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Calculation of the decay constants of scalar mesons

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The purpose of our work is to provide further evidence for the utility of our generalized Nambu–Jona-Lasinio model in describing the properties of light mesons. In this work we develop methods for the calculation of the decay constants of scalar mesons. Our model makes definite predictions of how scalar mesons should be organized into nonets and we use our calculation of the decay constants to support our predictions. The assignment of scalar mesons to various nonets is problematic, with various choices made in the literature for the $\sigma(500–600)$, $f_0(980)$, $a_0(980)$, $f_0(1370)$, $K_0^*(1430)$, and $f_0(1500)$. Maltman has suggested that one can gain information as to how these nonets should be chosen by determining the meson decay constants of the scalar mesons. In particular, he studies the $a_0(980)$, $a_0(1450)$, and $K_0^*(1430)$ resonances using QCD sum rule techniques and other methods. He concludes that the $a_0(980)$ and $K_0^*(1430)$ have similar decay constants and therefore have similar structure. (We have come to the same conclusion by an entirely different analysis, making use of our generalized Nambu–Jona-Lasinio (NJL) model, which includes a covariant model of confinement.) In the case of the $a_0(980)$ we obtain a result for the decay constant that is about 50% larger than that of Maltman, while for the $a_0(1450)$ and $K_0^*(1430)$ our results are somewhat smaller than his. Our results suggest that the $a_0(980)$ may have a significant $K\bar{K}$ component, which would reduce our theoretical value. The implication of our results and those of Maltman for the organization of scalar mesons into nonets is discussed.

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I. INTRODUCTION

In recent years we have constructed a generalized Nambu–Jona-Lasinio (NJL) model that includes a covariant model of confinement. We have made an extensive study of the spectra of light mesons in Refs. [1] and [2]. The parameters of the model were fixed in Ref. [1] when we described the pseudoscalar and vector nonets of states, so that when we extended our study to include the f_0 mesons in Ref. [2], the distribution of the scalar states in various nonets could not be chosen arbitrarily. For example the $a_0(980)$, $f_0(980)$, $K_0^*(1430)$, and $f_0(1370)$ were in the same nonet of $1\,^3P_0$ states (see Fig. 1). We found approximate ideal mixing, with the $f_0(1370)$ being largely an $s\bar{s}$ state. However, the physical situation is more complex, since the $f_0(1370)$ and $f_0(1500)$ resonances are expected to exhibit strong mixing with the scalar glueball. (That mixing was considered in Ref. [3], where our model was used.) Since the $f_0(1370)$ decays strongly to the 4π channel, such configuration mixing has to play a significant role, if our model has made the correct assignments. It is also possible that $q\bar{q}q\bar{q}$ states play an important role in the scalar spectrum, however, we do not consider that aspect of the problem in this work.

Other possible assignments of scalar states to various nonets are suggested in the literature. For example, Schechter and collaborators [4–7] place the $\sigma(500–600)$, $a_0(980)$, $f_0(980)$, and $\kappa(700–900)$ in the same nonet. Those assignments lead to various problems with the energetics as described in Ref. [7]. For example, one does not see the expected increase in energy when a u or d quark is replaced by an s quark in the passage from the $a_0(1450)$ to the $K_0^*(1430)$. These and other problems are discussed in Ref. [7] and a model involving configuration mixing of the $a_0(980)$ and $K_0^*(1430)$ with two other states, possibly of $q\bar{q}q\bar{q}$ character, is introduced in an attempt to overcome the problems raised by the abovementioned assignments of configurations. [In our analysis the $\sigma(500–600)$ and $\kappa(700–900)$ are ''dynamically generated'' states [8], which appear as poles in the $\pi\pi$ and πK T matrices, but which do not correspond to $q\bar{q}$ states.]

We find a suggestion made by Maltman of a procedure to help resolve these issues to be quite attractive [9,10]. He

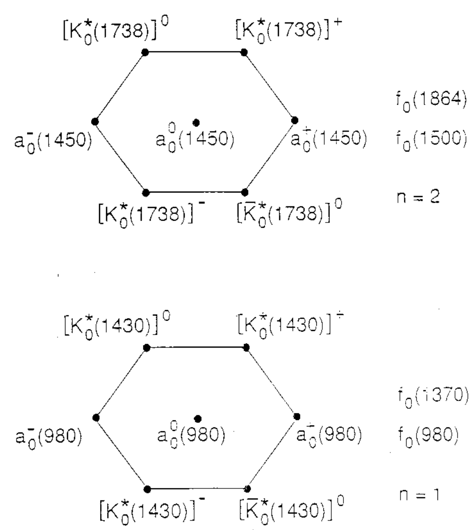

FIG. 1. The nonets of $1\,^3P_0$ and $2\,^3P_0$ scalar meson states obtained in Refs. [1] and [2] are shown. The $f_0(1864)$ and $K_0^*(1738)$ are predictions of our analysis with the $f_0(1864)$ being the $2\,^3P_0$ $s\bar{s}$ state.

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suggests that, if the $a_0(980)$ were a $K\overline{K}$ "molecule" [11], the state would have a very small decay constant compared to the $K_0^*(1430)$, which is generally thought to have a simple $q\overline{q}$ configuration with one strange quark. Maltman's study leads to the conclusion that the $a_0(980)$ and $K_0^*(1430)$ have similar structure. Since he obtains values for the $a_0(980)$, $a_0(1450)$, and $K_0^*(1430)$ decay constants, we decided that it would be of interest to calculate these decay constants in our generalized NJL model.

The organization of our work is as follows. In Sec. II we review some aspects of our model that are needed for our discussion. In Sec. III we describe how wave functions of $q\overline{q}$ scalar meson states are constructed in our model. In Sec. IV we present Maltman's definitions of the scalar meson decay constants and in Sec. V we show how our wave functions may be used to calculate the decay constants. Finally, in Sec. VI we compare our results with Maltman's values and provide some additional discussion and conclusions.

II. A GENERALIZED NAMBU–JONA-LASINIO MODEL

The Lagrangian of our model is

$$\begin{aligned}\mathcal{L}=&\overline{q}(i\not\partial-m^0)q+\frac{G_S}{2}\sum_{i=0}^{8}[(\overline{q}\lambda^i q)^2+(\overline{q}i\gamma_5\lambda^i q)^2]\\&-\frac{G_V}{2}\sum_{i=0}^{8}[(\overline{q}\gamma^\mu\lambda^i q)^2+(\overline{q}\gamma^\mu\gamma_5\lambda^i q)^2]\\&+\frac{G_D}{2}\{\det[\overline{q}(1+\gamma_5)q]+\det[\overline{q}(1-\gamma_5)q]\}\\&+\mathcal{L}_{\rm sensor}+\mathcal{L}_{\rm conf}.\end{aligned}\tag{2.1}$$

Here, the fourth term is the 't Hooft interaction, $\mathcal{L}_{\rm tensor}$ denotes interactions added to study tensor mesons, while $\mathcal{L}_{\rm conf}$ denotes our model of confinement [1,2]. In Eq. (2.1) m^0 is the current quark mass matrix $m^0=\mathrm{diag}(m_u^0,m_d^0,m_s^0)$, the λ_i $(i=1,\ldots,8)$ are the Gell-Mann matrices, and $\lambda_0=\sqrt{2/3}\,\mathbb{I}$, with $\mathbb{I}$ being the unit matrix in flavor space. In our previous work we had $G_S=11.83\,\mathrm{GeV}^{-2}$ and $G_D=86.39\,\mathrm{GeV}^{-2}$. The interaction strength in singlet states was $G_{00}=10.46\,\mathrm{GeV}^{-2}$ and the interaction strength in octet states was $G_{88}=12.46\,\mathrm{GeV}^{-2}$. (The deviation from ideal mixing induced by the 't Hooft interaction is described in Ref. [2].)

A novel feature of our model is the use of confinement vertex functions that satisfy an equation of the form shown in Fig. 2(a). There, V^C (dashed line) denotes our confining interaction [1,2]. Figure 2(b) shows the homogeneous equation for the vertex. That equation may be solved to obtain the $q\overline{q}$ wave functions bound by the confining field. Note that the vertex function that satisfies the inhomogeneous equation is singular at the energies where the homogeneous equation of Fig. 2(b) has a solution. In Fig. 2(c) we show a homogeneous equation that includes the effect of the short-range NJL interaction which is represented by the small filled circle.

The vertex of Fig. 2(a) satisfies the equation

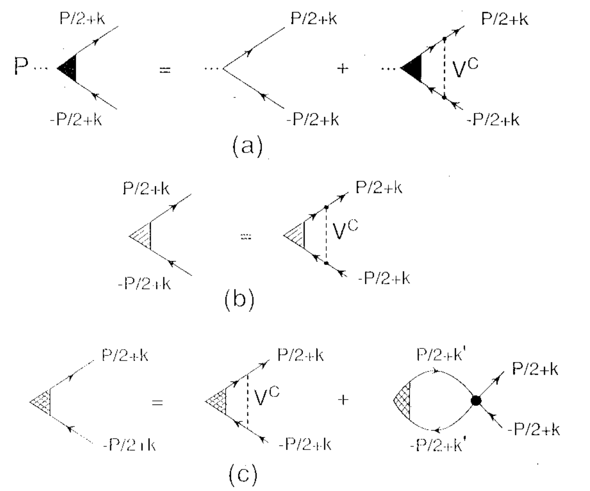

FIG. 2. (a) Schematic representation of the equation for the confinement vertex. Here V^C denotes the confining field [1,2]. (b) The homogeneous version of the equation shown in (a) is represented. (c) A representation of the homogeneous equation for the vertex that includes the effects of both confinement and the short-range NJL interaction is shown.

$$\overline{\Gamma}^S(P,k)=1-i\int\frac{d^4k'}{(2\pi)^4}[\gamma^\rho S(P/2+k')\overline{\Gamma}^S(P,k')\times S(-P/2+k')\gamma_\rho]V^C(\vec{k}-\vec{k}'),\tag{2.2}$$

where the γ^ρ matrices appear since we use Lorentz-vector confinement [1,2]. Here $S(p)=[\not p-m+i\eta]^{-1}$ is the quark propagator of a quark of mass m. The form of $V^C(\vec{k}-\vec{k}')$ is given in Ref. [2], for example.

In Fig. 3 we show the basic vacuum polarization integral of our NJL model $J^S(P^2)$, which is constructed using the

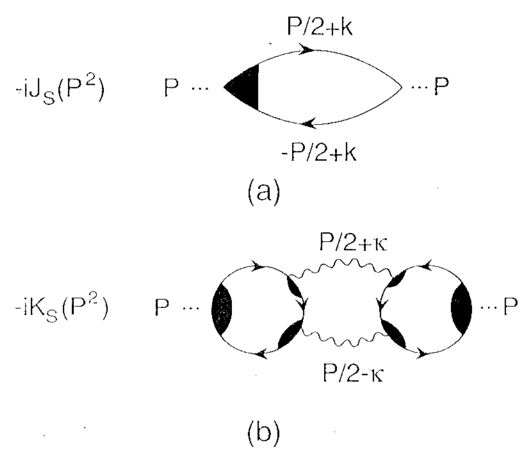

FIG. 3. (a) The diagram serves to define the function $-iJ^S(P^2)$. The triangular filled regions are the vertex functions shown in Fig. 2(a) [see Eq. (2.3)]. (b) The diagram represents the function $-iK^S(P^2)$ that describes polarization effects due to coupling to two-meson decay channels. The channels $\pi\pi$, $K\overline{K}$, $\eta\eta$, and $\eta\eta'$ were considered in Ref. [2] and values for $K^S_{\pi\pi}(P^2)$ and $K^S_{K\overline{K}}(P^2)$ were presented there.

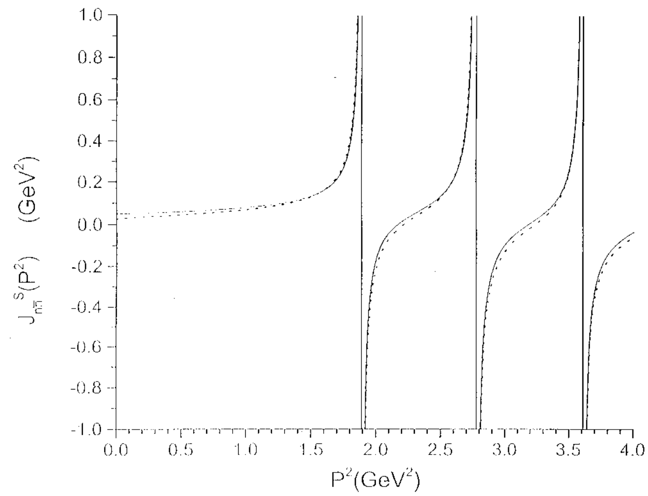

FIG. 4. The figure shows $J^S_{n\bar{n}}(P^2)$ calculated in the TDA (dotted line) and in the RPA (solid line). Here $m_u = m_d = 0.364$ MeV and $\alpha = 0.605$ GeV, $\kappa = 0.055$ GeV^2 and $\mu = 0.010$ GeV. The TDA result is obtained by dropping the second term in the brackets of Eqs. (2.3) and (2.4). In the RPA we use the complete expression.

vertex of Fig. 2(a). The function $J^S_{n\bar{n}}(P^2)$ is shown in Fig. 4 and $J^S_{s\bar{s}}(P^2)$ is shown in Fig. 5. Note that $J^S(P^2)$ is singular at the energies for which the homogeneous equation of Fig. 2(b) has a solution. For the study of the a_0 mesons only a single function $J^S(P^2)$ was needed [2]. In that case, in the meson rest frame ($\vec{P}=0$),

$$J^S(P^2) = -4n_c \int \frac{d^3k}{(2\pi)^3}\frac{\vec{k}^2}{E^2(\vec{k})} \times \left[\frac{\Gamma_S^{+-}(P^0,k)}{P^0-2E(\vec{k})} - \frac{\Gamma_S^{-+}(-P^0,k)}{P^0+2E(\vec{k})}\right] e^{-k^2/\alpha^2}. \tag{2.3}$$

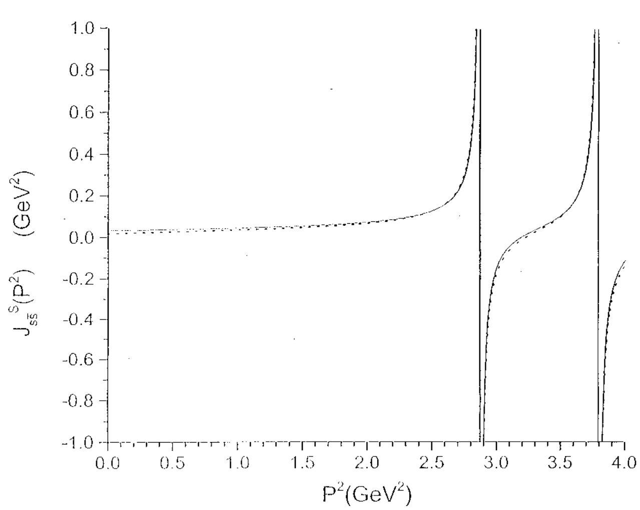

FIG. 5. The figure shows $J^S_{s\bar{s}}(P^2)$ calculated in the TDA (dotted line) and the RPA (solid line). Here $m_u = 364$ MeV and $m_s = 565$ MeV (see caption of Fig. 3).

We note that $\Gamma_S^{-+}(-P^0,k) = \Gamma_S^{+-}(P^0,k)$ so that we may write

$$J^S(P^2) = -4n_c \int \frac{d^3k}{(2\pi)^3}\frac{\vec{k}^2}{E^2(\vec{k})}\Gamma_S^{+-}(P^0,k) \times \left[\frac{1}{P^0-2E(\vec{k})} - \frac{1}{P^0+2E(\vec{k})}\right] e^{-k^2/\alpha^2}. \tag{2.4}$$

In the past we have regulated this integral using a sharp cutoff with $|\vec{k}| \leq \Lambda_3 = 0.622$ GeV [1]. In the present work we use a Gaussian cutoff in our analysis, since the sharp cutoff does not yield satisfactory wave functions. The function $\Gamma_S^{+-}(P^0,k)$ is obtained from the matrix $\overline{\Gamma}_S(P^0,k)$ using a procedure described in Ref. [1]. In Eq. (2.3) a factor of 2 arises from the flavor trace. We note that no small term $i\eta$ is needed in the denominator of the first term, since $\Gamma_S^{+-}(P^0,k)$ is equal to zero when $P^0 = 2E(\vec{k})$.

In Fig. 3(b) we show the polarization function $K^S(P^2)$, which was calculated in Ref. [2]. For a single channel, we may define a $q\bar{q}$ T matrix

$$t(P^2) = -\frac{G}{1-G[J^S(P^2)+K^S(P^2)]} \tag{2.5}$$

$$= -\frac{1}{G^{-1}-[J^S(P^2)+\mathrm{Re}\,K^S(P^2)] - i\,\mathrm{Im}\,K^S(P^2)}. \tag{2.6}$$

The bound state energies in the presence of the NJL interaction may be found by solving the equation

$$G^{-1}-[J^S(P^2)+\mathrm{Re}\,K^S(P^2)] = 0. \tag{2.7}$$

In our earlier work we calculated $\mathrm{Re}\,K^S(P^2)$ and $\mathrm{Im}\,K^S(P^2)$ for the $\pi\pi$, $K\bar{K}$, $\eta\eta$, and $\eta\eta'$ channels [2]. However, to fit the scalar spectrum we needed to introduce a parameter that represented contributions to $\mathrm{Re}\,K^S(P^2)$ from distant singularities and from the 4π channel. Recently, we have performed a new analysis of the kaon and its radial excitations. In that manner, we found that the coupling constant for octet states should be $G_{88} = 13.1$ GeV^{-2} and we will use that value for the study of the a_0 mesons. With our new determination of the coupling constant, we find that we do not need any adjustable parameters to fit the masses of the $a_0(980)$ and $f_0(980)$. However, the use of $G_S = 13.1$ GeV^{-2} overbinds the $K_0^*(1430)$ somewhat, so we use $G_S = 10.0$ GeV^{-2} to place the $K_0^*(1430)$ at 1350 MeV. In Ref. [1] we found K_0^* states at 1.416, 1.738, and 1.999 GeV [1]. The first and last of these states may be put into correspondence with the $K_0^*(1430)$ at $1429 \pm 4 \pm 5$ MeV and the $K_0^*(1950)$ at $1945 \pm 10 \pm 20$ [12]. We have recently discussed the evidence for a $K_0^*(1730)$ resonance in Ref. [13]. For example, in the analysis of πK scattering carried out by Jamin, Oller and Pich [14], a pole in the πK T matrix was found at $1731 - i147$ MeV, in addition to poles at $1450 - i142$ MeV and $1908 - i27$ MeV. Also, a K_0^* resonance at 1730 MeV is quite consistent with the phenomenological analysis of light meson spectra recently described in Ref. [15], and which we have discussed in Ref. [13]. Our results for the K_0^* states give us confidence that the strength (''string tension'') of the confining field introduced in Ref. [1] to fit the spectrum of pseudoscalar and vector meson states may be used without modification when we study the scalar mesons [1,2].

III. VERTEX FUNCTIONS AND WAVE FUNCTIONS OF THE GENERALIZED NJL MODEL

In this section we will discuss the case in which the quark and the antiquark have the same mass, leaving the more complicated case of unequal mass quarks for the Appendix. The function $\Gamma_S^{+-}(P^0,k)$ satisfies the equation [1,2]

$$\Gamma_S^{+-}(P^0,k)=1+\frac{1}{(2\pi)^2}\int k'^2dk'[2k'^2k^2V_0^C(k,k')+m^2kk'V_1^C(k,k')]\frac{1}{k^2E^2(\vec{k}')}\frac{\Gamma_S^{+-}(P^0,k')}{P^0-2E(\vec{k}')} \tag{3.1}$$

with $k=|\vec{k}|$, $k'=|\vec{k}'|$, and

$$V_l^C(k,k')=\frac{1}{2}\int_{-1}^{1}dxP_l(x)V^C(\vec{k}-\vec{k}'). \tag{3.2}$$

Here $x=\cos\theta$ and $P_l(x)$ is a Legendre polynomial. Note that no cutoff is needed for the momenta in Eq. (3.1).

It is useful to symmetrize the homogeneous version of Eq. (3.1) by multiplying by $k^2/E(\vec{k})$ from the left. We then define

$$\overline{\phi}_i(k)=\frac{k^2}{E(\vec{k})}\frac{\Gamma_i^{+-}(k)}{P_i^0-2E(\vec{k})}, \tag{3.3}$$

where i denotes a particular bound state solution. We find

$$[P_i^0-2E(\vec{k})]\overline{\phi}_i(k)=-\int dk'H_1^C(k,k')\overline{\phi}_i(k'), \tag{3.4}$$

where

$$H_1^C(k,k')=-\frac{1}{(2\pi)^2}\frac{kk'}{E(\vec{k})E(\vec{k}')}\times[2kk'V_0^C(k,k')+m^2V_1^C(k,k')]. \tag{3.5}$$

We may include the short-range NJL interaction, if we replace H_1^C in Eq. (3.4) by

$$H_1(k,k')=H_1^C(k,k')+H_1^{\mathrm{NJL}}(k,k'), \tag{3.6}$$

with

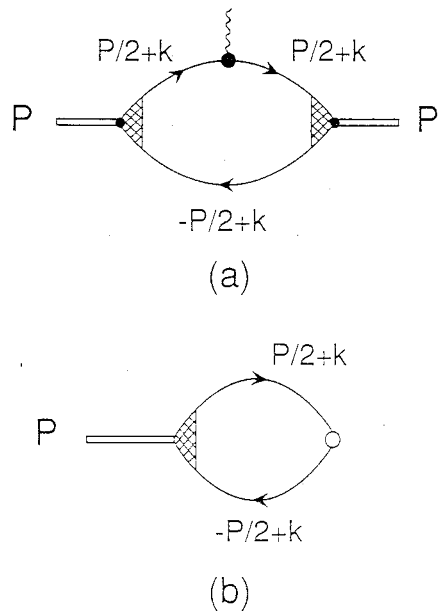

FIG. 6. (a) The normalization of the wave function may be carried out by requiring that the meson contain a single quark. (We use a relativistic normalization of the states $\langle\vec{P}'|\vec{P}\rangle=(2\pi)^3 2E_i(\vec{P})\delta^{(3)}(\vec{P}-\vec{P}')$ with $E_i(\vec{P})=[\vec{P}^2+M_i^2]^{1/2}$.) (b) The calculation of $M_i^2 f_i$ is indicated in a schematic fashion (see Secs. V and VI).

$$H_1^{\mathrm{NJL}}(k,k')=\frac{8n_C}{(2\pi)^2}\frac{k^2e^{-k^1/2\alpha^2}G_Sk'^2e^{-k'^2/2\alpha^2}}{E(\vec{k})E(\vec{k}')}. \tag{3.7}$$

Equation (3.7) contains a factor of 2 that arises from the flavor trace. In our recent work on scalar mesons [2], we used a Gaussian regulator with $\alpha=0.605$ GeV and we will continue to use that value in this work.

Once we obtain $\overline{\phi}_i(k)$, we define

$$\phi_i(k)=\frac{E(\vec{k})}{k^2}\overline{\phi}_i(k) \tag{3.8}$$

and

$$\phi_i^N(k)=\sqrt{N_i}\phi_i(k), \tag{3.9}$$

where $\sqrt{N_i}$ is a normalization factor that is obtained by requiring that the meson wave function contains only a single quark (see Fig. 6). We find

$$\frac{1}{N_i}=\frac{2n_C}{\pi^2}\left(\frac{1}{2M_i}\right)\int_0^\infty k^2dk\left(\frac{k^2}{E^2(k)}\right)|\phi_i(k)|^2, \tag{3.10}$$

where M_i is the mass of the meson. [Here, the states have been given a relativistic normalization, leading to the appearance of $2M_i$ in Eq. (3.10)].

In Fig. 7 the dotted line shows the nodeless function $\overline{\phi}(k)$, calculated with $H_1(k,k')=H_1^C(k,k')$, while the solid line is the result of the calculation with $H_1(k,k')=H_1^C(k,k')+H_1^{\mathrm{NJL}}(k,k')$. It is seen that adding the short-

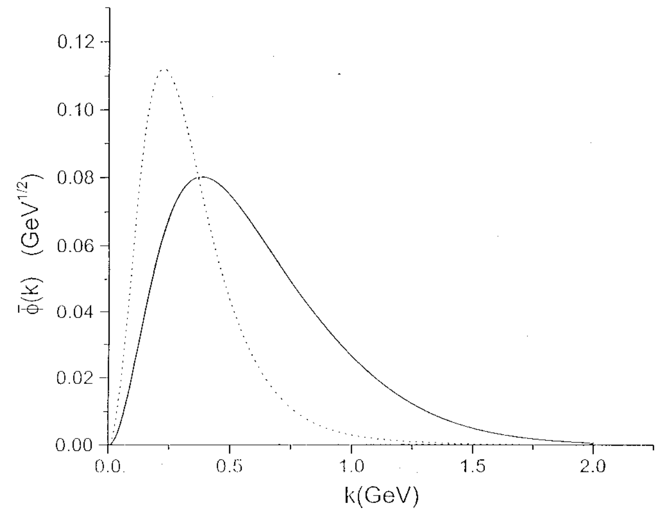

FIG. 7. The function $\overline{\phi}(k)$ relevant to the calculation of the decay constant of the $a_0(980)$ is shown for the case where $H_1(k,k')=H_1^C(k,k')$ (dotted line) and for the case in which $H_1(k,k')=H_1^C(k,k')+H^{\mathrm{NJL}}(k,k')$ (solid line). The use of the function given by the dotted line yields $m^2_{a_0(980)}f_{a_0(980)}=0.0222\,\mathrm{GeV}^3$, while the use of the function represented by the solid line yields $m^2_{a_0(980)}f_{a_0(980)}=0.0649\,\mathrm{GeV}^3$ (see Table I).

range NJL interaction admixes higher momentum components into the wave function and leads to a larger value of the decay constant (see Sec. V and Table I).

IV. DEFINITION OF SCALAR MESON DECAY CONSTANTS

Here we follow the definitions introduced by Maltman and use some of his notation [9,10]. Maltman studies the scalar correlator

$$\Pi(q^2)=i\int d^4x e^{iq\cdot x}\langle 0|\mathbf{T}(j(x)j^\dagger(0))|0\rangle, \tag{4.1}$$

TABLE I. Theoretical values of scalar meson decay constants. For the a_0 mesons we have $G_{88}=13.1\,\mathrm{GeV}^{-2}$ and $\alpha=0.605\,\mathrm{GeV}$. For the K_0^* mesons, we use $G_{88}=10.0\,\mathrm{GeV}^{-2}$.

	$M_i^2 f_i$ (GeV^3)	
Meson	$\alpha=0.605$ (GeV)	Maltman [9,10]
$a_0(980)$	0.0649[a]	0.0447 ± 0.0085[b]
$a_0(1450)$	0.0357	0.0647 ± 0.0123[b]
$a_0(1857)$[a]	0.0377	
$K_0^*(1430)$	0.0614	0.0842 ± 0.0045
$K_0^*(1730)$[c]	0.0425	
$K_0^*(1950)$	0.0414	

[a]The $a_0(1857)$ was predicted to exist in Ref. [1] (see Table IV of Ref. [1]).
[b]These values were obtained through the application of OCD sum rule techniques.
[c]We discuss evidence for a $K_0^*(1730)$ resonance in Ref. [13].

with $j^{du}(x)=\partial_\mu(\overline{d}\gamma^\mu u)$ or $j^{su}=\partial_\mu(\overline{s}\gamma^\mu u)$. He then defines

$$-i\langle 0|j^{su}(0)|K_0^*(1430)\rangle=f_{K_0^*(1430)}m^2_{K_0^*(1430)} \tag{4.2}$$

and

$$-i\langle 0|j^{du}(0)|a_0(980)\rangle=\hat{f}_{a_0(980)}m^2_{a_0(980)}. \tag{4.3}$$

In QCD, one has

$$\partial_\mu(\overline{q}^a\gamma^\mu q^b)=i(m_a^0-m_b^0)S^{ab} \tag{4.4}$$

with $S^{ab}=\overline{q}^a q^b$. It is then useful to redefine the a_0 decay constant using the relation

$$\left(\frac{m_s^0-m_u^0}{m_d^0-m_u^0}\right)\langle 0|j^{du}(0)|a_0(980)\rangle=f_{a_0(980)}m^2_{a_0(980)}. \tag{4.5}$$

If we consider the mesons of positive charge, we may use

$$-ij^{su}=(m_s^0-m_u^0)\overline{s}u \tag{4.6}$$

$$=(m_s^0-m_u^0)\frac{1}{\sqrt{2}}\overline{q}\left[\frac{\lambda_4-i\lambda_5}{\sqrt{2}}\right]q \tag{4.7}$$

and

$$-ij^{du}=(m_s^0-m_u^0)\frac{1}{\sqrt{2}}\overline{q}\left[\frac{\lambda_1-i\lambda_2}{\sqrt{2}}\right]q. \tag{4.8}$$

We take the value of m_s^0 from the work of Kambor and Maltman [16]. There one finds $m_s(1\,\mathrm{GeV})=158.6\pm18.7\pm16.3\pm13.3\,\mathrm{MeV}$, where the first error is statistical, the second is due to V_{us}, and the third is the theoretical error. We use the central value of $m_s=158.6\,\mathrm{MeV}$ and put $m_u^0=5.5\,\mathrm{MeV}$. Thus we take, $m_s^0-m_u^0=0.153\,\mathrm{GeV}$ for this work.

V. CALCULATION OF MESON DECAY CONSTANTS IN THE GENERALIZED NJL MODEL

With reference to Fig. 6(b), we proceed to calculate an expression for $M_i^2 f_i$. It is useful to use the following representation of the quark propagator:

$$S(\vec{k})=\frac{m}{E(\vec{k})}\left[\frac{\Lambda^{(+)}(\vec{k})}{k^0-E(\vec{k})+i\eta}-\frac{\Lambda^{(-)}(-\vec{k})}{k^0+E(\vec{k})-i\eta}\right], \tag{5.1}$$

where $\Lambda^{(+)}(\vec{k})$ and $\Lambda^{(-)}(-\vec{k})$ were defined in Refs. [1] and [2]. Thus, we have

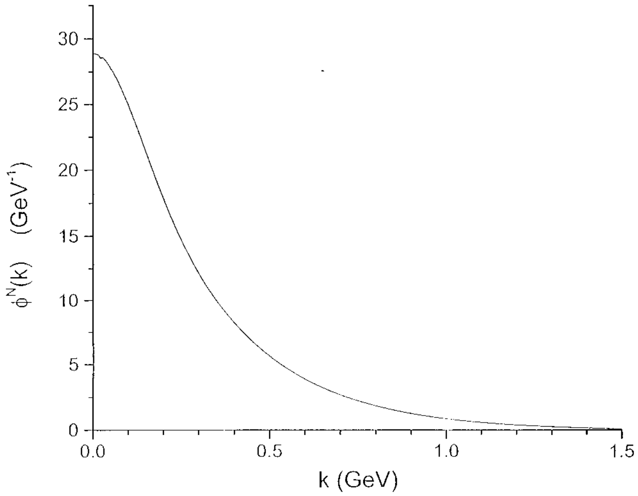

FIG. 8. The function $\phi_i^N(k)=\sqrt{N_i}\phi_i(k)=\hat{\Gamma}_i^{+-}(k)/[M_i-2E(k)]$ that is used in the calculation of $m^2_{a_0(980)}f_{a_0(980)}=0.0649\,\mathrm{GeV}^3$ is shown (see Table I).

$$M_i^2 f_i=-\frac{2n_C}{\sqrt{2}}(m_s^0-m_u^0)i\int\frac{d^4k}{(2\pi)^4}\Gamma_i^B(k)\left(\frac{m}{E(\vec{k})}\right)^2 \times\mathrm{Tr}\left[\frac{\Lambda^{(+)}(\vec{k})}{\frac{P^0}{2}+k^0-E(\vec{k})+i\eta}-\frac{\Lambda^{(-)}(-\vec{k})}{-\frac{P^0}{2}+k^0+E(\vec{k})-i\eta}\right], \tag{5.2}$$

where there is a factor of 2 from the flavor trace and the $\sqrt{2}$ is present in the denominator because of the form of Maltman's definition of the decay constant. Completing the k^0 integral in the lower-half place, we find

$$M_i^2 f_i=\frac{2n_C}{\sqrt{2}\pi^2}(m_s^0-m_u^0)\int_0^\infty k^2dk\left(\frac{k^2}{E^2(\vec{k})}\right)\left[\frac{\hat{\Gamma}_{B,i}^{+-}(k)}{M_i-2E(\vec{k})}\right] \tag{5.3}$$

$$=\frac{2}{\sqrt{2}}\frac{n_C}{\pi^2}(m_s^0-m_u^0)\int_0^\infty k^2dk\left(\frac{k^2}{E^2(\vec{k})}\right)\phi_i^N(k). \tag{5.4}$$

Here $\hat{\Gamma}_{B,i}^{+-}(k)$ is identified as the vertex that includes both the effects of confinement and the short-range NJL interaction. The nodeless function relevant to the calculation of the $a_0(980)$ decay constant is shown in Fig. 8, with

$$\phi_i^N(k)=\frac{\hat{\Gamma}_{B,i}^{+-}(k)}{M_i-2E(\vec{k})}, \tag{5.5}$$

in general.

VI. DISCUSSION AND CONCLUSIONS

As noted earlier, there is no consensus as to the organization of scalar mesons into nonets. We find Maltman's efforts in this direction particularly valuable, since his analysis is based in part upon the application of QCD sum rules. His results for the decay constants of the $a_0(980)$, $a_0(1450)$, and $K_0^*(1430)$ are given in Table I along with our values for the $1\,^3P_0$, $2\,^3P_0$, and $3\,^3P_0$ states of the a_0 and K_0^* resonances that were calculated in his work. It is gratifying to see that our results, calculated by an entirely different method, are in reasonable agreement with Maltman's values, although our value for the $a_0(980)$ is too large when compared his.

Maltman concludes that the $a_0(980)$ is not a $K\bar{K}$ "molecule," since such a structure would lead to a very small decay constant. [However, our result for the $a_0(980)$ decay constant does suggest that this resonance may have some $K\bar{K}$ component.] We have also stated that the $\sigma(500$–$600)$ and $\kappa(700$–$900)$ are "dynamically-generated" states [8]. These states appear as poles of the $\pi\pi$ and πK T matrices, respectively. However, there is no corresponding $q\bar{q}$ state, since the lowest lying poles of the $q\bar{q}$ T matrix correspond to the $a_0(980)$ and $f_0(980)$ [1,2]. (In the case of absolute confinement the $q\bar{q}$ T matrix is represented by only bound states, as we have discussed in detail in Ref. [17].) The identification of the $\sigma(500$–$600)$ and $\kappa(700$–$900)$ as "dynamically generated" states [8] is also consistent with the phenomenological model of light meson spectra introduced in Ref. [15].

If we accept Maltman's conclusion that the $a_0(980)$ is not a pure $K\bar{K}$ state, and also identify the $\sigma(500$–$600)$ as a "dynamically generated" state [8], we may conclude that it is consistent to describe the $a_0(980)$ and $K_0^*(1430)$ as having large $1\,^3P_0$ $q\bar{q}$ components. That is in agreement with the predictions of our model (see Fig. 1). Of course, it is not possible to prove that any state has a particular configuration. Such demonstrations are made within the context of a model in which one compares theoretical predictions with experiment or with other theories. We believe our model has passed the test we considered in this work.

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APPENDIX

In this appendix we generalize our equation for the case in which the quark and antiquark have different masses. Some of the relevant equations were given in Ref. [1]. We have

$$\Gamma_{S,ab}^{+-}(P^0,k)=1+\frac{1}{(2\pi)^2}\int k'^2dk'\int_{-1}^{1}dx \times\frac{A_1(k,k',x)}{E_a(\vec{k}')E_b(\vec{k}')}\frac{1}{C(\vec{k})} \times\frac{\Gamma_{S,ab}^{+-}(P^0,k')V^C(\vec{k}-\vec{k}')}{P^0-E_a(\vec{k}')-E_b(\vec{k}')}, \tag{A1}$$

with $x = \cos\theta$ and

$$C(\vec{k}) = [E_a(\vec{k})E_b(\vec{k}) + \vec{k}^2 - m_a m_b]. \tag{A2}$$

The rather lengthy expression for $A_1(k,k',x)$ is given as Eq. (6.6) in Ref. [1]. The homogeneous version of Eq. (A1) may be symmetrized by multiplying from the left by $k\sqrt{C(\vec{k})}/\sqrt{2E_a(\vec{k})E_b(\vec{k})}$. We define

$$\bar{\phi}_{ab}(k) = \frac{k\sqrt{C(\vec{k})}}{\sqrt{2E_a(\vec{k})E_b(\vec{k})}} \frac{\Gamma^{+-}_{S,ab}(k)}{P^0 - E_a(\vec{k}) - E_b(\vec{k})} \tag{A3}$$

such that

$$[P^0 - E_a(\vec{k}) - E_b(\vec{k})]\bar{\phi}_{ab}(k) = \frac{1}{(2\pi)^2}\int kk'dk' \int_{-1}^{1} dx \times \frac{A_1(k,k',x)V^C(\vec{k}-\vec{k}')}{\sqrt{E_{al}(\vec{k})E_b(\vec{k})E_a(\vec{k}')E_b(\vec{k}')}} \times \frac{1}{\sqrt{C(\vec{k})C(\vec{k}')}}\bar{\phi}_{ab}(k'). \tag{A4}$$

Here P^0 is the eigenvalue equal to the mass of the bound state. If we complete the integral over x, we have

$$[P^0 - E_a(\vec{k}) - E_b(\vec{k})]\bar{\phi}_{ab}(k) = \frac{1}{(2\pi)^2}\int kk'dk' [V_0^C(k,k')b_1(k,k') + kk'V_1^C(k,k')b_2] \frac{1}{\sqrt{C(\vec{k})C(\vec{k}')}} \times \frac{1}{\sqrt{E_a(\vec{k})E_b(\vec{k})E_a(\vec{k}')E_b(\vec{k}')}}\bar{\phi}_{ab}(k'), \tag{A5}$$

with

$$b_1(k,k') = [E_a(\vec{k}')E_b(\vec{k}') + \vec{k}'^2][E_a(\vec{k})E_b(\vec{k}) + \vec{k}^2] - m_a m_b\Big[E_a(\vec{k})E_b(\vec{k}) + E_a(\vec{k}')E_b(\vec{k}') + \vec{k}^2 + \vec{k}'^2 - m_a m_b - \frac{1}{2}E_a(\vec{k})E_b(\vec{k}') - \frac{1}{2}E_a(\vec{k}')E_b(\vec{k})\Big] - \frac{1}{2}m_a^2 E_b(\vec{k}')E_b(\vec{k}) - \frac{1}{2}m_b^2 E_a(\vec{k})E_a(\vec{k}') \tag{A6}$$

and

$$b_2\Big[m_a m_b + \frac{1}{2}m_a^2 + \frac{1}{2}m_b^2\Big]. \tag{A7}$$

In the limit $m_a = m_b = m$, we have

$$C(\vec{k}) \to 2\vec{k}^2, \tag{A8}$$

$$b_1(k,k') \to 4\vec{k}^2\vec{k}'^2, \tag{A9}$$

and

$$b_2 \to 2m^2. \tag{A10}$$

Thus, Eq. (A5) becomes

$$[P^0 - 2E(\vec{k})]\bar{\phi}(k) = \frac{1}{(2\pi)^2}\int \frac{kk'dk'}{E(\vec{k})E(\vec{k}')}[2kk'V_0^C(k,k') + m^2 V_1^C(k,k')]\bar{\phi}(k') \tag{A11}$$

[see Eqs. (3.4) and (3.5)].

Further, $H_1^C(k,k')$ for the case $m_a \neq m_b$ is

$$H_1^C(k,k') = -\frac{1}{(2\pi)^2}\frac{kk'}{\sqrt{E_a(\vec{k})E_b(\vec{k})E_a(\vec{k}')E_b(\vec{k}')}} \times \frac{1}{\sqrt{C(\vec{k})C(\vec{k}')}}[V_0^C(k,k')b_1(k,k') + kk'V_1^C(k,k')b_2] \tag{A12}$$

and $H_1^{\rm NJL}(k,k')$ is

$$H_1^{\rm NJL}(k,k') = \frac{8n_C}{(2\pi)^2} \times \frac{[G_S kk' e^{-k^2/2\alpha^2}e^{-k'^2/2\alpha^2}]\sqrt{C(\vec{k})C(\vec{k}')}}{2\sqrt{E_a(\vec{k})E_b(\vec{k})E_a(\vec{k}')E_b(\vec{k}')}}. \tag{A13}$$

The normalization factor is given by

$$\frac{1}{N_{i,ab}} = \frac{n_C}{2\pi^2}\left(\frac{1}{2M_1}\right)\int k^2 dk \times \left[\frac{2(E_a(\vec{k})E_b(\vec{k}) + \vec{k}^2) - 2m_a m_b}{E_a(\vec{k})E_b(\vec{k})}\right]|\phi_i(k)|^2, \tag{A14}$$

where

$$\phi_i(k) = \frac{\sqrt{2E_a(\vec{k})E_b(\vec{k})}}{k\sqrt{C(\vec{k})}}\bar{\phi}_i(k). \tag{A15}$$

We also define

$$\phi_i^N(k)=\sqrt{N_{i,ab}}\phi_i(k). \tag{A16}$$

The expression for the decay constant is

$$M_i^2 f_i=\frac{2n_C}{\sqrt{2}\pi^2}(m_s^0-m_u^0)\int k^2dk \times\left[\frac{E_a(\vec{k})E_b(\vec{k})+\vec{k}^2-m_am_b}{2E_a(\vec{k})E_b(\vec{k})}\right]\phi_{ab}^N(k), \tag{A17}$$

where

$$\phi_{ab}^N(k)=\frac{\hat{\Gamma}_{i,ab}^{+-}(k)}{M_i-E_a(\vec{k})-E_b(\vec{k})}. \tag{A18}$$

Here $\hat{\Gamma}_{i,ab}^{+-}(k)$ is the vertex that includes both the effects of confinement and the short-range NJL interaction.

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