Effect of spin on the quantum entropy of black holes

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By using the Newman-Penrose formalism and 't Hooft brick wall model, the quantum entropies of the Kerr-Newman black hole due to the Dirac and electromagnetic fields are calculated and the effects of the spins of the photons and Dirac particles on the entropies are investigated. It is shown that the entropies depend only on the square of the spins of the particles and the contribution of the spins is dependent on the rotation of the black hole, except that different fields obey different statistics.

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I. INTRODUCTION

Since Bekenstein and Hawking found that black hole entropy is proportional to the event horizon area by comparing black hole physics with thermodynamics and from the discovery of black hole evaporation [1–3], much effort has been devoted to the study of the statistical origin of black hole entropy [4–27]. 't Hooft [4] proposed a "brick wall" model (BWM) in which he argued that black hole entropy is identified with the statistical-mechanical entropy arising from a thermal bath of quantum fields propagating outside the horizon. In order to eliminate the divergence which appears due to the infinite growth of the density of states close to the event horizon, 't Hooft introduces a "brick wall" cutoff: a fixed boundary Σ_h near the event horizon within the quantum field does not propagate and the Dirichlet boundary condition was imposed on the boundary, i.e., the wave function $\phi=0$ for $r=r(\Sigma_h)$. The BWM has been successfully used in studies of statistical-mechanical entropy arising from scalar fields for static black holes [4,11,16,22] and the stationary axisymmetric black holes [23,24].

For the electromagnetic field case Kabat [28] studied entropy in Rindler space and found an unexpected surface term which corresponds to particle paths beginning and ending at the event horizon. This term gives a negative contribution to the entropy of the system and is large enough to make the total entropy negative at the equilibrium temperature. However, Iellici and Moretti [29] proved that the surface term is gauge dependent in the four-dimensional case and therefore can be discarded. Cognola and Lecca [30] studied the statistical-mechanical entropy in Reissner-Nordström black hole spacetime. They showed that there is no such surface term by applying the BWM, and they found that the leading term of the entropy for the electromagnetic fields is exactly twice the one for a massless scalar field. The result can be extended to the general static spherical static black holes even if we consider both leading and subleading corrections [31]. However, the question whether or not the result is valid for stationary axisymmetric black holes, say the Kerr and the Kerr-Newman black holes, still remains open.

On the other hand, Li [32] studied the entropy of the Dirac field in the Reissner-Nordström black hole by the BWM and he declared that the entropy depends on the linear term of the spins of the particles. However, the expressions presented in the Ref. [32] are only valid for each component of the Dirac field. The total entropy of the black hole due to the Dirac field does not include linear spins terms because the entropy should be the sum of the four components of the fields and then the terms for the spins cancel each other. Liu and Zhao [33] calculated the entropy of the Dirac field in the Kerr-Newman black hole but they did not consider the subleading terms. How do the quantum entropy relate to the spins of the Dirac particles also is an interesting question and should be studied deeply.

The purpose of this paper is to investigate effects of the spins of the photons and Dirac particles on the statistical entropy by deducing expressions of the statistical-mechanical entropy arising from the electromagnetic and the Dirac fields in the four-dimensional Kerr-Newman black holes. The paper is organized as follows. In Sec. II, the Dirac field equations are decoupled by introducing the Newman-Penrose formalism and then the quantum entropy of the Kerr-Newman black hole due to the Dirac field is calculated by using 't Hooft's BWM. In Sec. III, we deduce an expression of the statistical-mechanical entropy of the Kerr-Newman black hole arising from the electromagnetic field by using the same method as in Sec. II. The last section is devoted to discussions and conclusions.

II. STATISTICAL-MECHANICAL ENTROPY OF THE KERR-NEWMAN BLACK HOLE DUE TO THE DIRAC FIELD

We now try to find an expression of the statistical-mechanical entropy due to the quantum Dirac fields in the

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Kerr-Newman black hole. We first express the decoupled Dirac equation in Newman-Penrose formalism, then we seek the total number of modes with energy less than E, and after that we calculate a free energy. The statistical-mechanical entropy of the black hole is obtained by variation of the free energy with respect to the inverse temperature and setting $\beta=\beta_H$.

In Boyer-Lindquist coordinates, the metric of the Kerr-Newman black hole [34,35] is described by

$$g_{tt}=-\frac{\Delta-a^2\sin^2\theta}{\Sigma},\quad g_{t\varphi}=-\frac{a\sin^2\theta(r^2+a^2-\Delta)}{\Sigma},$$

$$g_{rr}=\frac{\Sigma}{\Delta},\quad g_{\theta\theta}=\Sigma,$$

$$g_{\varphi\varphi}=\left(\frac{(r^2+a^2)^2-\Delta a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta, \tag{2.1}$$

with

$$\Sigma=r^2+a^2\cos^2\theta,\quad \Delta=(r-r_+)(r-r_-), \tag{2.2}$$

where $r_\pm=M\pm\sqrt{M^2-Q^2-a^2}$, and r_+, M, Q, and a represent the radius of the event horizon, the mass, the charge, and the angular momentum per unit mass of the black hole, respectively.

In order to express the Dirac equation in the spacetime (2.1) in the Newman-Penrose formalism, we take covariant components of the null tetrad vectors as

$$l_\mu=\frac{1}{\Delta}(\Delta,\ -\Sigma,\ 0,\ -a\Delta\sin^2\theta),$$

$$n_\mu=\frac{1}{2\Sigma}(\Delta,\ \Sigma,\ 0,\ -a\Delta\sin^2\theta),$$

$$m_\mu=-\frac{\bar\rho}{\sqrt2}[ia\sin\theta,\ 0,\ -\Sigma,\ -i(r^2+a^2)\sin\theta],$$

$$\bar m_\mu=-\frac{\rho}{\sqrt2}[-ia\sin\theta,\ 0,\ -\Sigma,\ i(r^2+a^2)\sin\theta]. \tag{2.3}$$

The nonvanishing spin-coefficients can then be written as [36,37]

$$\rho=-\frac{1}{r-ia\cos\theta},\quad \beta=-\frac{\bar\rho\cot\theta}{2\sqrt2},\quad \pi=\frac{ia\rho^2\sin\theta}{\sqrt2},$$

$$\tau=-\frac{ia\rho\bar\rho\sin\theta}{\sqrt2},\quad \mu=\frac{\rho^2\bar\rho\Delta}{2},$$

$$\gamma=\mu+\frac{\rho\bar\rho(r-M)}{2},\ \alpha=\pi-\bar\beta. \tag{2.4}$$

The Dirac equation in a curved spacetime is given by

$$\sigma^i_{AB'}\mathcal{P}^A_{;i}+i\mu_0\bar{\mathcal{Q}}^{C'}\varepsilon_{C'B'}=0,$$

$$\sigma^i_{AB'}\mathcal{Q}^A_{;i}+i\mu_0\bar{\mathcal{P}}^{C'}\varepsilon_{C'B'}=0, \tag{2.5}$$

where $\sqrt2\mu_0$ is the mass of the particle which will be set to zero in this paper for simplicity, and the matrix $\sigma^i_{AB'}$ is defined as

$$\sigma^i_{AB'}=\frac{1}{\sqrt2}\begin{bmatrix} l^i & m^i\\ \bar m^i & n^i\end{bmatrix}. \tag{2.6}$$

When we let

$$\mathcal{P}^0=-\rho\psi_{11}e^{-i(Et-m\varphi)},\quad \mathcal{P}^1=\psi_{12}e^{-i(Et-m\varphi)},$$

$$\mathcal{Q}^0=\psi_{21}e^{-i(Et-m\varphi)},\quad \mathcal{Q}^1=-\bar\rho\psi_{22}e^{-i(Et-m\varphi)},$$

$$(E=\text{const}\quad m=\text{const}) \tag{2.7}$$

with

$$\psi_{11}=R_{-1/2}(r)\Theta_{-1/2}(\theta),\quad \psi_{12}=R_{+1/2}(r)\Theta_{+1/2}(\theta),$$

$$\psi_{21}=R_{+1/2}(r)\Theta_{-1/2}(\theta),\quad \psi_{11}=R_{-1/2}(r)\Theta_{+1/2}(\theta), \tag{2.8}$$

we get the following decoupled Dirac equations [36]:

$$\mathcal{D}_0\Delta\mathcal{D}^\dagger_{1/2}R_{+1/2}(r)=\lambda^2R_{+1/2}(r),$$

$$\Delta\mathcal{D}^\dagger_{1/2}\mathcal{D}_0R_{-1/2}(r)=\lambda^2R_{-1/2}(r),$$

$$\mathcal{L}^\dagger_{1/2}\mathcal{L}_{1/2}\Theta_{+1/2}(\theta)+\lambda^2\Theta_{+1/2}(\theta)=0,$$

$$\mathcal{L}_{1/2}\mathcal{L}^\dagger_{1/2}\Theta_{-1/2}(\theta)+\lambda^2\Theta_{-1/2}(\theta)=0, \tag{2.9}$$

with

$$\mathcal{D}_n=\frac{\partial}{\partial r}+\frac{iK_1}{\Delta}+2n\frac{r-M}{\Delta},$$

$$\mathcal{D}^\dagger_n=\frac{\partial}{\partial r}-\frac{iK_1}{\Delta}+2n\frac{r-M}{\Delta},$$

$$\mathcal{L}_n=\frac{\partial}{\partial\theta}+K_2+n\cot\theta,$$

$$\mathcal{L}^\dagger_n=\frac{\partial}{\partial\theta}-K_2+n\cot\theta, \tag{2.10}$$

where $K_1=(r^2+a^2)E-ma$ and $K_2=aE\sin\theta-m/\sin\theta$. The Eqs. (2.9) can be explicitly expressed as

$$\Delta\frac{d^2R_s}{dr^2}+3(r-M)\frac{dR_s}{dr}+\left[2s+4isrE+\frac{K_1^2-2isK_1(r-M)}{\Delta}-\lambda^2\right]R_s=0\quad\left(s=+\frac{1}{2}\right),$$

$$\Delta\frac{d^2R_s}{dr^2}+(r-M)\frac{dR_s}{dr}+\left[+4isrE+\frac{K_1^2}{\Delta}-\frac{2isK_1(r-M)}{\Delta}-\lambda^2\right]R_s=0\quad\left(s=-\frac{1}{2}\right),$$

$$\frac{d^2\Theta_s}{d\theta^2}+\cot\theta\frac{d\Theta_s}{d\theta}+\left[2maE-a^2E^2\sin^2\theta-\frac{m^2}{\sin^2\theta}+2asE\cos\theta+\frac{2sm\cos\theta}{\sin^2\theta}-s-s^2\cot^2\theta+\lambda^2\right]\Theta_s=0\quad\left(s=+\frac{1}{2}\right),$$

$$\frac{d^2\Theta_s}{d\theta^2}+\cot\theta\frac{d\Theta_s}{d\theta}+\left[2maE-a^2E^2\sin^2\theta-\frac{m^2}{\sin^2\theta}+2asE\cos\theta+\frac{2sm\cos\theta}{\sin^2\theta}+s-s^2\cot^2\theta+\lambda^2\right]\Theta_s=0\quad\left(s=-\frac{1}{2}\right);\tag{2.11}$$

here and hereafter $s=\pm\frac{1}{2}$ represents the spin of the Dirac particles. For explicit calculation of the free energy we adopt the following WKB approximation. We now rewrite the mode functions as

$$R_{\pm 1/2}(r)=\tilde{R}_{\pm 1/2}(r)e^{-ik_r r},$$

$$\Theta_{\pm 1/2}(\theta)=\tilde{\Theta}_{\pm 1/2}(\theta)e^{-ik_\theta\theta},\tag{2.12}$$

and suppose that the amplitudes $\tilde{R}_{\pm 1/2}(r)$ and $\tilde{\Theta}_{\pm 1/2}(\theta)$ are slowly varying functions:

$$\left|\frac{1}{\tilde{R}_{\pm 1/2}}\frac{d\tilde{R}_{\pm 1/2}}{dr}\right|\ll|k_r|,\quad\left|\frac{1}{\tilde{R}_{\pm 1/2}}\frac{d^2\tilde{R}_{\pm 1/2}}{dr^2}\right|\ll|k_r|^2,$$

$$\left|\frac{1}{\tilde{\Theta}_{\pm 1/2}}\frac{d\tilde{\Theta}_{\pm 1/2}}{dr}\right|\ll|k_\theta|,\quad\left|\frac{1}{\tilde{\Theta}_{\pm 1/2}}\frac{d^2\tilde{\Theta}_{\pm 1/2}}{dr^2}\right|\ll|k_\theta|^2.\tag{2.13}$$

Thus, from Eqs. (2.8) and (2.11) we obtain

$$\psi_{11}:\ k_{11}[E,m,k_s(\theta),r,\theta]^2=\frac{[(r^2+a^2)E-ma]^2}{\Delta^2}+\frac{1}{\Delta}\left(2maE-a^2E^2\sin^2\theta-\frac{m^2}{\sin^2\theta}-k_s(\theta)^2+2saE\cos\theta+\frac{2sm\cos\theta}{\sin^2\theta}+s-s^2\cot^2\theta\right)\quad\left(s=-\frac{1}{2}\right),\tag{2.14}$$

$$\psi_{12}:\ k_{12}[E,m,k_s(\theta),r,\theta]^2=\frac{[(r^2+a^2)E-ma]^2}{\Delta^2}+\frac{1}{\Delta}\left(2maE-a^2E^2\sin^2\theta-\frac{m^2}{\sin^2\theta}-k_s(\theta)^2+2saE\cos\theta+\frac{2sm\cos\theta}{\sin^2\theta}+s-s^2\cot^2\theta\right)\quad\left(s=+\frac{1}{2}\right),\tag{2.15}$$

$$\psi_{21}:\ k_{21}[E,m,k_s(\theta),r,\theta]^2=\frac{[(r^2+a^2)E-ma]^2}{\Delta^2}+\frac{1}{\Delta}\left(2maE-a^2E^2\sin^2\theta-\frac{m^2}{\sin^2\theta}-k_s(\theta)^2+2saE\cos\theta+\frac{2sm\cos\theta}{\sin^2\theta}-s-s^2\cot^2\theta\right)\quad\left(s=-\frac{1}{2}\right),\tag{2.16}$$

$$\psi_{22}:\ k_{22}[E,m,k_s(\theta),r,\theta]^2=\frac{[(r^2+a^2)E-ma]^2}{\Delta^2}+\frac{1}{\Delta}\left(2maE-a^2E^2\sin^2\theta-\frac{m^2}{\sin^2\theta}-k_s(\theta)^2+2saE\cos\theta+\frac{2sm\cos\theta}{\sin^2\theta}-s-s^2\cot^2\theta\right)\quad\left(s=+\frac{1}{2}\right).\tag{2.17}$$

Therefore, for each component ψ_{ij} of the Dirac field the number of modes with E, m, and k_θ takes the form [38]

$$n_{ij}[E,m,k_s(\theta)]=\frac{1}{\pi}\int d\theta\int_{r_H+h}^{L}dr k_{ij}[E,m,k_s(\theta),r,\theta];\tag{2.18}$$

here we introduce the 't Hooft brick-wall boundary condition. In this model the Dirac filed wave functions are cut off outside the horizon, i.e., $\psi_{ij}=0$ at Σ_h which stays at a small distance h from the event horizon r_+. There is also an infrared cutoff $\psi_{ij}=0$ at $r=L$ with $L\gg r_+$.

It is known that "a physical space" must be dragged by the gravitational field with an azimuth angular velocity Ω_H in the stationary axisymmetric space-time [39]. Apparently, a quantum Dirac field in thermal equilibrium at temperature $1/\beta$ in the Kerr-Newman black hole must be dragged too. Therefore, it is rational to assume that the Dirac field is rotating with angular velocity $\Omega_0=\Omega_H$ near the event horizon. For such an equilibrium ensemble of states of the Dirac field, the free energy can be expressed as

$$\begin{aligned}\beta F&=\int dm\int dp_\theta\int dn(E,m,p_\theta)\ln[1+e^{-\beta(E-\Omega_0 m)}]\\&=\int dm\int dp_\theta\int dn(E+\Omega_0 m,m,p_\theta)\ln(1+e^{-\beta E})\\&=-\beta\int dm\int dp_\theta\int\frac{n(E+\Omega_0 m,m,p_\theta)}{e^{\beta E}+1}dE\\&=-\beta\int\frac{n(E)}{e^{\beta E}+1}dE,\end{aligned}\tag{2.19}$$

with

$$n(E)=\sum_s\int dm\int dk_s(\theta)\int n_{ij}[E+\Omega_0 m,m,k_s(\theta)],\tag{2.20}$$

where the function $n(E)$ presents the total number of modes with energy less than E.

It is interesting to note that Eqs. (2.14)–(2.17) can be rewritten as

$$\begin{aligned}k_{11}=&\sqrt{\frac{-g_{rr}g_{\varphi\varphi}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}}\left\{(E-m\Omega)^2+\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left[\frac{k_s(\theta)^2}{g_{\theta\theta}}+\left(\frac{m}{\sqrt{g_{\varphi\varphi}}}-\frac{s\sqrt{g_{\varphi\varphi}}\cos\theta}{g_{\theta\theta}\sin^2\theta}\right)^2\right.\right.\\&\left.\left.+\frac{s^2}{g_{\theta\theta}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\cot^2\theta-\frac{s}{g_{\theta\theta}}(1-2aE\cos\theta)\right]\right\}^{1/2}\quad\left(s=-\frac{1}{2}\right),\end{aligned}\tag{2.21}$$

$$\begin{aligned}k_{12}=&\sqrt{\frac{-g_{rr}g_{\varphi\varphi}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}}\left\{(E-m\Omega)^2+\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left[\frac{k_s(\theta)^2}{g_{\theta\theta}}+\left(\frac{m}{\sqrt{g_{\varphi\varphi}}}-\frac{s\sqrt{g_{\varphi\varphi}}\cos\theta}{g_{\theta\theta}\sin^2\theta}\right)^2\right.\right.\\&\left.\left.+\frac{s^2}{g_{\theta\theta}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\cot^2\theta-\frac{s}{g_{\theta\theta}}(1-2aE\cos\theta)\right]\right\}^{1/2}\quad\left(s=+\frac{1}{2}\right),\end{aligned}\tag{2.22}$$

$$\begin{aligned}k_{21}=&\sqrt{\frac{-g_{rr}g_{\varphi\varphi}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}}\left\{(E-m\Omega)^2+\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left[\frac{k_s(\theta)^2}{g_{\theta\theta}}+\left(\frac{m}{\sqrt{g_{\varphi\varphi}}}-\frac{s\sqrt{g_{\varphi\varphi}}\cos\theta}{g_{\theta\theta}\sin^2\theta}\right)^2\right.\right.\\&\left.\left.+\frac{s^2}{g_{\theta\theta}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\cot^2\theta+\frac{s}{g_{\theta\theta}}(1+2aE\cos\theta)\right]\right\}^{1/2}\quad\left(s=-\frac{1}{2}\right),\end{aligned}\tag{2.23}$$

$$\begin{aligned}k_{22}=&\sqrt{\frac{-g_{rr}g_{\varphi\varphi}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}}\left\{(E-m\Omega)^2+\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left[\frac{k_s(\theta)^2}{g_{\theta\theta}}+\left(\frac{m}{\sqrt{g_{\varphi\varphi}}}-\frac{s\sqrt{g_{\varphi\varphi}}\cos\theta}{g_{\theta\theta}\sin^2\theta}\right)^2\right.\right.\\&\left.\left.+\frac{s^2}{g_{\theta\theta}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\cot^2\theta+\frac{s}{g_{\theta\theta}}(1+2aE\cos\theta)\right]\right\}^{1/2}\quad\left(s=+\frac{1}{2}\right).\end{aligned}\tag{2.24}$$

In the above equations function $\Omega \equiv -g_{t\varphi}/g_{\varphi\varphi}$ and its value on the event horizon is equal to Ω_H. Thus, we have

$$\begin{aligned} n_{11}(E) &= \frac{1}{\pi}\int d\theta \int_{r_+ + h}^{L} dr \int dm \int dk_s(\theta)\; k_{11}[E+\Omega_0 m, m, k_s(\theta)] \\ &= \frac{1}{3\pi}\int d\theta \int_{r_+ + h}^{r_E} \frac{dr\sqrt{-g}}{\left[\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left(1+\frac{g_{\varphi\varphi^2(\Omega-\Omega_0)^2}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\right]^2}\left\{E^2+\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\right. \\ &\quad \left.\times\left(1+\frac{g_{\varphi\varphi^2(\Omega-\Omega_0)^2}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\left[\frac{s^2}{g_{\theta\theta}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\cot^2\theta-\frac{s}{g_{\theta\theta}}(1-2aE\cos\theta)\right]\right\}^{3/2} \\ &\approx \frac{1}{3\pi}\int d\theta \int_{r_+ + h}^{r_E} \frac{dr\sqrt{-g}}{\left[\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left(1+\frac{g_{\varphi\varphi^2(\Omega-\Omega_0)^2}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\right]^2}\left\{E^3+\frac{3E}{2}\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\right. \\ &\quad \left.\times\left(1+\frac{g_{\varphi\varphi^2(\Omega-\Omega_0)^2}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\left[\frac{s^2}{g_{\theta\theta}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\cot^2\theta-\frac{s}{g_{\theta\theta}}(1-2aE\cos\theta)\right]\right\}, \end{aligned} \tag{2.25}$$

$$\begin{aligned} n_{21}(E) &= \frac{1}{\pi}\int d\theta \int_{r_+ + h}^{L} dr \int dm \int dk_s(\theta)\; k_{21}[E+\Omega_0 m, m, k_s(\theta)] \\ &= \frac{1}{3\pi}\int d\theta \int_{r_+ + h}^{r_E} \frac{dr\sqrt{-g}}{\left[\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left(1+\frac{g_{\varphi\varphi^2(\Omega-\Omega_0)^2}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\right]^2}\left\{E^2+\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\right. \\ &\quad \left.\times\left(1+\frac{g_{\varphi\varphi^2(\Omega-\Omega_0)^2}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\left[\frac{s^2}{g_{\theta\theta}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\cot^2\theta+\frac{s}{g_{\theta\theta}}(1+2aE\cos\theta)\right]\right\}^{3/2} \\ &\approx \frac{1}{3\pi}\int d\theta \int_{r_+ + h}^{r_E} \frac{dr\sqrt{-g}}{\left[\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left(1+\frac{g_{\varphi\varphi^2(\Omega-\Omega_0)^2}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\right]^2}\left\{E^3+\frac{3E}{2}\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\right. \\ &\quad \left.\times\left(1+\frac{g_{\varphi\varphi^2(\Omega-\Omega_0)^2}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\left[\frac{s^2}{g_{\theta\theta}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\cot^2\theta+\frac{s}{g_{\theta\theta}}(1+2aE\cos\theta)\right]\right\}, \end{aligned} \tag{2.26}$$

$n_{12}(E)$ takes same form as $n_{11}(E)$ but with different value, so does $n_{22}(E)$ with $n_{21}(E)$. In above calculations the integrations of the m and $k_s(\theta)$ are taken only over the value for which the square root of $k_{ij}[E+\Omega_0 m, m, k_s(\theta)]^2$ exists.

Inserting $n_{ij}(E)$ listed above into Eq. (2.20) we get

$$\begin{aligned} n(E) &= n_{11}(E)+n_{12}(E)+n_{21}(E)+n_{22}(E) \\ &= \frac{4}{3\pi}\int d\theta \int_{r_+ + h}^{r_E} \frac{dr\sqrt{-g}}{\left[\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left(1+\frac{g_{\varphi\varphi^2(\Omega-\Omega_0)^2}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\right]^2}\left\{E^3+\frac{3E}{2}\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\right. \\ &\quad \left.\times\left(1+\frac{g_{\varphi\varphi^2(\Omega-\Omega_0)^2}}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\left[\frac{s^2}{g_{\theta\theta}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\cot^2\theta\right]\right\}. \end{aligned} \tag{2.27}$$

We should note that although there are terms of s and s^2 in each component of the modes, n_{ij}, the total number of the modes only depend on quadratic terms s^2 since all linear terms of s counteracts each other.

Taking the integration of the r in Eq. (2.27) for the case $\Omega_0=\Omega_H$ we have

$$n(E)=\frac{4}{3\pi}\left(\frac{\beta_H}{4\pi}\right)^3\int d\theta\left\{\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left[\frac{1}{h}\frac{\partial g^{rr}}{\partial r}-C(r,\theta)\ln\frac{L}{h}\right]\right\}_{r_+}+\frac{2s^2E}{\pi}\left(\frac{\beta_H}{4\pi}\right)\int d\theta\left[\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\times\frac{\cot^2\theta}{g_{\theta\theta}}\right]_{r_+}\ln\frac{L}{h}, \tag{2.28}$$

with

$$C(r,\theta)=\frac{\partial^2 g^{rr}}{\partial r^2}+\frac{3}{2}\frac{\partial g^{rr}}{\partial r}\frac{\partial\ln f}{\partial r}-\frac{2\pi}{\beta_H\sqrt{f}}\left(\frac{1}{g_{\theta\theta}}\frac{\partial g_{\theta\theta}}{\partial r}+\frac{1}{g_{\varphi\varphi}}\frac{\partial g_{\varphi\varphi}}{\partial r}\right)-\frac{2g_{\varphi\varphi}}{f}\left[\frac{\partial}{\partial r}\left(\frac{g_{t\varphi}}{g_{\varphi\varphi}}\right)\right]^2, \tag{2.29}$$

where $f\equiv-g_{rr}(g_{tt}-g_{t\varphi^2}/g_{\varphi\varphi})$.

Substituting Eq. (2.28) into Eq. (2.19) and then taking the integration over E we find the total free energy

$$\beta F=\frac{-7}{16\times360}\left(\frac{\beta_H}{\beta}\right)^3\times\int d\theta\left[\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left(\frac{1}{h}\frac{\partial g^{rr}}{\partial r}-C(r,\theta)\ln\frac{L}{h}\right)\right]_{r_+}-\frac{s^2}{24}\left(\frac{\beta_H}{\beta}\right)\int d\theta\left[\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\times\frac{\cot^2\theta}{g_{\theta\theta}}\right]_{r_+}\ln\frac{L}{h}. \tag{2.30}$$

In order to simplify the expression we set $\delta^2=2\epsilon^2/15$ and $\Lambda^2=L\epsilon/h$ as we did in Refs. [22] and [16] [where $\delta=\int_{r_+}^{r_++h}\sqrt{g_{rr}}dr\approx2\sqrt{h/(\partial g^{rr}/\partial r)_{r_+}}$ is the proper distance from the horizon to Σ_h, ϵ is the ultraviolet cutoff, and Λ is the infrared cutoff [9,21]]. Using the formula $S=\beta^2(\partial F/\partial\beta)$ and noting that the area of the event horizon is given by $A_H=\int d\varphi\int d\theta\{\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\}_{r_H}$, we obtain the following expression of the entropy:

$$S_D=\frac{7A_H}{96\pi\epsilon^2}-\frac{7}{720}\times\int d\theta\left\{\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left[\frac{\partial^2 g^{rr}}{\partial r^2}+\frac{3}{2}\frac{\partial g^{rr}}{\partial r}\frac{\partial\ln f}{\partial r}-\frac{2\pi}{\beta\sqrt{f}}\left(\frac{1}{g_{\theta\theta}}\frac{\partial g_{\theta\theta}}{\partial r}+\frac{1}{g_{\varphi\varphi}}\frac{\partial g_{\varphi\varphi}}{\partial r}\right)-\frac{2g_{\varphi\varphi}}{f}\left(\frac{\partial}{\partial r}\frac{g_{t\varphi}}{g_{\varphi\varphi}}\right)^2\right]\right\}_{r_+}\times\ln\frac{\Lambda}{\epsilon}+\frac{s^2}{6}\int d\theta\left[\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\times\frac{\cot^2\theta}{g_{\theta\theta}}\right]_{r_+}\ln\frac{\Lambda}{\epsilon}. \tag{2.31}$$

Substituting the metric (2.1) into Eq. (2.31) and then taking the integrations of the θ we find that the statistical-mechanical entropy of the Kerr-Newman black hole due to the Dirac field is given by

$$S_D=\frac{7}{2}\left(\frac{A_H}{48\pi\epsilon^2}+\frac{1}{45}\left\{1-\frac{3Q^2}{4r_+^2}\times\left[1+\frac{r_+^2+a^2}{ar_+}\arctan\left(\frac{a}{r_+}\right)\right]\right\}\ln\frac{\Lambda}{\epsilon}\right)+\frac{s^2}{6}\left[1-\frac{r_+^2+a^2}{ar_+}\arctan\left(\frac{a}{r_+}\right)\right]\ln\frac{\Lambda}{\epsilon}, \tag{2.32}$$

where $A_H=4\pi(r_+^2+a^2)$ is the area of the event horizon.

III. STATISTICAL-MECHANICAL ENTROPY OF THE KERR-NEWMAN BLACK HOLE DUE TO THE ELECTROMAGNETIC FIELD

We now use the same procedure to study statistical-mechanical entropy due to quantum electromagnetic fields in the Kerr-Newman black hole. In the Newman-Penrose formalism, the antisymmetric Maxwell tensor $F_{\mu\nu}$ is replaced by the complex scalars ϕ_i [37]. Substituting spin coefficients into Maxwell equations $F_{[\mu\nu;\gamma]}=0$ and $F^{\mu\nu}_{;\nu}=0$, and then letting

$$\Phi_0=\phi_0=R_{+1}(r)\Theta_{+1}(\theta)e^{-i(Et-m\varphi)},$$

$$\Phi_2=\frac{2\phi_2}{\bar{\rho}^2}=R_{-1}(r)\Theta_{-1}(\theta)e^{-i(Et-m\varphi)}, \tag{3.1}$$

after some calculations we obtain the decoupled equations [36]

$$(\Delta\mathcal{D}_1\mathcal{D}_1^\dagger-2iEr)R_{+1}(r)=\lambda R_{+1}(r),\quad (\Delta\mathcal{D}_0^\dagger\mathcal{D}_0+2iEr)R_{-1}(r)=\lambda R_{-1}(r),$$

$$(\mathcal{L}_0^\dagger\mathcal{L}_1+2aE\cos\theta)\Theta_{+1}(\theta)=-\lambda\Theta_{+1}(\theta),\quad (\mathcal{L}_0\mathcal{L}_1^\dagger-2aE\cos\theta)\Theta_{-1}(\theta)=-\lambda\Theta_{-1}(\theta), \tag{3.2}$$

where functions $\mathcal{D}_n$, $\mathcal{D}_n^\dagger$, $\mathcal{L}_n$, and $\mathcal{L}_n^\dagger$ are defined by Eq. (2.10). The Eq. (3.2) can be explicitly shown as

$$\Delta\frac{d^2R_s}{dr^2}+4(r-M)\frac{dR_s}{dr}+\left[2s+4isrE+\frac{K_1^2}{\Delta}-\frac{2isK_1(r-M)}{\Delta}-\lambda\right]R_s=0\ \ (s=+1),$$

$$\Delta\frac{d^2R_s}{dr^2}+\left[+4isrE+\frac{K_1^2}{\Delta}-\frac{2isK_1(r-M)}{\Delta}-\lambda\right]R_s=0\ \ (s=-1),$$

$$\frac{d^2\Theta_s}{d\theta^2}+\cot\theta\frac{d\Theta_s}{d\theta}+\left[2maE-a^2E^2\sin^2\theta-\frac{m^2}{\sin^2\theta}+2asE\cos\theta+\frac{2sm\cos\theta}{\sin^2\theta}-s-s^2\cot^2\theta+\lambda\right]\Theta_s=0\ \ (s=+1),$$

$$\frac{d^2\Theta_s}{d\theta^2}+\cot\theta\frac{d\Theta_s}{d\theta}+\left[2maE-a^2E^2\sin^2\theta-\frac{m^2}{\sin^2\theta}+2asE\cos\theta+\frac{2sm\cos\theta}{\sin^2\theta}+s-s^2\cot^2\theta+\lambda\right]\Theta_s=0\ \ (s=-1), \tag{3.3}$$

where K_1 and K_2 possess the same value as that of the Dirac field. Taking the WKB approximation we know that $k_{(+1)}$ for Φ_0 and $k_{(-1)}$ for Φ_2 possesses the same form, which can be expressed as

$$k_s[E,m,k_s(\theta),r,\theta]^2=\frac{[(r^2+a^2)E-ma]^2}{\Delta^2}+\frac{1}{\Delta}\Bigg(2maE-a^2E^2\sin^2\theta-\frac{m^2}{\sin^2\theta}$$
$$-k_s(\theta)^2+2saE\cos\theta+\frac{2sm\cos\theta}{\sin^2\theta}+s-s^2\cot^2\theta\Bigg)$$
$$(s=+1\ \text{for}\ \Phi_0\ \text{and}\ s=-1\ \text{for}\ \Phi_2). \tag{3.4}$$

Therefore, for each component of the electromagnetic field the number of modes with E is

$$n_s(E)=\frac{1}{\pi}\int d\theta\int_{r_+ +h}^{L}dr\int dm\int dk_s(\theta)\ k_s[E+\Omega_0 m,m,k_s(\theta)]$$
$$=\frac{1}{3\pi}\int d\theta\int_{r_+ +h}^{r_E}\frac{dr\sqrt{-g}}{\left[\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left(1+\frac{g_{\varphi\varphi^2}(\Omega-\Omega_0)^2}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\right]^2}\Bigg\{E^2+\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)$$
$$\times\left(1+\frac{g_{\varphi\varphi^2}(\Omega-\Omega_0)^2}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\left[\frac{s^2}{g_{\theta\theta}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\cot^2\theta-\frac{s}{g_{\theta\theta}}(1+2aE\cos\theta)\right]\Bigg\}^{3/2}$$
$$\approx\frac{1}{3\pi}\int d\theta\int_{r_+ +h}^{r_E}\frac{dr\sqrt{-g}}{\left[\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left(1+\frac{g_{\varphi\varphi^2}(\Omega-\Omega_0)^2}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\right]^2}\Bigg\{E^3+\frac{3E}{2}\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)$$
$$\times\left(1+\frac{g_{\varphi\varphi^2}(\Omega-\Omega_0)^2}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\left[\frac{s^2}{g_{\theta\theta}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\cot^2\theta-\frac{s}{g_{\theta\theta}}(1+2aE\cos\theta)\right]\Bigg\}. \tag{3.5}$$

The integrations of the m and $k_s(\theta)$ are taken only over the value for which the square root of k_{ij}^2 exists. Therefore, the total number of modes of the electromagnetic field with E is given by

$$n(E)=n_{+1}(E)+n_{-1}(E)$$
$$=\frac{2}{3\pi}\int d\theta\int_{r_++h}^{r_E}\frac{dr\sqrt{-g}}{\left[\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left(1+\frac{g_{\varphi\varphi^2}(\Omega-\Omega_0)^2}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\right]^2}$$
$$\times\left\{E^3+\frac{3E}{2}\left(g_{tt}-\frac{g_{t\varphi}^2}{g_{\varphi\varphi}}\right)\left(1+\frac{g_{\varphi\varphi^2}(\Omega-\Omega_0)^2}{g_{tt}g_{\varphi\varphi}-g_{t\varphi}^2}\right)\right.$$
$$\left.\times\left[\frac{s^2}{g_{\theta\theta}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\cot^2\theta\right]\right\}. \tag{3.6}$$

The linear terms of the spin s are also eliminated in the total number of the modes. Carrying out the integration of the r in Eq. (3.6) for the case $\Omega_0=\Omega_H$ we obtain

$$n(E)=\frac{2}{3\pi}\left(\frac{\beta_H}{4\pi}\right)^3\int d\theta\left\{\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left[\frac{1}{h}\frac{\partial g^{rr}}{\partial r}-C(r,\theta)\ln\frac{L}{h}\right]\right\}_{r_+}$$
$$+\frac{s^2E}{\pi}\left(\frac{\beta_H}{4\pi}\right)\int d\theta\left[\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\right.$$
$$\left.\times\frac{\cot^2\theta}{g_{\theta\theta}}\right]_{r_+}\ln\frac{L}{h}, \tag{3.7}$$

where $C(r,\theta)$ is given by Eq. (2.29).

Substituting Eq. (3.7) into the formula of the free energy

$$\beta F=-\beta\int\frac{n(E)}{e^{\beta E}-1}, \tag{3.8}$$

and then taking the integration over E we find

$$\beta F=\frac{-1}{45\times 32}\left(\frac{\beta_H}{\beta}\right)^3$$
$$\times\int d\theta\left[\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left(\frac{1}{h}\frac{\partial g^{rr}}{\partial r}-C(r,\theta)\ln\frac{L}{h}\right)\right]_{r_+}$$
$$-\frac{s^2}{24}\left(\frac{\beta_H}{\beta}\right)\int d\theta\left[\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\right.$$
$$\left.\times\frac{\cot^2\theta}{g_{\theta\theta}}\right]_{r_+}\ln\frac{L}{h}, \tag{3.9}$$

from which we obtain the statistical-mechanical entropy

$$S_M=\frac{A_H}{24\pi\epsilon^2}-\frac{1}{180}\int d\theta\left\{\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left[\frac{\partial^2 g^{rr}}{\partial r^2}+\frac{3}{2}\frac{\partial g^{rr}}{\partial r}\frac{\partial\ln f}{\partial r}\right.\right.$$
$$-\frac{2\pi}{\beta\sqrt{f}}\left(\frac{1}{g_{\theta\theta}}\frac{\partial g_{\theta\theta}}{\partial r}+\frac{1}{g_{\varphi\varphi}}\frac{\partial g_{\varphi\varphi}}{\partial r}\right)$$
$$\left.\left.-\frac{2g_{\varphi\varphi}}{f}\left(\frac{\partial}{\partial r}\frac{g_{t\varphi}}{g_{\varphi\varphi}}\right)^2\right]\right\}_{r_+}\ln\frac{\Lambda}{\epsilon}+\frac{s^2}{6}$$
$$\times\int d\theta\left[\sqrt{g_{\theta\theta}g_{\varphi\varphi}}\left(1-\frac{g_{\varphi\varphi}}{g_{\theta\theta}\sin^2\theta}\right)\frac{\cot^2\theta}{g_{\theta\theta}}\right]_{r_+}\ln\frac{\Lambda}{\epsilon}. \tag{3.10}$$

By using the metric (2.1) and Eq. (3.10) and then taking the integrations of the θ we finally find the following expression for the statistical-mechanical entropy of the Kerr-Newman black hole due to the electromagnetic field

$$S_M=2\left(\frac{A_H}{48\pi\epsilon^2}+\frac{1}{45}\right.$$
$$\left.\times\left\{1-\frac{3Q^2}{4r_+^2}\left[1+\frac{r_+^2+a^2}{ar_+}\arctan\left(\frac{a}{r_+}\right)\right]\right\}\ln\frac{\Lambda}{\epsilon}\right)$$
$$+\frac{s^2}{6}\left[1-\frac{r_+^2+a^2}{ar_+}\arctan\left(\frac{a}{r_+}\right)\right]\ln\frac{\Lambda}{\epsilon}. \tag{3.11}$$

IV. DISCUSSION AND SUMMARY

The statistical entropies of the Kerr-Newman black hole arising from the Dirac and electromagnetic fields are studied. First, the null tetrad is introduced in order to decouple Dirac and Maxwell equations. Then, from the decoupled equations we find the total number of the modes of the fields by taking the WKB approximation. Last, the free energies are worked out and the expressions of the quantum entropies are presented by Eqs. (2.31) and (3.10) or explicitly by Eqs. (2.32) and (3.11). Several special properties of the entropies are listed in order.

(a) The entropies depend on the spins of the particles just in quadratic term s^2 except different spin field obey different statistics. We know from each component of the Dirac and electromagnetic fields (say, ψ_{11} or Φ_0) that the number of modes for every component field contains both terms of the s and s^2. However, the linear terms of s have eliminated each other when we sum up all components to get the total number of modes. Of course, if we study entropy for a single component of the Dirac or electromagnetic fields the result would include both the linear and quadratic terms of the spins.

(b) The contribution of the spin to the entropies is dependent on the rotation of the black hole or nonspherical symmetry of the spacetimes. For the static spherical symmetric black holes, such as the Reissner-Nordström and the Schwarzschild black holes, we know from the following limitation,

$$\lim_{a\to 0}\left[1-\frac{r_+^2+a^2}{ar_+}\arctan\left(\frac{a}{r_+}\right)\right]=0, \tag{4.1}$$

that the contribution of the spins of the particles in the results (2.32) and (3.11) vanishes. The result shows that the spins of the particles affect the statistical-mechanical entropy of the black hole only if interaction between the spins of the particles and the rotation of the black hole takes place for the Kerr-Newman black hole. We also know from the results (2.31) and (3.10) that the term of s^2 would be nonzero for a static nonspherical symmetric black hole, such as the Schwarzschild black hole with a cosmic string [40].

(c) The contribution of the spins of the particles to the quantum entropy of the Kerr black hole takes the same form as that of the Kerr-Newman black hole. Equations (2.32) and (3.11) show that the entropies of the Kerr black hole due to the Dirac and electromagnetic fields are, respectively, given by

$$S_{\text{Dirac, Kerr}}=\frac{7}{2}\left(\frac{A_H}{48\pi\epsilon^2}+\frac{1}{45}\ln\frac{\Lambda}{\epsilon}\right)+\frac{s^2}{6}\left[1-\frac{r_+^2+a^2}{ar_+}\arctan\left(\frac{a}{r_+}\right)\right]\ln\frac{\Lambda}{\epsilon}, \tag{4.2}$$

$$S_{\text{EM, Kerr}}=2\left(\frac{A_H}{48\pi\epsilon^2}+\frac{1}{45}\ln\frac{\Lambda}{\epsilon}\right)+\frac{s^2}{6}\left[1-\frac{r_+^2+a^2}{ar_+}\arctan\left(\frac{a}{r_+}\right)\right]\ln\frac{\Lambda}{\epsilon}. \tag{4.3}$$

The results are different from quantum entropy of the Kerr black hole caused by the scalar field which coincides with that of the Schwarzschild black hole [27], i.e., $S_{\text{scalar,Kerr}}=A_H/48\pi\epsilon^2+\frac{1}{45}\ln(\Lambda/\epsilon)$. We learn from Eqs. (2.32), (3.11), (4.2), and (4.3) that the contribution of the spins of the particles to the entropy of the Kerr black hole possesses exactly the same form as that of the Kerr-Newman black hole.

(d) For the static spherical symmetric black holes, such as the Reissner-Nordström and the Schwarzschild black holes, the quantum entropies arising from the Dirac field are 7/2 times that of the scalar field, and the entropy due to the electromagnetic field is exactly twice the one for a scalar field. The properties can be easily found from Eqs. (2.32) and (3.11) and clause (b) listed above. For the electromagnetic field the result agrees with that of Refs. [30,31] in which the Maxwell equations are expressed in terms of a couple of scalar fields satisfying Klein-Gordon-like equations.

In summary, starting from the Dirac and electromagnetic fields, the effects of the spins on the statistical-mechanical entropies of the Kerr-Newman, the Kerr, and the static black holes are investigated. It is shown that the quantum entropies of the black holes depend on the spins of the particles just in quadratic term of the spins, and the contribution of the spins of the photons and Dirac particles is dependent on the rotation of the black hole and the nonspherical symmetry of the spacetime since the terms for the spins in the entropy vanishes as the rotation of the black hole tends to zero and the spacetime becomes spherically symmetric.

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[1] J. D. Bekenstein, Lett. Nuovo Cimento Soc. Ital. Fis. **4**, 737 (1972); Phys. Rev. D **7**, 2333 (1973).
[2] S. W. Hawking, Nature (London) **248**, 30 (1974); Commun. Math. Phys. **43**, 199 (1975).
[3] J. Bekenstein, Phys. Rev. D **9**, 3292 (1974); R. Kallosh, T. Ortin, and A. Peet, *ibid.* **47**, 5400 (1993).
[4] G. 't. Hooft, Nucl. Phys. **B256**, 727 (1985).
[5] S. N. Solodukhin, Phys. Rev. D **52**, 7046 (1995).
[6] S. Carlip, Phys. Rev. Lett. **82**, 2828 (1999).
[7] S. Carlip, Class. Quantum Grav. **16**, 3327 (1999).
[8] V. P. Frolov and D. V. Fursaev, Class. Quantum Grav. **15**, 2041 (1998).
[9] S. N. Solodukhin, Phys. Rev. D **54**, 3900 (1996).
[10] G. Cognola, Phys. Rev. D **57**, 1108 (1998).
[11] A. Ghosh and P. Mitra, Phys. Rev. Lett. **73**, 2521 (1994).
[12] M. H. Lee and J. K. Kim, Phys. Lett. A **212**, 323 (1996).
[13] M. H. Lee and J. K. Kim, Phys. Rev. D **54**, 3904 (1996).
[14] J. Ho, W. T. Kim, and Y. J. Park, Class. Quantum Grav. **14**, 2617 (1997).
[15] J. Ho and G. Kang, Phys. Lett. B **445**, 27 (1998).
[16] Jiliang Jing, Int. J. Theor. Phys. **37**, 1441 (1998).
[17] A. O. Barvinsky, V. P. Frolov, and A. I. Zelnikov, Phys. Rev. D **51**, 1741 (1995).
[18] F. Belgiorno and S. Liberati, Phys. Rev. D **53**, 3172 (1996).
[19] V. P. Frolov, D. V. Fursaev, and A. I. Zelnikov, Phys. Lett. B **382**, 220 (1996).
[20] S. N. Solodukhin, Phys. Rev. D **52**, 7046 (1995).
[21] S. N. Solodukhin, Phys. Rev. D **56**, 4968 (1997).
[22] Jiliang Jing, Chin. Phys. Lett. **14**, 495 (1997).
[23] Jiliang Jing and Mu-Lin Yan, Phys. Rev. D **60**, 084015 (1999).
[24] Jiliang Jing and Mu-Lin Yan, Phys. Rev. D **61**, 044016 (2000).
[25] Jiliang Jing and Mu-Lin Yan, Phys. Rev. D **62**, 104013 (2000).
[26] Jiliang Jing and Mu-Lin Yan, Phys. Rev. D **63**, 024003 (2001).
[27] R. B. Mann and S. N. Solodukhin, Phys. Rev. D **54**, 3932 (1996).
[28] D. Kabat, Nucl. Phys. **B453**, 281 (1995).
[29] D. Iellici and V. Moretti, Phys. Rev. D **54**, 7459 (1996).
[30] G. Cognola and P. Lecca, Phys. Rev. D **57**, 1108 (1998).
[31] Jiliang Jing and Mu-Lin Yan, Int. J. Theor. Phys. **39**, 1687 (2000).
[32] Li Zhong-Heng, Phys. Rev. D **62**, 024001 (2000).
[33] Liu Wenbiao and Zhao Zheng, Phys. Rev. D **61**, 063003 (2000).

[34] R. P. Kerr, Phys. Rev. Lett. **11**, 237 (1963).
[35] E. T. Newman *et al.*, J. Math. Phys. **6**, 918 (1965).
[36] S. Chandrasekhar, *The Mathematical Theory of Black Hole* (Oxford University Press, Oxford, New York, 1992).
[37] M. Carmeli, *Classical fields: General Relativity and Gauge Theory* (Wiley, New York, 1982).
[38] T. Padmanabhan, Phys. Lett. B **173**, 43 (1986); Phys. Lett. A **136**, 203 (1989).
[39] Zhao Zheng and Gui Yuangxing, Acta. Astrophys. Sin. **3**, 146 (1983).
[40] M. Aryal, L. H. Ford, and A. Vilenkin, Phys. Rev. D **34**, 2263 (1986).