Correlation between gamma-ray bursts and gravitational waves

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(Received 20 November 2000; published 19 March 2001)

The cosmological origin of γ-ray bursts (GRB's) is now commonly accepted and, according to several models for the central engine, GRB sources should also emit at the same time gravitational wave bursts (GWB's). We have performed two correlation searches between the data of the resonant gravitational wave detector AURIGA and GRB arrival times collected in the BATSE 4B catalogue. No correlation was found and an upper limit $h_{\mathrm{RMS}} \leqslant 1.5 \times 10^{-18}$ on the averaged amplitude of gravitational waves associated with γ-ray bursts has been set for the first time.

DOI: 10.1103/PhysRevD.63.082002 PACS number(s): 04.80.Nn, 98.70.Rz

I. INTRODUCTION

Thirty years after their discovery, gamma-ray bursts (GRB's) are still the most mysterious objects in the Universe, as their properties have not yet been associated with any well-known object, while, for instance, pulsars have been almost immediately identified as final objects of the evolution of stars, and quasars have been well framed among galaxy nuclei in particular in the Seyfert class. Unfortunately ''standard'' astrophysical objects do not exist for GRB's. The ''nonstandard'' compact astrophysical objects which might reproduce the experimental characteristics of GRB's involve black holes (BH), neutron stars (NS), and massive stars.

The wide range of characteristics prevents the systematic classification of GRB's. Their energy spectrum is continuous and nonthermal, and covers the range between 1 keV and 1 MeV; their emission lasts from 10 ms to 10^3 s. The burst and transient source experiment (BATSE) has detected GRB's with an event rate of about one per day between the years 1991 and 2000.

One of the most important results of observations is that many GRB's are at cosmological distances [1]. This fact, recently confirmed also by the satellite BeppoSAX [2], which has detected some optical counterpart with redshifts close to $z=1$ [3–6], implies the emission of a huge amount of electromagnetic radiation: in a few seconds the GRB's release an energy of 10^{51}–10^{54} erg. On the other hand, this implies that the GRB's are rare events in a galaxy: in fact, as the rate of GRB events is of the order 1/day, their rate amounts to 1 event/10^6 years per galaxy [7].

The theoretical framework which gives a coherent explanation of the experimental data set of GRB's is the so-called ''*fireball*'' model: an ''inner engine'' produces a flux of relativistic energy of the order 10^{52} erg and it is extremely compact and clearly not visible; this relativistic flux of particles is a kind of ''fireball'' able to produce electromagnetic radiation in a optically thin shell of matter. The inner engine works for 1–3 days, producing the afterglow, but the bulk of the explosion lasts typically 10 s (with peaks around 0.5 and 30 s and a variation of six orders of magnitude, 10^{-3}–10^3 s) when the major part of the energy is emitted.

The inner engine progenitors are likely to be coalescing binary systems, such as NS-BH or NS-NS systems, or single stars that collapse into BH in supernovalike events (the so-called ''*collapsar*'' models [8]). If this is the correct explanation of the GRB's, then we would have compact astrophysical systems for which gravity plays an important role, and gravitational waves would be emitted [9–11].

This scenario has motivated us to investigate the behavior of the gravitational wave detector AURIGA during time spans which include the arrival time of a GRB burst. The AURIGA detector [12] is an Al5056 resonant bar of about 2.3 tons with a typical noise temperature of 7 mK and a bandwidth of about 1 Hz. The detector is sensitive to gravitational waves (g.w.) signals over 1-Hz bandwidth around each one of its two resonant frequencies, i.e., 913 and 931 Hz. The sensitivity of a gravitational wave burst (GWB) de-

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tector can be conveniently expressed by the quantity h_{min}, representing the minimum g.w. amplitude detectable at signal-to-noise (SN) ratio$=1$, which for the AURIGA detector is $h_{min} \sim 2-5\times 10^{-19}$. The AURIGA sensitivity is enough to detect NS-NS, BH-NS, and BH-BH mergers which take place within the Galaxy.

Previous work on GRB-GWB association appeared in the literature, specifically about a single GRB trigger [13], and about a set of GRB triggers [14–16,33]; here we present an experimental upper limit on such an association obtained with the AURIGA-BATSE data.

The plan of the paper is as follows: in Sec. II we discuss the two methods, coincidence and statistical searches, which we use for the association of the GRB's in the BATSE catalog with GWB's. The results obtained with the two methods are presented in Sec. III. In Sec. IV we analyze our results and discuss the possibilities opened by our methods for future searches.

II. SEARCH METHODS

The aim of our analysis is the search of an association between GRB and GWB arrival times within a time window W. The analysis has been carried out by means of two different procedures which have been already discussed in the literature: (i) the correlation method [17] which is based on the coincidence between the arrival time of candidate gravitational events selected over a given threshold and the GRB trigger and (ii) a statistical method [16] which relies on a hypothesis test on a statistical variable representing the mean energy of the gravitational detector at the GRB trigger.

The coincidence window plays an important role in both the analyses. In fact, the coincidence window should be wide enough to hold the delay distribution between GRB and GWB. We notice that its ''optimal'' value depends on the astrophysical sources and on the g.w. detector properties. If we assume that the GRB's are generated by internal shocks in the fireball, the delay between GRB and GWB is less than 1 sec [10] but there are still some uncertainties. Moreover, the delay is widened by the cosmological redshift. To be as much conservative as possible on the distribution of the delays we choose $W=5$ s. This value turns out to be also consistent with the requirements of the filtering procedures of the AURIGA detector. In fact, the search for g.w. bursts requires a filtering of the data by a Wiener-Kolmogorov filter matched to δ-like signals [18] and its characteristic time is the inverse of the detector bandwidth, i.e., ~ 1 s. This time, as we shall see below, establishes the time scale of the noise correlation of the filtered data.

In what follows, ''δ-like signal'' means any g.w. signal which shows a nearly flat Fourier transform at the resonance frequencies of the detector (913 and 931 Hz) over a ~ 1-Hz bandwidth. Therefore the metric perturbations $h(t)$ sensed by the AURIGA detector can be a large class of short signals (of millisecond duration) including the latest stable orbits of inspiralling NS-NS or NS-BH, the subsequent merging and the final ringdown [19] and the collapsar bursts [11] which could be expected signals associated with GRB's.

A. Coincidence search

The coincidence method has been successfully applied to the search of coincident excitations of different g.w. detectors [20,21]; much work has been devoted to exploit its potentialities and to develop robust estimates of the background of accidental, even in the presence of nonstationary event rates [17,21]. A g.w. candidate event is a local maximum of the filtered data corresponding to any excitation of the detector. From the AURIGA filtered output we extract a list of candidate events setting an adaptive threshold in their signal-to-noise ratio (SN ratio$=5$). The event lists used in this analysis belong to periods of satisfactory performance of AURIGA as described in Ref. [22]. It is worth noticing that the AURIGA event search checks each event against the expected signal template by means of a χ^2 test [23]. The event lists contain the information needed to describe a δ-like signal, namely, its time of arrival [in Coordinated Universal Time (UTC) units], the amplitude of the Fourier transform, and the detector noise level at that time. The BATSE data are taken from the 4B catalog by Meegan *et al.*, available on the Internet [24] and includes trigger time (in UTC units), right ascension and declination, error box, and other information about every GRB triggered.

We label with $t_A^{(i)}$ the estimate arrival time of the ith candidate g.w. event detected by AURIGA and with $t_\gamma^{(k)}$ the trigger time of the kth GRB detected by BATSE. A coincidence between the ith AURIGA event and the kth GRB trigger time is observed if $|t_A^{(i)}-t_\gamma^{(k)}| \leqslant W$, where W is the coincidence window.

The coincidences found have to be compared with the accidental coincidence background due to chance. Two standard methods to evaluate the probability of accidentals have been applied to the pair BATSE-AURIGA: (i) performing thousands time-shifts of the arrival time of one detector with respect to the other and looking for accidental coincidences at each shift [20]; (ii) assuming independent Poisson distribution of event times and using the mean measured rates to estimate the accidental rate n_a,

$$n_a = N_A N_\gamma \frac{\Delta T}{T}, \tag{1}$$

where N_A is the number of AURIGA events in the period T, N_γ the number of GRB events in the same period, and $\Delta T = 2W$ is the total amplitude of the coincidence window (see Appendix A for a proof of this basic relation). To avoid the problem of multiple coincidence within the same window we prefer to estimate the background with $W=1$ s and to scale our results with the help of Eq. (1) to $W=5$ s. The consistency of the two estimates ensures that the arrival times of the AURIGA-BATSE pair are distributed as Poisson random points even if the AURIGA event rate has been found to be not stationary [17,21].

B. Statistical search

The method of the statistical search has been proposed for a pair of interferometric detectors by Finn and co-workers [16]; here we slightly modify their approach to the case of a

single resonant detector. The data we use are the AURIGA filtered data obtained by means of the Wiener filter matched to a δ-like signal, without setting any threshold on their amplitudes. If a signal enters the detector at time t_0, its output can be written as

$$y(t)=hf(t-t_0)+\eta(t), \tag{2}$$

where $f(t-t_0)$ is the normalized signal template of the Wiener filter, h its amplitude [i.e., $f(0)=1$], t_0 its arrival time and $\eta(t)$ is a stochastic process with zero mean and correlation

$$\langle \eta(t),\eta(t')\rangle=\sigma_h^2 f(t-t'). \tag{3}$$

Here σ_h^2 is the variance of the filter output in the absence of any signal and $f(t)$ is a superposition of two exponentially damped oscillating functions [25] which can be approximately expressed as

$$f(t)\approx e^{-|t|/t_w}\cos(\omega_0 t)\cos(\omega_B t) \quad (\omega_0 t_w \gg 1), \tag{4}$$

where t_w is the Wiener filter decaying time (i.e., the inverse of the detector bandwidth), ω_0 a center carrier frequency and ω_B is an amplitude modulation frequency. Typical values for the AURIGA detector are $t_w \simeq 1$ s, $\omega_0 \simeq 920$ Hz, and $\omega_B \simeq 20$ Hz.

Let us define now the random variable X, which represents an averaged measure of the energy released by a g.w. signal impinging on the detector at time t_0,

$$X(t_0)=\frac{1}{2W}\int_{-W}^{W} dt|y(t-t_0)|^2, \tag{5}$$

where t_0 is the center of the time window. If an association between GRB and gravitational waves exists, the filtered output of the gravitational waves detector, in periods just prior the GRB (''*on-source*'' population) will differ (statistically) from the output at other times (''*off-source*'' population).

A statistically significant difference between *on*- and *off-source* populations clearly supports a GWB-GRB association. For each of the N_{on} GRB trigger we compute $X(t_\gamma^{(k)})$, $k=1,\ldots,N_{on}$ which forms the χ_{on} set of *on-source* events, and construct a complementary set χ_{off} with N_{off} *off-source* events using windows before and after the trigger. The sets χ_{on} and χ_{off} are samples drawn from the populations whose distributions we denote p_{on} and p_{off}.

The *off-source* events are taken in periods not correlated with the trigger time, at a distance in time greater than 10^3 sec from GRB the trigger, both before and after it; this should be sufficient to have a fair sample of the *off-source* events as any GRB-GWB association is reasonably excluded.

For windows W greater than the Wiener filter characteristic time, the central limit theorem implies that p_{off} is a normal distribution.

Now suppose that the GWB's fall within the window W opened around the GRB trigger time and that the SN ratio of the gravitational signal associated with the GRB (averaged over the source population) is smaller than one, then p_{on} is also a normal distribution with mean

$$\mu_{on}=\mu_{off}+E\left[\frac{1}{2W}\int_{-W}^{W} dt|hf(t-t_\gamma)|^2\right]$$
$$\simeq \mu_{off}+\left(\frac{t_w}{8W}\right)E[h^2] \quad (W\gg t_w), \tag{6}$$

where $E[\cdot]$ is the average over the astrophysical source population of GRB's.

The basic idea behind the statistical approach is a hypothesis testing where the null hypothesis $\mathcal{H}_0$ to test is the equivalence of the *off-source* and *on-source* distributions:

$$\mathcal{H}_0: \quad p_{off}(X)=p_{on}(X) \tag{7}$$

The rejection of $\mathcal{H}_0$ clearly supports a GWB-GRB association. Since p_{on} and p_{off} are normal and could differ only in their mean values, we can test $\mathcal{H}_0$ by the Student's t test [26].

The t statistic is defined from χ_{on} and χ_{off} by

$$t=\frac{\hat{\mu}_{on}-\hat{\mu}_{off}}{\Sigma}\sqrt{\frac{N_{on}N_{off}}{N_{on}+N_{off}}}, \tag{8}$$

$$\Sigma^2=\frac{(N_{on}-1)\hat{\sigma}_{on}^2+(N_{off}-1)\hat{\sigma}_{off}^2}{N_{on}+N_{off}-2}, \tag{9}$$

where $\hat{\mu}_{on}$ and $\hat{\mu}_{off}$ ($\hat{\sigma}_{on}^2$ and $\hat{\sigma}_{off}^2$) are the sample means (variances) of χ_{on} and χ_{off}, respectively.

The expected value of t averaged on the source population and on the filtered output of the detector is

$$\mu_t=E[t]=\left(\frac{t_w}{8W}\right)\frac{E[h^2]}{\sigma}\sqrt{\frac{N_{on}N_{off}}{N_{on}+N_{off}}}, \tag{10}$$

where $\sigma=E[\Sigma]$.

Let us consider the upper limit on $E[h^2]$ in the assumption $\mathcal{H}_0$ is true: in this case the most probable value of $E[h^2]$ is zero. From Eq. (10) we get

$$\left(\frac{t_w}{8W}\right)\frac{E[h^2]}{\sigma}\leq \mu_{t,max}\sqrt{\frac{N_{on}+N_{off}}{N_{on}N_{off}}}$$
$$=\begin{cases}\mu_{t,max}\sqrt{2/N_\gamma} & (N_{on}=N_{off}=N_\gamma)\\ \mu_{t,max}/\sqrt{N_{on}} & (N_{off}\gg N_{on})\end{cases}, \tag{11}$$

where $\mu_{t,max}$ is the upper limit. Assuming $\mathcal{H}_0$ true and $N_{off}\gg N_{on}$, we have

$$E[h^2]\leq h_{max}^2=\left(\frac{8W}{t_w}\right)\frac{\mu_{t,max}}{\sqrt{N_{on}}}\sigma; \tag{12}$$

the value of $\mu_{t,max}$ can be deduced by the selected confidence level. In this way we are able to set an upper limit on the average amplitude of gravitational signals associated with GRB's:

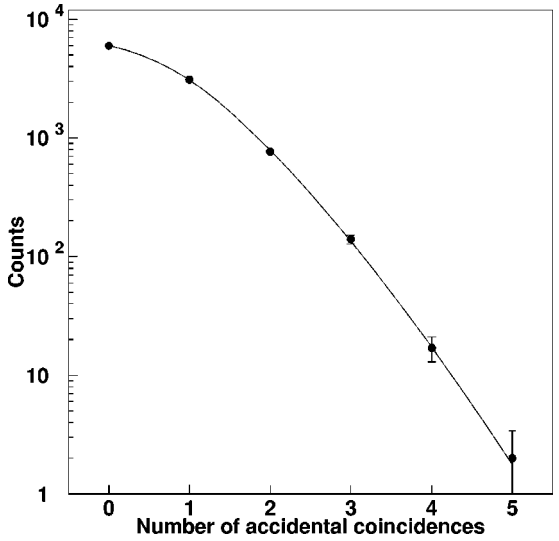

FIG. 1. Poisson fit of the data obtained with the shifts method.

$$h^2_{\mathrm{RMS}} \leqslant [1.4\times 10^{-18}]^2 \frac{W}{5s}\left(\frac{t_w}{1s}\right)^{-1} \times \frac{\mu_{t,max}}{1.96}\left(\frac{N_{on}}{100}\right)^{-1/2} \frac{\sigma}{[5\times 10^{-19}]^2}. \quad (13)$$

III. RESULTS

The AURIGA data used in the two analyses are relative to the years 1997 and 1998, and the number of GRB's which fall into the AURIGA data taking periods is 120.

A. Coincidence search

Within the window of 5 sec we have found two events in coincidence. This experimental result has to be compared with the number of coincidences due to chance.

The shifts method consists of 10^4 time shifts of the coincidence window; for each one we compute the number of coincidences; if the candidate GWB arrival times and the GRB triggers can both be modeled as Poisson random points [27], the number of accidental coincidences is fitted to a Poisson curve; from the fit we get the expected number of coincidences due to chance. The results are summarized in Fig. 1. From the fit we obtain a value of mean expected accidental coincidences of $n_a = 2.57 \pm 0.04$.

Another approach to evaluate the number of accidental coincidences is given by Eq. (1), which holds in case of Poisson random points [27]. The total number of AURIGA events and GRB triggers are, respectively, $N_A = 26816$, $N_\gamma = 120$, assuming $\Delta T = 2W = 10$ s and $T = 1.32\times 10^7$ s we get $n_a = 2.4 \pm 0.2$. The error on n_a can be easily estimated assuming the Poisson statistics for the fluctuations on N_A and N_γ, i.e., $\sqrt{N_A}$ and $\sqrt{N_\gamma}$, respectively. The two estimates of the accidental number of coincidences are in good agreement and demonstrate that the two event rates are uncorrelated. We conclude that the two coincidences found are due to chance.

TABLE I. Results of a Monte Carlo simulation of the statistical search. The column $P(t)$ is the probability that the estimated t is due to chance.

SN ratio of the generated signals	N_{on}	t	$P(t)$
3	120	8.1	$<10^{-9}$
2	120	3.4	4×10^{-4}
1	120	0.4	3×10^{-1}
1	1000	3.6	10^{-4}

B. Statistical search

We have first tested the method using a Monte Carlo simulation by adding N_{on} signals at fixed SN ratio over a Gaussian noise generated by means of a noise model with the same parameters of the AURIGA detector. The simulated detector output is then fed to the same data filtering procedures used for the AURIGA experimental data. We have generated 120 δ-like signals with SN ratio 3, 2, and 1 and 1000 with SN ratio 1 storing their true arrival times t_0. The signals have been superimposed to stationary Gaussian noise and we have then formed the *on* and *off* populations and calculated the t value. The probability $P(t)$ that the t value obtained is due to chance is reported in Table I. For signal with SN ratio$=3$ and SN ratio$=2$ this probability is very small and the value of t is statistically significant, showing a GRB-GWB association. On the other hand, for signals with SN ratio$=1$, we have to increase the number of GRB's to 1000 to get a statistical significance.

The χ_{off} and χ_{on} sets relative to the AURIGA-BATSE data between 1997 and 1998 are given in the histograms of Figs. 2 and 3, where the non-Gaussian tails are due to the nonstationarity of the AURIGA noise. The value of the Student's t test obtained is $t = 0.58$ (see Table II), which corresponds to a probability of 0.28 that it is due to chance.

As we conclude that there is no evidence of an association of GRB-GWB, we can put a constraint on gravitational ra-

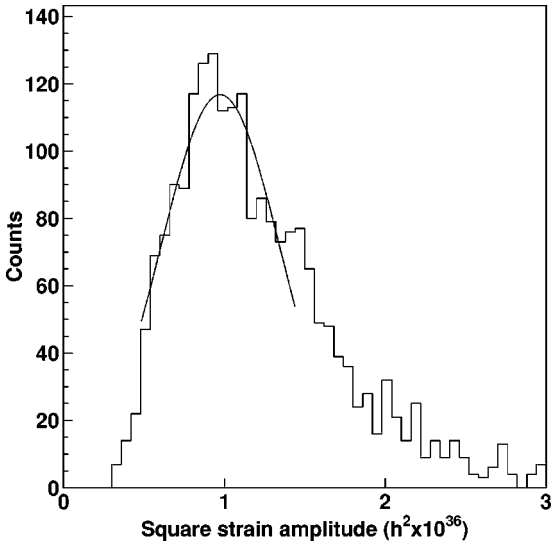

FIG. 2. Gaussian fit of the *off*-source set of the statistical search.

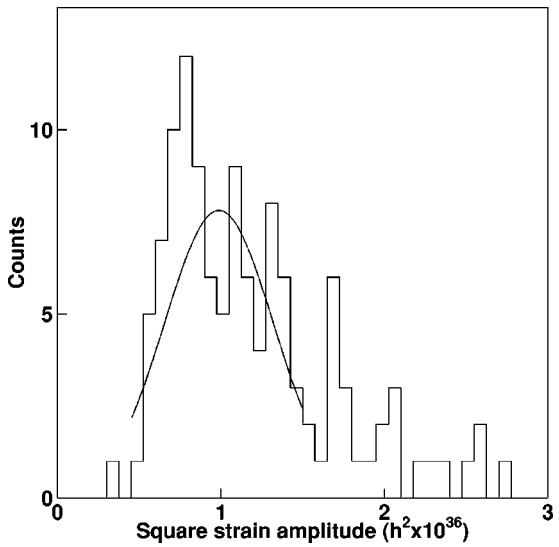

FIG. 3. Gaussian fit of the *on*-source set of the statistical search.

diation emitted from GRB's averaged over the source population, h_{RMS}. The value of $\mu_{t,max}$ can be found setting a confidence level CL and solving the equation

$$\int_{-\infty}^{\mu_{t,max}} f_d(t)dt = \mathrm{CL}, \tag{14}$$

where $f_d(t)$ is the distribution function for the student's t with d degrees of freedom. Notice that for high d, $f_d(t)$ tends to a normal curve with zero mean and unitary variance. If we choose CL=95% and set $d=N_{on}+N_{off}-2$, we get $\mu_{t,max}=1.65$. Using Eq. (13), we set the following upper limit on the averaged g.w. amplitude associated with GRB's:

$$h_{\mathrm{RMS}} \leqslant 1.5\times 10^{-18}. \tag{15}$$

IV. DISCUSSION

The two search methods reported in this paper show no evidence of a correlation between γ-ray bursts and gravitational waves. First, the two coincidences found with the coincidence search have no statistical significance and can be reconduced to chance. Next, the statistical method does not lead to a statistically significative value of the student's t. However we were able to put an upper limit on gravitational signals associated to the GRB's averaged over the source population, $h_{\mathrm{RMS}} \leqslant 1.5\times 10^{-18}$.

The existence of burstlike excitations (usually referred as non-Gaussian noise) in the AURIGA data is well known, and here we deal with this noise selecting the periods of time when the detector was operating in a satisfactory way. The vetoing procedure of data dominated by non-Gaussian noise has been set by the AURIGA data analysis [28] and the resulting duty cycle is about 40% of the total operating time. Care was also taken to cope with nonstationarities of the AURIGA noise that give rise to the non-Gaussian tails in Figs. 2 and 3 excluding such tails from the fits we used to estimate the statistical variable t. However, it is important to notice that even with the above limitations the analysis has reached a level of sensitivity which is astrophysically of some interest. In order to get better upper limits we have also explored the possibility of using in our analysis the incoming direction of GRB's but we have found no significant improvements (see Appendix B).

TABLE II. Results of the fits on the χ_{off} and χ_{on} sets with a Gaussian distribution.

	N	μ	σ
Off	2206	$(0.97\pm 0.02)\times 10^{-36}$	$(0.37\pm 0.02)\times 10^{-36}$
On	120	$(0.99\pm 0.06)\times 10^{-36}$	$(0.33\pm 0.06)\times 10^{-36}$

To increase the confidence in our results we have applied the nonparametric Mann-Whitney u test [29] to the sample sets of the statistical search, obtaining a second confirmation of the null hypothesis. Ranking the elements of the union set $\chi_{off}\oplus\chi_{on}$ in increasing order, we get a statistical parameter $z=1.59$, smaller than the critic value $z=1.95$ imposed by the fixed CL of 95%.

The upper limit we have set can be improved in the future simply by the increasing of common data taking periods of AURIGA (N_{on} increases) and the new experiment HETE-II (High Energy Transient Explorer [30]) which is going to substitute by now the wasted BATSE satellite in the GRB search. Another possibility to reach more astrophysically interesting sensitivities is the upgrade of the AURIGA detector which is now in progress [31]. The predicted sensitivity and bandwidth of the AURIGA detector equipped with the new read out system would be, respectively, $h_{min}\approx 8\times 10^{-20}$ and $t_w^{-1}\approx 10$ Hz. This sensitivity, together with an enhancement of noise stationarity and duty cycle of the detector, would correspond to the lowering of the upper limit h_{RMS} of about two orders of magnitude in one year of correlation analysis.

APPENDIX A

To derive Eq. (1) let us consider n points random distributed in a time interval $[0,T]$. The probability $P(\{k_a,t_a\})$ that k_a points lies in the time window $t_a=t_2-t_1$ is given by the binomial distribution [27] with probability $p=t_a/T$ that a single point lie in t_a. If $n\gg 1$ and $t_a\ll T$, using the Poisson theorem we get

$$P(\{k_a,t_a\}) = e^{-nt_a/T}\frac{(nt_a/T)^{k_a}}{k_a!}. \tag{A1}$$

For m not overlapping windows, it can be demonstrated that in the limit of $n\to\infty$, $T\to\infty$, and n/T constant, the probability of $\{k_1$ points in $t_1\}$, . . . ,$\{k_m$ points in $t_m\}$ is [27]

$$P(\{k_1,t_1\},\ldots,\{k_m,t_m\}) = \prod_{i=1}^{m} e^{-nt_i/T}\frac{(nt_i/T)^{k_i}}{k_i!} = \prod_{i=1}^{m} P(\{k_i,t_i\}) \tag{A2}$$

showing that the events $\{k_i$ points in $t_i\}$ and $\{k_j$ points in $t_j\}$ are independent for every i and j. Substituting $m=N_\gamma$, $t_i=2W=\Delta T$, and $n=N_A$ we get

$$P(\{k_1,\Delta T\},\ldots,\{k_{N_\gamma},\Delta T\})=e^{-N_\gamma N_A \Delta T/T}\left(\frac{N_A\Delta T}{T}\right)^{k}\prod_{i=1}^{N_\gamma}\frac{1}{k_i!}. \tag{A3}$$

We can now evaluate the probability $P(k)$ to have k coincidences, that can be obtained taking into account, at fixed k, all the possible sets $[k_i]=\{k_1,\ldots,k_{N_\gamma}\}$ with the constrain $\Sigma_{i=1}^{N_\gamma}k_i=k$; we get

$$\begin{aligned}P(k)&=\sum_{[k_i]}P(\{k_1,\Delta T\},\ldots,\{k_{N_\gamma},\Delta T\})\\&=e^{-N_\gamma N_A\Delta T/T}\left(\frac{N_A\Delta T}{T}\right)^{k}\sum_{[k_i]}\prod_{i=1}^{N_\gamma}\frac{1}{k_i!}\\&=e^{-N_\gamma N_A\Delta T/T}\left(\frac{N_A\Delta T}{T}\right)^{k}\frac{N_\gamma^k}{k!}.\end{aligned} \tag{A4}$$

Rearranging the factors in Eq. (A4) we get a Poisson distribution as in Eq. (1). The last equality can be easily demonstrated using the multinomial expansion relation [32]

$$\left(\sum_{i=1}^{N}x_i\right)^k=k!\sum_{[k_i]}\prod_{i=1}^{N}\frac{x_i^{k_i}}{k_i!} \tag{A5}$$

and substituting $x_i=1$, $i=1\cdots N$.

APPENDIX B

As the incoming direction of GRB's is known, one may wonder if a selection of the GRB's based on a cutoff on the AURIGA antenna pattern, averaged on the unknown polarizations of the GWB, could increase the sensitivity of our analysis.

The antenna pattern of a resonant bar detector is given by

$$F(\theta)=1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})^2=\sin^2(\theta), \tag{B1}$$

where $\hat{\mathbf{k}}$ and $\hat{\mathbf{z}}$ are unitary vectors parallel to the GRB direction and the antenna bar axis and θ is the angle between $\hat{\mathbf{k}}$ and $\hat{\mathbf{z}}$. Sources that fall outside the two cones, which are defined by the equation $\hat{\mathbf{k}}\cdot\hat{\mathbf{z}}\geqslant\cos(\xi)$, have a figure pattern $F(\theta)\geqslant\sin^2(\xi)$ and therefore the average energy associated with these g.w. sources is

$$\begin{aligned}E^{\xi}[h^2]&\propto\frac{15}{16}\int_{\Omega_\xi}F^2(\theta)\frac{d\Omega}{4\pi}\\&=\frac{75}{64}\left[\sin(\xi)+\frac{1}{6}\sin(3\xi)+\frac{1}{50}\sin(5\xi)\right],\end{aligned} \tag{B2}$$

where the solid angle Ω_ξ is defined by $F(\theta)\geqslant F(\xi)$. Moreover, as GRB's are isotropically distributed over the sky, the cutoff on the figure pattern decreases the number of available GRB: $N_{on}^{\xi}=N_{on}\sin(\xi)$. Therefore the net effect of this cutoff on the expected value of t in Eq. (10) is

$$\mu_t^{\xi}=\mu_t\frac{75}{64}\left[\sin(\xi)+\frac{1}{6}\sin(3\xi)+\frac{1}{50}\sin(5\xi)\right]\sin(\xi)^{1/2}. \tag{B3}$$

The function μ_t^{ξ}/μ_t is a continuously increasing function in the range $\xi\in[0,\pi/2]$, and $\mu_t^{\xi}/\mu_t=1$ at $\xi=\pi/2$ (i.e., the whole solid angle). We must conclude that a cutoff on the GRB direction does not enhance the sensitivity of the statistical search.

[1] C. A. Meegan *et al.*, Nature (London) **355**, 143 (1992).
[2] E. Costa *et al.*, Nature (London) **387**, 783 (1997).
[3] S. Kulkarni *et al.*, Nature (London) **393**, 35 (1998).
[4] M. R. Metzger *et al.*, Nature (London) **387**, 878 (1997).
[5] S. G. Djorgovski *et al.*, GCN notice 139 (1998).
[6] P. M. Vreeswijk *et al.*, astro-ph/9904286.
[7] T. Piran, Phys. Rep. **314**, 575 (1999).
[8] S. E. Woosley, in *Proceedings of the Fifth Huntsville Conference on Gamma-Ray Bursts*, edited by R. M. Kippen, R. S. Mallozzi, and V. Connaughton, AIP Conf. Proc. No. 526 (AIP, Melville, NY, 2000), pp. 555–564.
[9] C. S. Kochanek and T. Piran, Astrophys. J. Lett. **417**, L17 (1993).
[10] P. Meszaros and M. J. Rees, Astrophys. J. Lett. **430**, L93 (1994).
[11] C. L. Fryer, S. E. Woosley, and A. Heger, Astrophys. J. (to be published), astro-ph/0007176.
[12] G. A. Prodi *et al.*, in *Proceedings of the Second Edoardo Amaldi Conference on Gravitational Waves*, edited by E. Coccia, G. Pizzella, and G. Veneziano (World Scientific, Singapore, 1998), Vol. 4, pp. 148–158.
[13] L. Amati *et al.*, Astron. Astrophys. **138**, 605 (1999).
[14] P. Astone *et al.*, Astron. Astrophys., Suppl. Ser. **138**, 603 (1999).
[15] G. Modestino *et al.*, in *Frontier Objects in Astrophysics and Particle Physics*, edited by F. Giovannelli and G. Mannocchi (World Scientific, Singapore, 1998), p. 295.
[16] L. S. Finn, S. D. Mohanty, and J. D. Romano, Phys. Rev. D **60**, 121101 (1999).
[17] G. A. Prodi *et al.*, Int. J. Mod. Phys. D **9**, 237 (2000).
[18] A. Ortolan *et al.*, in *Proceedings of the Second Edoardo Amaldi Conference on Gravitational Waves*, [12], Vol. 4, pp. 204–215.
[19] E. E. Flanagan and S. A. Hughes, Phys. Rev. D **57**, 4535 (1998).
[20] E. Amaldi *et al.*, Astron. Astrophys. **216**, 325 (1989).
[21] Z. A. Allen *et al.*, Phys. Rev. Lett. **85**, 5046 (2000).
[22] L. Baggio *et al.*, Nucl. Phys. B (Proc. Suppl.) **70**, 537 (1999).
[23] L. Baggio *et al.*, Phys. Rev. D **61**, 102001 (2000).
[24] C. A. Meegan *et al.*, internet report at http://www.batse.msfc.nasa.gov/

[25] V. Crivelli-Visconti *et al.*, Phys. Rev. D **57**, 2045 (1998).
[26] G. W. Snedecor, *Statistical Methods*, 5th ed. (The Iowa State University Press, Ames, IA, 1962).
[27] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 3rd ed. (McGraw-Hill, Singapore, 1991).
[28] L. Baggio *et al.*, Int. J. Mod. Phys. D **9**, 251 (2000).
[29] H. B. Mann and D. R. Whitney, Ann. Math. Stat. **18**, 50 (1947).
[30] Internet: http://space.mit.edu/HETE
[31] J. P. Zendri *et al.*, in Proceedings of the XIV SIGRAV Congress on General Relativity and Gravitational Physics (World Scientific, Singapore, in press).
[32] M. Abramowitz and I. E. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1972), p. 823.
[33] J. Weber and B. Radak, Nuovo Cimento Soc. Ital. Fis., B **111**, 687 (1996).