Mode generating mechanism in inflation with a cutoff

Achim Kempf*
Institute for Fundamental Theory, Departments of Physics and Mathematics, University of Florida, Gainesville, Florida 32611
(Received 14 September 2000; published 23 March 2001)

In most inflationary models, space-time inflated to the extent that modes of cosmological size originated as modes of wavelengths at least several orders of magnitude smaller than the Planck length. Recent studies confirmed that, therefore, inflationary predictions for the cosmic microwave background perturbations are generally sensitive to what is assumed about the Planck scale. Here, we propose a framework for field theories on curved backgrounds with a plausible type of ultraviolet cutoff. We find an explicit mechanism by which during cosmic expansion new (comoving) modes are generated continuously.

DOI: 10.1103/PhysRevD.63.083514 PACS number(s): 98.80.Cq, 98.70.Vc

Several problems of standard big bang cosmology, such as the horizon and the flatness problems, can be explained under the assumption that the very early universe underwent a period of extremely rapid inflation, driven by the potential of some assumed inflaton field. In particular, the inflationary scenario is also able to explain the observed perturbations in the cosmic microwave background (CMB), namely as originating ultimately from quantum fluctuations of the inflaton field. Indeed, the inflationarily predicted Gaussianity and near scale invariance of the perturbations' spectrum closely matches the current experimental evidence; see e.g. [1].

However, it has also been pointed out that in typical inflationary models, such as simple models of chaotic inflation, space-time inflated to the extent that modes which are now of cosmological size originated as modes with wavelengths that were at least several orders of magnitude smaller than the Planck length. Until recently, reason to believe that the inflationary prediction of the CMB spectrum might be insensitive to structure at the Planck scale was provided by the analogy with black hole radiation, which suffers from a similar *trans*-Planckian problem: any asymptotic Hawking photon with a medium range frequency should have had a far *trans*-Planckian proper frequency close to the event horizon, even at distances from the horizon which are farther than a Planck length. This problem has been investigated in detail; see e.g. [2]. The current consensus appears to be that the derivation of Hawking radiation is indeed essentially robust to changes in the assumed Planck scale structure. Likely, the reason for this insensitivity is that there are basic thermodynamical arguments for the properties of black hole radiation.

In the case of inflation, however, there does not seem to exist any basic thermodynamic reason why the derivation of the particle production should be insensitive to what is assumed about the Planck scale. Indeed, recent studies (see [3,4]) independently found that the inflationary CMB prediction *is* in general sensitive. Those studies calculated the effects of *ad hoc* ultraviolet modified dispersion relations similar to those which had been shown not to affect black hole radiation.

Interestingly, this means that if the inflationary scenario is true, then the currently rapidly improving experimental evidence about the CMB could provide a window to at least some aspects of particle physics at energies as high as the Planck scale. Therefore, our aim here is to formulate quantum field theory on curved backgrounds with some plausible type of ultraviolet cutoff. The aim is to obtain a framework in which all the usual entities of interest such as propagators, correlators, vacuum fluctuations and eventually the CMB perturbation spectrum can be calculated explicitly and compared to experiment.

We will need good arguments for choosing a particular type of cutoff. One obvious option would be to choose the lattice cutoff. However, apart from breaking translation and rotation invariance, the lattice cutoff faces further problems on expanding space-times: As has been pointed out earlier (see e.g. [4]), if space-time were a discrete lattice with a spacing of say one Planck length, then it is not clear how during the expansion new discrete lattice points could be created continuously. Indeed, whatever form of natural ultraviolet cutoff we assume, we will need to address the following questions: How does the expansion continuously generate new modes? What is the initial vacuum of these modes? And, once this is clarified, which CMB perturbation spectrum is predicted?

Our starting point for choosing a type of cutoff is an observation made in [5]: The short-distance structure of space-time can only be one of very few types — if we make a certain assumption. The assumption is that the fundamental theory of quantum gravity will possess for each space-time coordinate a linear operator X^i whose formal expectation values $\langle X^i \rangle$ are real. The X^i may or may not commute. One can prove on functional analytic grounds that the short-distance structure of any such coordinate, considered separately, can only be continuous, discrete, or ''unsharp'' in one of two ways. All other cases are mixtures of these.

The mathematical origin of the two ''unsharp'' cases lies in the fact that such an operator is not necessarily self-adjoint and may be merely what is called a symmetric operator. If X^i is self-adjoint, then it is diagonalizable and its spectrum can only be discrete (i.e., point) or continuous. Correspondingly, the short-distance structure of the coordinate which is described by X^i is therefore also discrete or continuous (or a mixture, including, e.g., fractals). However, if X^i is merely symmetric, then, crucially, it is not diagonalizable. Such coordinates are unsharp because a merely symmetric operator

*Email address: kempf@phys.ufl.edu

X^i need not possess eigenvectors, and the eigenvectors of its adjoint $(X^i)^*$ need not be orthogonal. (Technically, the two unsharp cases are distinguished by the so-called deficiency indices of X^i being either equal or unequal; see [5].)

Given the generality of the argument, namely the fact that the sharpness or unsharpness of indeed *any* real entity described by a linear operator falls into this classification, it is not surprising that such unsharp short-distance structures do occur ubiquitously. For example, the aperture induced unsharpness of optical images is of this kind, as is, e.g., the unsharpness in the time resolution of band-limited electronic signals; see [6].

In fact, also a number of studies in quantum gravity and string theory point towards one of these unsharp cases: They point towards the case of coordinates X whose formal uncertainty $\Delta X(\phi)=\langle\phi|(X-\langle\phi|X|\phi\rangle)^2|\phi\rangle^{1/2}$ possesses a finite lower bound at a Planck or string scale, $\Delta X(\phi)\geqslant\Delta X_{min}=l_{Pl}$, where ϕ is any unit vector on which the operator X can act. (Technically, this is a case of equal deficiency indices.)

In a first-principles quantum gravity theory such as string theory this behavior may of course arise from a complicated dynamics where space-time is a derived concept. Nevertheless, it has been argued (see e.g. [7]) that, effectively, this short-distance structure can be modeled as arising from quantum gravity correction terms to the uncertainty relation

$$\Delta x\Delta p\geqslant\frac{1}{2}[1+\beta(\Delta p)^2+\cdots]. \tag{1}$$

The positive constant β implies a constant positive lower bound $\Delta x_{min}=\sqrt{\beta}$. Usually, β is assumed such that the cutoff Δx_{min} is at a Planck or string scale. As first discussed in [8], the type of uncertainty relation of Eq. (1) can then be viewed as arising from corrections to the canonical commutation relations:

$$[\mathbf{x},\mathbf{p}]=i(1+\beta\mathbf{p}^2\cdots). \tag{2}$$

This commutation relation can be represented, e.g., with the operators $\mathbf{x}$ and $\mathbf{p}$ acting on fields over some auxiliary variable ρ as $\mathbf{x}\phi(\rho)=i\partial_\rho\phi(\rho)$, $\mathbf{p}\phi(\rho)=\tan(\rho\sqrt{\beta})/\sqrt{\beta}$ on the space of fields $\phi(\rho)$ over the interval $[-\rho_{max},\rho_{max}]$, where $\rho_{max}=\pi/(2\sqrt{\beta})$, with scalar product $\langle\phi_1|\phi_2\rangle=\int_{-\rho_{max}}^{\rho_{max}}d\rho\,\phi_1^*(\rho)\phi_2(\rho)$. The requirement of symmetry of $\mathbf{x}$ (i.e., that all formal expectation values are real) yields the boundary conditions $\phi(\pm\rho_{max})=0$. As expected, $\mathbf{x}$ is not self-adjoint: what would be its (now normalizable) eigenvectors "$|x)$," namely the plane waves in ρ space, are not in its domain and are not orthogonal on the interval $[-\rho_{max},\rho_{max}]$. In the "position representation," the fields $\phi(x)=(x,\phi)=\pi^{-1/2}\beta^{1/4}\int d\rho e^{ix\rho}$ exhibit a finite minimum wavelength. Modes whose wavelengths are close to the minimum wavelength are energetically exceedingly expensive (p diverges).

It would be a drawback of our approach if it covered only this perfectly translation invariant case in which Fourier theory applies. Let us therefore also mention, for completeness, that in this class of unsharp short-distance structures the lower bound on the position uncertainty, ΔX_{min}, is in general some function of the position $\langle X\rangle$ around which one tries to localize the particle: $\Delta X_{min}=\Delta X_{min}(\langle X\rangle)$. In this case, the situation is no longer describable as an overall wavelength cutoff. But it has been shown in [6] that in all cases where $\Delta X_{min}(x)>0$, fields over such unsharp coordinates can be described as possessing an ultraviolet cutoff in the sense that they contain only a finite density of degrees of freedom. Namely, such fields can be reconstructed at all points if known only on a set of discrete points — if these points are sufficiently tightly spaced. The minimum spacing varies over space and is small where the minimum position uncertainty $\Delta X_{min}(x)$ is small and vice versa. Thus, fields over such unsharp coordinates, while being continuous, always possess regularity properties similar to fields over lattices.

We now begin by recalling that the calculation of inflationary scalar density perturbations effectively reduces in the simplest case to the study of a minimally coupled massless real scalar field on a fixed curved background space-time such as, e.g., de Sitter space. For simplicity, we will assume the case of spatial flatness. It is usually most convenient to choose comoving coordinates y and the conformal time coordinate η. The metric then reads $g=\mathrm{diag}(a(\eta)^2,-a(\eta)^2,-a(\eta)^2,-a(\eta)^2)$ with $a(\eta)$ being the scale factor, and the action takes the form

$$S=\int d\eta\, d^3y\ \frac{a(\eta)^2}{2}\left((\partial_\eta\phi)^2-\sum_{i=1}^{3}(\partial_{y^i}\phi)^2\right). \tag{3}$$

Since our aim is to introduce a short-distance cutoff in proper distances (while leaving the time coordinate as is), we transform the action into proper space coordinates x^i, to obtain

$$S=\int d\eta\, d^3x\ \frac{1}{2a}\times\left\{\left[\left(\partial_\eta+\frac{a'}{a}\sum_{i=1}^{3}\partial_{x^i}x^i-\frac{3a'}{a}\right)\phi\right]^2-a^2\sum_{i=1}^{3}(\partial_{x^i}\phi)^2\right\} \tag{4}$$

where the prime stands for ∂_η. By defining

$$(\phi_1,\phi_2):=\int d^3x\ \phi_1^*(x)\phi_2(x) \tag{5}$$

$$\mathbf{x}^i\phi(x):=x^i\phi(x) \tag{6}$$

$$\mathbf{p}^i\phi(x):=-i\partial_{x^i}\phi(x) \tag{7}$$

we can write the action in the form

$$S=\int d\eta\ \frac{1}{2a}\{(\phi,A^\dagger(\eta)A(\eta)\phi)-a^2(\phi,\mathbf{p}^2\phi)\} \tag{8}$$

where the operator $A(\eta)$ is defined as

$$A=\left(\partial_\eta+i\frac{a'}{a}\sum_{i=1}^{3}\mathbf{p}^i\mathbf{x}^i-\frac{3a'}{a}\right). \tag{9}$$

In Eq. (8), the fields are time dependent abstract vectors in a Hilbert space representation of the commutation relations

$$[\mathbf{x}^i,\mathbf{p}^j]=i\delta^{ij}. \tag{10}$$

This only means that, as in quantum mechanics, also in quantum field theory the commutation relations of Eq. (10) are setting the stage, albeit without the simple quantum mechanical interpretation of the $\mathbf{x}^i$ and $\mathbf{p}^i$ as observables. For example, to express the action in momentum space is to choose the spectral representation of the $\mathbf{p}^i$.

Our ansatz now is to the keep the scalar field action exactly as given in Eqs. (8),(9), but to modify the underlying three-dimensional position-momentum commutation relations, Eq. (10), for large momenta, such as to introduce the type of cutoff which we discussed above. While we will break Lorentz invariance by introducing the cutoff, we will maintain translation and rotation invariance through the ansatz

$$[\mathbf{x}^i,\mathbf{p}^j]=i[f(\mathbf{p}^2)\delta^{ij}+g(\mathbf{p}^2)\mathbf{p}^i\mathbf{p}^j] \tag{11}$$

and by requiring that $[\mathbf{x}^i,\mathbf{x}^j]=0=[\mathbf{p}^i,\mathbf{p}^j]$, for all $i,j=1,2,3$. As was first shown in [9], the Jacobi identities then relate the functions f and g as follows:

$$g=\frac{2f\partial_{p^2}f}{f-2p^2\partial_{p^2}f}. \tag{12}$$

The behavior of the functions f and g for p^2 small compared to the Planck momentum is required to be $f\rightarrow 1$ and $g\rightarrow 0$. The functions f and g are then unique to first order in β, namely $f=1+\beta p^2+O(\beta^2)$ and $g=2\beta+O(\beta^2)$. We note that corresponding to the ambiguity in choosing f there is also an ordering ambiguity of the $\mathbf{x}^i$ and $\mathbf{p}^j$ in the action of Eqs. (8),(9). Equation (12) shows that g may develop singularities. We avoid this by choosing, e.g., $g=2\beta$. This then yields from Eq. (12) that $f(p^2)=2\beta p^2/(\sqrt{1+4\beta p^2}-1)$. A convenient Hilbert space representation of the new commutation relations is on fields $\phi(\rho)$ over auxiliary variables ρ^i,

$$\mathbf{x}^i\phi(\rho)=i\partial_{\rho^i}\phi(\rho) \tag{13}$$

$$\mathbf{p}^i\phi(\rho)=\frac{\rho^i}{1-\beta\rho^2}\phi(\rho), \tag{14}$$

with scalar product

$$(\phi_1,\phi_2)=\int_{\rho^2<\beta^{-1}}d^3\rho\,\phi_1^*(\rho)\phi_2(\rho). \tag{15}$$

The symmetry of the $\mathbf{x}^i$ requires the boundary condition $\phi(\rho^2=1/\beta)=0$. The $\mathbf{x}^i$ are not self-adjoint: Their would-be eigenvectors ''$|x)$,'' i.e., the now normalizable plane waves in ρ space, are not in their domain and are not orthogonal. The finiteness of ρ space, $\rho^2_{max}=\beta^{-1}$, implies a finite minimum wavelength $\lambda_{min}=2\pi\sqrt{\beta}$ for the fields $\phi(x)=(x,\phi)=\sqrt{3/(4\pi)}\beta^{3/4}\int d^3\rho\,\phi(\rho)e^{ix\rho}$ over position space and a corresponding finite minimum position uncertainty.

The action, as given in its abstract form, Eqs. (8),(9), with the new commutation relations Eqs. (11) underlying, can now be written in the ρ representation

$$S=\int d\eta\int_{\rho^2<\beta^{-1}}d^3\rho\;\frac{1}{2a}\times\left\{\left|\left(\partial_\eta-\frac{a'}{a}\frac{\rho^i}{1-\beta\rho^2}\partial_{\rho^i}-\frac{3a'}{a}\right)\phi\right|^2-\frac{a^2\rho^2|\phi|^2}{(1-\beta\rho^2)^2}\right\}. \tag{16}$$

The presence of ρ derivatives means that the ρ modes are coupled. Fortunately, it is still possible to find new variables $(\tilde{\eta},\tilde{k})$, namely $\tilde{\eta}=\eta$, $\tilde{k}^i=a\rho^i\exp(-\beta\rho^2/2)$, in which the $\tilde{k}$ modes decouple. As is readily verified,

$$\partial_\eta-\frac{a'}{a}\frac{\rho^i}{1-\beta\rho^2}\partial_{\rho^i}=\partial_{\tilde{\eta}}.$$

We will use the common index notation $\phi_{\tilde{k}}$ for those decoupling modes. The realness of the field $\phi(x)$ then translates through $\phi(\rho)^*=\phi(-\rho)$ into $\phi^*_{\tilde{k}}=\phi_{-\tilde{k}}$. We observe that the $\tilde{k}$ modes coincide with the usual comoving modes that are obtained by scaling, $k^i=ap^i$, only on large scales, i.e., only for small ρ^2, i.e., only for small momentum eigenvalues p^2. Conversely, this means that the comoving k modes only decouple at large proper distance scales, but do couple at small scales.

The action now reads

$$S=\int d\eta\int_{\tilde{k}^2<a^2/e\beta}d^3\tilde{k}\mathcal{L} \tag{17}$$

with

$$\mathcal{L}=\frac{1}{2}\nu\left\{\left|\left(\partial_\eta-3\frac{a'}{a}\right)\phi_{\tilde{k}}(\eta)\right|^2-\mu|\phi_{\tilde{k}}(\eta)|^2\right\} \tag{18}$$

where we defined

$$\mu(\eta,\tilde{k})\coloneqq-\frac{a^2\,\mathrm{plog}(-\beta\tilde{k}^2/a^2)}{\beta[1+\mathrm{plog}(-\beta\tilde{k}^2/a^2)]^2} \tag{19}$$

$$\nu(\eta,\tilde{k})\coloneqq\frac{\exp[-\frac{3}{2}\,\mathrm{plog}(-\beta\tilde{k}^2/a^2)]}{a^4[1+\mathrm{plog}(-\beta\tilde{k}^2/a^2)]} \tag{20}$$

and where the function plog, the ''product log,'' being the inverse of the function $x\mapsto xe^x$, allowed us to express ρ^2 in terms of η and $\tilde{k}^2$ through

$$\rho^2=-\beta^{-1}\mathrm{plog}(-\beta\tilde{k}^2/a^2). \tag{21}$$

Before we proceed, let us remark that this action reduces for $\beta\rightarrow 0$ to the standard action

$$S_{\beta=0}=\int d\eta d^3k \ \frac{1}{2a^4}\left\{\left|\left(\partial_\eta-3\frac{a'}{a}\right)\phi\right|^2-k^2|\phi|^2\right\}. \tag{22}$$

To see that this is the standard action, recall that Fourier transforming and scaling commute only up to a scaling factor. We Fourier transformed the proper positions and then scaled. Thus, our ϕ differs by a factor of a^3 from the ϕ obtained as usual by first scaling and then Fourier transforming.

The equation of motion for the action of Eqs. (17),(18) is

$$\phi''_{\tilde{k}}+\frac{\nu'}{\nu}\phi'_{\tilde{k}}+\left[\mu-3\left(\frac{a'}{a}\right)'-9\left(\frac{a'}{a}\right)^2-\frac{3a'\nu'}{a\nu}\right]\phi_{\tilde{k}}=0. \tag{23}$$

We recognize the damped harmonic oscillator form, with its friction term and with its variable mass term in which the contributions from the momentum and from the expansion compete. Comparing this modified mode equation to the modified wave equations of the pioneering works [3,4] we note certain differences in its derivation and in its form:

First, in [3,4] the new mode equations are derived essentially by transforming comoving momentum variables k into proper momentum variables $p=k/a(\eta)$, by then applying a nontrivial dispersion relation $F(k/a(\eta))$ and by finally transforming back into effective comoving momentum variables $k_{eff}=a(\eta)F(k/a(\eta))$. Our approach here has been similar in spirit. However, we here transformed not only the momentum *variables* but the entire action and wave equation from comoving to proper coordinates and back. Since the transformation to proper coordinates is time dependent, even the time derivatives thereby pick up terms, such as e.g. in the first quadratic term of Eq. (4). These terms involve spatial coordinates and derivatives, and therefore spatial momenta. As Eqs. (9),(11) show, in our approach the short-distance structure modification entered through those terms into the derivation of the modified mode equation (23). Recall also that at small distances we here distinguish the comoving momenta k from the momenta $\tilde{k}$ which are defined as those variables in terms of which the wave equation actually decouples into independent modes.

Second, our equation (23) can be transformed, as usual, to new variables in which the friction term is absent. This is being pursued in forthcoming work. Let us note already that even in this formulation a new feature of our approach remains, namely the fact that in our case each mode possesses its own starting time.

We now continue our investigation by noting that the field $\pi_{\tilde{k}}(\eta)$, canonically conjugate to $\phi_{\tilde{k}}(\eta)$, reads

$$\pi_{\tilde{k}}(\eta)=\nu\phi'_{-\tilde{k}}(\eta)-3\nu\frac{a'}{a}\phi_{-\tilde{k}}(\eta) \tag{24}$$

(recall that the canonical conjugate of the Fourier transform is the complex or Hermitian conjugate of the Fourier transform of the canonical conjugate). We can now use that $\phi'_{\tilde{k}}(\eta)=\nu^{-1}\pi_{-\tilde{k}}(\eta)+3(a'/a)\phi_{\tilde{k}}(\eta)$ to express the Hamiltonian

$$H=\int_{\tilde{k}^2<a^2/e\beta}d^3\tilde{k} \ \ [\pi_{\tilde{k}}(\eta)\phi'_{\tilde{k}}(\eta)-\mathcal{L}] \tag{25}$$

explicitly in terms of ϕ and π. We quantize by imposing the commutation relation $[\hat{\phi}_{\tilde{k}}(\eta),\hat{\pi}_{\tilde{r}}(\eta)]=i\delta^3(\tilde{k}-\tilde{r})$. (A path integral formulation of field theories over unsharp coordinates has been developed in [10].) The Heisenberg equations $\hat{\phi}'_{\tilde{k}}=i[\hat{H},\hat{\phi}_{\tilde{k}}]$ and $\hat{\pi}'_{\tilde{k}}=i[\hat{H},\hat{\pi}_{\tilde{k}}]$ then yield Eq. (23) as an equation for operator-valued fields.

We can now answer the first question which we raised in the beginning, namely by which mechanism new modes are generated: Automatically, the quantum field $\hat{\phi}(\eta,x)$ and the quantum Hamiltonian $\hat{H}$ of Eq. (25) contain the $\tilde{k}$ mode, i.e., the fields $\hat{\phi}_{\tilde{k}}(\eta)$ and $\hat{\pi}_{\tilde{k}}(\eta)$, only after the $\tilde{k}$ mode's ''creation'' time, $\eta_c(\tilde{k})$, which is when $a(\eta)$ has grown enough so that $\tilde{k}^2<a^2/e\beta$, i.e., when the proper wavelength of the $\tilde{k}$ mode becomes larger than λ_{min}. The action of $\hat{H}$ on the Hilbert subspace of modes with $\tilde{k}^2>a^2/(e\beta)$ is zero; i.e., the time evolution operator leaves the respective Hilbert subspace invariant.

Technically, the kernel of the Hamiltonian (its eigenspace to eigenvalue 0) is infinite dimensional and shrinks during cosmic expansion. Conversely, during a cosmic contraction the kernel enlarges. It should be interesting to calculate the correspondingly changing zero-point energy of the Hamiltonian as it picks up or loses modes. In this scenario, we live quite literally in a universe which resembles ''Hilbert's hotel'' (which can welcome guests even if full, because it has an infinite number of rooms).

We can solve for the dynamics of the quantized field $\hat{\phi}$ as usual, by using a complex classical solution ϕ to write the $\tilde{k}$ mode of the quantum field $\hat{\phi}$ as

$$\hat{\phi}_{\tilde{k}}(\eta)=[a_{\tilde{k}}\phi_{\tilde{k}}(\eta)+a^\dagger_{-\tilde{k}}\phi^*_{-\tilde{k}}(\eta)]. \tag{26}$$

The time independence of $a_{\tilde{k}}$ and $a^\dagger_{\tilde{k}}$ guarantees that $\hat{\phi}$ solves the equation of motion. Imposing $[a_{\tilde{k}},a^\dagger_{\tilde{r}}]=\delta^3(\tilde{k}-\tilde{r})$ guarantees that the field commutation relation is obeyed — if ϕ obeys $\nu(\eta,\tilde{k})[\phi_{\tilde{k}}(\eta)\phi'^*_{\tilde{k}}(\eta)-\phi^*_{-\tilde{k}}(\eta)\phi'_{-\tilde{k}}(\eta)]=i$. Normally, this Wronskian condition does not determine ϕ and therefore $\hat{\phi}$ uniquely; i.e., the choice of a classical complex solution and correspondingly the choice of a quantum vacuum are not unique. Here, however, each $\tilde{k}$ mode automatically possesses a creation time, $\eta_c(\tilde{k})$, at which both ν,μ and the $\tilde{k}$ mode's equation of motion are singular. This opens the interesting possibility, answering our second question from the beginning, that the requirement of the regularity of ϕ or other physical quantities at this singularity determines the vacuum uniquely.

Once the vacuum is fixed, it is then possible to calculate arbitrary quantum field theoretic entities, such as the magnitude of $\langle 0|\hat{\phi}^\dagger_{\tilde{k}}\hat{\phi}_{\tilde{k}}|0\rangle$ after horizon crossing, which yields the prediction for the CMB perturbation spectrum. This was our third question, and it can certainly be addressed at least numerically. The results should be very interesting to compare with the standard inflationary prediction of near scale invariance and canonically too large amplitude.

The author is happy to thank J. C. Niemeyer, J. R. Klauder, Ch. Thorn, K. Bering and W. H. Kinney for valuable suggestions and comments.

[1] J. A. Peacock, *Cosmological Physics* (Cambridge University Press, Cambridge, England, 1998); W. H. Kinney, A. Melchiorri, and A. Riotto, Phys. Rev. D **63**, 023505 (2001).

[2] W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981); G. 't Hooft, Nucl. Phys. **B256**, 727 (1985); R. Brout, S. Massar, R. Parentani, and P. Spindel, Phys. Rev. D **52**, 4559 (1995); R. Brout, C. Gabriel, M. Lubo, and P. Spindel, *ibid.* **59**, 044005 (1999); A. Kempf, J. Math. Phys. **41**, 2360 (2000).

[3] J. Martin and R. H. Brandenberger, Phys. Rev. D (to be published), hep-th/0005209; R. H. Brandenberger and J. Martin, astro-ph/0005432.

[4] J. C. Niemeyer, astro-ph/0005533.

[5] A. Kempf, in *Proceedings of the 36th International School of Subnuclear Physics*, Erice, Italy, 1998, edited by A. Zichichi (World Scientific, Singapore, 2000), Course 36, p. 613.

[6] A. Kempf, Phys. Rev. Lett. **85**, 2873 (2000).

[7] D. J. Gross and P. F. Mende, Nucl. Phys. **B303**, 407 (1988); D. Amati, M. Ciafaloni, and G. Veneziano, Phys. Lett. B **216**, 41 (1989); L. J. Garay, Int. J. Mod. Phys. A **10**, 145 (1995); E. Witten, Phys. Today **49**(4), 24 (1996); G. Amelino-Camelia, J. Ellis, N. E. Mavromatos, and N. Nanopoulos, Mod. Phys. Lett. A **12**, 2029 (1997); S. de Haro, J. High Energy Phys. **10**, 023 (1998); D. V. Ahluwalia, Phys. Lett. A **275**, 31 (2000).

[8] A. Kempf, J. Math. Phys. **35**, 4483 (1994).

[9] A. Kempf, J. Phys. A **30**, 2093 (1997).

[10] A. Kempf, J. Math. Phys. **38**, 1347 (1997); A. Kempf and G. Mangano, Phys. Rev. D **55**, 7909 (1997).