Noncommutative geometry and the Higgs boson masses

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We study a noncommutative generalization of the standard electroweak model proposed by Balakrishna, Gürsey, and Wali [Phys. Lett. B **254**, 430 (1991)] that is formulated in terms of the derivations $\mathrm{Der}_2(M_3)$ of a three-dimensional representation of the $su(2)$ Lie algebra of weak isospin. The linearized Higgs field equations and the scalar boson mass eigenvalues are explicitly given. A light Higgs boson with a mass around 130 GeV together with four very heavy scalar bosons are predicted.

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I. INTRODUCTION

In spite of its observational successes, the standard model of electroweak interactions cannot yet be considered as a fundamental theory because the scalar boson sector, unlike the gauge sector involving the fermions and the gauge bosons, has to be written down in an *ad hoc* way and not by gauge principles. Furthermore, the unavoidable Higgs scalar has not been observed and there is no way to predict its mass. In this connection a remarkable attempt at unifying gauge fields and Higgs scalars was suggested by Connes [1], making use of the tools of noncommutative differential geometry. The formalism involves three steps. First, a spectral triplet $(\mathcal{D},\mathcal{H},\mathcal{A})$ is introduced, consisting of the (generalized) Dirac operator $\mathcal{D}$ that acts on a Hilbert space of states $\mathcal{H}$, together with an associative C^* algebra $\mathcal{A}$ also acting on $\mathcal{H}$. Next, $\mathcal{A}$ is related to the algebra of complex-valued functions on space-time in the commutative case, whereas in more complicated settings, in which the gauge groups are non-Abelian, $\mathcal{A}$ has to be replaced by the tensor product $\mathcal{A}=C^\infty(V)\otimes M_n$ with an appropriate matrix algebra. Finally, the construction of the Yang-Mills Lagrangian is done by replacing the Dixmier trace instead of integration. Within the above scheme, a generalization of the standard electroweak model in noncommutative geometry can be given as a gauge theory with a built-in spontaneous symmetry breakdown mechanism. This way, it is not only the Higgs sector that arises naturally, but also the correct hypercharge assignments acquire a natural meaning. The earliest model along these lines is due to Connes and Lott [2]. Several other attempts have followed since then [3–5]. Here we wish to reexamine the Higgs boson masses in a model proposed by Balakrishna, Gürsey, and Wali (BGW) [6]. In this approach the Yang-Mills and Higgs fields occur on equal footing and the Higgs potential consisting of a sum of complete squares appears already shifted onto an absolute minimum. Thus, both the gauge boson and Higgs boson masses can be predicted in terms of two mass scales, each related to one of the $SU(2)_I\times U(1)_Y$ gauge symmetry groups.

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II. MATHEMATICAL FRAMEWORK

In order to study the bosonic sector alone it is enough to deal with the tensor product space $\mathcal{A}=C^\infty(V)\otimes M_n$ so that $\mathcal{A}$ can be regarded as the set of matrix-valued functions on the space-time manifold V and is itself a C^* algebra. The differential calculus of this space has been studied in [7]. It is also possible to identify the vector fields of $\mathcal{A}$ with a restricted set of derivations of M_n rather than the algebra of all derivations of M_n. We have this extra freedom because $\mathrm{Der}(M_n)$ is not a module over M_n. Here the Lie subalgebra $\mathrm{Der}_2(M_3)$ generated by a three-dimensional representation of $su(2)$ is used rather than the Lie algebra $\mathrm{Der}(M_3)$ of all derivations of M_3. Exterior derivation, connection, and curvature are defined as in [6], but with some modifications [7]. The dimension of $\mathrm{Der}_2(M_3)$ is $2^2-1=3$. Hence we may take as the generators of M_3 the first three Gell-Mann matrices τ_1,τ_2,τ_3 and the generators of the U-spin and V-spin subalgebras along with the identity τ_0 which we identify with $Y+\frac{2}{3}$ where Y is the hypercharge $\tau_8/\sqrt{3}$. The generators of the U-spin and V-spin subalgebras are

$$U_\pm=\frac{1}{2}(\tau_6\pm i\tau_7),\quad U_3=\frac{1}{2}[U_+,U_-],$$

$$V_\pm=\frac{1}{2}(\tau_4\pm i\tau_5),\quad V_3=\frac{1}{2}[V_+,V_-]. \tag{2.1}$$

The choice of derivations is dictated by which symmetries we want unbroken at the end. In electroweak theory electromagnetic $U_{em}(1)$ whose generator is $\tau_0+\tau_3$ is unbroken. Among the above generators of M_3 only the generators of the U-spin subalgebra commute with $\tau_0+\tau_3$, so we define our derivations as

$$e_a(f)=m_a[\lambda_a,f],\quad f\in M_3, \tag{2.2}$$

where a runs through the indices $(+,-,3)$ and

$$\lambda_\pm=\frac{U_\pm}{\sqrt{2}},\quad \lambda_3=U_3,$$

$$m_\pm=m,\quad m_3=\frac{m^2}{M}. \tag{2.3}$$

Here m and M are two mass scales that have to be introduced into the theory to keep the dimensions correct. In defining the derivations we use the fact that all derivations of M_n are inner and hence they are in the form $e_a = ad(\lambda_a)$. They obey the commutation relations

$$[e_a, e_b] = \frac{m_a m_b}{m_c} C^c_{ab} e_c, \tag{2.4}$$

where the structure constants C^c_{ab} are

$$C^3_{+-} = -C^3_{-+} = 1, \quad C^-_{3+} = -C^-_{+3} = 1, \quad C^+_{-3} = -C^+_{3-} = 1, \tag{2.5}$$

and all others are zero.

We can now define the exterior derivative exactly as in [8], but with the set of derivations in $\mathrm{Der}_2(M_3) \subseteq \mathrm{Der}(M_3)$:

$$df(e_a) = e_a(f). \tag{2.6}$$

This means in particular that

$$d\lambda^a(e_b) = m_b[\lambda_b, \lambda^a], \tag{2.7}$$

where the indices are lowered and raised by the group metric

$$g_{ab} = -\mathrm{Tr}(\lambda_a \lambda_b). \tag{2.8}$$

We define the set of one-forms $\Omega_2{}^1(M_3)$ to be the set of all elements of the form $f\,dg$ or $dg\,f$ with f and g in $\mathcal{A}$ subject to the relations $d(fg) = df\,g + f\,dg$. Here the subindex 2 refers to the fact that we are using the derivation algebra $\mathrm{Der}_2(M_3)$. The set $d\lambda^a$ forms a system of generators of $\Omega_2{}^1(M_3)$ as a left or right module but it is not a convenient one since $\lambda^a\, d\lambda^b \neq d\lambda^b \lambda^a$. However, there is another system of generators completely characterized by the equations

$$\theta_\pm(e_\mp) = 1, \quad \theta_\pm(e_3) = 0,$$

$$\theta_3(e_\mp) = 0, \quad \theta_3(e_3) = 1. \tag{2.9}$$

They are related to $d\lambda^a$ by the equations

$$d\lambda^a = m_b C_{bc}{}^a \theta^b \lambda^c, \tag{2.10}$$

and they satisfy the structure equations

$$d\theta^a = C_{bc}{}^a \frac{m_b m_c}{m_a} \theta^b \wedge \theta^c. \tag{2.11}$$

The θ^a's commute with all elements of M_3.

Let us choose a basis $\theta^\alpha_\beta dx^\beta$ of $\Omega^1(V)$ over V and suppose e_α be the Pfaffian derivations dual to θ^α. Set $i = (\alpha, a)$, $1 \leqslant i \leqslant 4+3=7$, and introduce $\theta^i = (\theta^\alpha, \theta^a)$ as generators of $\Omega^1(\mathcal{A})$ as a left or right $\mathcal{A}$ module and $e_i = (e_\alpha, e_a)$ as a basis of $\mathrm{Der}_2(\mathcal{A})$ as a direct sum:

$$\Omega^1(\mathcal{A}) = \Omega^1_h \oplus \Omega^1_v, \tag{2.12}$$

where

$$\Omega^1_h = \Omega^1(V) \otimes M_n, \quad \Omega^1_v = \mathcal{C}^\infty(V) \otimes \Omega^1(M_n). \tag{2.13}$$

Thus the exterior derivative df of an element f of $\mathcal{A}$ can be written as the sum of its vertical and horizontal parts:

$$df = d_h f + d_v f. \tag{2.14}$$

From the basis elements θ^a we can construct a one-form θ in Ω^1_v, that is,

$$\theta = -m_a \lambda_a \theta^a, \tag{2.15}$$

which satisfies the zero-curvature condition

$$d\theta + \theta^2 = 0. \tag{2.16}$$

III. GAUGE FIELDS

The gauge potential, which is an element of $\Omega^1(V)$ for a trivial $U(1)$ bundle, can be generalized to the noncommutative case as an anti-Hermitian element of $\Omega^1(\mathcal{A})$. Let ω be such an element of $\Omega^1(\mathcal{A})$. We can write it then as

$$\omega = A + \theta + \Phi, \tag{3.1}$$

where

$$A = -igA_\alpha \theta^\alpha \in \Omega^1_h(\mathcal{A}),$$

$$\Phi = g\phi_a \theta^a \in \Omega^1_v(\mathcal{A}), \tag{3.2}$$

and θ as in Eq. (2.15). Here g is the coupling constant of the theory. ϕ_a here are interpreted as Higgs fields.

The gauge transformations of the trivial $U(1)$ bundle over V are the unitary elements of $C^\infty(V)$. By analogy, we will choose the group of local gauge transformations as the group of unitary elements $\mathcal{U}$ of $\mathcal{A}$—that is, the group of invertible elements $u \in \mathcal{A}$ satisfying $uu^* = 1$. Here * is the * product induced in $\mathcal{A}$ and $\mathcal{A}$ is considered as the set of functions on V with values in GL_n. An element of $\mathcal{U}$ defines a map of $\Omega^1(\mathcal{A})$ into itself of the form

$$\omega' = g^{-1}\omega g + g^{-1}\, dg. \tag{3.3}$$

We define

$$\theta' = g^{-1}\theta g + g^{-1}\, d_v g, \tag{3.4}$$

$$A' = g^{-1} A g + g^{-1}\, d_h g, \tag{3.5}$$

and so ϕ transforms under the adjoint action of $\mathcal{U}$:

$$\phi' = g^{-1}\phi g. \tag{3.6}$$

θ is invariant under the gauge transformations and hence ω' is again of the form (3.1). The curvature two-form Ω and the field strength F are defined as usual:

$$\Omega = d\omega + \omega^2, \quad F = d_h A + A^2, \tag{3.7}$$

with components

$$\Omega = \frac{1}{2}\Omega_{ij}\theta^i \wedge \theta^j, \quad F = \frac{1}{2}F_{\alpha\beta}\theta^\alpha \wedge \theta^\beta. \tag{3.8}$$

We find

$$\Omega_{\alpha\beta}=F_{\alpha\beta},$$

$$\Omega_{\alpha a}=g\mathcal{D}_\alpha\phi_a=g(\partial_\alpha\phi_a-ig[A_\alpha,\phi_a]),$$

$$\Omega_{ab}=g^2[\phi_a,\phi_b]-g\frac{m_a m_b}{m_c}C_{ab}^{\ \ c}\phi_c. \tag{3.9}$$

As we shall see the term Ω_{ab} is responsible for the Higgs potential.

Given the curvature two-form, we can write down the usual gauge-invariant Yang-Mills Lagrangian density four-form:

$$\mathcal{L}=-\frac{1}{2g^2}\,\mathrm{Tr}(\Omega_{ij}\Omega^{ij}). \tag{3.10}$$

In terms of the components of Ω, $\mathcal{L}$ becomes

$$\mathcal{L}=-\frac{1}{2g^2}\,\mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta})-\mathrm{Tr}(\mathcal{D}_\alpha\phi_a\mathcal{D}^\alpha\phi^a)+V(\phi), \tag{3.11}$$

where the Higgs potential $V(\phi)$ is given by

$$V(\phi)=-\frac{1}{2g^2}\,\mathrm{Tr}(\Omega_{ab}\Omega^{ab}). \tag{3.12}$$

From the form of Ω_{ab} in Eq. (3.9) we see that $V(\phi)$ vanishes for values

$$\phi_a=0,\quad \phi_a=\frac{m_a}{g}\lambda_a. \tag{3.13}$$

For the second vacuum configuration above, the second term on the right-hand side of Eq. (3.11) becomes

$$g^2\,\mathrm{Tr}([A_\alpha,m_a\lambda_a][A^\alpha,m_a\lambda^a]). \tag{3.14}$$

This expression is quadratic in the potential and hence it gives a mass to the vector bosons. This means we have a naturally built-in Higgs mechanism.

IV. HIGGS BOSON MASSES

In what follows we assume a Minkowskian space-time and work in Cartesian coordinates. Therefore we take $e_\alpha=\partial_\alpha$ and $\theta^\alpha=dx^\alpha$. Hence we have

$$d_h=dx^\alpha\partial_\alpha. \tag{4.1}$$

In this model there are three independent Higgs fields:

$$\phi_+=\frac{H^\dagger}{\sqrt{2}},\quad \phi_-=\frac{H}{\sqrt{2}},\quad \phi_3=\Delta+\frac{m^2}{2Mg}(2\tau_0-1), \tag{4.2}$$

where

$$H=H_+V_++H_0U_+,$$

$$\Delta=\frac{1}{2}(\Delta_0\lambda_0+\Delta_a\lambda_a). \tag{4.3}$$

By using the metric components (2.8) we see that

$$\phi^+=-2\phi_-,\quad \phi^-=-2\phi_+,\quad \phi^3=-2\phi_3. \tag{4.4}$$

For the gauge potential we will write

$$A=-igA_\mu dx^\mu=-ig\frac{1}{2}(B_\mu\lambda_0+W_{\mu a}\lambda_a)dx^\mu, \tag{4.5}$$

where B and W are going to be identified as the weak gauge bosons.

Using the field components above we can write the connection one-form directly from Eq. (3.1):

$$\omega=A+\frac{g}{\sqrt{2}}H\theta_-+\frac{g}{\sqrt{2}}H^*\theta_++g\Delta\theta_3-\frac{m}{\sqrt{2}}U_+\theta_- -\frac{m}{\sqrt{2}}U_-\theta_++\frac{m^2}{4M}(\lambda_0+\lambda_3)\theta_3. \tag{4.6}$$

The next step is to construct the curvature two-form

$$\Omega=\frac{1}{2}\Omega_{\mu\nu}dx^\mu dx^\nu+\Omega_{\mu+}dx^\mu\theta_-+\Omega_{\mu-}dx^\mu\theta_+ +\Omega_{\mu3}dx^\mu\theta_3+\Omega_{+-}\theta_-\theta_++\Omega_{+3}\theta_-\theta_3+\Omega_{3-}\theta_3\theta_+. \tag{4.7}$$

From Eq. (3.9) we can see that

$$\Omega_{\mu\nu}=F_{\mu\nu},$$

$$\Omega_{\mu+}=\frac{g}{\sqrt{2}}\mathcal{D}_\mu H,\quad \Omega_{\mu-}=\Omega^*_{\mu+},$$

$$\Omega_{\mu3}=g\mathcal{D}_\mu\Delta, \tag{4.8}$$

where

$$\mathcal{D}_\mu=\partial_\mu-ig[A_\mu,] \tag{4.9}$$

and the remaining three terms are

$$\Omega_{+-}=\frac{g^2}{2}[H,H^*]-gM\Delta-m^2\lambda_0+\frac{m^2}{2},$$

$$\Omega_{+3}=-\frac{g^2}{\sqrt{2}}\Delta H,\quad \Omega_{3-}=\Omega^*_{+3}. \tag{4.10}$$

These can also be found directly from Eqs. (3.9) and the definitions (4.2) and (4.4). We write down the Lagrangian as before and obtain

$$\mathcal{L}=-\frac{1}{2g^2}\mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta})+2\,\mathrm{Tr}(\mathcal{D}_\alpha H\mathcal{D}^\alpha H^\dagger)+2\,\mathrm{Tr}(\mathcal{D}_\alpha\Delta\mathcal{D}^\alpha\Delta^\dagger)+V(H,\Delta), \tag{4.11}$$

where the Higgs potential is

$$\frac{1}{8g^2}V(H,\Delta)=\frac{1}{8}\left[H^\dagger H-\frac{m^2}{g^2}\right]^2+\frac{1}{4}\left[\frac{1}{2}H^\dagger H-\frac{M}{g}\Delta_0-\frac{m^2}{g^2}\right]^2+\frac{1}{4}\left[\frac{1}{2}H^\dagger\sigma_a H-\frac{M}{g}\Delta_a\right]^2+\frac{1}{8}H^\dagger(\Delta_0+\Delta_a\sigma_a)^2H. \tag{4.12}$$

Above H is written as a two-component column vector with complex entries H_+ and H_0 and σ_a are the Pauli spin matrices. The vacuum configuration can be determined either directly from the minimum of the above potential, which is a sum of squares, or from Eq. (3.13), to be

$$H_0=\frac{m}{g},\quad H_+=0,\quad \Delta_0=\Delta_3=-\frac{m^2}{2Mg},\quad \Delta_{1,2}=0, \tag{4.13}$$

where only the electromagnetism survives symmetry breaking.

In this model we have considered our structure group $SU(2)_I\times U(1)_Y$ as a subgroup of $U(3)$ and hence their coupling constants g and g' merge to the same value. As a consequence, the Weinberg angle is obtained from the standard relation

$$\sin^2\theta_w=\frac{g^2}{g^2+g'^2}=\frac{1}{2}.$$

The mass spectrum of the model can be found easily. The masses of the W and Z bosons are found from Eq. (3.14) to be

$$M_W=m\sqrt{1+\frac{m^2}{2M^2}},\quad M_Z=\sqrt{2}m.$$

To find the mass spectrum of the Higgs sector, on the other hand, we first write down the linearized field equations [9]:

$$d\star dH_1+2m^2\left(1+\frac{m^2}{2M^2}\right)H_1-2Mm\left(1+\frac{m^2}{2M^2}\right)\Delta_1=0$$

$$d\star dH_2+2m^2\left(1+\frac{m^2}{2M^2}\right)H_2+2Mm\left(1+\frac{m^2}{2M^2}\right)\Delta_2=0$$

$$d\star dH_3+8m^2H_3-2Mm(\Delta_0-\Delta_3)=0$$

$$d\star dH_4=0$$

$$d\star d\Delta_0-2MmH_3+2M^2\left(1+\frac{m^2}{2M^2}\right)\Delta_0-m^2\Delta_3=0$$

$$d\star d\Delta_3+2MmH_3+2M^2\left(1+\frac{m^2}{2M^2}\right)\Delta_3-m^2\Delta_0=0$$

$$d\star d\Delta_1+2M^2\left(1+\frac{m^2}{2M^2}\right)\Delta_1-2Mm\left(1+\frac{m^2}{2M^2}\right)H_1=0$$

$$d\star d\Delta_2+2M^2\left(1+\frac{m^2}{2M^2}\right)\Delta_2+2Mm\left(1+\frac{m^2}{2M^2}\right)H_2=0, \tag{4.14}$$

where

$$H_1=H_++H_+^*,\quad H_2=(H_+-H_+^*)/i,$$

$$H_3=H_0+H_0^*,\quad H_4=(H_0-H_0^*)/i.$$

The diagonalization of the mass matrix that is read from linearized Higgs field equations yields the mass eigenvalues

$$0,0,0,2M^2,\quad \left(3m^2+\frac{m^4}{M^2}+2M^2\right),\quad \left(3m^2+\frac{m^4}{M^2}+2M^2\right),$$

$$(5m^2+M^2+\sqrt{9m^4+2m^2M^2+M^4}),$$

$$(5m^2+M^2-\sqrt{9m^4+2m^2M^2+M^4}).$$

The value of the Weinberg angle and the above masses imply that the ρ parameter

$$\rho=\frac{M_W^2}{M_Z^2\cos^2\theta_W}=1+\frac{m^2}{2M^2}.$$

Experimentally ρ is very close to 1 so we must have $M \gg m$. Thus at the mass scale M we obtain three zero-mass eigenvalues that refer to Goldstone modes which would be absorbed by weak intermediate bosons to become massive, one *light* Higgs boson with mass $\sqrt{2}m$, and all the remaining scalar masses converge to $\sqrt{2}M$ as we take $M\gg m$.

It is now possible to predict the values of these masses at the electroweak scale $E_Z\sim m$ by considering the renormalization flow of the coupling constants g, g' and the Higgs self-coupling constant λ down from the scale M to the scale m and also using the fact that $\lambda=g^2/4$ from the Higgs potential (4.12) [6]. The relevant renormalization group equations are given by [10]

$$16\pi^2\frac{dg}{dt}=-\frac{19}{6}g^3, \tag{4.15}$$

$$16\pi^2\frac{dg'}{dt}=\frac{41}{6}g'^3, \tag{4.16}$$

$$16\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 - 3\lambda(3g^2 + g'^2) + \frac{3}{8}[2g^4 + (g^2 + g'^2)^2]. \tag{4.17}$$

We solve Eqs. (4.15) and (4.16) and set $g = g'$ and $\lambda = \frac{1}{4}g^2$ at the scale M. This implies

$$\frac{1}{g^2(\mu)} - \frac{1}{g'^2(\mu)} = \frac{60}{48\pi^2} \ln \frac{\mu}{M} \tag{4.18}$$

at an arbitrary mass scale μ. We fix g and g' at the scale $\mu = E_Z = 91$ GeV by their measured values $g(E_Z) = 0.4234$ and $g'(E_Z) = 0.1278$. This choice drives the Weinberg angle to its experimental value 0.23 at the scale $\mu = E_Z$. We also find that we should have $M \sim 5\times 10^{20}$ GeV to start with. Inserting what we found back into Eqs. (4.15) and (4.16) we obtain

$$g^2(M) = g'^2(M) = 4\lambda(M) = 0.49. \tag{4.19}$$

The remaining equation (4.17) can be solved numerically by feeding in the solutions of Eqs. (4.15) and (4.16), yielding the result $\lambda(E_Z) = 0.14$. From the standard model,

$$\frac{m_H^2(\mu)}{m_Z^2(\mu)} = \frac{8\lambda(\mu)}{g^2(\mu) + g'^2(\mu)}, \tag{4.20}$$

which is already satisfied at scale M. This relation gives the mass of the Higgs particle at the electroweak scale as $m_H(E_z) = 130$ GeV. However, the actual determination of the physical mass should take into consideration radiative corrections. But it is well known that [11]

$$m_H(\mu) = m_H^{pole}[1 + \delta(\mu)], \tag{4.21}$$

where $\delta(\mu)$ referring to the radiative corrections are very small at the scale $\mu = E_Z$. Therefore we may conclude $m_H^{pole} \sim m_H(E_Z) \sim 130$ GeV.

V. CONCLUDING COMMENTS

The noncommutative extension of the electroweak model proposed by Balakrishna, Gürsey, and Wali [6] where the space-time is extended by the Pauli matrices themselves is both intiutive and comparatively simple to study. It predicts a *light* Higgs boson with a mass around 130 GeV together with four very heavy Higgs bosons.

The model may be generalized in several directions. In fact a supersymmetric generalization [12] as well as a grand unification scheme [13] had already been discussed. We think it would not be unreasonable to contemplate an effective field theory approach to the noncommutative electroweak models. In a first attempt, we consider, to the lowest possible order, the following cubic term in the Higgs potential:

$$\frac{\alpha}{3!g^3}\mathrm{Tr}(\Omega_{ab}\Omega^{bc}\Omega_{cd}g^{ad}), \tag{5.1}$$

which contributes as

$$\frac{\alpha g^3}{4}\left\{\left[\sum_{i=0}^{3}\Delta_i(H^\dagger\sigma_i H)\right]^2 + H^\dagger H\left[H^\dagger\left(\sum_{i=0}^{3}\Delta_i\sigma_i\right)^2 H\right]\right\} + \frac{\alpha g^2}{4}M\left[H^\dagger\left(\sum_{i=0}^{3}\Delta_i\sigma_i\right)^3 H\right] - \frac{\alpha g}{2}m^2 H^\dagger\left(\sum_{i=0}^{3}\Delta_i\sigma_i\right)^2 H. \tag{5.2}$$

It can be checked that the vacuum configuration (4.13) makes the above expression vanish. This does not necessarily mean that with the inclusion of effective terms the complete Higgs potential cannot acquire a distinct set of vacuum expectation values. The possibility remains open at present.

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