Electroweak symmetry breaking by strong supersymmetric dynamics at the TeV scale

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We construct models in which electroweak symmetry is spontaneously broken by supersymmetric strong dynamics at the TeV scale. The order parameter is a composite of scalars, and the longitudinal components of the W and Z are strongly coupled bound states of scalars. The usual phenomenological problems of dynamical electroweak symmetry breaking are absent: the sign of the S parameter unconstrained in strongly interacting SUSY theories, and fermion masses are generated without flavor-changing neutral currents or large corrections to the ρ parameter. The lightest neutral Higgs scalar can be heavier than M_Z without radiative corrections from standard-model fields. All the mass scales in the model can be naturally related in low-scale models of supersymmetry breaking. The μ problem can also be solved naturally, and the model can incorporate perturbative unification of standard-model gauge couplings with intermediate thresholds.

I. INTRODUCTION

Understanding the origin of electroweak symmetry breaking is without question the most important open problem in particle physics. On the experimental side, despite a wealth of precision data that shows convincingly that the electroweak interactions are described by a spontaneously broken $SU(2)_W \times U(1)_Y$ gauge theory, we still have no direct information about the dynamics of electroweak symmetry breaking. On the theoretical side, there are only a handful of mechanisms known for electroweak symmetry breaking that can naturally explain the enormous hierarchy between the weak scale $M_W \sim 100$ GeV and more fundamental scales such as the unification scale $M_{\text{GUT}} \sim 10^{16}$ GeV and the Planck scale $M_{\text{Planck}} \sim 10^{18}$ GeV. The oldest idea is that new QCD-like strong dynamics near the weak scale are responsible for electroweak symmetry breaking [1]. This idea, known as "technicolor," is currently out of favor because of phenomenological problems and the difficulty of constructing compelling models. Perhaps the most attractive and well-studied idea is supersymmetry (SUSY) [2]. Most recently, there has been a great deal of interest in the idea that the fundamental Planck scale is near the weak scale, thus obviating the hierarchy problem. In such scenarios the observed weakness of gravity compared to the weak interactions is explained by the presence of large extra dimensions felt only by gravity [3] or by the effects of gravitational curvature in extra dimensions [4].

In this paper, we consider a new class of models in which electroweak symmetry is broken by strong *supersymmetric* dynamics at the TeV scale. Supersymmetry is assumed to be broken softly at the weak scale, but this breaking is small enough to be viewed as a perturbation on the strong dynamics. Electroweak symmetry is broken by a vacuum expecta-

tion value (VEV) for a composite operator made of scalars arising from a nonperturbative "deformed moduli space" [5]. The Nambu-Goldstone bosons that become the longitudinal components of the W and Z are composites of scalars. In this sense, the mechanism can be viewed as the "superpartner" of the technicolor mechanism, in which the condensate and the longitudinal component of the gauge bosons are fermion composites. We therefore call this mechanism "S-color," where the "S" stands for "super" or "scalar." We will show that these models elegantly avoid all of the problems of technicolor models, and compare favorably with other SUSY models in terms of naturalness and simplicity. Most importantly, these models give a viable and well-motivated scenario for strongly-coupled supersymmetric physics at the TeV scale. The models have many interesting signatures, including a nonminimal Higgs sector, nonstandard Yukawa couplings, and an approximately supersymmetric spectrum of strong resonances in the TeV region.

It is interesting to compare these models with nonsupersymmetric technicolor models. Technicolor models have difficulty generating fermion masses without generating large flavor-changing neutral currents [6]. The models we consider have no problem with fermion masses because they contain an elementary Higgs multiplet that gets a VEV by mixing with the composite fields of the S-color sector. The fermion masses therefore arise from ordinary Yukawa couplings, and the usual Glashow-Iliopoulos-Maiove (GIM) mechanism suppresses flavor changing neutral currents (FCNC's).[1] Also, technicolor models generally give rise to large positive contributions to the electroweak S parameter from strong resonances in the TeV region [9]. In the models we consider, the sign of S is not determined by any currently known method. Other radiative corrections are also naturally under control.

Compared to more traditional SUSY models, these mod-

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[1]Technicolor models with a GIM mechanism [7] or "walking" [8] can be constructed, but the models are very complicated and require nontrivial dynamical assumptions.

els also have a number of attractive features. For example, the μ problem can be solved by the S-color dynamics. Also, the lightest neutral Higgs boson can be significantly more massive than M_Z due to mixing with the composite states. Perhaps the least appealing feature of these models is that the SUSY breaking masses must be close to the S-color scale, even though they do not originate from the S-color dynamics. We will show that this can be natural if the S-color group is near a conformal fixed point and is driven away from the fixed point by low-scale SUSY breaking, for example from gauge-mediated SUSY breaking. We will present a model that incorporates this mechanism, together with a dynamical solution to the μ problem and gauge unification, all without excessive complication.

A model very similar to the ones considered here was discussed in Ref. [11], which appeared while this paper was in progress. However, the model of Ref. [11] has a massless fermion with couplings to the Z, and is therefore ruled out.[2] Also, electroweak radiative corrections are not discussed in Ref. [11]. However, the idea that there can be strong approximately supersymmetric dynamics at the TeV scale appears for the first time in Ref. [11].

The models presented here also have some similarities with ''bosonic technicolor'' models [10], which involve both SUSY and strong dynamics near the TeV scale. However, in bosonic technicolor, SUSY breaking scalar masses are large compared to the strong dynamical scale, so the dynamics that breaks electroweak symmetry is completely nonsupersymmetric. Therefore, in bosonic technicolor the spectrum of strong resonances at the TeV scale is nonsupersymmetric, and the S parameter is unsuppressed and positive.

This paper is organized as follows. In Sec. II, we analyze a minimal (but realistic) model that illustrates the main features of the idea. In Sec. III, we consider an extension of the model analyzed in Sec. II that incorporates a solution to the μ problem. In Sec. IV, we estimate the electroweak radiative corrections in this class of models. In Sec. V, we discuss a mechanism that can explain the coincidence of the S-color scale and the scale of soft SUSY breaking, and account for gauge coupling unification. Section VI contains some speculations on phenomenology and our conclusions.

II. A MINIMAL MODEL

We now present a simple model that illustrates the main features of the mechanism. The non-Abelian symmetries of the model are

$$\mathrm{SU}(2)_{\mathrm{SC}} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R, \qquad (2.1)$$

where $\mathrm{SU}(2)_{\mathrm{SC}}$ is the S-color gauge group, $\mathrm{SU}(2)_L$ is the weak gauge group, and we only gauge the $\mathrm{U}(1)_Y$ subgroup of $\mathrm{SU}(2)_R$ which is generated by the τ_3 generator. The fields are

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[2]The massless fermion can be avoided in the model of Ref. [11] by assuming a different structure for the VEV's and introducing additional B-type soft masses. See Sec. II.

$$T_L \sim (\square,\square,\mathbf{1}), \quad T_R \sim (\square,\mathbf{1},\square), \quad H \sim (\mathbf{1},\square,\square), \quad (2.2)$$

and two singlets S_L, S_R. The field H therefore contains a pair of doublet Higgs fields, and the fields T_L and T_R are a supersymmetric version of minimal technicolor [1]. The theory has a tree-level superpotential

$$W = \lambda_L S_L T_L T_L + \lambda_R S_R T_R T_R + \lambda_H H T_L T_R + \tfrac{1}{2}\mu HH. \qquad (2.3)$$

These terms break all global U(1) symmetries, which is important for avoiding massless fermions or axions. The gauge symmetries allow the addition of a superpotential term

$$\Delta W = \lambda'_H H T_L (T_R \tau_3) \qquad (2.4)$$

that violates custodial SU(2). We will ignore this term for simplicity when discussing the effective potential, but we will return to it when we discuss electroweak radiative corrections. The elementary Higgs fields, H, are also assumed to have Yukawa couplings to the quark and lepton fields. These are important for generating the quark and lepton masses, but they do not play a role for the vacuum structure as long as the squark and slepton fields do not get VEV's.

The strong S-color dynamics has a global SU(4) symmetry that is broken only by standard-model gauge interactions and trilinear superpotential couplings. Under SU(4), the S-colored fields transform as a fundamental:

$$T^j = \begin{pmatrix} T_L \\ T_R \end{pmatrix} \sim \square, \quad j = 1, \ldots, 4. \qquad (2.5)$$

The $\mathrm{SU}(2)_{\mathrm{SC}}$ group has a deformed moduli space [5]. This means that below the scale Λ where the theory becomes strong, the light degrees of freedom correspond to the ''meson'' fields $M^{jk} \propto T^j T^k$:

$$M^{jk} = -M^{kj} = \begin{pmatrix} B_L \epsilon & \Pi \\ -\Pi^T & B_R \epsilon \end{pmatrix}, \quad \epsilon \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad (2.6)$$

subject to the constraint

$$\mathrm{Pf}(M) = B_L B_R - \det(\Pi) \neq 0. \qquad (2.7)$$

Under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ the composite fields transform as

$$\Pi \sim (\square,\square), \quad B_L \sim (\mathbf{1},\mathbf{1}), \quad B_R \sim (\mathbf{1},\mathbf{1}). \qquad (2.8)$$

In order to be realistic, this theory must incorporate soft SUSY breaking.[3] Since the strong dynamics is responsible for breaking electroweak symmetry, the required soft SUSY

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[3]It is amusing to note that if we omit the μ term in Eq. (2.3), then this model dynamically breaks SUSY [12]. However, this cannot be the only source of SUSY breaking since it gives very small (gauge-mediated) masses to standard-model gauginos and scalars. Even if we add soft SUSY breaking by hand, the model without the μ term gives rise to an ''extra'' Goldstino that couples to the Z. Therefore, we must complicate the model to ensure that it does *not* dynamically break SUSY.

breaking terms are not much smaller than the dynamical scale of the S-color dynamics. However, we will see that naive dimensional analysis (NDA) [15,16] indicates that it is sensible to treat soft SUSY breaking as a perturbation.

Denote the scale where the S-color dynamics becomes strong by Λ. In a normalization where the composite fields have kinetic terms of order 1, the quantum constraint can be written [16]

$$\det(\Pi) - B_L B_R = \tfrac{1}{2} f^2, \qquad (2.9)$$

and the effective superpotential is

$$W_{\mathrm{eff}} = f[\lambda_L S_L B_L + \lambda_R S_R B_R + \lambda_H H \Pi] + \tfrac{1}{2} \mu H H, \qquad (2.10)$$

where $f = \Lambda/4\pi$. We have used our freedom to normalize the fields to set various coefficients to 1; in this normalization, all of the unknown strong interaction coefficients appear in the effective Kähler potential for the composite fields.

SU(4) symmetry and NDA tells us that the effective Kähler potential is

$$K_{\mathrm{eff}} = f^2 k \left(\frac{\mathrm{tr}(M^\dagger M)}{2 f^2} \right)$$

$$= f^2 k \left(\frac{\Pi_0^\dagger \Pi_0 + \Pi_A^\dagger \Pi_A + B_L^\dagger B_L + B_R^\dagger B_R}{f^2} \right), \quad (2.11)$$

where k is an unknown order-1 function. We know that $k' > 0$ for all field values in order that the theory has a positive kinetic term in the SUSY limit.

From the above, we see that we require $f \sim 100\,\mathrm{GeV}$, which implies $\Lambda \sim 1\,\mathrm{TeV}$. The soft masses must be of order $m_{\mathrm{soft}} \sim 100\,\mathrm{GeV}$, so NDA implies that SUSY breaking perturbations are suppressed by $m_{\mathrm{soft}}/\Lambda \sim 1/4\pi$. Some of our results rely on NDA, so it is reassuring to note that NDA for soft SUSY breaking is known to work well in supersymmetric theories where exact results are available [13]. Note also that in QCD, the strange quark mass breaks SU(3) flavor symmetry by an amount $m_{\mathrm{strange}}/\Lambda_{\mathrm{QCD}} \sim (100\,\mathrm{MeV})/(1\,\mathrm{GeV})$, a perturbation just as large as the one we are contemplating. The fact that SU(3) is a useful approximate symmetry in QCD is thus further support that the expansion we are performing is sensible.

At this point, there is no explanation for the near coincidence of the scales f and m_{soft}. Also, the μ term must be put in by hand, and must be the same order as m_{soft}. In Sec. III we will discuss extensions of this model that can address these issues. However, the present model gives a simple and realistic illustration of the mechanism we are proposing.

To solve the quantum constraint, we write

$$\Pi^j{}_k = \frac{1}{\sqrt{2}} (\Pi_0 \mathbf{1}_2 + i \Pi_A \tau_A)^j{}_k, \qquad H^j{}_k = \frac{1}{\sqrt{2}} (H_0 \mathbf{1}_2 + i H_A \tau_A)^j{}_k, \qquad (2.12)$$

where τ_A $(A = 1,2,3)$ are the Pauli matrices. This gives

$$H\Pi = H_0 \Pi_0 + H_A \Pi_A, \qquad \det(\Pi) = \tfrac{1}{2}(\Pi_0^2 + \Pi_A \Pi_A), \qquad (2.13)$$

etc. Solving Eq. (2.9) for Π_0 gives

$$\Pi_0 = (f^2 + 2 B_L B_R - \Pi_A \Pi_A)^{1/2}. \qquad (2.14)$$

We therefore parametrize the moduli space by B_L, B_R, and Π_A; this parametrization is nonsingular for all vacua where $\langle \Pi_0 \rangle \neq 0$. In this way we obtain the unconstrained effective superpotential

$$W_{\mathrm{eff}} = f\{\lambda_L S_L B_L + \lambda_R S_R B_R + \lambda_H [H_0 \Pi_0 + H_A \Pi_A]\}$$

$$+ \tfrac{1}{2} \mu (H_0^2 + H_A H_A), \qquad (2.15)$$

where Π_0 is eliminated using Eq. (2.14). Similarly, Π_0 should also be eliminated in the effective Kähler potential Eq. (2.11).

We now discuss the vacua in the SUSY limit. The H_0, H_A, and Π_A equations of motion give, respectively,

$$H_0 = -\frac{\lambda_H f}{\mu} \Pi_0, \qquad (2.16)$$

$$f \lambda_H \Pi_A = -\mu H_A, \qquad (2.17)$$

$$H_0 \Pi_A = H_A \Pi_0, \qquad (2.18)$$

where Π_0 is given by Eq. (2.14). Substituting Eq. (2.17) into Eq. (2.18) reproduces Eq. (2.16), so we find three flat directions. The moduli space of vacua includes a subspace where $\mathrm{SU(2)}_L \times \mathrm{SU(2)}_R \to \mathrm{SU(2)}$, i.e., the gauge symmetry breaks as $\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y \to \mathrm{U(1)}_{em}$. In these vacua, electroweak symmetry is broken in the correct pattern in the SUSY limit, and the three flat directions are associated with the Nambu-Goldstone bosons of the symmetry breaking.

To obtain a realistic model we must include soft SUSY breaking with $m_{\mathrm{soft}} \sim \Lambda/4\pi$. We must then check that there are choices for the fundamental soft masses where electroweak symmetry is broken. An important point is that the potential has no global U(1) symmetries, so there is no danger of obtaining a weak-scale axion. The potential is not calculable in this model, because when we include soft SUSY breaking the potential depends on the full functional form of the effective Kähler potential, parametrized by the function k defined in Eq. (2.11). This is simply because in units where $f = 1$, the Kähler potential is an order-1 function of an order-1 argument. Derivatives of the Kähler potential appear multiplicatively in the potential, and do not affect the VEV's in the SUSY limit. However, for $m_{\mathrm{soft}} \sim f$, these multiplicative corrections are parametrically as important as the soft mass contributions to the potential. Without knowledge of the Kähler potential we cannot determine rigorously whether vacua of the desired form exist. However, given the large number of free parameters in the soft masses, it is

reasonable to assume that there are vacua that break electroweak symmetry in the desired fashion.[4]

We now turn to the fermion masses. We first neglect gaugino masses. In the presence of a nontrivial Kähler potential and SUSY breaking, the fermion mass matrix is proportional to

$$m_{ab} = \langle W_{ab} \rangle + \langle K_{ab}{}^c \rangle \langle F_c^\dagger \rangle, \tag{2.19}$$

where we denote the fields by Φ^a and $W_a = \partial W/\partial \Phi^a$, $W^a = \partial W/\partial \Phi_a^\dagger$, etc. The physical fermion mass matrices are given by matrix products of m_{ab} and $\langle K^a{}_b \rangle$, but the important fact for our purposes is that massless fermions are present if and only if $\det(m) = 0$.[5] The determinant of m is thus an important diagnostic, and we find that it is nonzero for general VEV's. The precise expression depends on the form of the effective Kähler potential, and is complicated and unenlightening; for example, if we assume that the Kähler potential is $\text{tr}(M^\dagger M)$, and assume $\langle \Pi_A \rangle = 0$, $\langle B_L \rangle = \langle B_R \rangle = 0$, we find

$$\det(m) = -\mu f^7 \lambda_H^3 \lambda_L^2 \lambda_R^2 (\mu \langle H_0 \rangle + \lambda_H f)^3. \tag{2.20}$$

This vanishes for SUSY vacua [see Eq. (2.16)], but is nonzero (and nonsingular) for general VEV's. When we include the gauginos, there are mass terms that mix the gauginos with some of the fermions above, as part of the SUSY Higgs effect. These mixing mass terms are of order M_Z, so the nonvanishing of the determinant above shows that there are no massless fermions in the limit $g_{1,2} \to 0$. This is important because it shows that there are no fermions whose mass comes entirely from the SUSY Higgs effect, so there is no reason that all fermions cannot be heavier than M_Z. The nonzero electroweak gauge couplings can in principle give rise to light fermions, but only for special parameter choices. We conclude that the fermion masses do not present a phenomenological problem for this model.

An undesirable feature of the present model is that it contains an explicit "μ term." This term must be of order the weak scale: if μ is large compared to the weak scale, the elementary Higgs fields H decouple and we do not generate quark and lepton masses; if μ is too small we have light fermions [see Eq. (2.20)]. (The only difference from the μ problem in the MSSM is that the present model can break electroweak symmetry for any value of μ.) In the next section, we show that a simple modification of the model can solve this problem.

We close this section by considering what happens when $\lambda_L = \lambda_R = 0$. Then the singlets S_L, S_R decouple, and we ob-

tain the model of Ref. [11]. This model has an anomaly-free global $U(1)_{SB} \times U(1)_R$ symmetry, where the $U(1)_{SB}$ is "S-baryon" number. The $U(1)_R$ symmetry is broken explicitly by soft SUSY breaking terms. Reference [11] assumes that $\langle B_L \rangle = \langle B_R \rangle = 0$ in order to avoid spontaneously breaking the $U(1)_{SB}$. In this case, we find that there is a massless "baryon" fermion in the spectrum. This fermion has unsuppressed couplings to the Z, and is therefore ruled out. The massless fermion can be avoided by allowing $\langle B_L \rangle$, $\langle B_R \rangle \neq 0$. In order to avoid a weak-scale axion, $U(1)_{SB}$ must be broken explicitly. This can be done by adding the B-type soft SUSY breaking mass terms $T_L T_L + \text{H.c.}$, $T_R T_R + \text{H.c.}$ to the potential. The origin of these terms in a specific model of SUSY breaking may be difficult to understand, since there are no terms with the same symmetries in the SUSY part of the theory.

III. AN IMPROVED MODEL

We can eliminate the μ problem simply by replacing the μ term with a cubic interaction:[6]

$$W = \lambda_L S_L T_L T_L + \lambda_R S_R T_R T_R + \lambda_H H T_L T_R + \tfrac{1}{2} y (S_L + S_R) H H. \tag{3.1}$$

The symmetry between the S_L and S_R cubic couplings is not essential; it merely simplifies the form of the VEV's in the model. We can also include further cubic interactions for the singlets S_L and S_R, but these do not lead to qualitatively different results. Note that all global $U(1)$ symmetries are broken.

In the SUSY limit, the VEV's are determined by

$$\frac{\partial W}{\partial S_L} = f \lambda_L B_L + \frac{y}{2} (H_0 H_0 + H_A H_A),$$

$$\frac{\partial W}{\partial S_R} = f \lambda_R B_R + \frac{y}{2} (H_0 H_0 + H_A H_A),$$

$$\frac{\partial W}{\partial B_L} = f \lambda_L S_L - \frac{f \lambda_H H_0 B_R}{\Pi_0},$$

$$\frac{\partial W}{\partial B_R} = f \lambda_R S_R - \frac{f \lambda_H H_0 B_L}{\Pi_0},$$

$$\frac{\partial W}{\partial H_0} = f \lambda_H \Pi_0 + y (S_L + S_R) H_0,$$

$$\frac{\partial W}{\partial H_A} = f \lambda_H \Pi_A + y (S_L + S_R) H_A,$$

$$\frac{\partial W}{\partial \Pi_A} = f \lambda_H \left(H_A - \frac{H_0 \Pi_A}{\Pi_0} \right). \tag{3.2}$$

Using the H_0 equation we can see that the Π_A and H_A equa-

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[4]In the limit where the superpartners decouple this model becomes minimal technicolor. In minimal technicolor, the vacuum aligns to break electromagnetism, due to effects of standard-model gauge loops. In the present model, the superpotential couplings as well as standard-model gauge couplings break the accidental global symmetries of the strong dynamics. There is no reason to think that the problems of minimal technicolor are present in our model.

[5]This assumes that the Kähler potential is nonsingular.

[6]This is similar in spirit to the next-to-minimal supersymmetric standard model [14].

tions are equivalent, so we again find three flat directions. Solving the remaining equations for the special case $\langle H_A \rangle = 0$, we obtain

$$\langle H_0 \rangle = \left(\frac{2\lambda_L \lambda_R}{9y^2} \right)^{1/4} f,$$

$$\langle S_L \rangle = \langle S_R \rangle = \pm \lambda_H \left(\frac{2}{9y^2 \lambda_L \lambda_R} \right)^{1/4} f,$$

$$\langle B_L \rangle = - \left(\frac{\lambda_R}{18y^2 \lambda_L} \right)^{1/2} f,$$

$$\langle B_R \rangle = - \left(\frac{\lambda_L}{18y^2 \lambda_R} \right)^{1/2} f. \tag{3.3}$$

We see that there are points on the moduli space where electroweak symmetry is broken in the correct pattern. In addition, the nonzero VEV for the singlets gives an effective μ term for the Higgs doublets.

The inclusion of soft SUSY breaking proceeds as for the simpler model above. The details are not enlightening, and will not be given here. We expect that there is a vacuum with the desired properties for reasonable choices of soft masses.

We also computed the determinant of the fermion mass matrix to check that there are no light fermions. As in the previous model, we find that the fermion determinant is nonzero and unsuppressed for nonsupersymmetric VEV's. The discussion is similar to that for the simpler model, but the expressions are more complicated.

IV. ELECTROWEAK RADIATIVE CORRECTIONS

We now discuss electroweak radiative corrections in these models. This is particularly interesting because it is generally believed that models in which electroweak symmetry is broken by strong dynamics are strongly constrained by the electroweak S parameter [9]. However, we show that in the present model this is not the case. We will show that other radiative corrections are also small.

A. The S parameter

The S parameter can be viewed as a gauge kinetic mixing term between $SU(2)_L$ and $U(1)_Y$ gauge groups in an effective theory below the scale Λ where electroweak symmetry is broken:

$$\mathcal{L}_{\text{eff}} = -\frac{S}{16\pi} gg' F_L^{\mu\nu} F_{\mu\nu Y}. \tag{4.1}$$

In our model, the leading contribution to S comes from operators of the form

$$\frac{c}{\Lambda^2} \int d^4\theta \, \text{tr}[\bar{\nabla}\nabla(M^\dagger e^V)\bar{\nabla}\nabla M] + \text{H.c.}, \tag{4.2}$$

where ∇_α is the gauge covariant SUSY derivative, and $c \sim 1$ by NDA. Putting in VEV's for M we find

$$S \sim \frac{c}{\pi} \sim \pm 0.3. \tag{4.3}$$

Note that there are no large multiplicity factors, since there is only a single electroweak doublet charged under a strongly coupled $SU(2)$ gauge group.

The most recent Particle Data Group analysis of precision electroweak data gives $S = -0.16 \pm 0.14$ for the standard model with $m_{h^0} = M_Z$, and $S = -0.26 \pm 0.14$ for $m_{h^0} = 300$ GeV [17]. We see that the estimates above are compatible with the data, provided that the sign of S can be negative.

It is therefore very encouraging that the sign of the contribution Eq. (4.3) to S is not determined by the usual arguments in QCD-like theories, because of the crucial role played by scalars in these theories. Clearly we cannot use QCD data to directly estimate S in these theories. An alternative approach in QCD-like theories uses the Weinberg sum rules [18] together with the less rigorous resonance saturation ''approximation'' to estimate S. The first step is to use the operator product expansion (OPE) to find the short-distance behavior of the current-current correlation function relevant for S,

$$\int d^4 x \, e^{ip\cdot x} \langle 0 | T J_{LA}^\mu(x) J_{RB}^\nu(0) | 0 \rangle$$

$$\sim \frac{g^{\mu\nu} - p^\mu p^\nu/p^2}{p^2} \langle 0 | (T_L^\dagger \tau_{LA} T_L)(T_R^\dagger \tau_{RB} T_R) | 0 \rangle + \cdots,$$
$$\tag{4.4}$$

where $T_{L,R}$ are the scalar components of the S-colored fields. In QCD, the leading operator on the right-hand side is a quartic fermion term, and the correlation function behaves as $1/p^4$ rather than $1/p^2$. This implies that only the first Weinberg sum rule holds in the present class of theories. The Weinberg sum rules can be written in terms of spectral density functions for vector and axial-vector channels [18]. Making the assumption that these sum rules are approximately saturated by the lowest-lying single particle intermediate states with vector and axial-vector quantum numbers, the first and second sum rules yield

$$f_\rho^2 - f_A^2 = f_\pi^2, \tag{4.5}$$

$$f_\rho^2 m_\rho^2 - f_A^2 m_A^2 = 0. \tag{4.6}$$

With these assumptions, S is given by [19]

$$S = 4\pi \left(\frac{f_\rho^2}{m_\rho^2} - \frac{f_A^2}{m_A^2} \right). \tag{4.7}$$

Using both sum rules Eqs. (4.5) and (4.6), one one can show that $m_\rho < m_A$, $f_A < f_\rho$ and hence that $S > 0$. However in a SUSY theory with only the first Weinberg sum rule Eq. (4.5) we can reach no conclusion as to the sign of S. Abandoning the saturation approximation we have even less information.

Yet another approach to determining the sign of S is to apply Vafa-Witten [20] positivity arguments to current-

current correlators. However, in spite of significant efforts along these lines, the sign of S has not been determined in this way for a QCD-like theory [21]. Furthermore, the Vafa-Witten arguments generally break down in theories (such as supersymmetric theories) that include scalars with Yukawa couplings. We conclude that there is no reason to believe that S cannot be negative in this class of models.

In the remainder of this section, we will argue that the F term contributions to S are much smaller than the contribution of Eq. (4.3). This is a somewhat surprising result, because there is an operator

$$\frac{1}{\Lambda^2} \int d^2\theta \, (W^\alpha)^j{}_k (W_\alpha)^l{}_n \epsilon_{jlrs} M^{kn} M^{rs} + \text{H.c.} \quad (4.8)$$

that is invariant under all symmetries, and that gives a non-zero value for S. NDA implies that the contribution to S from this operator is the same as Eq. (4.3), so the conclusions above would not be affected if this operator is present. The remainder of this subsection is therefore primarily of theoretical interest, and the reader interested mainly in the results is urged to skip to the next subsection.

We begin by classifying the possible F terms that can contribute to S. For this, it is important to keep track of the SU(4) symmetry of the strong dynamics. The elementary Higgs fields and composite "meson" fields fall into SU(4) representations

$$\Sigma_{jk} = \begin{pmatrix} \lambda_L S_L \epsilon & \lambda_H H \\ -\lambda_H H^T & \lambda_R S_R \epsilon \end{pmatrix} \sim \bar{\Box}, \quad M^{jk} = \begin{pmatrix} B_L \epsilon & \Pi \\ -\Pi^T & B_R \epsilon \end{pmatrix} \sim \Box, \quad (4.9)$$

while the SU(2)$_L \times$ SU(2)$_R$ field strengths are in the SU(4) adjoint:

$$(W_\alpha)^j{}_k \sim \begin{pmatrix} W_{L\alpha} & 0 \\ 0 & W_{R\alpha} \end{pmatrix} \sim \mathbf{Ad}. \quad (4.10)$$

In this notation, the tree-level superpotential of the model of Sec. III is

$$W = \Sigma_{jk} M^{jk} + y^{jklnpq} \Sigma_{jk} \Sigma_{ln} \Sigma_{pq}, \quad (4.11)$$

where y^{jklmpq} contains the cubic term in Eq. (3.1). In addition, there is an anomaly-free U(1)$_R$ symmetry with

$$R(M) = 0, \quad R(\Sigma) = 2, \quad R(W_\alpha) = 1, \quad R(y) = -4. \quad (4.12)$$

This SU(4)$\times$U(1)$_R$ symmetry strongly constrains the form of operators that can appear in the effective theory.

We now show that the only operator allowed by these symmetries that can contribute to S at the level of Eq. (4.3) is Eq. (4.8). We first consider contributions that do not vanish in the SUSY limit. In the SUSY limit, any operator that contributes to a gauge kinetic mixing must have the form

$$\int d^2\theta \, (W^\alpha)^j{}_k (W_\alpha)^l{}_n \mathcal{F}^{kn}{}_{jl}(M, \Sigma, y) + \text{H.c.} \quad (4.13)$$

We can expand $\mathcal{F}$ in a power series in y, and NDA tells us that the terms proportional to powers of y are suppressed by powers of $1/4\pi$. We therefore consider only terms independent of y.[7] $\mathcal{F}$ cannot depend on Σ^{-1} because it must have a smooth limit as $\Sigma \to 0$. [$\mathcal{F}$ can depend on both M and M^{-1} to construct invariants since $\text{Pf}(M) \neq 0$ ensures that M is invertible.] Therefore U(1)$_R$ invariance then does not allow any dependence of the y-independent part of $\mathcal{F}$ on Σ. We are left with terms where $\mathcal{F}$ is a function of M only. It is not hard to see that Eq. (4.8) is the only possibility, taking into account the quantum constraint.

However, we claim that the operator Eq. (4.8) cannot be present in the theory because it cannot arise from the theory with an additional massive flavor. Consider an SU(2) gauge theory with 6 fundamentals, and a mass term $W = m_{jk} M^{jk}$ that gives mass to 2 fundamentals. Near the origin of moduli space, this theory has a weakly-coupled "s-confined" [5] description in terms of the unconstrained meson fields M^{jk} and a dynamically generated superpotential $W_{\text{dyn}} \propto \text{Pf}(M)$. The theory has ananomaly-free U(1)$_R$ symmetry with $R(M) = \frac{2}{3}$, $R(m) = \frac{4}{3}$. This forbids all terms of the form

$$\int d^2\theta \, (W^\alpha)^j{}_k (W_\alpha)^l{}_n \mathcal{F}^{kn}{}_{jl}(M, m) + \text{H.c.} \quad (4.14)$$

where $\mathcal{F}$ is nonsingular in the limit $m \to 0$. For example, the term

$$\int d^2\theta \, (W^\alpha)^j{}_k (W_\alpha)^l{}_n \epsilon_{jlpqrs} M^{kn} M^{pq} (m^{-1})^{rs} \quad (4.15)$$

is invariant under all symmetries, and reduces to Eq. (4.8) if the matrix m_{jk} has rank 2, but it has a singular limit as $m \to 0$. Therefore, it cannot appear in the effective theory below the scale Λ_6, where the theory with 6 fundamentals becomes strong.

We now turn to the possibility that the operator Eq. (4.8) is generated in the theory with 6 fundamentals when we integrate out the massive modes. For $m \ll \Lambda_6$ we can integrate out the massive modes using the confined description. The operator Eq. (4.8) does not appear at any order in the perturbative expansion, as follows from conventional perturbative nonrenormalization theorems. We believe that there are no nonperturbative corrections to the matching that can give Eq. (4.8), since the effective theory is simply a weakly coupled Wess-Zumino model. This argument is valid only for $m \ll \Lambda$, but there can be no phase transitions as a function of a holomorphic coupling in the SUSY limit, so we conclude that the operator Eq. (4.8) vanishes even for $m \gg \Lambda$. We can summarize this by saying that the operator Eq. (4.8) in the theory with 4 fundamentals is not generated despite the fact that it is invariant under all symmetries; we do not have a generic superpotential in the ultraviolet theory (with 6 fundamentals).

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[7]There are contributions such as $\mathcal{F}^{jk}{}_{ln} \sim y^{jkpqrs} \Sigma_{pq} \Sigma_{rs} (M^{-1})_{ln}$ that are allowed by all symmetries.

There are additional contributions to the S parameter from SUSY breaking terms such as

$$\int d^4\theta (W^\alpha)^j{}_k (W_\alpha)^l{}_n (M^\dagger)_{jl} (\Sigma^\dagger)^{kn} + \text{H.c.} \quad (4.16)$$

However, NDA shows that these are highly suppressed because the VEV's of fields and SUSY breaking parameters are of order $\Lambda/4\pi$, while the mass scale that suppresses such higher dimension operators is Λ.

B. The T parameter

We now turn to the isospin-breaking T parameter. In an effective Lagrangian language, T is proportional to the difference between the W_3 and $W^\pm$ mass term obtained by integrating out the S-color states at the TeV scale. The only large contribution from the S-color dynamics comes from the isospin breaking term Eq. (2.4), which gives a shift in the Z mass of order

$$\Delta M_Z^2 \sim \frac{(\lambda_H' f)^2}{16\pi^2}. \quad (4.17)$$

Comparing this to the shift induced by the top quark, we obtain

$$\frac{T_{\text{SC}}}{T_{\text{top}}} \sim \frac{(\lambda_H' f)^2}{m_{\text{top}}^2}. \quad (4.18)$$

For $\lambda_H' \sim 1$, the S-color contribution is as large as the top contribution, but we can easily obtain an acceptable contribution for moderately small values of λ_H'.[8]

V. FURTHER MODEL BUILDING

Why is $f \sim m_{\text{soft}}$? It could be a coincidence just like $f_\pi \sim m_{\text{strange}}$. However, it is not difficult to construct models where the strong interaction scale of S-color is fixed by SUSY breaking. The idea is that the S-color gauge dynamics is near a strongly coupled fixed point, and is perturbed away from this fixed point by SUSY breaking, similar to "postmodern" technicolor theories [22]. For example, in our model, $SU(2)_{\text{SC}}$ would have an infrared fixed point if there were four or five flavors rather than two. If the additional S-colored fields are electroweak singlets and receive masses somewhat larger than m_{soft}, then the S-color gauge coupling rapidly becomes strongly coupled near the scale m_{soft}.

For an explicit example of this type of model, consider a theory with four S-color flavors, one electroweak doublet and two electroweak singlets, as well as a gauge singlet X with a superpotential term

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[8]Note that f is somewhat smaller than $v = 246$ GeV, since electroweak symmetry breaking is distributed between four Higgs doublets.

$$\Delta W = \frac{\kappa}{3} X^3. \quad (5.1)$$

Suppose that X gets a negative soft mass squared of order $m_{\text{soft}}^2 \sim (100 \, \text{GeV})^2$. (We will discuss how this can happen below.) The potential for X is

$$V \sim -m_{\text{soft}}^2 |X|^2 + |\kappa|^2 |X|^4, \quad (5.2)$$

with a minimum at $\langle X \rangle \sim m_{\text{soft}}/\kappa$. For small κ, X gets a VEV larger than m_{soft}. If X has Yukawa couplings to the S-colored electroweak singlets, then they can be integrated out and the S-color gauge coupling will rapidly go from fixed point behavior to strong coupling. This threshold is not supersymmetric, but $\langle F_X \rangle / \langle X \rangle \sim m_{\text{soft}}$. This SUSY breaking feeds into S-colored superpartners through S-color gauge mediation, but this gives corrections smaller than m_{soft}.

One might also wonder what becomes of unification since the S-color particles we have added to the MSSM are not in complete $SU(5)$ multiplets. There are many ways to achieve unification with the addition of further particles. An attractive possibility is that these may be responsible for gauge-mediating SUSY breaking to the ordinary superpartners. A simple example is to add a vector-like right-handed up quark to the model of this section. With the S-color particles responsible for electroweak symmetry breaking described above this makes an approximate $SU(5)$ multiplet. The SM gauge couplings unify if the vector-like quark mass is near 10^6 GeV. If the vector-like quark is the gauge messenger of SUSY breaking for the usual superpartners then the spectrum of superpartner masses will differ from the usual scenarios, since the messengers are not complete $SU(5)$ multiplets.

VI. CONCLUSIONS

We have presented models that dynamically break electroweak symmetry via strong supersymmetric ("S-color") dynamics. Our analysis of the dynamics is based on exact nonperturbative results in supersymmetric gauge theories, and is therefore on a firm theoretical foundation. The failure of the second Weinberg sum rule shows that the S parameter can have either sign, and is thus potentially consistent with precision electroweak data. One of our models gives a solution to the "μ problem." Simple extensions of these models can relate the supersymmetry breaking scale to the S-color scale and allow for gauge coupling unification.

The phenomenology of these models is very exciting. The spectrum contains the MSSM spectrum with two extra (composite) Higgs doublets. In addition, the theory is strongly coupled with a rich spectrum of supersymmetric strong interaction resonances in the TeV range, and therefore will exhibit anomalous WW scattering. Yukawa couplings are larger than in the MSSM (or the standard model), since electroweak symmetry breaking is distributed between composite and fundamental Higgs doublets. We hope that this approach to electroweak symmetry breaking will stimulate further theoretical and experimental work on the possibility of strong dynamics at the TeV scale.

ACKNOWLEDGMENTS

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