

# Instabilities in neutrino-plasma density waves

Luís Bento

Centro de Física Nuclear da Universidade de Lisboa, Av. Prof. Gama Pinto 2, 1649-003 Lisboa, Portugal

(Received 12 December 2000; published 12 March 2001)

One examines the interaction and possible resonances between supernova neutrinos and electron plasma waves. The neutrino phase space distribution and its boundary regions are analyzed in detail. It is shown that the boundary regions are too wide to produce nonlinear resonant effects. The growth or damping rates induced by neutrinos are always proportional to the neutrino flux and  $G_F^2$ .

DOI: 10.1103/PhysRevD.63.077302

PACS number(s): 13.15.+g, 14.60.Lm, 97.60.Bw

## I. INTRODUCTION

The study of the interactions between neutrinos and plasma waves has received a great deal of attention. Early works of Bingham *et al.* [1,2] claimed that an intense neutrino flux passing through a plasma is capable of producing unstable modes of electron density waves causing a significant transfer of energy from neutrinos to the mantle of a supernova. If true, this could constitute a sort of realization of the Wilson explosion mechanism. However, their description of the weak interactions in the neutrino-electron system was not satisfactory.

More recently [3,4], the full standard model quantum field theory of electroweak interactions was applied to establish the dynamics of the excitations of the electromagnetic field and electron and neutrino current density distributions [Eqs. (27)–(32) of Ref. [4]]. From that we derived the modification on the dispersion relation of electron density waves due to a neutrino flow. Our analysis of the conditions of neutrino emission in supernovae led us to the conclusion that they do not satisfy the necessary requirements to generate the unstable waves and growing rates predicted by Bingham *et al.* [1,2]. Subsequent works [5,6] gave some support to this conclusion. However, the controversy seems to persist on this specific issue [7]. We report here a detailed analysis concerning the neutrino phase space distribution and implications for plasma waves.

## II. NEUTRINO INDUCED INSTABILITIES

The dispersion relation of electron density waves, also called plasmons, is modified by weak interactions when a neutrino flow passes through the plasma. Let  $\omega_{pl}(\mathbf{k})$  designate the wave frequency as a function of the wave vector  $\mathbf{k}$  in the absence of neutrinos and in the plasma collisionless limit. We assume the background medium to be static and *spatially* homogeneous within the time and length scales characteristic of the plasma waves of interest. The momentum distribution is assumed isotropic for electrons but not for neutrinos, as they have an almost unique direction at far distances from the supernova core. The electron plasma is considered non-relativistic. In these conditions a stream of neutrinos (antineutrinos) modifies the dispersion relation as follows [3,4] ( $\hbar = c = 1$ ):

$$\omega^2 - \omega_{pl}^2 = \Gamma A \omega_{pl}^2, \quad (1)$$

$$\Gamma = 2 G_F^2 c_V'^2 \frac{n_e n_\nu}{m_e \bar{E}_\nu}, \quad (2)$$

where  $c_V' = 1/2 + 2 \sin^2 \theta_W \approx 0.96$  for  $\nu_e$  ( $\bar{\nu}_e$ ),  $G_F$  is the Fermi constant,  $n_e$ ,  $n_\nu$  are the electron and neutrino number densities, respectively, and

$$A = \left(1 - \frac{\omega^2}{\mathbf{k}^2}\right)^2 \frac{\mathbf{k}^2 \bar{E}_\nu}{\omega_p^2 n_\nu} \times \int d^3 p_\nu \frac{f_\nu}{E_\nu} \frac{\mathbf{k}^2 - (\mathbf{k} \cdot \mathbf{v}_\nu)^2}{(\omega - \mathbf{k} \cdot \mathbf{v}_\nu)^2 - (\omega^2 - \mathbf{k}^2)^2 / 4 E_\nu^2} \quad (3)$$

is a dimensionless quantity.  $f_\nu$  is the neutrino or antineutrino distribution function in momentum space,  $\int f_\nu d^3 p_\nu = n_\nu$ , and  $\bar{E}_\nu$  is a typical neutrino energy. Finally,  $\omega_p^2 = 4 \pi \alpha n_e / m_e$ .

The expression  $A$  admits in general a classical approximation because the frequency and wave number are much smaller than the single particle energy  $E_\nu$ . In fact  $\omega$  is around the magnitude of the plasma frequency,  $\omega_p$ , and  $k$  is limited above by the Debye wave number,  $k_D = \omega_p \sqrt{m_e / T_e}$  at a temperature  $T_e$  [8]. In the classic limit  $A$  is approximated as

$$A = \left(1 - \frac{\omega^2}{\mathbf{k}^2}\right)^2 \frac{\mathbf{k}^2 \bar{E}_\nu}{\omega_p^2 n_\nu} \int d^3 p_\nu \frac{f_\nu}{E_\nu} \frac{\mathbf{k}^2 - (\mathbf{k} \cdot \mathbf{v}_\nu)^2}{(\omega - \mathbf{k} \cdot \mathbf{v}_\nu)^2}, \quad (4)$$

or, after integrating by parts,

$$A = - \left(1 - \frac{\omega^2}{\mathbf{k}^2}\right)^2 \frac{\mathbf{k}^2 \bar{E}_\nu}{\omega_p^2 n_\nu} \int d^3 p_\nu \frac{\mathbf{k} \cdot \partial f_\nu / \partial \mathbf{p}_\nu}{\omega - \mathbf{k} \cdot \mathbf{v}_\nu}, \quad (5)$$

an expression directly related to classic kinetic theory [3,4].

The frequency shift due to weak interactions is in general extremely small. The factor  $G_F^2 n_e n_\nu / m_e E_\nu$  is about  $3 \times 10^{-28}$  for electron and neutrino densities as high as  $n_e = 10^{29} \text{ cm}^{-3}$  and  $n_\nu = L_\nu / 4 \pi r^2 \bar{E}_\nu = 1.8 \times 10^{30} \text{ cm}^{-3}$  at radius  $r = 300 \text{ km}$  for a neutrino energy luminosity  $L_\nu = 10^{52} \text{ erg/s}$  and  $\bar{E}_\nu = 10 \text{ MeV}$ . The claim has been [1,2,7] that the shift on the imaginary part of the frequency,  $\gamma = \text{Im}\{\omega - \omega_{pl}\}$ , is not suppressed by  $G_F^2$  but rather by a smaller power of  $G_F$  for certain wave modes that are resonantly enhanced by powers of  $(\omega - \mathbf{k} \cdot \mathbf{v}_\nu)^{-1} \sim \gamma^{-1} \gg \omega_{pl}^{-1}$ . That would be the case if

all the neutrinos had exactly the same velocity vector,  $\mathbf{v}_\nu$ . Then Eq. (4) would give  $A \sim \omega_{pl}^2/\gamma^2$  for the resonant modes and the dispersion relation (1) complex solutions  $\omega(\mathbf{k})$  with growth rates  $\gamma \sim \Gamma^{1/3} \omega_{pl}$ , around  $10^{10}$  s for the parameters shown above. This corresponds to the reactive instability put forward by Bingham *et al.* [1,2].

However, that calculation is not realistic because the neutrinos do not have exactly the same direction of motion. As emphasized in Refs. [3,4], no matter how far the neutrinos are from the core, there is always an angular spread proportional to the neutrinosphere radius,  $R$ ,  $2\alpha_\nu \approx 2R/r$ . This causes a variation of  $\omega - \mathbf{k} \cdot \mathbf{v}_\nu$  proportional to  $\alpha_\nu \omega_{pl}$ , orders of magnitude higher than any conceivable value for the width  $|\gamma|$  of a resonance due to neutrino interactions. This means that  $\omega_{pl} - \mathbf{k} \cdot \mathbf{v}_\nu$  changes sign over the neutrino momentum distribution and only a vanishing small fraction of the neutrinos lie on the resonance. Such a situation is similar to Landau damping [8,9] in an electron plasma and the imaginary part of  $A$  can be calculated by replacing  $(\omega - \mathbf{k} \cdot \mathbf{v}_\nu)^{-1}$  with  $-i\pi\delta(\omega_{pl} - \mathbf{k} \cdot \mathbf{v}_\nu)$ . It yields a neutrino contribution to  $\gamma$  proportional to  $G_F^2$  given by Eq. (15), first obtained by Hardy and Melrose [10] through other methods. The only possible exceptions, that we want to analyze here, are resonances located at some boundary of the phase space occupied by neutrinos.

One kinematic boundary is the velocity direction parallel to  $\mathbf{k}$  (when  $\mathbf{k}$  lies inside the neutrino velocity cone): it corresponds to the minimum angle between  $\mathbf{v}_\nu$  and  $\mathbf{k}$  and minimum value of  $\omega_{pl} - \mathbf{k} \cdot \mathbf{v}_\nu$ . The other boundary is the upper limit on the angle between  $\mathbf{v}_\nu$  and the radial direction,  $\theta \leq \alpha_\nu \approx R/r$ , due to the finite size of the neutrino sphere. Ideally, the distribution function would be discontinuous at  $\theta = \alpha_\nu$  but that is not true as we will see.

Consider first the case of a resonance at  $\mathbf{v}_\nu$  parallel to  $\mathbf{k}$ , i.e.,  $\omega_{pl} = kv_\nu$ . Then, the factor  $(1 - \omega^2/k^2)^2$  in  $A$  vanishes unless the neutrinos are massive in which case  $v_\nu \approx 1 - m_\nu^2/2E_\nu^2$ . Let  $v_0 = \omega_{pl}/k$  be the exact resonant speed. Then  $(1 - \omega^2/k^2) \approx m_\nu^2/E_0^2$  and  $\omega_{pl} - \mathbf{k} \cdot \mathbf{v}_0$  is positive throughout the neutrino angular distribution. The problem is, the very mass that makes  $1 - \omega^2/k^2$  different from zero also causes a neutrino speed variation over the energy spectrum and a change of sign in  $\omega_{pl} - \mathbf{k} \cdot \mathbf{v}_\nu$ . In fact,  $\omega_{pl} - kv_\nu$  is negative for energies larger than  $E_0$ . Only energies obeying  $\omega_{pl} - kv_\nu \lesssim |\gamma|$  can participate in the resonance of width  $|\gamma|$ . The standard deviation  $\Delta E_\nu$  in the energy spectrum of supernova neutrinos [11,12] is comparable to the average energy  $\bar{E}_\nu$ ,  $\Delta E_\nu \approx \bar{E}_\nu/2.5$ , which implies a deviation  $\Delta v_\nu \approx 2m_\nu^2/5\bar{E}_\nu^2$ . In order that a significant fraction of the energy spectrum contributes to the resonance it is necessary that  $\Delta v_\nu \lesssim |\gamma|/\omega_{pl}$ , but that puts an upper limit on the neutrino mass,  $m_\nu^2/\bar{E}_\nu^2 \lesssim 5|\gamma|/\omega_{pl}$ , and, quite remarkably, on the factor

$$1 - \omega^2/k^2 \approx \frac{m_\nu^2}{E_0^2} \lesssim \frac{5|\gamma|}{\omega_{pl}}. \quad (6)$$

That simply washes out the resonance because the numerator  $(1 - \omega^2/k^2)^2$  in  $A$  becomes suppressed by  $\gamma^2$ . To put it in

another way, if there is a resonance for  $\mathbf{v}_\nu$  parallel to  $\mathbf{k}$ , the energy spectrum is always too broad to prevent  $\omega_{pl} - kv_\nu$  from changing sign and departing from the resonance width.

The other phase space boundary is the upper limit on the angle  $\theta$  between  $\mathbf{v}_\nu$  and the radial direction,  $\alpha_\nu \approx R/r$  ( $R$  is the neutrino sphere radius and  $r$  is the distance from the supernova center). If the resonance lies on  $\theta = \alpha_\nu$ , i.e.,  $\omega_{pl} = k \cos \alpha_\nu$ , then  $\omega_{pl} - \mathbf{k} \cdot \mathbf{v}_\nu$  is essentially negative over the neutrino angular distribution. However, the distribution does not fall abruptly to zero at the polar angle  $\alpha_\nu$ . There are two reasons for this. First, the neutrino sphere radius depends on the  $\nu$  energy: the interaction cross sections increase with the  $\nu$  energy and consequently more energetic neutrinos suffer the last scattering in regions of lower density, farther from the center. Second, scattering is a statistical process by nature. Particles with the same energy suffer the last scattering at different radii according to a certain statistical distribution dependent upon the particular chemical and density profile of the medium (see the Appendix). Both factors imply that the neutrino sphere has a considerable thickness and so has the neutrino angular aperture. This fact changes the way the distribution function depends on  $\theta$ .

Let  $R_E$  and  $\Delta R_E$  be the average neutrino sphere radius and statistical uncertainty for neutrinos with well-defined energy  $E_\nu = E$  and  $\alpha_E = R_E/r$ ,  $\Delta \alpha_E = \Delta R_E/r$  the respective angular aperture and uncertainty. Assuming axial symmetry around the radial direction, the distribution function only depends on the energy and polar angle:  $f_\nu = f_\nu(E, \theta)$ . Its derivative with respect to  $\theta$  can be modeled as

$$\frac{\partial f_\nu}{\partial \theta}(E, \theta) = -\frac{g(E)}{\sqrt{2\pi}\Delta\alpha_E} e^{-[(\theta - \alpha_E)^2/2\Delta\alpha_E^2]}, \quad (7)$$

with  $g(E) \approx f_\nu(E, 0)$ . It corresponds to a distribution function practically constant in the interval  $\theta < \alpha_E - 2\Delta\alpha_E$  and dropping to zero at  $\theta > \alpha_E + 2\Delta\alpha_E$ . In addition to  $\Delta\alpha_E$ , the distribution function is also smoothed by the dependence of  $\alpha_E$  on the neutrino energy, which makes a total angular width  $\Delta\alpha_\nu = \Delta\alpha_E + d\alpha_E/dE_\nu \Delta E_\nu$ , centered on the polar angle  $\alpha_\nu$ . The consequence of this is that  $\omega_{pl} - \mathbf{k} \cdot \mathbf{v}_\nu$  may change sign and depart from the resonance still within the angular boundary  $\theta \approx \alpha_\nu \pm 2\Delta\alpha_\nu$ , if the resonance is not wide enough.

Let  $\mathbf{v}_0$  be a particular vector of the resonance surface (defined by  $\mathbf{k} \cdot \mathbf{v}_\nu = \omega_{pl}$ ) situated in the angular boundary, i.e., with polar angle  $\theta_0$  close to  $\alpha_\nu$ , and azimuthal angle  $\phi_0$ . The angular coordinates of a generic velocity vector  $\mathbf{v}_\nu$  and wave vector  $\mathbf{k}$  are denoted as  $(\theta, \phi)$  and  $(\theta_k, \phi_k)$ , respectively. Without loss of generality,  $\phi_k = 0$ . The product  $\mathbf{k} \cdot \mathbf{v}_\nu$  and its variation from  $\mathbf{v}_0$  are given by

$$\mathbf{k} \cdot \mathbf{v}_\nu = kv_\nu(\cos \theta_k \cos \theta + \sin \theta_k \sin \theta \cos \phi), \quad (8)$$

$$\delta \mathbf{k} \cdot \mathbf{v}_\nu = k_\theta v_0 \delta \theta + k_\phi v_0 \sin \theta_0 \delta \phi, \quad (9)$$

where  $k_\theta = k(-\cos \theta_k \sin \theta_0 + \sin \theta_k \cos \theta_0 \cos \phi_0)$  and  $k_\phi = -k \sin \theta_k \sin \phi_0$  represent the components of  $\mathbf{k}$  along the directions  $\mathbf{e}_\theta$  and  $\mathbf{e}_\phi$ , respectively. With  $k_\nu = \mathbf{k} \cdot \mathbf{v}_0/v_0$  they obey the identity  $k_\theta^2 + k_\phi^2 + k_\nu^2 = 1$ . The angular displacement

$$\delta\theta = \pm \frac{\gamma \cos \beta}{\sqrt{k^2 v_0^2 - (\mathbf{k} \cdot \mathbf{v}_0)^2}}, \quad (10)$$

$$\sin \theta_0 \delta\phi = \pm \frac{\gamma \sin \beta}{\sqrt{k^2 v_0^2 - (\mathbf{k} \cdot \mathbf{v}_0)^2}}, \quad (11)$$

with  $\tan \beta = k_\phi / k_\theta$ , causes a variation  $\delta \mathbf{k} \cdot \mathbf{v}_\nu = \pm \gamma$ . Notice that  $\mathbf{k} \cdot \mathbf{v}_0 = \omega_{pl}$  and  $\sin \theta_0 \approx \sin \alpha_\nu$ . If that displacement satisfies  $|\delta\theta| \ll \Delta\alpha_\nu$  and  $|\delta\phi| \ll \pi$ , then  $\omega_{pl} - \mathbf{k} \cdot \mathbf{v}_\nu$  goes away to both sides of the resonance for velocity directions well inside the range  $\theta \approx \alpha_\nu \pm 2\Delta\alpha_\nu$  and therefore well inside the neutrino distribution. Of course,  $\omega_{pl} - \mathbf{k} \cdot \mathbf{v}_\nu$  would have a definite sign if  $\theta_0 > \alpha_\nu \pm 2\Delta\alpha_\nu$  but then, the resonance would exist where the neutrino distribution function drops to zero, i.e., there are no neutrinos.

Our estimates of the neutrino sphere depth (see the Appendix) give  $\Delta\alpha_\nu$  varying between  $0.11\alpha_\nu$  and  $0.03\alpha_\nu$  at different stages of supernova evolution and  $\alpha_\nu = R/r$  between 0.1 and 0.03 at  $r = 300$  km. On the other hand,  $|\gamma|$  is orders of magnitude below  $10^{-9}\omega_{pl}$ . This implies that the displacements calculated above satisfy

$$|\delta\theta|/\Delta\alpha_\nu \lesssim 10^3 |\gamma|/(\omega_{pl} \sin \theta_{k\nu}), \quad (12)$$

$$|\delta\phi|/\pi \lesssim 10 |\gamma|/(\omega_{pl} \sin \theta_{k\nu}), \quad (13)$$

and therefore are well inside the neutrino angular boundary. The contrary would require a very small angle between  $\mathbf{k}$  and  $\mathbf{v}_0$ ,  $\theta_{k\nu} < 10^{-6}$  for  $|\gamma|/\omega_{pl} < 10^{-9}$ , which leads to a huge suppression factor  $(1 - \omega^2/k^2)^2 \approx \theta_{k\nu}^4$  in  $A$ , which in turn further suppresses the values of  $\gamma$  and so on, in other words, it falls in the case  $\mathbf{v}_0$  parallel to  $\mathbf{k}$  treated above in the first place.

The lesson from all this is that the contribution,  $\gamma_\nu$ , of neutrino weak interactions to the imaginary part of the wave frequency  $\omega(\mathbf{k})$ , is too small to produce resonances where the quantity  $A$  is inversely proportional to some power of  $\gamma_\nu$ . On the contrary,  $A$  is independent of  $\gamma_\nu$  and

$$\gamma_\nu = \frac{1}{2} \Gamma \omega_{pl} \text{Im}\{A\}. \quad (14)$$

Applying Landau prescription [8,9] to Eq. (5), the result is

$$\frac{\gamma_\nu}{\omega_{pl}} = \frac{G_F^2 c_V'^2}{4\alpha k^2} (k^2 - \omega_{pl}^2)^2 \int d^3 p_\nu \delta(\omega_{pl} - \mathbf{k} \cdot \mathbf{v}_\nu) \mathbf{k} \cdot \frac{\partial f_\nu}{\partial \mathbf{p}_\nu}, \quad (15)$$

the same as obtained by Hardy and Melrose [10] from the study of stimulated emission and absorption of plasmons by neutrinos.  $\gamma_\nu$  is proportional to  $G_F^2$  and suppressed by  $G_F^2 n_e n_\nu / m_e \bar{E}_\nu$ , down to  $\sim 10^{-28}$  even for electron and neutrino densities as high as  $n_e = 10^{29} \text{ cm}^{-3}$  and  $n_\nu \sim 10^{30} \text{ cm}^{-3}$  at 300 km from the supernova center. The other point is, the rate  $\gamma_\nu$  does not drive the evolution of plasma waves because it is many orders of magnitude smaller than the damping rate caused by electron-ion collisions,  $\gamma_c$ , only two or three orders of magnitude below  $\omega_{pl}$  for such high density plasmas [8,9].

More important than the growth rate is to know the energy transferred from neutrinos to the plasma. Keeping in mind that the overall wave growth is shut down by electron-ion collisional damping, the energy transferred per unity of time to a single mode is  $2\gamma_\nu(\mathbf{k})\omega_{pl}(\mathbf{k})$ . Assuming that the plasma waves obey a thermal equilibrium Bose-Einstein distribution, cutoff at the Debye wave number  $k_D$ , the total energy transferred per unity of time and volume is

$$\dot{\rho} = 2 \int \frac{d^3 k}{(2\pi)^3} \frac{\gamma_\nu(\mathbf{k})\omega_{pl}}{e^{\omega_{pl}/T_e} - 1}. \quad (16)$$

Integrating in time and volume and dividing by the total neutrino energy,  $4\pi r^2 n_\nu E_\nu \Delta t$ , one obtains the fraction of total energy transferred per neutrino,  $\Delta E/E_\nu = \int dr \dot{\rho} / E_\nu n_\nu$ .

For the sake of argument let us assume that some new interaction produces a contribution like  $A$  in Eq. (4), but without the factor  $(1 - \omega_{pl}^2/k^2)^2$ , so that a resonance is possible for  $\mathbf{k}$  parallel to  $\mathbf{v}_\nu$  ( $\omega_{pl} = k$ ) making  $A \propto \gamma^{-1}$ . The collisionless dispersion relation (1) gives a growth rate  $\gamma \propto \Gamma^{1/2} \omega_{pl}$  that is proportional to  $G_F$  rather than  $G_F^2$ . However, these resonant modes are limited to the very thin shell  $k = \omega_{pl} \pm \gamma$  in  $\mathbf{k}$  space. That makes an extra factor of  $\gamma$  in the integration of Eq. (16) and the transferred energy goes as  $\gamma^2$ , proportional to  $G_F^2$  not to  $G_F$ . Taking only the main factors, one obtains  $\Delta E/E_\nu \sim G_F^2 T_e \omega_{pl}^6 E_\nu^{-2} r$ , about  $10^{-18}$  for the same numbers used before and  $T_e = 100$  KeV. This is very far from the few percent needed for a supernova explosion. It discourages the application of nonstandard weak interactions seeking for nonlinear resonant effects.

### III. CONCLUSIONS

We analyzed different instabilities that could possibly emerge from the interaction between neutrinos and electron plasma waves in a supernova environment. The hypothetical neutrino induced resonances are too narrow to embrace the total angular spread of the neutrino stream due to the finite size of the neutrino sphere [3,4]. But they are also too narrow to contain the boundary regions of the neutrino phase space distribution. When the resonant neutrino velocity vector is parallel to the wave vector, a neutrino mass is needed and the energy spectrum is too broad to keep the neutrino speed,  $v_\nu \approx 1 - m_\nu^2/2E_\nu^2$ , inside the resonance. When the resonant velocities are in the boundary of the velocity angular distribution, the depth of the neutrino sphere is too large to make an angular boundary abrupt enough. In both cases the boundaries of the neutrino phase space distribution are too wide to prevent a departure from the poles to both sides of the resonances. As a result, nonlinear resonant effects are not possible and the growth or damping rates are always linear in the neutrino flux and  $G_F^2$ . They correspond to a balance between stimulated Čerenkov emission and absorption of plasma waves by neutrinos [10], analogous to Landau damping. The energy that could possibly be transferred from neutrinos to plasma waves seems to be vanishing small even for nonstandard neutrino interactions.

## ACKNOWLEDGMENTS

This work was supported in part by FCT under Grant No. PESO/P/PRO/1250/98.

## APPENDIX: NEUTRINO SPHERE DEPTH

Neutrinos like any other form of radiation are not emitted from an ideal surface but from a shell with a certain depth. The neutrino “optical” depth at a point of radius  $r$  can be defined as the Roseland depth,  $\tau(r) = \int_r^\infty dr/\lambda(r)$ , where  $\lambda^{-1} = k\rho = \sigma n$  is the inverse mean free path, a function of the opacity  $k$  and density  $\rho$ , or cross section  $\sigma$  and number density of target particles,  $n$ .  $\tau(r)$  is a measurement of the number of collisions the neutrinos suffer moving from the radius  $r$  to infinity. The neutrino sphere can be arbitrarily defined as the surface, of radius  $R$ , where  $\tau = 2/3$ , but the bulk of neutrino last scatterings spreads between the  $\tau = 1$  and  $\tau = 1/3$  surfaces. The way this translates as a radial distribution depends on the chemical and density profile of the

medium. For an exponentially decreasing density near the neutrino sphere,  $\rho(r) \propto 10^{-r/l}$ , the radial separation between  $\tau = 1$  and  $\tau = 1/3$  is approximately  $2\Delta R \approx l/2$ . Numerical simulations [13] show that the length scale  $l$  varies from around  $R/5$  to  $R/20$  at different stages of supernova. Then,  $\Delta R$  is about  $R/20$  to  $R/80$ .

The other source of radial spread is the opacity or cross section dependence on the neutrino energy [12], typically  $k \propto E_\nu^2$ . At constant optical depth  $\tau$ , the neutrino sphere radius increases with the energy at a rate that we estimate as  $\delta R/\delta E_\nu = 2l/E_\nu \ln 10$ . Since the neutrino energy spectrum [11,12] has a standard deviation  $\Delta E_\nu \approx \bar{E}_\nu/2.5$  we obtain a radial deviation  $\Delta R \approx l/3 \approx R/15 - R/60$ . The joint effect of statistical fluctuations and energy spectrum is a neutrino sphere depth varying between  $2\Delta R \approx 0.23R$  and  $0.06R$ . This implies a finite width in the boundary of the neutrino angular aperture at large distances from the supernova core as discussed in the text.

- 
- [1] R. Bingham, J. M. Dawson, J. J. Su, and H. A. Bethe, Phys. Lett. A **193**, 279 (1994); R. Bingham, H. A. Bethe, J. M. Dawson, P. K. Shukla, and J. J. Su, *ibid.* **220**, 107 (1996).
  - [2] J. M. Dawson, in *Proceedings of the Second Workshop on New Worlds in Astroparticle Physics*, Faro, Portugal, 1998, edited by A. M. Mourão, M. Pimenta, and P. Sá (World Scientific, Singapore, 1999).
  - [3] L. Bento, in *Proceedings of the Second Workshop on New Worlds in Astroparticle Physics* [2], hep-ph/9912533.
  - [4] L. Bento, Phys. Rev. D **61**, 013004 (2000).
  - [5] J. F. Nieves, Phys. Rev. D **61**, 113008 (2000).
  - [6] H.-Th. Elze, T. Kodama, and R. Opher, Phys. Rev. D **63**, 013008 (2001).
  - [7] L. O. Silva, R. Bingham, J. M. Dawson, J. T. Mendonça, and P. K. Shukla, Phys. Rev. Lett. **83**, 2703 (1999).
  - [8] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics*, Vol. 10 of Course of Theoretical Physics (Butterworth-Heinemann, Oxford, 1997).
  - [9] D. B. Melrose, *Instabilities in Space and Laboratory Plasmas* (Cambridge University Press, Cambridge, England, 1986).
  - [10] S. J. Hardy and D. B. Melrose, Astrophys. J. **480**, 705 (1997).
  - [11] R. Mayle, J. R. Wilson, and D. N. Schramm, Astrophys. J. **318**, 288 (1987); A. Burrows, D. Klein, and R. Gandhi, Phys. Rev. D **45**, 3361 (1992).
  - [12] A. Burrows, Annu. Rev. Nucl. Sci. **40**, 181 (1990).
  - [13] J. R. Wilson and R. W. Mayle, “The Nuclear Equation of State: Part A,” in *Proceedings of a NATO ASI*, Peníscola, Spain, 1989, edited by W. Greiner and H. Stöcker (Plenum, New York, 1989).