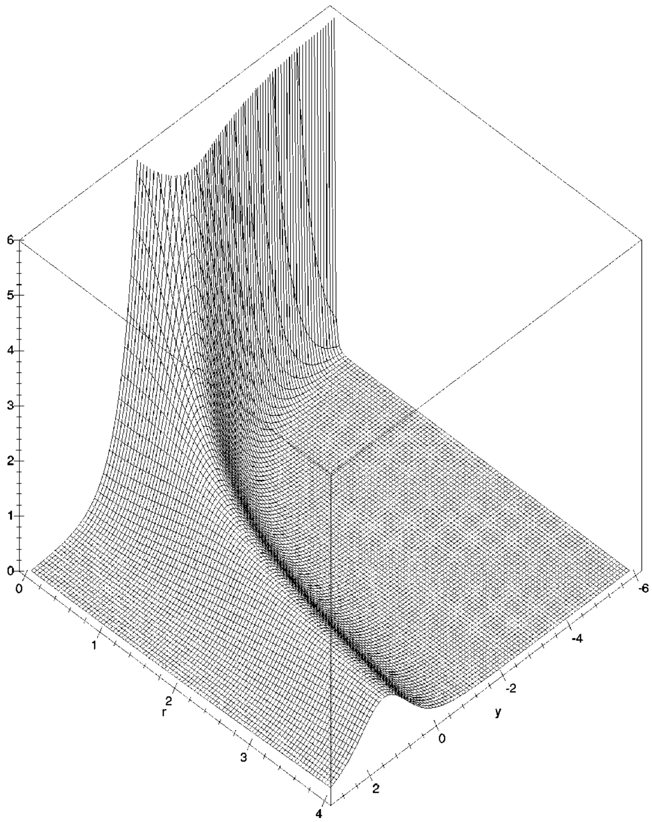


FIG. 1. A plot of the energy function $\mathcal{E}(r,y)$.

This has a maximum when $e^y|Z|=1$, so we can take this as the surface to which the domain wall is asymptotic at large $r=|z|$. This implies that for large $r$ the domain wall is asymptotic to the surface

$$y=\log r, \tag{41}$$

unless $\Sigma_i\lambda_i=0$, in which case $y\sim\log r^2$.

To proceed we shall now focus on the one-lump case with $Z=1/z$. In this case the energy density is

$$\mathcal{E}(r,y)=\frac{4(1+r^2)e^{2y}}{(e^{2y}+r^2)^2}. \tag{42}$$

This function is plotted in Fig. 1 for a range of the independent variables $r$ and $y$. The function $\mathcal{E}(r,y)$ has no extrema (except in the $r\to\infty$ limit discussed above) but some understanding of the solution near $r=0$ can be had by considering the extrema of the cross-sectional energy density $\mathcal{E}(r)$ at fixed $y$. As already noted, the solution for fixed $y$ is a lump that interpolates between $X=1$ at $r=0$ and $X=1$ at $r=\infty$. The function $\mathcal{E}(r)$ has an extremum at $r=0$ and, if

$$y>y_*\equiv\log\sqrt{2}, \tag{43}$$

at

$$r=r_*(y)\equiv\sqrt{e^{2y}-2}. \tag{44}$$

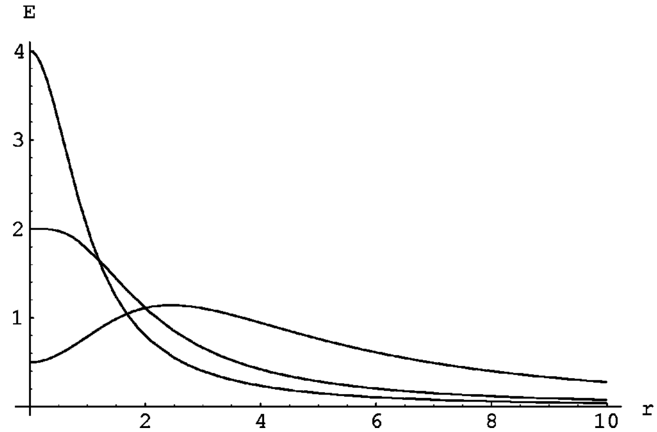


FIG. 2. A plot of $\mathcal{E}(r,y)$ for three fixed values of y with $y>y_*$, $y=y_*$ and $y<y_*$, specified by the values of $\mathcal{E}$ at $r=0$ given by 1/2, 2 and 4, respectively.

When $y>y_*$ the extremum at $r=0$ is a minimum and the extremum at $r=r_*$ is a maximum. The cross-sectional energy density is therefore ring shaped for sufficiently large $y$. The radius of the ring shrinks as $y$ increases; this is the advertised ''spike,'' which is essentially hollow for $y>y_*$. The radius of the ring shrinks to zero at $y=y_*$ and for $y<y_*$ the only extremum of $\mathcal{E}(r)$ is a maximum at $r=0$. The solution remains non-singular in that the energy density remains everywhere smooth and finite, but the cross-sectional lump is no longer ring shaped. The hollow ''spike'' for $y>y_*$ is ''filled in'' for $y<y_*$, as one might expect from the fact that the domain wall has a finite width, of order 1 in our units. This behavior is shown in Fig. 2 in which $\mathcal{E}$ is plotted as a function of $r$ for values of $y>y_*$, $y=y_*$ and $y<y_*$. A natural interpretation of this result is that the string is actually attached to the wall at the point at which $e^{2y}=2$, the wall being deformed by the string's tension just so as to meet the string end point at this distance.

For $y\gg y_*$ it is natural to interpret $r_*\sim e^y$ as the size of the cross-sectional lump. This implies that we have a domain wall with a shape that is again given by $y\sim\log r$, consistent with the asymptotic behavior as $r\to\infty$ that we found earlier. But we also wish to determine the shape for $y\ll y_*$. One way to do this would be to determine the size of the cross-sectional lump as a function of $y$. Since the energy density is centered at $r=0$ for $y\ll y_*$ the size is not related to the position of the maximum of $\mathcal{E}$ for fixed $y$, as it is for $y\gg y_*$. Naively, we might define the size as

$$\langle r\rangle\equiv\int_0^\infty(2\pi r)dr\, r\mathcal{E}(r), \tag{45}$$

but this integral diverges for the simple lump solution with $Z=\lambda/z$. In fact the integral of $\mathcal{E}$ also diverges. Both divergences may be removed by considering a multi-lump solution of the form (39) with $\Sigma_i\lambda_i=0$ but the value of $\langle r\rangle$ is then more naturally interpreted as the mean distance between the constituent lumps (as discussed for the $Q$ lump in [7]).

Since the size of an individual lump is really determined by the energy density for small $r$, we shall proceed by first noting that

$$\mathcal{E}\approx\frac{4e^{2y}}{(e^{2y}+r^2)^2} \qquad (46)$$

for $r\ll 1$. This has a finite integral over the $z$ plane, and $\langle r\rangle$ is also finite. In fact,

$$\langle r\rangle=\text{const}\times e^{y}. \qquad (47)$$

The constant depends on the particular ''regularization'' used. Its value will not be important to us but one may note that we could have defined the shape of the spike in terms of the surface on which $\mathcal{E}(y)$ is a maximum for fixed $r$. This surface is $r=e^{y}$, so we have agreement with the result of considering $\mathcal{E}(r)$ for fixed $y$ if we set the constant in Eq. (47) to unity.

The final conclusion of this analysis is that the shape of the spike for $y\ll y_*$ is given by

$$y=\log r, \qquad (48)$$

just as it was for $y\gg y_*$. A cutoff at a distance $\delta$ from this singularity therefore corresponds to a distance $l$ from the wall with $l$ related to $\delta$ by

$$-\log\delta=\sqrt{1-v^2}\,l+\text{const}. \qquad (49)$$

We now return to the formula (37) for the energy density on the wall. Let us again take $Z=1/z$ and integrate over the $z$ plane, with IR cutoff at $r=R$ and UV cutoff at $r=\delta$. We find that

$$E=2\gamma(\pi R^2+2\pi\log R)+4\pi l+\text{const}+\cdots \qquad (50)$$

where we have used the relation (49) to convert the $\delta$ dependence to a dependence on $l$, and the terms omitted vanish in the limit of $\delta\to 0$. The $R^2$ term can be considered as the vacuum energy of the domain wall. The $\log R$ term is the expected IR divergent energy of a global vortex in $D=3$. The term linear in $l$ can be interpreted as the energy in a string of length $l$ and tension

$$T_{string}=4\pi. \qquad (51)$$

This is precisely the tension of the singular lump string, so the natural interpretation is that the kink lump provides a $D=4$ spacetime description of a normally singular lump string ending on a $D=4$ $Q$-kink domain wall.

Note that all of the above discussion applies for any value of the parameter $v<1$, in particular for $v=0$, which yields the static kink-lump solution. We now turn to the limiting case of $v=1$. In this limit the tube-like mid-section of the lump string gets stretched out, with the wall itself, and the ''solid spike'' region, being pushed off to infinity. We then have an infinite straight $Q$-lump string, i.e., a string with a $Q$-lump core and cross-sectional energy density

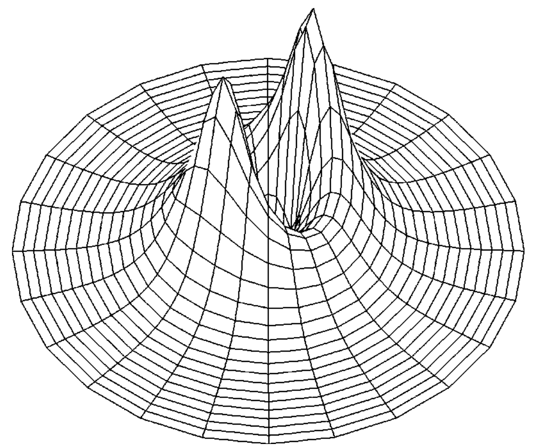

FIG. 3. A plot of the energy density $\mathcal{E}(z)$ for a charge-2 $Q$ lump.

$$\mathcal{E}=\frac{4(|Z|^2+|Z'|^2)}{(1+|Z|^2)^2}. \qquad (52)$$

As noted by Abraham, the integrated cross-sectional energy, i.e. The $Q$-lump string tension, is infinite for a single lump, but is finite for a multi-lump solution with $\Sigma_i\lambda_i=0$. For example, for any complex constant $a$ the choice

$$Z=\frac{1}{z-a}-\frac{1}{z+a} \qquad (53)$$

leads to a non-singular and finite energy charge-2 $Q$-lump solution. We refer to [7] for detailed properties of multi $Q$ lumps, but a plot of the energy density for the above charge-2 $Q$-lump solution is shown for $a=1/2$ in Fig. 3.

## IV. EFFECTIVE D-BRANE DESCRIPTION

As mentioned in the introduction, the new kink-lump solution of massive HK sigma models that we have found and studied here is similar in some respects to the bion solution of the DBI field equations describing the fluctuations of a type IIA superstring theory D2-brane [5]. However, the proper analogy of the kink lump in this context would be to a type IIA *supergravity* solution in which a string ends on a D2-brane, because only in this case would the D2-brane be part of the solution. In this sense, the proper sigma-model analogue of the bion is found by asking whether the 1/4 supersymmetric kink-lump solution can be be understood as a 1/2 supersymmetric solution on the effective $D=3$ field theory governing the fluctuations of the kink domain wall. Indeed, it can be understood this way, as we now describe.

The general static kink solution is given implicitly by

$$X=\pm\tanh[(x-x_0)\mp aX], \quad \varphi=\varphi_0, \qquad (54)$$

where $x_0$ and $\varphi_0$ are two real collective coordinates, with $\varphi_0\sim\varphi+2\pi$. Identification of the collective coordinates as the coordinates of the space transverse to an infinite planar membrane, and the fact that the kink solution preserves 1/2 of the eight sigma-model supersymmetries implies that the kink has an effective description as a supermembrane in a

$D=5$ $\mathbb{E}^{1,3}\times S^1$ spacetime [3]. To see this we allow the collective coordinates to become smooth functions of the world volume coordinates $\xi^i$ $(i=0,1,2)$ to arrive at the world volume fields

$$\phi(\xi)\equiv x_0(\xi), \quad \sigma(\xi)\equiv\varphi_0(\xi), \tag{55}$$

which may be identified with the physical (transverse) boson fields of the supermembrane in the gauge in which three world volume fields taking values in $\mathbb{E}^{(1,2)}$ are identified with the coordinates of an $\mathbb{E}^{(1,2)}$ subset of the $D=5$ spacetime. The physical world volume fields thus determine the position of a membrane in the $\mathbb{E}^{(1,3)}\times S^1$ spacetime. The symmetries of the kink solution then imply that the low energy effective action for these fields is that of the $D=5$ supermembrane [16]. As the kink domain wall tension equals 2 in our mass units, the bosonic action is

$$I=-2\int d^3\xi\sqrt{-\det(g_{ij}+\partial_i\sigma\partial_j\sigma)} \tag{56}$$

where $g_{ij}$ is the metric induced from the $D=4$ Minkowski metric. In a physical gauge it is given by

$$g_{ij}=\eta_{ij}+\partial_i\phi\partial_j\phi. \tag{57}$$

Because $\sigma$ is periodically identified, $d\sigma$ is the dual of a $U(1)$ world volume 2-form field strength. The dual field theory is just the $D=4$ DBI action (for the same reasons that the D2-brane action is dual to the $D=11$ supermembrane action in a $\mathbb{E}^{(1,9)}\times S^1$ background [4]). The bosonic action is

$$I=-2\int d^3\xi\sqrt{-\det(g_{ij}+F_{ij})} \tag{58}$$

where the on-shell relation of the BI two-form field strength $F$ to $\sigma$ is given by

$$\sqrt{-\det g}g^{ij}\partial_j\sigma=\frac{1}{2}\sqrt{1+(\partial\sigma)^2}\,\varepsilon^{ijk}F_{jk} \tag{59}$$

where $(\partial\sigma)^2=g^{ij}\partial_i\sigma\partial_j\sigma$. Note that the solution of the supermembrane equations with $d\phi=0$ and $d\sigma=v\,dt$ corresponds to a solution of the DBI equations with $d\phi=0$ and

$$F=-\frac{v}{\sqrt{1-v^2}}d\xi^1\wedge d\xi^2, \tag{60}$$

so that $B\equiv F_{12}$ is a constant related to $v$ by

$$\sqrt{1+B^2}=\gamma(v). \tag{61}$$

The above discussion for the static kink domain walls can be generalized to stationary solutions by expanding the above DBI action about a non-zero but constant magnetic background field $B$ given by Eq. (60). We begin with a formula of [10] for the physical gauge DBI energy density $\mathcal{H}$. For static 2-brane configurations this formula is

$$\mathcal{H}^2=4[(1+|\mathbf{E}|^2)(1+B^2)+(\mathbf{E}\cdot\nabla\phi)^2+|\nabla\phi|^2] \tag{62}$$

where $\mathbf{E}$ is the electric field. Assuming that $B$ is constant, and related to the constant $\gamma(v)$ by Eq. (61), we may rewrite this as

$$\mathcal{H}^2=4(\gamma\pm\mathbf{E}\cdot\nabla\phi)^2+4|\gamma\mathbf{E}\mp\nabla\phi|^2. \tag{63}$$

Following the argument of [10] we deduce the bound

$$\int d^2\sigma[\mathcal{H}-2\gamma]\geq 2\left|\int d^2\sigma\mathbf{E}\cdot\nabla\phi\right| \tag{64}$$

with equality when

$$\gamma\mathbf{E}=\pm\nabla\phi. \tag{65}$$

This implies that $\phi$ is harmonic and we may choose the unit point charge solution

$$\phi=\gamma\log r, \tag{66}$$

for which $\mathbf{E}=\mathbf{e}_r/r$ where $\mathbf{e}_r$ is a unit vector directed radially outwards.[1]

To perform the integral of $\mathbf{E}\cdot\nabla\phi$ we introduce an IR cutoff at $r=R$ and a UV cutoff at $r=\delta$. The total energy of the point charge solution is then

$$H=2\gamma[\pi R^2-\pi\delta^2]+2\left|\phi(R)\oint_{r=R}\mathbf{dS}\cdot\mathbf{E}+\phi(\delta)\oint_{r=\delta}\mathbf{dS}\cdot\mathbf{E}\right| \tag{67}$$

where $\mathbf{dS}$ is an outward pointing line element on a curve enclosing the origin. The integrals are easily done, with the result that

$$H=2\gamma\pi R^2+4\pi[\phi(R)-\phi(\delta)]+\cdots, \tag{68}$$

where the terms neglected vanish in the limit that $\delta\to 0$. Using the formula (66) and the fact that $-\phi(\delta)=l$, where $l$ is distance from the 2-brane, we find that

$$H=2\gamma[\pi R^2+2\pi\log R]+4\pi l+\cdots \tag{69}$$

in complete agreement with the formula (50) for the energy of a $Q$-kink lump. The agreement confirms that we have correctly identified the DBI action as the effective action of the kink domain wall and that we have correctly identified the 1/2 supersymmetric bion solution of the latter with the 1/4 supersymmetric kink-lump solution of the sigma model.

## V. M-THEORETIC INTERPRETATION

We shall provide the 1/4 supersymmetric sigma model solitons discussed here with an M-theoretic interpretation by

[1]For $v=0$ this solution corresponds to the $D=5$ supermembrane configuration $\phi+i\sigma=-\log\zeta$, where $(t,\zeta)$ ($\zeta$ complex) parametrize the membrane's world volume. For $v=0$ similar solutions are well known in string theory, e.g. as a D4-brane ending on an NS5-brane [17]. In this case we have a sigma-model lump string ending on a sigma-model kink membrane.

showing that they yield solutions of the M5-brane equations of motion in a $D=11$ supergravity background with vanishing 4-form field strength and 11-metric:

$$ds^2=-dT^2+ds^2(\mathbb{E}^4)+dS^2+ds^2(HK_4)+dZ^2. \quad (70)$$

We take the Killing vector field

$$l=\frac{\partial}{\partial S} \quad (71)$$

to generate a $U(1)$ isometry, and $HK_4$ to be a multi-center 4-metric of the type considered above. This has an M-theory interpretation as the metric produced by M monopoles (situated at the centers of the 4-metric).

We now consider a 5-brane in this background. As the background breaks half the supersymmetry of the M-theory vacuum, the field theory on the M5-brane has a $D=6$ (1,0) supersymmetry and the field content splits into a tensor multiplet and a hypermultiplet. There is a consistent truncation to the hypermultiplet sector, and we shall perform this truncation. For an appropriate choice of the M5-brane vacuum the low energy field theory is then a massless $D=6$ sigma model with a multi-center HK 4-manifold as its target space. The massive $D=5$ sigma model is then found as the effective field theory on an M5-brane wrapped on a particular combination of $S^1$ cycles in the background, as described for the M2-brane in [11]. A further trivial double-dimensional reduction yields the massive $D=4$ sigma model discussed above.

To specify the needed M5-brane configuration we begin by taking $(\mathbf{Y},W)$ to be the $\mathbb{E}^4$ coordinates and $X^I$ ($I=1,2,3,4$) the $HK_4$ coordinates. A 5-brane configuration is then specified by giving the 11 spacetime coordinates $X^M=(T,\mathbf{Y},S,W,X^I,Z)$ as functions of the six world volume coordinates $(t,\mathbf{y},s,w)$. Six of these functions may be chosen so as to fix the world volume diffeomorphism invariance of the five-brane action. We shall make the ''physical gauge'' choice

$$T=t, \quad \mathbf{Y}=\mathbf{y}, \quad S=s, \quad W=w. \quad (72)$$

This leaves $(X^I,Z)$ as the physical world volume fields, specifying the deformation of the 5-brane in the transverse 5-space from the vacuum configuration in which $Z$ and $X^I$ are constant. We will set $Z$ to a constant as its fluctuations belong to the fields of the tensor multiplet that we are discarding. We are left with the four world volume scalar fields $X^I$ (and their superpartners). In principle, these fields are functions of all six world volume coordinates, but we will impose invariance under shifts of $w$. This leaves us with the $D=5$ fields $X^I(\xi)$ (and their superpartners) where

$$\xi^\mu=(t,\mathbf{y},s). \quad (73)$$

Eventually we will impose the constraint

$$\partial_s X^I \equiv \partial_4 X^I=\zeta^I \quad (74)$$

where $\zeta$ is the tri-holomorphic KVF of the HK 4-metric, thus reducing the effective field theory to a massive $D=4$ supersymmetric sigma model. What we will now show is that the $Q$-kink-lump solution of this effective field theory defines an M5-brane configuration preserving 1/4 supersymmetry.

The number of supersymmetries preserved by a given M5-brane configuration is the dimension of the space of solutions for the constant spinor $\epsilon$ to the condition [18,11]

$$\Gamma\epsilon=\epsilon \quad (75)$$

where $\epsilon$ is a Killing spinor of the background, and $\Gamma$ is an 11-dimensional Dirac matrix function to be specified below. In the present case, there are 16 linearly independent Killing spinors, satisfying

$$\Gamma_{1234}\epsilon=\epsilon. \quad (76)$$

The Killing spinors have the form $\epsilon=f\epsilon_0$ for a universal function $f$ and constant spinor $\epsilon_0$. As $f$ cancels from Eq. (75) we may replace $\epsilon$ by $\epsilon_0$ in this equation; having done so we may then drop the suffix on $\epsilon$ to arrive back at Eq. (75) but with $\epsilon$ now taken to be a *constant* spinor satisfying the constraint (76).

To specify $\Gamma$ we begin by taking the $D=11$ Dirac matrices to be

$$\Gamma_M=(\gamma_\mu,\gamma_5,\Gamma_I,\gamma_*\Gamma_{1234}) \quad (77)$$

where

$$\gamma_*=\gamma_{01234}\gamma_5. \quad (78)$$

Thus $\gamma_5^2=\gamma_*^2=1$, and the remaining non-zero anticommutators are

$$\{\gamma_\mu,\gamma_\nu\}=2\eta_{\mu\nu}, \quad \{\Gamma_I,\Gamma_J\}=2G_{IJ}. \quad (79)$$

The matrix $\Gamma$ for bosonic M5-brane configurations with vanishing world volume 3-form field strength is then

$$(\sqrt{-\det g})\Gamma=\frac{1}{5!}\varepsilon^{\mu\nu\rho\lambda\sigma}\partial_\mu X^M\partial_\nu X^N\partial_\rho X^P\partial_\lambda X^Q\partial_\sigma X^R\,\Gamma_{MNPQR}\gamma_5 \quad (80)$$

where $g$ is the world volume 6-metric. In the physical gauge (and with $Z=0$) it is block diagonal with components diag$(g_{\mu\nu},1)$, where

$$g_{\mu\nu}=\eta_{\mu\nu}+\partial_\mu X^I\partial_\nu X^J G_{IJ}. \quad (81)$$

We can now rewrite the condition (75) as

$$(\sqrt{-\det g})\epsilon=\left(1-\gamma^\mu\partial_\mu X^I\Gamma_I-\frac{1}{2}\gamma^{\mu\nu}\partial_\mu X^I\partial_\nu X^J\Gamma_{IJ}+\cdots\right)\gamma_*\epsilon. \quad (82)$$

In principle, the right hand side includes terms up to 4th order in $\partial X$ (recall that there are only 4 sigma-model fields

$X^I$) but terms higher than second order vanish for a configuration such as the kink lump that depends on only two of the four sigma-model fields.

Since Eqs. (17) and (18) are linear in $(\partial X)$, the supersymmetry preservation condition (82) must be satisfied order by order. At zeroth order we have

$$\gamma_* \epsilon = \epsilon, \tag{83}$$

which tells us that the vacuum state of the M5-brane is a 1/2 supersymmetric M-theory configuration. The constraints (76) and (83) preserve 1/4 of the 32 supersymmetries of the M-theory vacuum; that is, they preserve 8 supersymmetries, which is the expected number for the vacuum of a supersymmetric HK sigma model. At first order we have

$$\gamma^\mu \partial_\mu X^I \Gamma_I \epsilon = 0. \tag{84}$$

Because the sigma model is obtained by retaining the terms quadratic in $\partial X$ in a series expansion of the 5-brane action, Eq. (84) is equivalent to the field theory condition for preservation of supersymmetry, as we shall verify below.

The higher order terms in Eq. (82) are now identities. The analysis is similar to that of [11]. We first note that Eq. (84) implies

$$\gamma^{\mu\nu} \partial_\mu X^I \partial_\nu X^J \Gamma_{IJ} = -\eta^{\mu\nu} \partial_\mu X^I \partial_\nu X^J G_{IJ}, \tag{85}$$

which in turn implies that Eq. (82) is satisfied if, and only if,

$$\det(\eta + G) = \left(1 + \frac{1}{2} \operatorname{tr} G\right)^2. \tag{86}$$

That this is indeed satisfied follows from the fact that the rank of $G$ cannot exceed 2 because there are only two ''active'' fields $(X, \varphi)$.

We now apply the above result to the $Q$-kink lump to confirm that it preserves 1/4 supersymmetry. By use of the $Q$-kink-lump equations, Eq. (84) can be shown to be equivalent to two further conditions on $\epsilon$. One is

$$\gamma^{23} \Gamma \Gamma_\varphi \epsilon = \sigma \epsilon \tag{87}$$

where we have used $\Gamma_I = (\mathbf{\Gamma}, \Gamma_\varphi)$ and set $U^{-1} \mathbf{n} \cdot \mathbf{\Gamma} = \Gamma$, so that $\Gamma^2 = \Gamma_\varphi^2 = 1$. This is the ''lump'' condition which, by itself, preserves 1/2 of the 8 sigma model supersymmetries. The other condition is

$$\Gamma_v \epsilon = -\epsilon \tag{88}$$

where

$$\Gamma_v \equiv v \gamma^{04} \pm \sqrt{1 - v^2}\, \gamma^{14} \Gamma \Gamma_\varphi. \tag{89}$$

Note that $\Gamma_v^2 = 1$ and $[\Gamma_v, \gamma^{23} \Gamma \Gamma_\varphi] = 0$, so this additional condition reduces the supersymmetry to 1/4 of the sigma model vacuum. Note that this is true even if $v = 1$, in which case the $Q$-kink lump reduces to the $Q$ lump. Thus, both the $Q$ lump and the $Q$-kink lump define M5-brane configurations preserving 1/16 of the supersymmetry of the M-theory vacuum, corresponding to 1/4 of the supersymmetry of the sigma model vacuum.

## VI. DISCUSSION

We have seen that much of the physics of D-branes can appear in a purely field theoretic context. It is natural to ask whether the D-brane analogy can be stretched further. One obvious question is whether non-Abelian symmetry enhancement occurs for coincident kink domain walls. The first point to appreciate here is that not every model with kink solutions will have static multi-kink solutions. In the simple 2-center model considered here there are not even multi-kink configurations. To get multi-kink configurations one needs either (i) a *multi-center* target space 4-metric or (ii) a higher-dimensional target space metric. In the first case, a model with co-linear centers has obvious multi-kink configurations, but *no static* multi-kink solutions because the kinks repel (as will be shown elsewhere [19]). This behavior can also occur in string theory [20], where it is attributable to non-Abelian instanton effects in the $D=3$, $N=2$ super Yang-Mills (SYM) theory on the branes. This suggests that a similar non-Abelian gauge theory interpretation may be possible for sigma-model D-branes.

We chose to set $\mu = 1$ and $g = 1$ throughout most of the paper. If one reinstates them, one finds, for example, that the DBI action (58) becomes

$$I = -\frac{2\mu}{g^2} \int d^3\xi \sqrt{-\det(g_{ij} + \mu^{-1} F_{ij})} \tag{90}$$

and the wall and string tensions become

$$T_{wall} = 2\mu/g^2, \quad T_{string} = 4\pi/g^2. \tag{91}$$

Recall that the $Q$-kink lump was recovered by expanding about a constant background magnetic field $B$, and the $Q$ lump was obtained in a limit corresponding to infinite $B$. Following [21] one can rescale $\mu$ and $g$ in this limit to end up with a non-commutative $D=3$ gauge theory. This suggests that the sigma-model $Q$ lump may have an alternative description as a non-commutative soliton.

A difference between the $D=5$ supermembrane of relevance to sigma-model D-branes and the M2-brane of relevance to string theory D-branes is that the $D=5$ membrane can be viewed as an $S^1$ wrapped $D=6$ 3-brane (whereas the M2-brane has no analogous $D=12$ precursor). The $D=4$ D2-brane is thus a 3-brane in a $D=6$ spacetime of the form $\mathbb{E}^{(1,3)} \times T^2$, which has been wrapped on a homology cycles of the 2-torus. This is to be expected from the fact that $D=5$ is the maximal dimension for massive HK sigma models while we considered only the $D=4$ models. The kink is a 3-brane of the $D=5$ massive sigma model and a sigma model lump is a 2-brane. The kink lump solution thus lifts to a solution of the $D=5$ model representing a 2-brane with a string boundary on the 3-brane.

Finally, we note that the results of Sec. V can be stated in terms of calibrations. Recall that the lump solution of the massless sigma model corresponds to a Kähler calibrated two surface in four dimensions [22]. We now have a similar interpretation of the kink lump of the massive sigma model as a Kähler calibrated 4-surface in six dimensions. The $Q$-kink lump, on the other hand, is not a calibrated 4-surface, strictly speaking, because it is time dependent. This kind of ''time-dependent calibration'' has been discussed in [13] and we suggest the terminology ''$Q$ calibration.'' As we have seen, $Q$ calibrations are *stationary*, but not necessarily static, minimal energy surfaces. The $Q$-kink lump is therefore a Kähler $Q$-calibrated 4-surface in six dimensions. It reduces for $v=1$ to the $Q$ lump, which is a Kähler $Q$-calibrated two-surface in four dimensions.

## ACKNOWLEDGMENTS

P.K.T. would like to thank Taichiro Kugo and Nobuyoshi Ohta for helpful discussions, and the ESI (Vienna), the YI (Kyoto) and the Kyoto University Department of Physics for their hospitality. D.T. is supported by EPSRC. R.P. thanks Trinity College, Cambridge for financial support. All authors are supported in part by PPARC through SPG ♯ 613.

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