Atom made from charged elementary black hole

V. V. Flambaum and J. C. Berengut
School of Physics, University of New South Wales, Sydney, 2052, Australia
(Received 10 January 2000; published 19 March 2001)

It is believed that there may have been a large number of black holes formed in the very early universe. These would have quantized masses. A charged ''elementary black hole'' (with the minimum possible mass) can capture electrons, protons, and other charged particles to form a ''black hole atom.'' We find the spectrum of such an object with a view to laboratory and astronomical observations of them. There is no limit to the charge of the black hole, which gives us the possibility of observing $Z>137$ bound states and transitions at a lower continuum. Negatively charged black holes can capture protons. For $Z>1$, the orbiting protons will coalesce to form a nucleus (after β decay of some protons to neutrons), with a stability curve different from that of free nuclei. In this system there is also the distinct possibility of single quark capture. This leads to the formation of a colored black hole that plays the role of an extremely heavy quark interacting strongly with the other two quarks. Finally we consider atoms formed with much larger black holes.

DOI: 10.1103/PhysRevD.63.084010 PACS number(s): 04.70.−s, 36.90.+f

I. INTRODUCTION

Fluctuations in the density of the very early universe led parts with extremely high density to collapse and form black holes [1–3]. The density of such objects would be greatly reduced in models of the early universe that include inflation, but they do not have to disappear entirely. These particles would help explain the mass deficit of the universe.

A black hole is essentially an elementary particle in the sense that it is completely described by a set of quantum numbers and can have no detectable internal structure (the famous ''black holes have no hair'' theory; see, e.g., [4]). Recently it has been demonstrated that the black hole mass is quantized in units of the Planck mass $M_p=(\hbar c/G)^{1/2}=1.2\times10^{19}$ GeV$=2\times10^{-5}$ g [5] (see also [6–9]). One intriguing reason for the quantization of black holes comes from a classical formula. The horizon area of Kerr-Newman black holes is

$$A=4\pi[(M+\sqrt{M^2-a^2-Q^2})^2+a^2], \tag{1}$$

where a is the angular momentum divided by the mass M, and Q is the charge of the black hole. This implies that for a given a and Q there is a minimum mass in order to avoid the singularity of the radical. In ordinary units (the previous equation was written in gravitational units where $\hbar=c=G=1$) one obtains [8]

$$M_{min}=M_p[Z^2\alpha/2+\sqrt{Z^4\alpha^2/4+J(J+1)}]^{1/2}, \tag{2}$$

where $\alpha=e^2/\hbar c$; Ze and J are the black hole charge and spin. A black hole with spin cannot have a mass smaller than $0.93M_p$. For spinless black holes this equation gives the minimal mass $M_{min}=M_p\sqrt{\alpha}=0.085M_p$. This classical consideration does not take into account the mass renormalization problem. One cannot completely exclude that after the ''renormalization'' the minimal mass of the charged black hole may be as small as the electron mass.

Such elementary black holes may appear in the very early universe or as a result of the ''evaporation'' of heavier black holes. We know that black holes radiate with a discrete spectrum in a blackbody envelope (Hawking radiation [10]), but we cannot say whether a final elementary black hole vanishes completely or what the lifetime of such a process would be. For example, it may not be ''easy'' for a black hole with $J=1/2$ (or any half-integer spin) to decay since any such decay involves violation of lepton or baryon number (e.g., black hole $\to e^-+\gamma$; however, if one views this decay as a ''disconnection'' of the black hole ''inner'' universe from our universe, no separate conservation laws for each ''universe'' should be assumed until they are totally disconnected). We will assume that the elementary black holes do not undergo a final radiation.

The first thing we would like to do is discuss a method to search for elementary black holes. An elementary charged black hole moving through matter at less than a few thousand km s^{-1} can capture electrons in the same way as an ordinary atomic nucleus, creating a ''black hole atom'' [3]. One can search for spectral lines in these systems. Normal nuclei are unstable for very large Z, but a black hole can have any charge at all. Furthermore, they can have a negative charge, giving rise to whole new types of systems. In fact just about any electrically charged particle can be bound in a black hole atom.

We would like to know whether or not we can do experiments on black holes in a laboratory. Obviously a neutral black hole can simply fall between atoms in the floor of our laboratory and to the center of the Earth, since it is small and the only force acting on it is gravity. However, if we have a charged black hole, then there is electromagnetic repulsion between atoms and the black hole which may be large enough to keep it in the laboratory. The contact Coulomb force between a neutral atom and the neutral black hole atom (with a charged black hole nucleus) is $\sim e^2/a_0^2\simeq10^{-7}$ N where a_0 is the Bohr radius. This equation is just the Coulomb force on the radius of an external electron orbit. Surprisingly, the force due to gravity M_pg is also $\sim10^{-7}$ N for an elementary black hole with the Planck mass. This means that a black hole has a large probability to ''tunnel'' between the atoms and fall through. A lighter spinless black hole may be an exception since the classical equation (2) gives in this case the minimal mass $M_p/12$. The electromagnetic force in this case is an order of magnitude larger than the gravitational force, and such a black hole may stay for some time on the surface of the Earth. In this situation it may be reasonable to do both laboratory and astronomical observations.

We discuss the ''atomic'' spectrum of an elementary black hole in Sec. II with a view to the observation of black hole atoms and verification of their existence. We calculate the ''isotopic'' shifts of the black hole lines relative to the usual atomic lines. This shift is relatively large. For example, for hydrogen $2p$-$1s$ transitions the frequency shift is equal to 44.8 cm^{-1}. This shift is far beyond a typical linewidth in cosmic and laboratory spectra. For heavier atoms and ions the shift is smaller (e.g., for Mg II $3p_{3/2}$-$3s$ transtions it is 1.378 cm^{-1}). However, it is still much larger than the present accuracy of the frequency measurements (for example, for strong electric dipole transitions in Mg II the accuracy is better than 10^{-2} cm^{-1} and the small isotopic separations between ^{24}Mg, ^{25}Mg, and ^{26}Mg lines have been measured [11]). This suggests an obvious strategy to search for black hole atoms. One has to look for very weak lines at certain distances from the strongest atomic and ionic lines. If the isotopic shifts for the ''normal'' atomic lines have been measured, it is possible to calculate these distances very accurately. It seems natural to start this search in hydrogen spectra where the shifts are maximal and can be very accurately calculated. However, other atoms may have the advantage of having spectra in a more convenient optical region of frequencies.

Note that as a reuslt of the large mass, the black hole spectral lines do not have thermal Doppler broadening and hyperfine structure. This also may help in identification of such lines.

Note that the upper limit on the concentration of black holes follows from an estimate of their mass density. The Planck mass M_p is 10^{18} times larger than the proton mass. If we assume that the minimal mass of elementary black holes is $M_p/12$ and that the dark matter is 100 times heavier than the hydrogen matter and consists of elementary black holes only, we obtain that the abundance of the elementary black holes in cosmic space does not exceed 10^{-15} of the hydrogen abundance (one can compare this with the abundance of uranium, 3×10^{-13}). The limit on the concentration of the elementary black holes may become stronger if one considers other effects—see, for example, the explanation of the deficit of solar neutrinos based on the catalysis of nuclear reactions in the Sun by charged black holes [12]. This may practically exclude the possibility of the observation of black holes in the spectra of very distant objects. We must rely on the observation of close objects (such as Sun spectra) or laboratory data. We should also recall another motivation to search for atoms with superheavy nuclei and shifted atomic lines which is related to so-called ''strange matter'' that have nuclei made from the ''normal'' (up, down) and strange quarks.

It is not excluded that an orbiting electron or proton could fall into the central black hole and neutralize its charge [3]. This may give some limits on the lifetime. An attempt to address this problem can be found in Sec. III. In a naive picture of a ''classical'' black hole the lifetime of low-Z electronic atoms is many orders of magnitude larger than the age of the universe, but it decreases exponentially with Z. This ''classical'' estimate tells us that primordial black holes would now not have a charge greater than about $Z=70$. However, quantum gravity may introduce a large unknown coefficient into this estimate and significantly shift this border.

We also discuss a few questions which may be of theoretical interest. In Sec. IV we discuss the K-shell states of black hole atoms for $Z>137$, where there is a well-known singularity in the equations. While single particle solutions of the Dirac equation for high Z have been known for some time, it is worth rexamining these because now we have a physical system where the critical field is realized. We show in this section that the ground state reaches the lower continuum ($E=-mc^2$) before $Z=138$ which means that there is no room for negative energy bound states (recall that for $Z=1/\alpha\simeq137$ the relativistic energy $E\simeq0$) and hence spontaneous positron emission will occur for all $Z>137$. We also consider a similar problem for a charged scalar particle where a singularity appears for $Z>68$.

If a black hole has a negative charge, then it can become a ''protonic'' atom. The strong force between protons leads to a peculiar ground state characterized by a nucleus orbiting our elementary black hole (Sec. V). It leads to a new stability curve for the captured nucleus, shifted towards more protons (some of the protons still undergo β decay to neutrons).

Furthermore, in the protonic atom the black hole could gain a color charge by capturing a single quark. This naturally leads to an extension of the ''no hair'' theorem (which states that any black hole can be characterized by its mass, electric charge, and spin) to include color. Discussion of the solutions of the Einstein-Yang-Mills equations for colored black holes can be found, e.g., in Ref. [13]. In Sec. VI we present a naive ''classical'' estimate of the lifetime of these systems and find that a black hole with color charge (which may be called a ''superheavy quark'') will persist for 10^7 yr. Again, quantum gravity may change this conclusion.

We should note that single quark capture may be forbidden if the energy of the colored black hole is higher than that of the electrically charged black hole. In this case the lifetime of the protonic black hole can be very long [it contains an extra factor $(r_g/fm)^6\sim10^{-150}$ where the Planck length $r_g=\hbar/M_pc=1.6\times10^{-33}$ cm, $fm=10^{-13}$ cm] since the probability of all three quarks to be near the black hole horizon is very small. The same argument may be valid for the electron black hole atom if the electron is not an elementary particle, i.e., consists of ''prequarks.'' Therefore, one may view the lifetime calculations in the present work as estimates of the minimal lifetimes.

Finally, in Sec. VII, we briefly consider atoms formed when much heavier (with masses $\sim10^{12}$ kg) black holes capture electrons. The electric charge is shown to neutralize very quickly, but there still remains the possibility of short-lived gravitationally bound systems.

II. BLACK HOLE SPECTRUM

The spectrum of a single electron in a pure central Coulomb potential is given by

$$E=-\frac{mZ^2e^4}{2\hbar^2n^2}, \tag{3}$$

where n is the principal quantum number and Z is the charge at the center. If we have many electrons, then the outer electron energy is usually described by the Rydberg formula

$$E=-\frac{mZ_a^2e^4}{2\hbar^2\nu^2}, \tag{4}$$

where ν is an effective principal quantum number, and Z_a-1 is the ion charge (Z_a is the charge that the outer electron ''sees''; for neutral atoms $Z_a=1$). To search for black holes we should calculate the level shifts from normal atoms with the same charge. The normal atom spectra thus provide us with a calibration point.

Normal atom spectra are shifted from the ''ideal'' spectra because of the finite mass and volume of their nuclei. In an elementary black hole atom, however, the mass of the nucleus is practically infinite (over 10^{18} proton masses) and its volume is zero (around 10^{-60} of nuclear volume). Thus we wish to find the energy levels of normal atoms in terms of the ideal (black hole) spectra plus energy shifts due to the finite mass and volume of their nucleus. To improve the accuracy of the calculations one can use experimental data for isotopic shifts in normal atoms.

The theory of isotopic shifts is presented in numerous books (see, e.g., [14]). However, it would be instructive to present some results here with a particular application to the black hole line shifts. The mass shift is given by

$$\Delta E_m=E_A-E_\infty=E_\infty\cdot\left(\frac{\mu}{m}-1\right)(1+S), \tag{5}$$

where m is the electron mass, $\mu=mM/(m+M)$ is the reduced mass, M is the mass of the nucleus, E_A is the energy of atomic level, E_∞ is the energy of the black hole level (corresponding to infinite M), and S is the correction due to the specific mass shift which exists in many-electron atoms. The contribution of the specific shift to the transition frequencies can be calculated or extracted from the experimental data on isotopic shifts (see below).

The volume shift is given by

$$\Delta E_V=-e\int(\phi-Ze/r)\psi^2(r)dV, \tag{6}$$

where ϕ is the Coulomb nuclear potential. While this integral is formally extended to all space, it is practically zero outside of the nucleus. For relativistic s-wave electrons,

$$\Delta E_V\simeq E_\infty\frac{6}{\nu}\frac{\gamma+1}{\gamma(2\gamma+1)(2\gamma+3)[\Gamma(2\gamma+1)]^2}\left(2Z\frac{r_0}{a_0}\right)^{2\gamma}, \tag{7}$$

where $\gamma=\sqrt{1-(Z\alpha)^2}$, $a_0=\hbar^2/me^2$ is the Bohr radius, $r_0=1.2A^{1/3}$ is the nuclear radius, and A is the mass number of nucleus. To calculate the parameter ν, we simply use Eq. (4), since E is the known ionization energy [15]. Estimates of the normal mass shifts and volume shifts of electron levels in atoms relative to ''ideal'' black hole atoms are presented in Figs. 1 and 2.

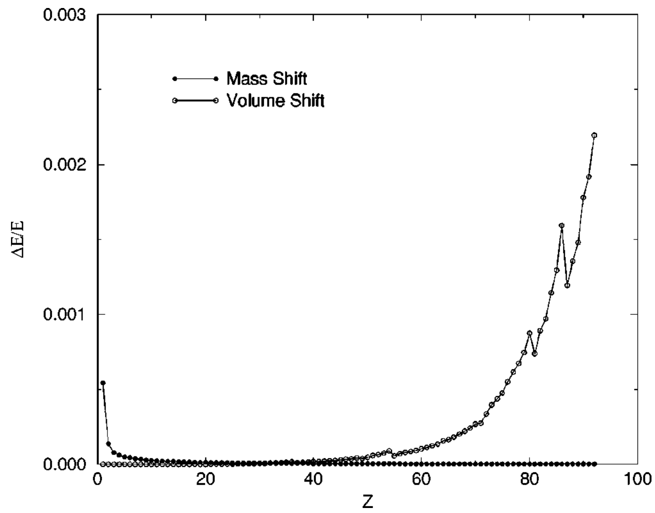

FIG. 1. Mass shifts and s-wave volume shifts in atoms of varying Z (see also Fig. 2).

Consider two examples. In hydrogen the volume shift is very small. We will specifically look at the $2p_{3/2}\rightarrow 1s_{1/2}$ transition. The spectral data is given in [17], which gives, for hydrogen and deuterium,

$$\omega_H=82259.279\ \ \mathrm{cm}^{-1},$$

$$\omega_D=82281.662\ \ \mathrm{cm}^{-1}.$$

Using the nuclear masses given in [16] we obtain

$$\omega_\infty-\omega_H=44.801\ \ \mathrm{cm}^{-1}. \tag{8}$$

It turns out that this shift is the same for the $2p_{1/2}\rightarrow 1s_{1/2}$ transition.

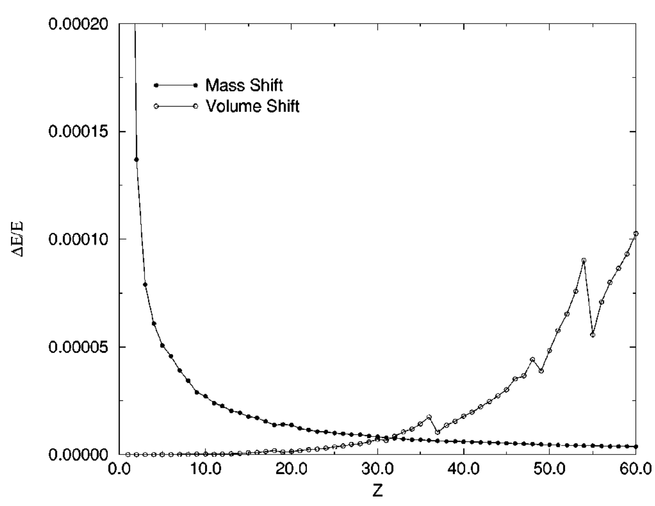

FIG. 2. Mass shifts and s-wave volume shifts in atoms of varying Z. The effects are equal at about $Z=32$.

Now consider Mg II. For magnesium we must include the volume shift. The two isotopes used are ^{24}Mg and ^{26}Mg. From Eq. (5) we then obtain

$$\omega_{26}-\omega_{24}=\omega_\infty\left(\frac{\mu_{26}}{m}-\frac{\mu_{24}}{m}\right)(1+S)-\Delta E_{V26}+\Delta E_{V24}, \tag{9}$$

$$\omega_\infty-\omega_{24}=\omega_\infty\left(1-\frac{\mu_{24}}{m}\right)(1+S)+\Delta E_{V24}. \tag{10}$$

Data for the MgII 2796 line (corresponding to the $3p_{3/2}\rightarrow 3s_{1/2}$ transition) have been obtained [18] which give

$$\omega(^{24}\mathrm{MgII}2796)=35760.834\pm 0.004\ \ \mathrm{cm}^{-1},$$

$$\Delta\nu(^{26}\mathrm{Mg}-{}^{24}\mathrm{Mg})=3.050\pm 0.1\ \ \mathrm{GHz},$$

$$\omega_{26}-\omega_{24}=0.1017\pm 0.003\ \ \mathrm{cm}^{-1}.$$

Now using the formula for the volume shift, Eq. (7), we obtain

$$\Delta E_{V24}=0.0315\ \ \mathrm{cm}^{-1},$$

$$\Delta E_{V26}=0.0333\ \ \mathrm{cm}^{-1}.$$

And so Eqs. (9),(10) give

$$\omega_\infty-\omega_{24}=1.378\ \ \mathrm{cm}^{-1}. \tag{11}$$

III. LIFETIME

There is a finite probability that electrons orbiting a black hole atom enter the black hole. This ''direct capture'' mechanism leads to a lifetime for the black hole atom. A solution of this problem would require the application of quantum gravity theory. However, in any theory the capture probability should be proportional to the density of the electron wave function near the origin which follows from the solution of the Dirac equation in the area of a weak gravitational field. This density is a very strong function of the black hole charge Z. The variation of this density from $Z=1$ to $Z=130$ exceeds 50 orders of magnitude. In this situation even a naive estimate of the lifetime may be instructive. Of course, there is an unknown coefficient in this estimate which is to be determined by quantum gravity.

The simplest estimate of the capture probability is given by the product of the black hole horizon area $4\pi r_g^2$ and flux for a particle moving with speed c near this horizon, $j=c|\psi_s(0)|^2$:

$$\frac{1}{\tau}=w\sim|\psi_s(0)|^2 4\pi r_g^2 c. \tag{12}$$

Now the K-shell electron is the most likely to be captured. Using relativistic electron wave function from [21] we obtain

$$w\sim\frac{4r_g^2cZ^3}{a_0^3}\left(\frac{a_0}{2Zr_g}\right)^{2(1-\gamma)}. \tag{13}$$

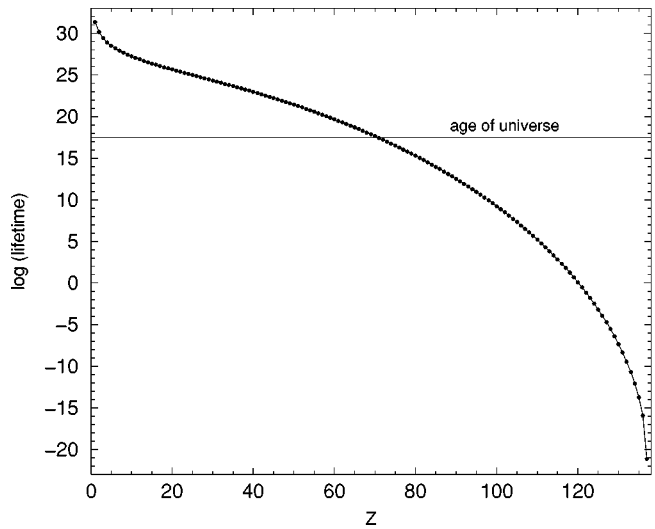

FIG. 3. Lifetime of K-shell electrons as a function of Z. On the y axis is plotted $\log_{10}[\mathrm{lifetime\ (sec)}]$ because the order of magnitude is of interest.

A complete picture is given in Fig. 3. In all numerical estimates we assume that the mass of the black hole is equal to M_p. For $Z=1$ this gives lifetime about 10^{22} yr, for $Z=70$, about the age of the universe. Thus, elementary black holes formed at the big bang with $Z\gtrsim 70$ would have captured electrons in the K shell, reducing the charge of the black hole. This process would have continued until the present day. Since the lifetime exponentially decreases with Z, we would not expect black holes with Z larger than about 70 to exist today.

Note that after the electron has been captured by the black hole, the quantization rules of the black hole (see the Introduction) are no longer obeyed. Thus the black hole must undergo some process to correct itself, such as radiate.

To conclude this section we would like to stress again that the estimates presented here must be multiplied by some unknown coefficient determined by quantum gravity. In this situation it may be reasonable to speak only about the relative probabilities of electron capture by black holes with different charges (if the quantum gravity coefficient is not zero). Indeed, applying standard quantum field theory on a curved background to such systems gives the prediction of an extremely high luminosity in Hawking radiation. However, by definition for such systems, one cannot treat their interactions with their surroundings by the standard laws of physics (treating gravity as classical), e.g., by quantum field theory on a curved background (or as is done in this paper, by quantum theory on a flat background which would be a good approximation if the standard quantum field theory on a curved background were valid). Thus, we possibly cannot use the classical capture cross section of an electron by a black hole even for an order of magnitude estimate.

It is not justified to use standard field theory to analyze the interaction of matter with such objects; that would be little better than guesswork. For example, if a remnant accretes an electron, does Hawking radiation turn on again, allowing it to reemit radiation? In particular, the decay rates computed in Sec. III are thus suspect.

This paper also assumes that there are stable electrically charged remnants. Naively, if a neutral Planck mass remnant accretes an electron, say, then one would expect it to decay again back to neutrality on a time scale of roughly the order of the Planck time. However, one cannot be certain everything decays within the unknown laws of quantum gravity.

IV. CRITICALLY CHARGED BLACK HOLES

When a black hole forms it inherits the (conserved) quantum numbers of the particles used to form it. This means that most black holes are formed with some nonzero electric charge Z. The evaporation process should reduce this charge; however, a final charge can still be large. There are some results which are not possible in normal atoms but which may be observed in black hole atoms. For example there are two well-known singularities in the single particle solution of the Dirac equation for a Coulomb field, corresponding to a particular Z (see, e.g., [22]). These single particle solutions are very good approximations for the K-shell (ground state) energy because these electrons are closest to the nucleus and only very lightly screened by the other electrons.

The first singularity occurs when $Z\alpha$ becomes greater than 1 (Sec. IV A). This singularity is removable when the finite size of the nucleus is taken into account. The second occurs when the ground state energy reaches the lower Dirac sea (Sec. IV B). We call the charge corresponding to this the supercritical charge.

A. First critical charge

The solution of the Dirac equation for a Coulomb field, $V=-Z\alpha/r^2$, has a singularity at $Z\alpha=1$ (or $Z=137.04$). At this point the parameter $\gamma=\sqrt{1-(Z\alpha)^2}$ and the ground state energy $\varepsilon=m\gamma$ become imaginary. In fact nothing much should change when this critical point is passed in a system with a finite nucleus. The energy ε becomes negative, but this is entirely allowable.

The physical reason for this singularity is that when $Z\alpha>1$ the particle can ''fall to the center.'' The effective potential $U(r)$, which arises when the Dirac equation is squared and put into a Schrödinger-equation-like form, behaves, for a Coulomb field, in a singular manner, $U(r)\approx[j(j+1)-Z^2\alpha^2]r^{-2}$ as $r\to 0$. This leads, as it does nonrelativistically, to all bound state wave functions having an infinite number of nodes when $Z\alpha>j+1/2$.

To determine the level energies it is necessary to specify the potential $V(r)$ and a boundary condition at zero. In any physical system this is obtained by an alteration of the potential at small r, due to the finite size of the nucleus or, in our case, the black hole. The form of the potential at small r is not important when $Z\alpha<1$ (in fact in our case the black hole should not significantly affect the energy levels even at $Z=137$, since the scale of the cutoff, $r_0=10^{-35}$ m, is so small). Yet it does become very important when $Z\alpha>1$.

A lot of interesting physics comes into play when the energy of the K-shell electron goes below zero (but not into the lower continuum). The electrons will have an energy $\varepsilon=-\text{const}\times m$. Therefore it is energetically favorable for the electron to undergo a ''beta decay'' into heavier and heavier particles, such as a muon, tauon, or perhaps some new grand unification particle. This process would be very fast, and in fact it may be possible for this to occur before the particle is captured by the black hole.

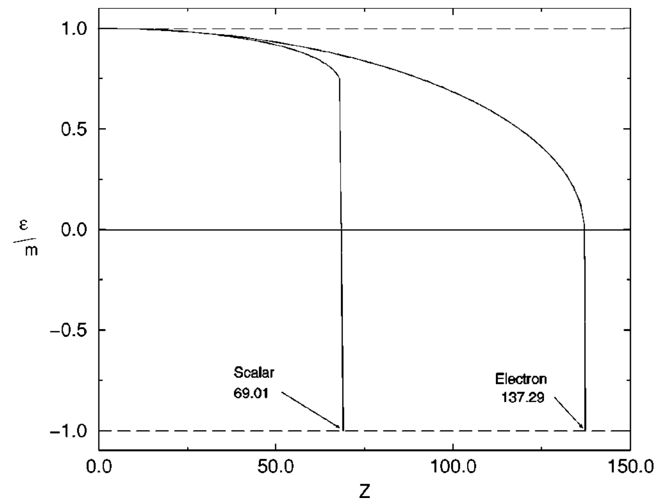

FIG. 4. K-shell electron and scalar particle energy as a function of Z. The electron energy reaches zero at $Z\alpha=1$ or $Z=137.036$. In the case where the nucleus is an elementary black hole, the energy reaches the Dirac sea ($\varepsilon/m=-1$) at $Z_c=137.29$, implying that there are no negative energy bound states. In the scalar particle case the energy reaches the Dirac sea very close to $Z_c=69$, so we cannot easily tell whether there is a bound state with negative energy or not.

Unfortunately in an elementary black hole system none of these effects are realized because the ground state energy falls into the Dirac sea before $Z=138$ (Sec. IV B; see Fig. 4), and hence there are no negative energy bound states. But the critical value of Z is dependent on the radius of the black hole, and hence it may be possible for bound states with $-m<\varepsilon<0$ to exist in heavier black holes.

B. Supercritical charge

If a black hole has a charge larger than some critical value Z_c, then the K-shell electron energy will reach the lower continuum (the lower Dirac sea) corresponding to energy $\varepsilon=-mc^2$. This corresponds to a binding energy of $-2mc^2$. This field can spontaneously polarize the vacuum to create an electron-positron pair.

In Appendix A we follow the method outlined by Popov [23] to find that the supercritical charge $Z_c=137.29$. Thus when $Z=138$ we are already in the lower continuum, and so we can conclude that there are no K-shell electron bound states with energy $-1<\varepsilon<0$.

When $Z>137$, then, the field can spontaneously polarize the vacuum to create an electron-positron pair. The electron goes into the K shell and the positron goes to infinity (in an ordinary nucleus there would be spontaneous emission of two positrons, after which the effective charge of the nucleus decreases by two units, corresponding to filling of the K

shell). In the black hole atom the electron would immediately fall into the black hole, decreasing its charge by 1.

This process has a finite lifetime, since the positron must tunnel to infinity to overcome the positive potential it would feel from the black hole. The spontaneous emission of positrons would continue until the charge of the black hole fell below $Z=138$. The ground states with $\varepsilon>0$ also have a finite lifetime, so the charge of the black hole then continues to fall until the charge is neutralized with the lifetimes described in Sec. III.

It is also interesting to consider the binding between black holes and charged scalar particles: Higgs boson, π meson, etc. For a scalar particle, the Klein-Gordon equation in the Coulomb field has a singularity at $Z\alpha=1/2$ (or $Z=68.5$). After this we must take into account the finite size of the nucleus. We wish to know at which point the energy reaches the lower continuum, to see whether there are any bound states with negative energy (in much the same fashion as was done for the electron case).

The calculation to find the critical charge in the case of the scalar particle is done in Appendix B, again following the method of Popov [23]. It is found that the bound state energy of the lowest shell equals $-mc^2$ at $Z_c=69.001$. Because of uncertainty in the size and boundary condition of the black hole, we cannot, from these results, tell if there is a $Z=69$ bound state or not. Anyway, it would be a very short-lived state.

In a case of a finite-size particle such as a π meson the low-Z Klein-Gordon equation may be treated in a similar fashion to the ''finite-size nucleus'' problem with the radius of π meson instead of nuclear radius. However, accurate high-Z results may be obtained only by solution of the three-body problem and depend on the strong interactions between the quarks. For example, one of the quarks can be rapidly captured by a black hole and they can form a ''superheavy quark'' interacting with a remaining quark—see the next section devoted to a protonic black hole.

V. PROTONIC BLACK HOLE ATOMS

If a black hole has a negative charge, then it can capture positively charged particles including protons and alpha particles. At first glance, one might think that this would form a ''protonic black hole atom,'' something akin to the usual atoms, but with orbiting protons instead of electrons. But in fact the physics of this system is quite different.

A. Ground state

Consider a protonic black hole. Because of the strong nuclear force, the ground state of this system would consist of a ''nucleus'' of protons (some of which may decay into neutrons) orbiting a black hole. For a singly charged black hole, there would simply be a single bound proton and nothing very exciting occurs (until it decays—see Sec. VI).

It is known that for two nucleons there are no bound isotopic triplet states. This means that in a system of two protons orbiting a black hole of charge $Z=-2$, one of the protons should decay via the weak interaction as

$$p^{+}\rightarrow n+e^{+}+\nu \tag{14}$$

and thus achieve a stable deuterium nucleus configuration. However, this is not the end of the story. The positron would be emitted with a large kinetic energy and so it would only be weakly bound if at all. Thus the system will still have an effective charge of -1, which can attract another proton. This would join with the deuterium nucleus and form a ^{3}He nucleus.

Also it is conceivable that the doubly charged black hole could pick up an extra neutron, if a sufficiently slow one chanced by the orbiting nucleus. Then the tritium nucleus would be formed. This would certainly be unstable, as it is usually, and decay to ^{3}He.

We now have the idea that the negatively charged black hole can have a nucleus orbiting it, having a certain number of neutrons and protons which follow the nuclear stability curve. But we have not considered the effect of the Coulomb field on this nucleus. The binding energy of the nucleus will decrease when protons decay to neutrons because the neutrons are not charged. This may provide enough energy in some cases to form a bound state with an unusually high number of protons. So the stability curve is effectively pushed towards the proton side.

B. Lifetime

The lifetime found from Eq. (13) is proportional to $m^{-2\gamma-1}$, where m is the mass of the orbiting particle. This means that a single proton orbiting around an elementary particle of charge $Z=1$ has a decay probability $(m_p/m_e)^3=1836^3$ times larger, leading to a lifetime of $\tau=3\times10^{12}$ yr.

But the mass of our particle will be that of the orbiting nucleus, which must in turn follow the new stability curve for a nucleus orbiting a black hole (Sec. V A). Thus the lifetimes will decrease even faster than they do in the electron case because the entire nucleus mass is of importance, not the mass of just one proton. However, if we consider black holes with only one proton, the lifetime is equal to the age of the universe for $Z=5$.

There is an additional complication to the lifetime, considered in Sec. VI.

VI. COLOR CHARGE IN BLACK HOLES

We have established in Sec. V that negatively charged black holes can have orbiting protons. We even estimated the probability that the bound protons fall into a black hole. But we did not consider that protons are not elementary particles, but are made up of three quarks, each with a different color charge. This means that as a result of the extremely small size of the elementary black hole, it is not the entire proton which is captured, but a single quark.

The black hole then obtains the color of the quark it captured, and becomes a ''superheavy quark.'' This would form a strongly bound state with the remaining two quarks of the original proton. While the other two quarks will eventually fall into the black hole, this will happen with a finite lifetime

$\tau \sim 10^7$ yr. Here we assumed that the quark wave function in the proton has radius $R \sim 0.5$ fm and $\psi(0)^2 = 1/(4\pi R^3/3)$.

VII. HEAVIER BLACK HOLES

Let us discuss briefly nonelementary black holes. Because the density fluctuations in the early universe would have occurred on all scales [2], there may conceivably have been black holes formed with much larger masses. The smaller ones of these would have evaporated into elementary black holes. The minimal mass of a black hole that would not have evaporated entirely by now is $M \sim 5 \times 10^{11}$ kg [2]. In this section we will consider black holes of mass $M \sim 10^{12}$ kg. Such an object has a radius of 1.5 fm.

These objects are interesting to study because at this mass they have gravitational fields comparable to the electric field, as well as sizes comparable to that of ordinary nuclei. Unfortunately there is another complication in determining orbits—the constant flux of Hawking radiation being emitted will interfere with any particle orbits.

Consider a singly charged black hole. Neglecting the gravitational potential and using Eq. (13) for the capture probability with a new Schwarzschild radius of 1.5 fm, we obtain the lifetime

$$\tau_{\mathrm{Coulomb}} = 5 \times 10^{-11} \ \text{sec}.$$

Obviously increasing the charge makes this smaller still, as does including the gravity term. Thus we can conclude from this and the fact that, if it was charged, evaporation would favor neutralization of the charge, that any charge of the black hole would have been neutralized by now.

Having concluded that there would be no electric charge on black hole atoms, we turn our attention to gravitational atoms. These would consist of particles (we will confine ourselves to electrons, but any particle would do) orbiting heavy black holes of the mass discussed. The potential is

$$U_{\mathrm{grav}} = -\frac{GMm}{r} = -\frac{g}{r}. \tag{15}$$

In relativistic units, $g = 1/520$ for electrons. The radius of the ground state for such a system is approximately $a = 1/g = 2 \times 10^{-11}$ m$=0.2$ nm. This does not take into account the continual radiation of the black hole which can interact with the particles and may potentially destroy the bound states.

It is interesting that there is no limit to the number of particles which can join the gravitational atom provided the particles are neutral or there is a balance between opposite charge particles (unlike an electromagnetic atom where the charge becomes screened). This gives rise to a new class of quantum many-body problems covering the area between hydrogens atom and the solar system.

VIII. CONCLUSION

We have seen that elementary black holes with charge can capture electrons and form bound states similar to those of an ordinary atom. The electronic spectra of these black hole atoms can be searched.

The electrons orbiting a black hole can fall into it. This ''direct capture'' leads to a finite lifetime for black hole atoms. This was found to be many times the age of the universe for black holes with small charges, but decreasing exponentially with increasing *Z*. From these calculations we concluded that primordial black holes would today have charges no larger than $Z \approx 70$.

The black hole atoms may give rise to new physical phenomena. When $Z > 137$ ($Z > 68$ for scalar particles) we have a physical realization of the much theorized supercritical Coulomb fields, where the single particle *K*-shell energy becomes negative. We found that these energies immediately drop into the lower continuum $\varepsilon < -mc^2$. Although the ground state falls into the Dirac sea at $Z\alpha = 1.002$, the upper states ($p_{3/2}$, d, etc.) do not until after $Z\alpha = |\kappa|$ (see [22]). Therefore it remains as an interesting question whether there are any states with negative energy (including mc^2) in the upper bound states. If there are, then there is still the possibility of the beta decay of electrons to muons, tauons, and so on.

We also discussed protonic black hole atoms which are formed with negatively charged elementary black holes. The ground state of such systems is a nucleus orbiting a black hole, with the nuclear stability curve pushed towards the proton side. Because protons are not elementary particles, the black holes would capture just one of the quarks of the proton (or of any constituent nucleon in the orbiting nucleus). This leads to a black hole with a color charge. The lifetime of this ''superheavy quark'' was found to be about 10^7 yr.

Finally we briefly considered much heavier black holes (with masses of $\sim 10^{12}$ kg) and found that their charge is neutralized very rapidly. There remains the possibility of gravitational atoms. However, because of Hawking radiation and the capture of the electron, such atoms may be very short lived.

ACKNOWLEDGMENTS

We are grateful to M. Yu. Kuchiev for valuable discussions. This work was supported by the Australian Research Council.

APPENDIX A: SUPERCRITICAL CHARGE: ELECTRON CASE

Here we proceed to find the supercritical *Z*, following the method outlined by Popov [23]. To find Z_c we need to solve the Dirac equation for $\varepsilon = -1$ (we use relativistic units $\hbar = c = m = 1$). We write the potential as

$$V(r) = \begin{cases} -\xi/r, & r > r_g, \\ -\xi/r_g, & 0 < r < r_g, \end{cases} \tag{A1}$$

where $\xi = Z\alpha$. This assumes that the wave function can extend inside the black hole, and that the charge of the black hole is concentrated entirely on the surface of it. The Dirac equation can be expressed [19,20]

$$\frac{dF}{dr}=-\frac{\kappa}{r}F+(1+\varepsilon-V)G,$$

$$\frac{dG}{dr}=(1-\varepsilon+V)F+\frac{\kappa}{r}G, \tag{A2}$$

where $F(r)=rf(r)$ and $G(r)=rg(r)$, with f and g the radial wave functions; $-\kappa$ is the eigenvalue of the operator $K=\beta(L\cdot\sigma+1)$, conserved in a spherically symmetric field. For K-shell electrons, $\kappa=-1$.

We eliminate G from Eq. (A2) to obtain the second order differential equation

$$F''+\frac{V'}{1+\varepsilon-V}\left(F'+\frac{\kappa}{r}F\right)+\left[(\varepsilon-V)^2-1-\frac{\kappa(\kappa+1)}{r^2}\right]F=0. \tag{A3}$$

The wave function outside the black hole can be obtained from this, setting $\varepsilon=-1$ and $V=-\xi/r$ where ξ now becomes its critical value when the bound state reaches the lower continuum, ξ_c:

$$r^2F''+rF'+(\xi^2-\kappa^2-2\xi r)F=0. \tag{A4}$$

Transforming this equation by $x=\sqrt{8\xi r}$ we obtain the Bessel equation

$$x^2F''+xF'-(x^2-\nu^2)F=0, \tag{A5}$$

with $\nu=2\sqrt{\xi^2-\kappa^2}$. This leads to the solution (for $\kappa=-1$ and $\xi>1$)

$$F=\text{const}\times K_{i\nu}(x)=\text{const}\times K_{i\nu}(\sqrt{8\xi r}), \tag{A6}$$

where $K_{i\nu}(x)$ is the MacDonald function of imaginary order (a Bessel function of the second kind). The second independent solution $I_{i\nu}(\sqrt{8\xi r})$ is unacceptable because of its growth at infinity.

Now we must choose a boundary condition at $r=r_g$, hence specifying completely the wave function F and allowing us to find ξ_c. For the potential considered in Eq. (A1), we equate the logarithmic derivatives of the wave functions inside and outside the black hole (for details see [23]) and obtain the transcendental equation

$$xK'_{i\nu}(x)=2\xi\cot\xi K_{i\nu}(x). \tag{A7}$$

Solving this for $r_g=1.6\times10^{-35}$ m$=4\times10^{-23}$ in relativistic units used we obtain $\xi=Z_c\alpha=1.00187$, corresponding to a critical charge

$$Z_c=137.29. \tag{A8}$$

Thus we conclude that there are no (K-shell) bound states with energy $-1<\varepsilon<0$ because when $Z=138$ we are already in the lower Dirac sea.

One may think that the boundary condition chosen is somewhat artificial, but in fact the actual boundary condition chosen is not important compared to the scale of r_g. Consider, for example, a very general (and incomplete) boundary condition $K_{i\nu}(\sqrt{8\xi r_g})=$const. The constant should necessarily be positive because the ground state should have no nodes. The maximum value of ξ will be realized when a node does exist at the boundary. So let us try $K_{i\nu}(\sqrt{8\xi r_g})=0$. This gives us a value of $\xi_c=1.00199$ corresponding to $Z_c=137.31$. So our conclusion that no negative energy bound states exist is valid.

APPENDIX B: SUPERCRITICAL CHARGE: SCALAR CASE

We can discuss the case of a pointlike scalar particle in a Coulomb field in an analogous manner to the electron case [23]. First, we must solve the Klein-Gordon equation, outside the nucleus

$$\varphi_l''+\left(\varepsilon^2-1+\frac{2\varepsilon\xi}{r}+\frac{\xi^2-l(l+1)}{r^2}\right)\varphi_l=0. \tag{B1}$$

We can solve this in its present form, but it is easier if we set $\varepsilon=-1$ immediately, since we are interested in finding ξ_c, where the K-shell energy meets the lower continuum. The wave function in this case has the form

$$\varphi_l(r)=\sqrt{r}K_{i\mu}(\sqrt{8\xi R}), \tag{B2}$$

$$\mu=2\sqrt{\xi^2-\left(l+\frac{1}{2}\right)^2}. \tag{B3}$$

If we again use the cutoff potential for $V(r)$, then we obtain for ξ_c the transcendental equation (for the lowest level $n=1$, $l=0$)

$$xK'_{i\nu}(x)=(2\beta\cot\beta-1)K_{i\nu}(x),$$

$$\nu=\sqrt{4\xi^2-1},$$

$$\beta=\sqrt{\xi(\xi-2R)},$$

$$x=\sqrt{8\xi R}. \tag{B4}$$

Solving this numerically we obtain that $\xi_c=0.50353$ which means that the energy of the scalar particle reaches the Dirac sea at $Z_c=69.001$. Because of the uncertainties in the size and boundary condition of the black hole, we cannot tell if there is a bound state with negative energy at $Z=69$.

[1] Ya. B. Zel'dovich and I. D. Novikov, Astron. Zh. **43**, 758 (1966) [Sov. Astron. **10**, 602 (1966)].
[2] C. W. Misner, in *Batelle Rencontres in Mathematics and Physics*, edited by B. S. DeWitt and B. Wheeler (Benjamin, New York, 1968).
[3] S. Hawking, Mon. Not. R. Astron. Soc. **152**, 75 (1971).
[4] W. Israel, in *300 Years of Gravitation*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1987).
[5] J. D. Bekenstein and V. F. Mukhanov, Phys. Lett. B **360**, 1 (1995).
[6] J. D. Bekenstein, Lett. Nuovo Cimento Soc. Ital. Fis. **11**, 467 (1974).
[7] V. Mukhanov, Pis'ma Zh. Éksp. Teor. Fiz. **44**, 50 (1986) [JETP Lett. **44**, 63 (1986)].
[8] P. Mazur, Gen. Relativ. Gravit. **19**, 1173 (1987).
[9] I. B. Kriplovich, Phys. Lett. B **431**, 19 (1998).
[10] S. W. Hawking, Nature (London) **248**, 30 (1974); Commun. Math. Phys. **43**, 199 (1975).
[11] R. E. Drullinger, D. J. Wineland, and J. C. Bergsquist, Appl. Phys. **22**, 365 (1980).
[12] E. M. Drobyshevski, Mon. Not. R. Astron. Soc. **282**, 211 (1996).
[13] D. Maison, in *Proceedings of the 8th M. Grossman Meeting on General Relativity*, edited by Tsvi Piran (World Scientific, Singapore, 1999), p. 530.
[14] I. I. Sobel'man, *Introduction to Theory of Atomic Spectra* (Nauka, Moscow, 1977).
[15] G. Aylward and T. Findlay, *S.I. Chemical Data* (Wiley, New York, 1994).
[16] C. M. Lederer and V. S. Shirley, *Table of Isotopes* (Wiley, New York, 1978).
[17] *Selected Tables of Atomic Spectra*, NSRDS-NBS 3 (U.S. Department of Commerce, Washington, D.C., 1972), Sec. 6.
[18] R. E. Drullinger, D. J. Wineland, and J. C. Bergquist, Appl. Phys. **22**, 265 (1980).
[19] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevsky, in *Relativistic Quantum Theory*, edited by L. D. Landau and E. M. Lifshitz, Theoretical Physics, Vol. 4 (Pergamon, New York, 1971).
[20] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon, New York, 1965).
[21] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), Vol. 1.
[22] Ya. B. Zel'dovich and V. S. Popov, Usp. Fiz. Nauk. **14**, 403 (1972) [Sov. Phys. Usp. **14**, 673 (1972)].
[23] V. S. Popov, Yad. Fiz. **12**, 429 (1970) [Sov. J. Nucl. Phys. **12**, 235 (1971)].