# Black holes with topologically nontrivial AdS asymptotics

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Asymptotically locally AdS black hole geometries of dimension $d\geqslant 3$ are studied for nontrivial topologies of the transverse section. These geometries are static solutions of a set of theories labeled by an integer $k \in\{1,2,\ldots,[(d-1)/2]\}$ which possess a unique globally AdS vacuum. The transverse sections of these solutions are $d-2$ surfaces of constant curvature $\gamma$, normalized to $\gamma=\pm 1,0$ allowing for different topological configurations. The thermodynamic analysis of these solutions reveals that the presence of a negative cosmological constant is essential to ensure the existence of stable equilibrium states. In addition, it is shown that these theories are holographically related to $[(d-1)/2]$ different conformal field theories at the boundary.

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## I. INTRODUCTION

Several formal arguments suggest that at a fundamental level the cosmological constant $(\Lambda)$ should be negative, in spite of recent observations that favor a positive effective value at a cosmic scale. As shown by Hawking and Page [1], the presence of a negative $\Lambda$ makes it possible for black holes to reach thermal equilibrium with a heat bath. Moreover, a negative cosmological constant is required to obtain a correct definition of Noether charges representing the mass and angular momentum [2,3]. Even in a spacetime with vanishing $\Lambda$, the correct results are obtained provided the construction is carried out with $\Lambda<0$ and taking the $\Lambda\to 0$ limit at the end. In this sense, $\Lambda$ acts as a regulator allowing the canonical ensemble as well as Noether charges, to be well defined.

On the other hand, black holes with $\Lambda>0$ do not admit a global definition of time, which prevents the existence of a positive energy theorem. This fact is related with the nonexistence of a locally supersymmetric extension of gravity for positive $\Lambda$ [4].

The *topological censorship theorem* [5] states that in asymptotically flat spacetimes only spherical horizons can give rise to well defined causal structure for a black hole. This is circumvented by the presence of a negative cosmological constant, in which case well defined black holes with locally flat or hyperbolic horizons have been shown to exist [6–8]. This kind of black hole with topologically nontrivial AdS asymptotics is relevant in testing the AdS conformal field theory (CFT) correspondence [9] in the cases where the thermal CFT is defined on backgrounds of different topologies such as $S^1\times\mathbb{R}^{d-2}$, $S^1\times S^{d-2}$, or $S^1\times H^{d-2}$ [10–12]. As will be shown, these backgrounds correspond to the asymptotic regions of the solutions discussed below.

As observed in Ref. [13] the renormalization group equation for a CFT at the boundary should be obtained from the radial Hamiltonian constraint of a gravitation theory with higher powers of the curvature.

In dimensions higher than 4, in addition to Einstein's theory, there exist a host of sensible gravity actions which contain higher powers of the curvature (for a recent review, see, e.g., Ref. [14] and references therein). A particular class of these theories gives rise to second order field equations for the metric, which possess spherical black hole solutions with a well defined asymptotically AdS behavior [15]. In this work, that family of black hole solutions is extended to geometries with locally flat and hyperbolic horizons, including different nontrivial asymptotic topologies. The thermal behavior of this class of theories around these solutions is analyzed in detail and is shown to be holographically connected with different thermal CFT's.

## II. AdS GRAVITY THEORIES IN HIGHER DIMENSIONS

A minimally sensible gravity theory should be described by an action leading to second order field equations for the metric in order to avoid problems with causality in the classical theory, or ghosts at the quantum level. The so called Lanczos-Lovelock gravities are the only purely metric theories that satisfy this requirement [16], although they present several problems. For each dimension, they possess a number of arbitrary constants. As a consequence, their field equations allow the existence of spherically symmetric black holes with negative energy, as well as positive energy solutions with naked singularities. Moreover, these solutions do not have a unique asymptotic behavior and can even spontaneously jump between distinct geometries [17–19].

These problems can be overcome by requiring the theories to possess *a unique* cosmological constant, which strongly restricts the arbitrary coefficients in the Lanczos-Lovelock actions. Hence one is led to a set of theories which, for each dimension $d$, have a number of Lagrangians labeled by an integer $k$ which represents the highest power of curvature in the Lagrangian. The action reads

$$I_k = \kappa \int \sum_{p=0}^{k} c_p^k L^{(p)}, \tag{1}$$

where $L^{(p)}$ is given by

$$L^{(p)} = \epsilon_{a_1 \cdots a_d} R^{a_1 a_2} \cdots R^{a_{2p-1} a_{2p}} e^{a_{2p+1}} \cdots e^{a_d}, \tag{2}$$

and

$$c_p^k = \begin{cases} \dfrac{l^{2(p-k)}}{(d-2p)} \dbinom{k}{p}, & p \leqslant k, \\ 0, & p > k, \end{cases} \tag{3}$$

with

$$1 \leqslant k \leqslant \left[ \frac{d-1}{2} \right], \tag{4}$$

where $[x]$ stands for the integer part of $x$.

Unlike a generic Lanczos-Lovelock theory, which would be obtained for arbitrary coefficients $c_p^k$, the action $I_k$ possesses only two fundamental constants, $\kappa$ and $l$, related to the gravitational constant $G_k$ and the cosmological constant $\Lambda$ through[1]

$$\kappa = \frac{1}{2(d-2)!\Omega_{d-2} G_k}, \tag{5}$$

$$\Lambda = -\frac{(d-1)(d-2)}{2l^2}, \tag{6}$$

where $\Omega_{d-2}$ is the surface area of a unit $(d-2)$-sphere.

The field equations read

$$\epsilon_{b a_1 \cdots a_{d-1}} \bar{R}^{a_1 a_2} \cdots \bar{R}^{a_{2k-1} a_{2k}} e^{a_{2k+1}} \cdots e^{a_{d-1}} = 0, \tag{7}$$

$$\epsilon_{a b a_3 \cdots a_d} \bar{R}^{a_3 a_4} \cdots \bar{R}^{a_{2k-1} a_{2k}} T^{a_{2k+1}} e^{a_{2k+2}} \cdots e^{a_{d-1}} = 0, \tag{8}$$

where $\bar{R}^{ab} = R^{ab} + 1/l^2 e^a e^b$ and $T^a$ is the torsion 2-form.

Note that the Einstein-Hilbert action in $d$ dimensions is obtained by setting $k=1$ in Eq. (1). This is the only possible choice in three and four dimensions, while in five or more dimensions there are other inequivalent theories with $k \geqslant 2$. In the case $d=5$ and $k=2$ the Lagrangian can be cast as the Euler-Chern-Simons form for the AdS group [20]. The Euler-Chern-Simons form is obtained from Eq. (1) in any odd dimension for the maximum allowed value $k=(d-1)/2$. The locally supersymmetric extension of these last kind theories with negative cosmological constant is known to exist for any odd dimension, in particular for $d=11$ [21].

An illustrative example of the kind of theories considered here, is the $k=2$ case. This action, which exists only for $d>4$, is written in tensor components as

$$I_2 = \frac{-2(d-3)!\kappa}{l^2} \int d^d x \sqrt{-g} \left[ \frac{l^2 \mathfrak{R}^2}{2(d-3)(d-4)} + R - \Lambda \right], \tag{9}$$

where $\mathfrak{R}^2$ stands for the Gauss-Bonnet density,

$$\mathfrak{R}^2 := (R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2). \tag{10}$$

In this paper, only torsion-free solutions will be considered, so that Eq. (8) is trivially satisfied.

## III. TOPOLOGICAL SOLUTIONS

Let us consider $d$-dimensional static spacetimes whose spatial sections are foliated along the radial direction by $(d-2)$-dimensional transverse surfaces $\Sigma_\gamma$ of constant curvature $\gamma$. In terms of Schwarzschild-like coordinates, the metric can be written as

$$ds^2 = -N^2(r) f^2(r) dt^2 + \frac{dr^2}{f^2(r)} + r^2 d\sigma_\gamma^2, \tag{11}$$

where $-\infty < t < \infty$, and $0 \leqslant r < \infty$ is the radial coordinate for which $r \to \infty$ defines the asymptotic region. The arc length $d\sigma_\gamma^2$ corresponds to the distance on $\Sigma_\gamma$. Substituting the ansatz (11) in the field Eqs. (7), leads to the following equations for $N(r)$ and $f^2(r)$:

$$\frac{dN}{dr} = 0, \tag{12}$$

$$\frac{d}{dr} \left( r^{d-1} \left[ F_\gamma(r) + \frac{1}{l^2} \right]^k \right) = 0, \tag{13}$$

where the function $F_\gamma(r)$ is given by

$$F_\gamma(r) = \frac{\gamma - f^2(r)}{r^2}. \tag{14}$$

By virtue of Eqs. (12) and (13), $N$ is a constant, which can be chosen as 1 (see Appendix Sec. 1), and

$$f^2(r) = \gamma + \frac{r^2}{l^2} - \alpha \left( \frac{2\mu G_k}{r^{d-2k-1}} \right)^{1/k}, \tag{15}$$

respectively, where $\alpha = (\pm 1)^{k+1}$.

The constant $\gamma$ can be normalized to $\pm 1,0$ by an appropriate rescaling of the coordinates. Thus the local geometry of $\Sigma_\gamma$ is a sphere, a plane[2] or a hyperboloid:

[1]The gravitational constant has natural units given by $[G_k] = (\text{length})^{d-2k}$, and $l$ corresponds to the AdS radius. In this work the conventions of Ref. [15] are followed.

[2]For $\gamma=0$ it is necessary that at least one direction of $\Sigma_0$ be compact, otherwise the integration constant $\mu$ could be rescaled away.

$$\Sigma_\gamma \text{ locally} = \begin{cases} S^{d-2}, & \gamma=1, \\ \mathbb{R}^{d-2}, & \gamma=0, \\ H^{d-2}, & \gamma=-1. \end{cases}$$

The solution (15) may vanish for some values of $r$. The largest zero of $f^2(r)$ corresponds to the outer horizon $r_+$ which allows to express the integration constant $\mu$ as

$$\mu = \frac{r_+^{(d-2k-1)}\left(\gamma + \frac{r_+^2}{l^2}\right)^k}{2G_k}, \tag{16}$$

and is related to the mass $M$ through

$$\mu = \frac{\Omega_{d-2}}{\Sigma_{d-2}} M + \frac{1}{2G_k}\delta_{d-2k,\gamma}. \tag{17}$$

Here $\Sigma_{d-2}$ is the volume of the transverse space, and $\Omega_{d-2}$ corresponds to the volume of $S^{d-2}$. Note that the mass is shifted with respect to the integration constant $\mu$ only for $d-2k=1=\gamma$, which corresponds to a spherically symmetric solution in Chern-Simons (CS) theory (see Appendix Sec. 1). Summarizing, for a fixed value of the label $k$ in $d$ dimensions, the action $I_k$ in Eq. (1) is extremized by the metric

$$ds^2 = -\left[\gamma + \frac{r^2}{l^2} - \alpha\left(\frac{2G_k\mu}{r^{d-2k-1}}\right)^{1/k}\right]dt^2 + \frac{dr^2}{\left[\gamma + \frac{r^2}{l^2} - \alpha\left(\frac{2G_k\mu}{r^{d-2k-1}}\right)^{1/k}\right]} + r^2 d\sigma_\gamma^2, \tag{18}$$

with $\alpha = (\pm 1)^{k+1}$, whose asymptotic behavior is locally AdS, for any topology of $\Sigma_\gamma$.

Note that the $\mu=0$ solution,

$$ds^2 = -\left(\gamma + \frac{r^2}{l^2}\right)dt^2 + \frac{dr^2}{\left(\gamma + \frac{r^2}{l^2}\right)} + r^2 d\sigma_\gamma^2, \tag{19}$$

is a locally AdS manifold, which is a common solution to the Eqs. (7) and (8) for any value of $k$, in particular for Einstein's theory $(k=1)$. As discussed in Refs. [7,8], when the transverse section is locally hyperbolic $(\gamma=-1)$, although the metric (19) possesses a horizon at $r_+=l$, it may not describe a black hole. If the transverse section $\Sigma_{-1}$ has topology $\mathbb{R}^{d-2}$, Eq. (19) is not a black hole, but it could be one provided suitable identifications are performed on $\Sigma_{-1}$, analogous to the Bañados-Teitelboim-Zenelli (BTZ) solution [22,23].

Note that for theories with odd $k$, the line element (18) is real for all values of the integration constants, however, for even $k$ only positive $\mu$ is allowed. In what follows it is shown that this metric describes black holes if $f^2(r)$ has at least one zero, and they are naked singularities otherwise. It is apparent from Eq. (18) that the theory with $d-2k-1=0$ must be treated separately.

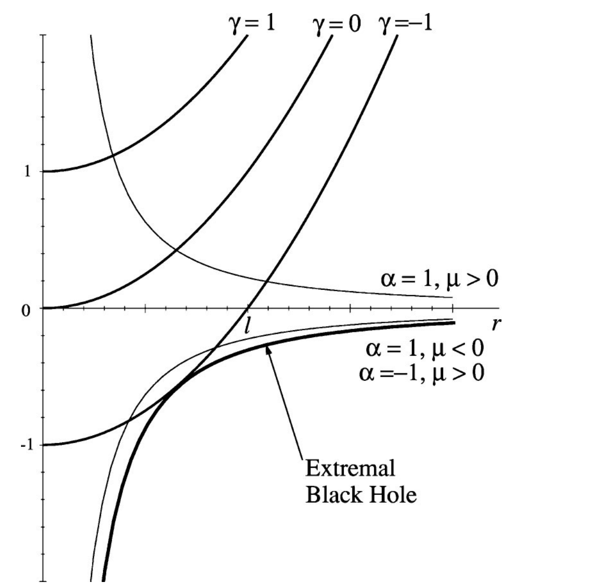


FIG. 1. The horizons are located at the zeros of $f^2(r)$, which occur at the intersections of the parabolas $(\gamma + r^2/l^2)$ and the functions $\alpha[2G_k\mu/(r^{d-2k-1}]^{1/k}$. These are displayed for $\gamma=0,\pm 1$, and different values of $\alpha$ and $\mu$. There exists a single horizon for $\alpha=1$ and $\mu\geq 0$. Two horizons arise either for $\alpha=1$ and $\mu_c<\mu<0$, or for $\alpha=-1$ and $0<\mu<\mu_c$. In the extreme case, both horizons coalesce for $\mu=\mu_c$.

### A. Generic theories: $d-2k\neq 1$

In Fig. 1, the zeros of $f^2(r)$ correspond to the intersections of the parabolas $(\gamma + r^2/l^2)$ and the functions $\alpha[2G_k\mu/(r^{(d-2k-1)}]^{1/k}$, for $\gamma=0,\pm 1$, and different values of $\alpha$ and $\mu$, respectively.

It is necessary to consider separately the theories of even and odd $k$. The range of $\mu$ for which black holes exist is summarized for different values of $\gamma$ and $k$ in Table I.

For $\gamma=-1$ there is a critical value $\mu_c$ given by

$$\mu_c = \frac{(-1)^k l^{d-2k-1}}{2G_k}\sqrt{\frac{(d-2k-1)^{d-2k-1}(2k)^{2k}}{(d-1)^{d-1}}}, \tag{20}$$

which separates topological black holes with hyperbolic transverse section from naked singularities.

*γ=1*: This case corresponds to the static, spherically symmetric black holes analyzed in Ref. [15]. They have a single event horizon, provided $\mu>0$. For any integer $k$ such that

TABLE I. Generic theories $(d-2k\neq 1)$.

| | $\gamma=1$ | $\gamma=0$ | $\gamma=-1$ |
|---|---|---|---|
| Odd $k$ $(\alpha=1)$ | $\mu>0$ | $\mu>0$ | $\mu\geq\mu_c$ : $\mu_c<0$ |
| Even $k$ | $\mu>0$ | $\mu>0$ | $\alpha=1$ : $\mu\geq 0$<br>$\alpha=-1$ : $\mu_c\geq\mu\geq 0$ |

$1\leqslant k<(d-1)/2$, these solutions share a common causal structure with the Schwarzschild-AdS$_4$ black hole solution of Einstein's theory $(k=1)$.

*$\gamma$=0*: In this case, the set of geometries described by Eq. (18) possess locally flat transverse sections $\Sigma_0$, which are assumed to be orientable. The metric describes a topological black hole for each $k$, with a unique event horizon located at

$$r_+=(2\mu G_k l^{2k})^{1/(d-1)}, \tag{21}$$

for any positive $\mu$. Considering that at least one of the transverse directions must be compact, this solution can be cast as a black $(d-3)$ brane,[3] whose world sheet could be further wrapped. The vacuum configuration $(\mu=0)$ is given by the locally AdS manifold, described by the metric

$$ds^2=-\frac{r^2}{l^2}dt^2+\frac{l^2}{r^2}dr^2+r^2 d\sigma_0^2, \tag{22}$$

which has no analogous in the vanishing cosmological constant limit.

*$\gamma$=−1*: In this case, the set of metrics in Eq. (18) describe topological black holes which possess a different behavior, depending on whether $k$ is odd or even. For all values of $k$, the exceptional case $\mu=0$ possesses a horizon at $r_+=l$, which could be a black hole depending on the topology of the transverse section $\Sigma_{-1}$. Both subcases—odd and even $k$—must be distinguished.

*Generic theories with odd k:* For theories with odd $k$, if $\mu\geqslant 0$ there is always a single horizon of radius $r_+\geqslant l$. The causal structure is the same as that of the case $\gamma=1$, discussed above.

It is noteworthy that black hole solutions with negative mass densities can also exist for odd $k$. If $\mu_c<\mu<0$, with $\mu_c$ given by Eq. (20), the singularity at $r=0$ is surrounded by two horizons, $r_-$ and $r_+$, and the causal structure is analogous to that of the Reissner-Nordström solution with negative cosmological constant. For the critical value $\mu=\mu_c$ both horizons coalesce at

$$r_c=l\sqrt{\frac{d-2k-1}{d-1}}, \tag{23}$$

which corresponds to the extremal solution. The critical radius $r_c$ is the smallest possible size of the outer horizon for the black holes within this family.

*Generic theories with even k:* For each even $k$ there are two families of solutions labeled by $\alpha=\pm 1$ in Eq. (18) with positive mass.

The branch with $\alpha=1$, describes black holes for $\mu\geqslant 0$ with a single horizon at $r_+\geqslant l$, and with the usual causal structure. The bound $\mu=0$ is saturated by the metric (19).

The black hole solutions belonging to the branch with $\alpha=-1$, have an unusual mass range, bounded above and below by $\mu_c\geqslant\mu\geqslant 0$, which in terms of the horizon radius means $r_c\leqslant r_+\leqslant l$. The positive upper bound $\mu_c$ is given by Eq. (20). For $\mu_c>\mu>0$ the solutions have two horizons $r_-$ and $r_+$, however, unlike the standard solutions, as the mass increases, $r_+$ decreases. At first sight this might seem to contradict the second law of thermodynamics, but this is not the case. The configuration $\mu=0$ will be excluded on thermodynamic grounds, as will be shown in Sec. IV.

The extreme case, $\mu=\mu_c$ corresponds to the limit in which the horizons merge at $r_c$ given by Eq. (23), which is the smallest possible radius also in this case.

It is worth noting that if one considers a fixed mass parameter in the range $\mu_c\geqslant\mu>0$, there exist two different topological black hole solutions, corresponding to the $\alpha=+1$ and $\alpha=-1$ branches, whose horizon radii are larger and smaller than $l$, respectively.

[3]This solution can also be interpreted as a $(d-2)$-brane with at least one spatial direction wrapped up.

TABLE II. Chern-Simons theories.

| | $\gamma=1$ | $\gamma=0$ | $\gamma=-1$ |
|---|---|---|---|
| Odd $k$ $(\alpha=1)$ | $\mu\geqslant\frac{1}{2G_k}$ | $\mu>0$ | $\mu\geqslant-\frac{1}{2G_k}$ |
| even $k$ | $\mu\geqslant\frac{1}{2G_k}$ | $\mu>0$ | $\alpha=1$ : $\mu\geqslant 0$<br>$\alpha=-1$ : $\frac{1}{2G_k}\geqslant\mu\geqslant 0$ |

### B. Chern-Simons theories: $d=2k+1$

In these theories, the functions $\alpha[2G_k\mu/(r^{d-2k-1})]^{1/k}$ degenerate into horizontal straight lines and therefore $f^2(r)$ in Eq. (15) possesses only a simple zero at

$$r_+=l\sqrt{\alpha(2G_k\mu)^{2/(d-1)}-\gamma}.$$

This means that these solutions are black holes with a unique event horizon. Again, it is necessary to distinguish the theories with even and odd $k$, corresponding to dimensions $d=4n+1$ and $d=4n-1$, respectively. Table II shows the allowed range of $\mu$ for which black holes exist $(r_+\geqslant 0)$.

As in the previous case, black holes with different values of $\gamma$ are analyzed separately.

*$\gamma$=1*: The spherical black holes were discussed in Ref. [24], and in further detail in Ref. [15]. As seen in Eq. (17), the lower bound $\mu=1/2G_k$ corresponds to the zero mass black hole $(M=0)$, which is separated by a mass gap from AdS space time $(M=-1/2G_k)$. These black holes have a common causal structure with the $(2+1)$-dimensional solution [25].

*$\gamma$=0*: As for the generic theories $(d-2k\neq 1)$, the locally flat transverse section $\Sigma_0$, is assumed to be orientable with at least one compact direction. In that theory, the metric (18) describes a black $(d-3)$ brane, whose horizon is located at $r_+=l(2G_k\mu)^{1/(d-1)}$, as is obtained from Eq. (21) for $d-2k=1$. Unlike the $\gamma=1$ case, the black brane vacuum $(\mu=0)$ corresponds to the same metric as in the generic case given by Eq. (22), and there is no energy gap.

*$\gamma$=−1:* As in the generic theories, solutions (18) describe topological black holes for the range of masses included in

Table II, except that for $\mu=0$, the metric (19) may or may not be a black hole, depending on the topology of $\Sigma_{-1}$. Unlike the generic theory, these family of topological black holes possess a single event horizon $r_+$ even for negative values of $\mu$.

The minimum size of this kind of black holes is $r_c=0$, as can be seen from Eq. (23), whose critical mass parameter is given by

$$\mu_c=\frac{(-1)^k}{2G_k}. \tag{24}$$

As in the generic case, the massive solutions of Chern-Simons (CS) theories have different features, for odd and even $k$.

*CS theories with odd k (d=4n−1)*: The solution (18) describes black holes with a single event horizon for $\mu\geqslant\mu_c=-1/2G_k$, and naked singularities otherwise.

*CS theories with even k (d=4n+1)*: Two families of solutions with positive mass labeled by $\alpha=\pm 1$ are obtained.

The branch with $\alpha=+1$, describes black holes with $\mu\geqslant 0$ and $r_+\geqslant l$, where the bound $\mu=0$ is saturated by Eq. (19).

The mass range of the black holes with $\alpha=-1$ is bounded above and below by $0\leqslant\mu\leqslant\mu_c=1/2G_k$, which in terms of the horizon radius means $l\geqslant r_+\geqslant 0$. Note that for $\alpha=-1$, the mass is a decreasing function of $r_+$.

If the mass parameter is in the range $\mu_c\geqslant\mu>0$, two inequivalent topological black hole solutions are found, corresponding to the branches $\alpha=\pm 1$, as in the generic theory.

Note that the static $2+1$ black hole is obtained from Eq. (18) for $\gamma=1$ as well as for $\gamma=0$, because in three dimensions the transverse section degenerates to $S^1$.

### C. Vanishing cosmological constant limit

The full set of topological black hole metrics discussed here approach asymptotically a locally AdS space time with radius $l$, whose curvature at the boundary satisfies $R^{ab}\rightarrow -l^{-2}e^a e^b$. Hence the asymptotically flat limit is obtained for $l\rightarrow\infty$, instead of taking the vanishing limit of the volume term ($c_0^k\rightarrow 0$). The vanishing cosmological constant limit of the solutions in Eq. (18) coincides with the solutions of the $l\rightarrow\infty$ limit of the action $I_k$, or equivalently, taking the same limit in the field Eqs. (7) and (8), which amounts to replacing $\bar{R}^{ab}$ by $R^{ab}$ [15].

The asymptotically flat limit of Eq. (18) is given by

$$ds^2=-\left[\gamma-\alpha\left(\frac{2G_k\mu}{r^{d-2k-1}}\right)^{1/k}\right]dt^2+\frac{dr^2}{\gamma-\alpha\left(\frac{2G_k\mu}{r^{d-2k-1}}\right)^{1/k}}+r^2 d\sigma_\gamma^2. \tag{25}$$

Hence, in case of vanishing $\Lambda$, these metrics describe black holes only for the spherically symmetric solutions in the non-CS case ($\gamma=1$ and $d-2k-1\neq 0$), with an event horizon located at $r_+=(2G_kM)^{1/(d-2k-1)}$.

Some of the topological black holes discussed here have been previously reported elsewhere. The case of Einstein-Hilbert action—corresponding to $k=1$ in our analysis—was extensively studied in Refs. [7] and [8]. The topological black holes corresponding to $k=1$ possess a geometry resembling just those found for the actions $I_k$ in Eq. (18) with odd $k$; in fact, they possess the same causal structure. However, as is shown below, the thermodynamic behavior corresponding to the Einstein-Hilbert case differs from the other odd $k$ theories.

The theories with $k=[(d-1)/2]$ were studied in Ref. [26] for odd $k$, which correspond to Chern-Simons and Born-Infeld theories in dimensions $d=4n-1,\ 4n$, respectively.

## IV. THERMODYNAMICS

### A. Temperature and specific heat

The black hole temperature is defined in the standard way as $\beta=1/\kappa_B T$, where $\kappa_B$ is the Boltzmann constant, and the period $\beta=4\pi(df^2/dr|_{r_+})^{-1}$ is found by demanding regularity of the Euclidean solution at the horizon. Thus

$$T=\frac{(d-1)}{4\pi\kappa_B l^2}\frac{r_+^2+\gamma r_c^2}{kr_+}, \tag{26}$$

where $r_c$ is the critial radius defined in Eq. (23). Note that for CS theories the temperature has the universal expression

$$T_{\rm CS}=\frac{r_+}{2\pi\kappa_B l^2}, \tag{27}$$

which does not depend on $d$ or $\gamma$. For generic theories with $\gamma=0$, the temperature is also a linear function of $r_+$, and this result is approximated for $r_+\gg l$ in all the other cases.

The specific heat $C=dM/dT$ is given by

$$C=k\frac{2\pi\kappa_B}{G_k}\frac{\Omega_{d-2}}{\Sigma_{d-2}}r_+^{d-2k}\left(\gamma+\frac{r_+^2}{l^2}\right)^{k-1}\left[\frac{r_+^2+\gamma r_c^2}{r_+^2-\gamma r_c^2}\right], \tag{28}$$

which for $r_+\gg l$ grows like $C\sim r_+^{d-2}$. Combining formulas (26) and (28) with the mass parameter $\mu(r_+)$ defined in Eq. (16), it is possible to investigate whether these topological black holes can reach thermal equilibrium with a heat bath at temperature $T_B$.

### B. Thermal equilibrium

*γ=1:* The thermodynamic equilibrium of spherically symmetric black holes ($\gamma=1$) was discussed in Ref. [15]. In this theory, for $d-2k-1\neq 0$, the temperature (26) has a minimum $T_c=\sqrt{(d-2k-1)(d-1)}/2\pi\kappa_B kl$ at $r_+=r_c=l\sqrt{(d-2k-1)}/d-1$. The specific heat (28) is positive for $r_+>r_c$, and has the opposite sign for $r_+<r_c$; and near the critical radius behaves as $C\sim(r_+-r_c)^{-1}$, signaling the existence of a phase transition. Two generic situations may occur.

(i) $T_B>T_c$: In this case there are two possible equilibrium states of radii $r_u$ (unstable) and $r_s$ (locally stable), with $r_u$

$<r_c<r_s$. Thus, if the initial state has $r_+<r_u$, the black hole cannot reach the equilibrium because it evaporates until its final stage. Otherwise, for $r_+>r_u$, the black hole evolves towards an equilibrium configuration[4] at $r_+=r_s$.

(ii) $T_B<T_c$: Under this assumption, a black hole cannot reach a stable equilibrium state and is doomed to evaporate.

In the special case of $d-2k=1$ (CS), the specific heat (28) is always positive, hence the equilibrium configuration is always reached, independently of the initial black hole state and for any finite temperature $T_B$.

*$\gamma$=0:* In case of black holes with a locally flat transverse section ($\gamma=0$), the temperature (26) grows linearly with $r_+$

$$T=\frac{1}{k}\frac{(d-1)}{4\pi\kappa_B}\frac{r_+}{l^2}, \tag{29}$$

and the specific heat is

$$C=k\frac{2\pi\kappa_B}{G_k l^{2k-2}}\frac{\Omega_{d-2}}{\Sigma_{d-2}}r_+^{d-2},$$

which implies that, independently of the initial black hole state, thermal equilibrium at some $r_+=r_s$, is always reached for any finite temperature of the heat bath $T_B$.

*$\gamma=-1$:* For all theories labeled with different *k*, the temperature of the topological black holes with hyperbolic transverse sections is a monotonically increasing function of $r_+$ which vanishes at the smallest possible size for a black hole, $r_+=r_c$. This is consistent with the fact that for the extremal solution, $r_+=r_c$, the Euclidean $r-t$ plane has the topology of a cylinder and hence $\beta$ is arbitrary.

The massless topological black hole in Eq. (19) has a horizon at $r_+=l$ and a universal temperature given by

$$T_l=\frac{1}{2\pi\kappa_B l}. \tag{30}$$

The specific heat (28) has a simple zero at $r_+=r_c$ and a zero of order $k-1$ at $r_+=l$. This second root is a local minimum for odd *k* and a saddle point[5] for even *k*. Thus the approach to equilibrium depends on the parity of the integer *k*.

### 1. Theories with odd *k*

For $k\neq1$, as depicted in Figs. 2 and 3, the temperature is a strictly increasing function of $\mu$ and the specific heat is a non-negative for the allowed range of $r_+$. This implies that equilibrium with a heat bath at temperature $T_B$ is always reached for any initial black hole state. Moreover, since the specific heat vanishes for the local minimum at $r_+=l$, the topological black hole behaves as a ''volatile'' system near the massless configuration, as there is a sudden increase in temperature with an infinitesimal increase in $\mu$.

For the Einstein-Hilbert (EH) action ($k=1$) the specific heat neither vanishes nor has a minimum at $r_+=l$, as seen in

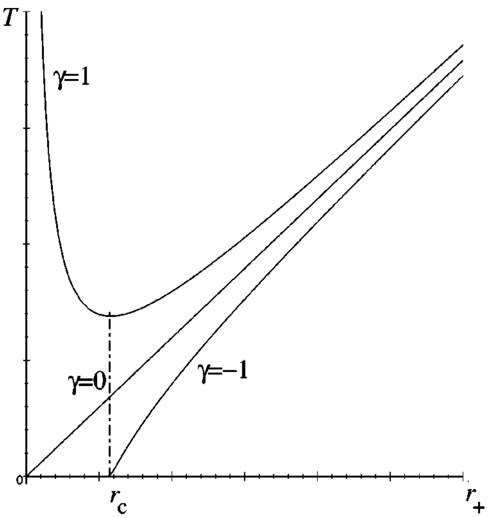


FIG. 2. The temperature as a function of $r_+$ is depicted for generic theories with $\gamma=0,\pm1$. For $\gamma=1$ the temperature has a minimum at $r_+=r_c$. When $\gamma=0$ the temperature is a linear function of $r_+$. For $\gamma=-1$ the temperature is an increasing function of $r_+$ that vanishes at $r_+=r_c$. For $r_+\gg l$ the temperature grows linearly with $r_+$ for all cases. For CS theories the three curves are replaced by the $\gamma=0$ straight line with a universal slope.

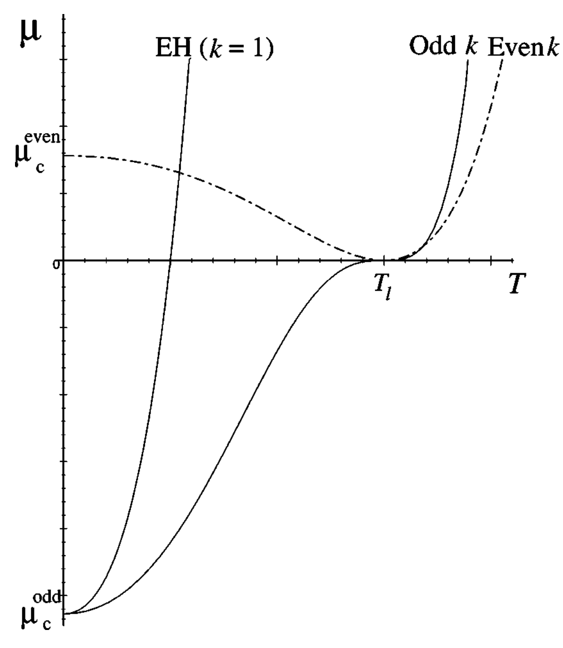


FIG. 3. The mass parameter ($\mu$) as function of the temperature is depicted for $\gamma=-1$ solutions. For even *k*, $\mu$ has a local maximum at $T=0$ and an absolute minimum at $T_l$. For odd *k*, the mass parameter has an absolute minimum at $T=0$ and an inflexion point at $T_l$ for $k\neq1$. The specific heat vanishes at these critical points. For even *k* the specific heat is negative for $T<T_l$. For odd $k\neq1$, the inflexion point at $T=T_l$ signals the existence of a ''volatile point.'' For the EH case ($k=1$) there is not such volatile state, since the specific heat has an absolute minimum at $T=0$.

[4]Curiously, the minimum size for which a spherical black hole can be at equilibrium with a heath bath ($r_c$) corresponds to the smallest size of a topological black hole with hyperbolical transverse section.

[5]Except for $k=2$, in which case *C* has a simple zero at $r_+=l$.

Fig. 3. This means that equilibrium with a heat bath is also attained for any initial configuration, and there is no ''volatility point'' at all.

#### *2. Theories with even k*

As shown in Fig. 3, the topological black holes in all theories with even $k$ have positive specific heat for $r_+>l$—corresponding to the branch with $\alpha=+1$—while $C<0$ for $r_c<r_+<l$ (for $\alpha=-1$). The zero of the specific heat at $r_+=l$, which corresponds to the massless topological black hole in Eq. (19) at temperature $T_l$, gives rise to two different scenarios depending whether the heat bath temperature is above or below $T_l$.

(i) $T_B<T_l$: If the initial state of the topological black hole is at a temperature above $T_B$, both branches ($\alpha=\pm 1$) reduce their masses, reaching a stable vacuum configuration out of thermal equilibrium at temperature $T_l$ and zero mass. On the contrary, if the initial state is at $T<T_B$ (which can only occur for the branch $\alpha=-1$), the black hole increases its mass tending towards the extreme state with $T=0$ and $\mu=\mu_c$. Hence the configuration at thermal equilibrium ($T=T_B$) is unstable.

(ii) $T_B>T_l$: If the initial state of the topological black hole belongs to the upper branch ($\alpha=+1$ and $T>T_l$), the equilibrium with the heat bath is always attained, in agreement with the positive specific heat of this branch. Conversely, a black hole in the lower branch will move away, reducing its temperature and increasing its mass, towards the extreme configuration. Note that now the vacuum configuration $\mu=0$ is unstable.

#### *3. Thermodynamics and topology fixing for $\gamma=-1$*

As mentioned in Sec. III for the $\gamma=-1$ case, the massless state (19) could be construed as a black hole or not, depending on the topology of the transverse section. The above thermodynamic analysis shows that the state $\mu=0$ admits a standard black hole interpretation for odd $k$ but not for even $k$.

To see this, consider a nearly massless black hole in vacuum (in a heat bath at zero temperature): in a theory with even $k$ both branches ($\alpha=\pm 1$) approach a final state at $\mu=0$ and $T=T_l$ which cannot loose energy further as that would make the metric complex. In this sense, this final state cannot be interpreted as a standard black hole. On the other hand, for theories with odd $k$ the $\mu=0$ configuration behaves as a volatile state which radiates violently, decaying into a negative mass black hole.

Thus the thermodynamics provides a criterion to restrict the topology of the transverse section $\Sigma_{-1}$: for odd $k$, the transverse section must have a topology such that the geometry for $\mu=0$ *is* a black hole; on the contrary, for even $k$, the topology of the transverse section must be chosen so that the massless solution *is not* a black hole.

### C. Entropy

An analytic expression for entropy as a function of the horizon radius $r_+$, can be obtained in the semiclassical approximation from the Euclidean version of the action $I_k$ (see the Appendix). Alternatively, the entropy can be obtained from the first law of thermodynamics, $dM=TdS$, as

$$\frac{dS_k}{dr_+}=\frac{\Sigma_{d-2}}{T\Omega_{d-2}}\frac{d\mu}{dr_+}, \tag{31}$$

which, upon substitution of Eqs. (16) and (26), yields

$$\frac{dS_k}{dr_+}=k\frac{2\pi\kappa_B\Sigma_{d-2}}{\Omega_{d-2}G_k}r_+^{(d-2k-1)}\left(\gamma+\frac{r_+^2}{l^2}\right)^{k-1}. \tag{32}$$

This expression shows that the entropy is a monotonically increasing function of $r_+$ for all cases, except if $k$ is even and $\gamma=-1$, in which case it is decreasing in the range $r_c<r<l$.

For $\gamma=1,0$ the entropy $S_k(r_+)$ is given by

$$S_k=\kappa_B\frac{2k\pi\Sigma_{d-2}}{\Omega_{d-2}G_k}\int_{r_{\min}}^{r_+}r^{(d-2k-1)}\left(\gamma+\frac{r^2}{l^2}\right)^{k-1}dr, \tag{33}$$

for which the lower integration limit is chosen as $r_{\min}=0$, so that the vacuum ($r_+=0$) has vanishing entropy. For $\gamma=-1$, expression (33) is valid also for theories with odd $k$, provided $r_+>r_c$.

In the exceptional case (even $k$, $\gamma=-1$), Eq. (32) implies that the entropy attains an absolute minimum at the vacuum configuration ($r_+=l$). Hence the lower integration limit is naturally chosen as $r_{\min}=l$ in order to have non-negative entropy.[6] Superficially, the fact that in the range $r_c<r_+<l$, the entropy is a decreasing function of $r_+$, would seem to violate the second law of thermodynamics. However, this range of $r_+$ corresponds to the branch with $\alpha=-1$, for which the mass is also a decreasing function of $r_+$ and hence $\partial S/\partial M=1/T>0$ as shown in Fig. 2.

All solutions with $\gamma=0$ obey an ''area law'' for all $k$:

$$S_k=\kappa_B\frac{2\pi k}{(d-2)\Omega_{d-2}G_k l^{2(k-1)}}A, \tag{34}$$

which in standard units is

$$S_k=k\frac{G}{G_k l^{2(k-1)}}S_{EH},$$

where the Einstein Hilbert entropy reads $S_{EH}=(\kappa_B/\tilde{G})(A/4)$, and $A=\Sigma_{d-2}r_+^{d-2}$ is the horizon area. It is important to note that for $\gamma\pm 1$ the entropy (33) approaches the area law (34) in the limit $r_+\gg l$.

[6]Note that in this case the entropy vanishes for $r_+=l$, where the temperature is nonzero $[T=(2\pi\kappa_B l)^{-1}]$. This situation is not completely new, as it is found for instance in stringy dilatonic black holes [27]. These configurations are physically acceptable provided the black hole has a mass gap, a condition which is met by the solutions presented here.

### D. Canonical ensemble

For spherically symmetric black holes in the four-dimensional Einstein theory, it was shown in Ref. [1] that the canonical ensemble is well defined provided a negative cosmological constant is included. Those arguments can be extended to higher dimensions and for theories described by the action (1) [15]. It can be similarly shown that the canonical ensemble is also well defined for the whole set of topological black hole solutions considered here.

In the present case the partition function is given by

$$Z_\gamma^k(\beta)=\sum_\alpha \int \rho_\alpha(M)e^{-\beta M}dM, \tag{35}$$

where the sum extends over $\alpha=\pm 1$ only for even $k$ and $\gamma=-1$, while for all other cases $\alpha=1$ only. Integrating in $r_+$, this expression reads

$$Z_\gamma^k(\beta)=\int_{r_c}^{\infty}\rho(r_+)e^{-\beta M}\left|\frac{\partial M}{\partial r_+}\right|dr_+ \tag{36}$$

for all cases. As the density of states is given by $\rho(r_+)=\exp(S_k/\kappa_B)$, the convergence of (36) depends on the behavior of $S_k$ and $M$ for $r_+\gg l$. Combining Eqs. (16) and (34), the integral (36) can be seen to converge for all $k$ and $\gamma$. In fact, the integrand of Eq. (36) takes the convergent form $\exp[-\beta M+aM(d-2)/(d-1)]$ $(a>0)$ for $\Lambda=-l^2\neq 0$, whereas for $\Lambda=0$, it behaves as $\exp[-\beta M+aM(d-2k)/(d-2k-1)]$ and diverges for all $k\geq 1$.

### E. Connection with thermal CFT's

In the context of the Maldacena's AdS/CFT duality conjecture (see, e.g., Ref. [9] and references therein), a $d$-dimensional Euclidean gravity theory with asymptotic AdS behavior can be described by a suitable thermal conformal field theory on its boundary [10]. The fact that the actions $I_k$—defined in Eq. (1)—describes up to $[(d-1)/2]$ inequivalent gravity theories in the bulk, would imply the existence of an equal number of different $(d-1)$-dimensional dual CFT's at the boundary. The asymptotic behavior of these gravity theories, which can be read from the metrics (18), should be reconstructed from this set of CFT's at the boundary through the UV/IR relation. This relation states that different radial positions are mapped to different field theory scales, in such a way that the infrared effects in the bulk correspond to ultraviolet effects on the theory at the boundary [28].

In particular, certain type of CFT renormalization group equation can be generated by the action of the radial Hamiltonian constraint in the bulk [13]. It is expected that deviations from the proposed renormalization group flow should result from modifying the Hamiltonian constraints by the inclusion of higher curvature terms in the action. Thus the set of actions given by $I_k^d$ enhances the repertoire of theories which could provide a concrete holographic interpretation of gravity.

The different asymptotic regions of the black holes discussed here provide inequivalent background spacetimes where the corresponding dual CFT's are realized [11,12]. Thus CFT's defined on $S^1\times\mathbb{R}^{d-2}$, $S^1\times S^{d-2}$, or $S^1\times H^{d-2}$ are connected with black holes in the bulk for $\gamma=0$, 1, or $-1$, respectively.

Some insight about the correspondence can be gained by looking at the thermodynamic quantities in the simplest case corresponding to a thermal CFT on a flat background, that is, on $S^1\times\mathbb{R}^{d-2}$. In this case, conformal invariance is sufficient to fix the energy to be of the form

$$E_{CFT}=\sigma_{SB}V_{CFT}\beta_{CFT}^{1-d}, \tag{37}$$

where $\sigma_{SB}$ is the Stefan-Boltzmann constant. Using the first law of thermodynamics, the entropy can be written as[7]

$$S_{CFT}=\frac{d-1}{d-2}\sigma_{SB}^{1/(d-1)}V_{CFT}\left(\frac{E_{CFT}}{V_{CFT}}\right)^{(d-2)/(d-1)}, \tag{38}$$

The precise expression for $\sigma_{SB}$ is determined by the dynamical structure of the specific CFT considered, and is a growing function of the number of degrees of freedom. In terms of the AdS/CFT correspondence, this CFT can be viewed as defined on the boundary of the Euclidean space for the metric (18) with $\gamma=0$ at a large fixed radius $r_0\gg r_+$. Hence $V_{CFT}$ corresponds to the volume of the transverse section, given by

$$V_{CFT}=\Sigma_{d-2}r_0^{d-2}. \tag{39}$$

Correspondingly, the temperature at $r_0$ is given by the redshifted black hole temperature as

$$T_{CFT}=\frac{l}{r_0}T,$$

where $T$ is given by Eq. (29). Consequently, the energy $E_{CFT}$ in Eq. (37) corresponds to the red-shifted black hole mass,

$$E_{CFT}=\frac{l}{r_0}M, \tag{40}$$

provided the Stefan-Boltzmann constant is given by

$$\sigma_{SB}=\frac{1}{2\Omega_{d-2}}\left(\frac{4\pi k}{d-1}\right)^{d-1}\frac{l^{d-2k}}{G_k}. \tag{41}$$

Plugging expressions (37), (39) and (41) into Eq. (38), allows expressing $S_{CFT}$ in terms of the black hole mass density,

$$S_{CFT}=k\left(\frac{l^{2(d-k-1)}}{G_k}\right)^{1/(d-1)}\frac{2\pi\kappa_B}{(d-2)}\frac{\Sigma_{d-2}}{\Omega_{d-2}}(2\mu)^{(d-2)/(d-1)}, \tag{42}$$

which precisely matches the black hole entropy $S_k$ for $\gamma=0$ in Eq. (34).

[7]The form of $S$ can also be inferred demanding the entropy to be extensive and conformally invariant.

It is worth noting that the entropy grows linearly with $k$, the highest power of curvature in the action. Moreover, Eq. (41) relates the integer $k$ to the number of degrees of freedom of the corresponding CFT at strong coupling. For the case of standard five-dimensional supergravity ($k=1$), which is conjectured to be dual to four dimensional SYM with $\mathcal{N}=4$ at large $N$, the entropy relation (42) is reproduced up to a numerical factor [10].

Finally, note that the entropy matching between black holes with $\gamma=0$ and CFT's on a flat background $S^1\times\mathbb{R}^{d-2}$ is exact for all values of $\beta$, but it is not necessarily so for CFT's defined on $S^1\times S^{d-2}$ and $S^1\times H^{d-2}$, i.e., $\gamma=1$ and $-1$, respectively. Although the exact expression for the entropy of a CFT on a generic curved background is unknown, an approximate result can be established for $\gamma=\pm 1$ in the limit $\beta\to 0$. In fact, for $\gamma=\pm 1$, the curvature of the transverse section is $\pm 1/r_0^2$ and, by conformal invariance, the entropy should be a function of $\beta/r_0$ only. Hence the large $r_0$ limit is equivalent to $\beta\to 0$ and therefore the high temperature limit is reproduced if the horizon radius is very large[8] ($r_+\gg l$). Thus it is concluded that the entropy of a CFT and that of a black hole approach the same expression given by Eq. (34) in the high temperature limit, provided $\sigma_{SB}$ is chosen as in Eq. (41).

## V. SUMMARY AND COMMENTS

Static black hole-like geometries, possessing topologically nontrivial AdS asymptotics have been found as solutions of a family of gravity theories which admit a unique global AdS vacuum. These theories and their corresponding solutions are classified by an integer $k$, which is the highest power of curvature in the Lagrangian. These solutions are further labeled by the constant $\gamma=\pm 1,0$, representing the curvature of the transverse section.

### Locally spherical transverse section

The case $\gamma=1$ leads to a natural splitting between generic and CS theories ($d-2k=1$). In the first case, the causal and thermodynamic properties resemble those of the Schwarzschild-AdS black hole. In the CS case, black holes behave like the $2+1$ solution.

### Locally flat transverse section

The case $\gamma=0$ corresponds to (un)wrapped black branes, for all values of $d$, $k$ and $M>0$ exhibiting the same causal structure as a Schwarzschild-AdS black hole, but whose thermodynamic properties are analogous to those of a $2+1$-dimensional black hole. Therefore they possess a single event horizon, their temperature is a linear function of $r_+$, and hence they reach thermal equilibrium with a heat bath at any temperature. Moreover, the entropy follows an area law, $S_k=\kappa_B[2\pi k/(d-2)\Omega_{d-2}G_k l^{2(k-1)}]A$.

### Locally hyperbolic transverse section

The case $\gamma=-1$ naturally leads to a splitting between theories with even and odd $k$ which are treated separately. In addition, the Einstein-Hilbert action ($k=1$) and the CS cases exhibit a special behavior.

(i) *Odd k.* In this case, solutions with non-negative mass have a single horizon radius larger than $l$ and their causal structure is analogous to that of the $\gamma=1$ case discussed above. For $M<0$, in the generic case there are two horizons with the same causal structure as the Reissner-Nordstrom AdS black hole, but where $r_+$ and $r_-$ cannot be independently adjusted because they are functions of a single parameter ($\mu$). The extremal case corresponds to the lower bound for both mass and $r_+$ ($M\geqslant M_c<0$ and $r_+\geqslant r_c$). The temperature is a strictly increasing function of the mass and hence the specific heat is non-negative for the entire physical range ($r_+\geqslant r_c$). This means that equilibrium with a heat bath can always be reached. The specific heat vanishes at $r_+=l$, signaling the existence of ''volatile'' configurations near the massless state. The Einstein theory ($k=1$) is singled out in this respect, since its specific heat neither vanishes nor has a minimum at the massless configuration and therefore exhibits no volatile behavior.

(ii) *Even k.* These theories possess an interesting set of black hole solutions with hyperbolic transverse section. In this case, there exist two independent branches for a given mass: the branch with $\alpha=1$ describes single horizon black holes with $r_+\geqslant l$. They have non-negative mass and the usual causal structure. The other branch ($\alpha=-1$), has a nonstandard mass range $\mu_c\geqslant\mu>0$, and the corresponding range of horizon radius is $r_c\leqslant r_+<l$. On the other hand, solutions belonging to this latter branch present two horizons and curiously, $r_+$ is a decreasing function of the mass, unlike the standard black holes. The extreme case corresponds to the smallest possible size of the horizon radius ($r_c$), which has the largest possible mass ($\mu=\mu_c$).

The following remarks on the thermodynamics are in order.

Topological black holes with hyperbolic transverse section and even $k$ can reach thermal equilibrium only if the temperature of the bath is higher than that of the massless ($r_+=l$) configuration ($T_B>T_l$), and if the initial state of the black hole belongs to the upper branch ($\alpha=+1$ and $T>T_l$). Otherwise, the fate of the black hole is to approach either the vacuum ($\mu=0$), or the extremal configuration ($\mu=\mu_c$), as discussed in Sec. IV B.

It is remarkable that for $\gamma=-1$, thermodynamics restricts the topology of the transverse section $\Sigma_{-1}$: for even $k$ it must be such that the configuration $\mu=0$ is not a black hole, whereas for odd $k$, the massless configuration must be a black hole, which for instance can be obtain through suitable identification in the covering space of the transverse section.

For CS theories the temperature has a universal linear dependence on $r_+$ for all $d$ and $\gamma$.

Solutions with $\gamma=-1$, of CS theories always possess a

[8] In generic theories (non-CS) theories with $\gamma=1$, the limit $\beta\to 0$ can be obtained for $r_+\ll l$. This branch, however, is thermodynamically unstable and this fact could be interpreted as a confined phase in the CFT [10].

single horizon and have qualitative thermodynamic behavior for even and odd $k$.

The canonical ensemble is well defined for all values of the parameters $d$, $k$, $\mu$, and $\gamma$, provided a negative cosmological constant is present. Otherwise, the partition function diverges.

In the vanishing cosmological constant limit, only the spherically symmetric solutions $(\gamma=1)$ with $d-2k\neq 1$ are black holes.

*Holography:* The topological black hole solutions found here shed some light on holography in the sense of the AdS/CFT correspondence. It has been shown that the black hole thermodynamics for $\gamma=0$ can be described in terms of a CFT at the boundary, for all the theories considered here. For $\gamma=\pm 1$ the matching occurs for $r_+ \gg l$. Thus Einstein's theory is not the only one which admits a holographic interpretation, but the whole set of gravitational theories presented here do. The exact matching with a CFT is achieved provided the value of the Steffan-Boltzmann constant is a fixed function of $d$ and $k$ given by Eq. (41). Since $\sigma_{SB} \sim k^{d-1}$, the number of degrees of freedom in the CFT must increase with the power of the curvature in the bulk gravitational theory. Hence, the AdS/CFT correspondence, in this sense, suggests the existence of $[(d-1)/2]$ inequivalent $(d-1)$-dimensional dual CFT's, one for each action $I_k$, enlarging the options for concrete holographic interpretation of gravity. In particular, in five dimensions there are two gravitation actions within this family (EH and CS), which are candidates in equal footing to realize the AdS/CFT correspondence. One could speculate that for CS gravity, the asymptotic dynamics would be described by some higher dimensional generalization of the WZW model (see, e.g., Refs. [29–32]). Thus, WZW models may be relevant to count the microstates responsible for the entropy of these black holes.

Topological black hole metrics in eleven dimensions with $k=5$ are also solutions of a supergravity theory, described in terms of a CS action with gauge group $0Sp(32|1)$ [21]. Furthermore, it can be shown that some of them admit Killing spinors [33]. This claim might seem surprising as no local supersymmetric extension exists for the EH action with cosmological constant in eleven dimensions [34].

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## APPENDIX: MASS AND ENTROPY FROM BOUNDARY TERMS

### 1. Mass

The aim of this appendix is to establish the relationship between the integration constant $\mu$, appearing in the solutions (18), and the mass. In the Hamiltonian approach, the gravitational action is

$$I_T = I_G + B, \tag{A1}$$

where $I_G$ is the canonical action in phase space,

$$I_G = \int d^d x (\pi^{ij}\dot{g}_{ij} - N^{\perp}H_{\perp} - N^i H_i), \tag{A2}$$

and $B$ is a boundary term, which is required in order to guarantee that the action attain an extremum on shell [35]. Here $H_\mu$ are the Hamiltonian generators of space-time diffeomorphisms.

Replacing the ansatz (11) into the action, allows to obtain a one-dimensional minisuperspace model whose action,

$$I_T = \Delta t \frac{\Sigma_{d-2}}{\Omega_{d-2}} \int \frac{N}{2} \frac{d}{dr}\left\{ \frac{r^{d-1}}{G_k}\left[ F_\gamma(r) + \frac{1}{l^2}\right]^k \right\} dr + B, \tag{A3}$$

is a functional of the fields $N := N^{\perp}(r) f^{-2}(r)$, and $f^2(r)$, with $F_\gamma(r) = [\gamma - f^2(r)]/r^2$. The field equations obtained from Eq. (A3) reproduce Eqs. (12) and (13). The bulk term vanishes on the field equations, so that the variation of the action (A3) on shell, is the boundary term

$$\delta I_T = \Delta t \frac{\Sigma_{d-2}}{\Omega_{d-2}} \int \frac{d}{dr}\left( N \frac{r^{d-1}}{2G_k} \delta\left[ F_\gamma(r) + \frac{1}{l^2}\right]^k \right) dr + \delta B, \tag{A4}$$

which means that the action is stationary on the black hole solution provided

$$\delta B = -\Delta t N_\infty \frac{\Sigma_{d-2}}{\Omega_{d-2}} \delta\mu, \tag{A5}$$

and consequently, the boundary term to be added is

$$B = -\Delta t N_\infty \frac{\Sigma_{d-2}}{\Omega_{d-2}} \mu + B_0,$$

where $B_0$ is an arbitrary constant without variation. This allows identifying the mass as

$$M = \frac{\Sigma_{d-2}}{\Omega_{d-2}} (\mu - \mu_0), \tag{A6}$$

where the lapse at infinity $(N_\infty)$ has been chosen equal to 1. In order to avoid naked singularities with positive mass, the additive constant $\mu_0$ is set equal to zero for all cases except for spherical black holes solutions of Chern Simons theories $(d=2k+1$ and $\gamma=1)$, that is, $\mu_0 = (1/2G_k)\delta_{d-2k,\gamma}$.

An alternative way to obtain the mass and angular momentum for gravity theories with asymptotically locally AdS behavior in even dimensions $(d=2n)$, has been recently proposed [2,3]. This construction is fully covariant and background independent. This provides an independent check of formula (A6), which is summarized here. The demand on the action $I_k$ to have an extremum for asymptotically locally AdS space times fixes the boundary term that must be added to Eq. (1) as the integral of the Euler density with a fixed coefficient [15],

$$I_T = I_k + \kappa \alpha_n \int \mathcal{E}_{2n}, \tag{A7}$$

where

$$\alpha_n = c_n^k := \frac{(-1)^{n+k+1} l^{2(n-k)}}{2n \binom{n-1}{k}}. \tag{A8}$$

The invariance of Eq. (A7) under diffeomorphisms provides a conserved current through Noether theorem, $d{*}J = 0$. Assuming that the asymptotic region of the manifold is $\partial \mathcal{M} = R \times \Sigma_\gamma$, the conserved charge associated with diffeomorphisms $x^\mu \to x^\mu + \xi^\mu$ is

$$Q[\xi] = \int_{\Sigma_\gamma} \xi^\mu \omega_\mu^{ab} \mathcal{T}_{ab}, \tag{A9}$$

where $\mathcal{T}_{ab}$ is the functional derivative of the total Lagrangian in Eq. (A7) with respect to the curvature

$$\mathcal{T}_{ab} := \frac{\delta L_T}{\delta R^{ab}}. \tag{A10}$$

The mass is obtained from Eq. (A9) for $\xi = \partial_t$, without making further assumptions about the matching with a background geometry or its topology. Thus the mass for the topological black holes given by Eq. (18) is

$$M = Q[\partial_t] = \mu \frac{\Sigma_{d-2}}{\Omega_{d-2}}, \tag{A11}$$

in agreement with the result obtained from the Hamiltonian formalism in even dimensions.

### 2. Entropy

In the semiclassical approximation the partition function is given by $Z \approx e^{-I_E}$, where $I_E$ is the Wick rotated version of the action (A3) given by

$$I_E = -\beta \frac{\Sigma_{d-2}}{\Omega_{d-2}} \int_{r_+}^{\infty} \frac{N}{2} \frac{d}{dr} \left\{ \frac{r^{d-1}}{G_k} \left[ F_\gamma(r) + \frac{1}{l^2} \right]^k \right\} dr + B_E. \tag{A12}$$

The on shell value of $I_E$ is given by $B_E$ and therefore the Helmholtz free energy, $F = I_E/\beta = M - S/(\kappa_B \beta)$, is completely determined by the boundary term, where $(df^2/dr)|_{r_+} = 4\pi\beta^{-1}$. The boundary term $B_E$ is also fixed requiring the action to have an extremum on the Euclidean form of the geometry, which covers only the exterior section of the black hole $(r > r_+)$. Its variation is now given by

$$\delta B_E = \beta \delta M - k \frac{2\pi \Sigma_{d-2}}{\Omega_{d-2} G_k} r_+^{(d-2k-1)} \left( \gamma + \frac{r_+^2}{l^2} \right)^{k-1} \delta r_+.$$

This implies that the variation of the entropy, as function of the horizon radius, reads

$$\delta S_k = k \frac{2\pi \kappa_B \Sigma_{d-2}}{\Omega_{d-2} G_k} r_+^{(d-2k-1)} \left( \gamma + \frac{r_+^2}{l^2} \right)^{k-1} \delta r_+. \tag{A13}$$

This relation can be integrated to yield a closed expression for entropy as a function of $r_+$, given in Eq. (33).


[1] S. W. Hawking and D. N. Page, Commun. Math. Phys. **87**, 577 (1983).
[2] R. Aros, M. Contreras, R. Olea, R. Troncoso, and J. Zanelli, Phys. Rev. Lett. **84**, 1647 (2000).
[3] R. Aros, M. Contreras, R. Olea, R. Troncoso, and J. Zanelli, Phys. Rev. D **62**, 044002 (2000).
[4] L. F. Abbott and S. Deser, Nucl. Phys. **B195**, 76 (1982).
[5] J. L. Friedman, K. Schleich, and D. M. Witt, Phys. Rev. Lett. **71**, 1486 (1993).
[6] J. P. S. Lemos, Phys. Lett. B **353**, 46 (1995).
[7] L. Vanzo, Phys. Rev. D **56**, 6475 (1997).
[8] D. R. Brill, J. Louko, and P. Peldan, Phys. Rev. D **56**, 3600 (1997).
[9] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, Phys. Rep. **323**, 183 (2000).
[10] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[11] D. Birmingham, Class. Quantum Grav. **16**, 1197 (1999).
[12] R. Emparan, C. V. Johnson, and R. C. Myers, Phys. Rev. D **60**, 104001 (1999).
[13] V. Balasubramanian, E. Gimon, and D. Minic, J. High Energy Phys. **05**, 014 (2000).
[14] R. Troncoso and J. Zanelli, Class. Quantum Grav. **17**, 4451 (2000).
[15] J. Crisostomo, R. Troncoso, and J. Zanelli, Phys. Rev. D **62**, 084013 (2000).
[16] C. Lanczos, Ann. Math. **39**, 842 (1938); D. Lovelock, J. Math. Phys. **12**, 498 (1971).
[17] J. T. Wheeler, Nucl. Phys. **B268**, 737 (1986).
[18] D. G. Boulware and S. Deser, Phys. Rev. Lett. **55**, 2656 (1985).
[19] C. Teitelboim and J. Zanelli, Class. Quantum Grav. **4**, L127 (1987).
[20] A. H. Chamseddine, Phys. Lett. B **233**, 291 (1989); Nucl. Phys. **B346**, 213 (1990).
[21] R. Troncoso and J. Zanelli, Phys. Rev. D **58**, 101703(R) (1998).
[22] M. Banados, M. Henneaux, C. Teitelboim, and J. Zanelli, Phys. Rev. D **48**, 1506 (1993).
[23] We thank L. Vanzo and J. Louko for clarifying this point to us. See also R. Emparan, J. High Energy Phys. **06**, 036 (1999).
[24] M. Banados, C. Teitelboim, and J. Zanelli, Phys. Rev. D **49**, 975 (1994).

[25] M. Bañados, C. Teitelboim, and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992).
[26] R. G. Cai and K. S. Soh, Phys. Rev. D **59**, 044013 (1999).
[27] C. F. E. Holzhey and F. Wilczek, Nucl. Phys. **B380**, 447 (1992).
[28] L. Susskind and E. Witten, ''The holographic bound in anti-de Sitter space,'' hep-th/9805114.
[29] V. P. Nair and J. Schiff, Phys. Lett. B **246**, 423 (1990).
[30] M. Bañados, L. Garay, and M. Henneaux, Nucl. Phys. **B476**, 611 (1996).
[31] O. Chandía, R. Troncoso, and J. Zanelli, in *Trends in Theoretical Physics II*, edited by H. Falomir, R. E. Gamboa-Saravi, and F. Schaposnik, AIP Conf. Proceedings No. 484 (AIP, Woodbury, NY, 1999), hep-th/9903204.
[32] J. Gegenberg and G. Kunstatter, Phys. Lett. B **478**, 327 (2000).
[33] R. Aros, C. Martínez, R. Troncoso, and J. Zanelli (work in progress).
[34] K. Bautier, S. Deser, M. Henneaux, and D. Seminara, Phys. Lett. B **406**, 49 (1997).
[35] T. Regge and C. Teitelboim, Ann. Phys. (N.Y.) **88**, 286 (1974).