

Charmless two body hadronic decays of the Λ_b baryon

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Using a theoretical framework based on the next-to-leading order QCD improved effective Hamiltonian, we have estimated the branching ratios and asymmetry parameters for the two body charmless nonleptonic decay modes of the Λ_b baryon, i.e., $\Lambda_b \rightarrow p(\pi/\rho)$, $p(K/K^*)$, and $\Lambda(\pi/\rho)$, within the framework of generalized factorization. The nonfactorizable contributions are parametrized in terms of the effective number of colors, N_c^{eff} . So, in addition to the naive factorization approach ($N_c^{eff} = 3$), here we have taken two more values for N_c^{eff} ; i.e., $N_c^{eff} = 2$ and ∞ . The baryonic form factors at maximum momentum transfer (q_m^2) are evaluated using the nonrelativistic quark model and the extrapolation of the form factors from q_m^2 to the required q^2 value is done by assuming pole dominance. The obtained branching ratios for $\Lambda_b \rightarrow p\pi$, pK processes lie within the present experimental upper limit.

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I. INTRODUCTION

The principal interest in the study of weak decays of bottom hadrons in the context of the standard model (SM) lies in the fact that they provide valuable information on the weak rotation matrix—the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In fact b decays determine five of its matrix elements: V_{cb} , V_{ub} , V_{td} , V_{ts} , and V_{td} . The dominant decay modes of bottom hadrons are those involving $b \rightarrow c$ transitions. There are also rare decay modes which proceed through the CKM suppressed $b \rightarrow u$ spectator tree diagram and/or $b \rightarrow s(b \rightarrow d)$ penguin amplitudes with, in general, both QCD and electroweak penguin diagrams participating. The study of exclusive charmless nonleptonic bottom decays is of great interest for several reasons. First of all, they proceed in general through the W -loop diagrams, the so-called penguin diagrams without CKM suppression and through the CKM suppressed spectator diagrams. Thus the salient feature in charmless bottom decays is that the loop graphs are as important as the tree graphs. In some cases the loop graphs may even be dominant over the tree graphs. Furthermore, as most of these decays proceed through more than one amplitude with different CKM phases, there will in general be interference, and so there is an opportunity to observe direct CP violation. Hence the analysis and measurement of charmless hadronic b decays will enable us to understand the QCD and electroweak penguin effects as well as the origin of CP violation in the standard model and provide a powerful tool of seeing physics beyond the SM.

Recently, there has been remarkable progress in the study of exclusive charmless bottom meson decays both experimentally and theoretically. Experimentally, CLEO [1] has discovered many new two body decay modes

$$B \rightarrow \eta' K^\pm, \quad \eta' K^0, \quad \pi^\pm K^\mp, \quad \pi^0 K^\pm, \quad \rho^0 \pi^\pm, \omega K^\pm \quad (1)$$

and found possible evidence for $B \rightarrow \phi K^*$. Moreover, CLEO has provided new improved upper limits for many other decay modes. With B factories Babar and Belle starting to collect data, many exciting years in the arena of B physics and

CP violation are expected to come. Theoretically many significant improvements and developments have taken place over the past years. For example, a next-to-leading order effective Hamiltonian for current-current operators and QCD as well as electroweak penguin operators have become available. The renormalization scheme and scale problems with the factorization approach for matrix elements can be circumvented by employing scale- and scheme-independent Wilson coefficients. Incorporating all these improved results, the exclusive two body charmless hadronic decays of B mesons and their CP asymmetries have been extensively studied in Refs. [2–6].

It is also interesting to study the charmless nonleptonic decays of the bottom baryon system. Recently some data on the bottom baryon Λ_b have appeared. For instance, OPAL has measured its lifetime and the production branching ratio for inclusive semileptonic decay [7]. Furthermore, measurements of the nonleptonic decay $\Lambda_b \rightarrow \Lambda J/\psi$ have also been reported [8]. Certainly we expect more data in the bottom baryon sector in the near future.

In this paper we would like to study the charmless hadronic decays of the Λ_b baryon, i.e., $\Lambda_b \rightarrow p(\pi/\rho)$, $p(K/K^*)$, and $\Lambda_b \rightarrow \Lambda(\pi/\rho)$. Experimentally, only upper limits on the branching ratios for rare Λ_b decay modes $\Lambda_b \rightarrow p\pi$ and $\Lambda_b \rightarrow pK$ have been observed [9]. The standard theoretical framework to study the nonleptonic Λ_b decays is based on the effective Hamiltonian approach, which allows us to separate the short- and long-distance contributions in these decays using the Wilson operator product expansion [10]. QCD perturbation theory is then used in deriving the renormalization-group improved short-distance contributions [11]. This program has now been carried out up to and including next-to-leading order terms [12,13]. But the long-distance part in the two body decays $B_i \rightarrow B_f M$ [where $B_i(B_f)$ are the initial (final) baryons and M is the final pseudoscalar or vector meson] involves the transition matrix element $\langle B_f M | O_i | B_i \rangle$, where O_i is an operator in the effective Hamiltonian. Calculation of these matrix elements from first principles is not yet possible and hence some approximation has to be adopted to deal with these matrix elements. The

one we use here is based on the idea of factorization in which the final state interactions have to be absent and hadronic matrix elements in the $B_i \rightarrow B_f M$ transition factorize into a product of two comparatively tractable matrix elements, one involving the form factors and the other, the decay constant. Motivated by the phenomenological success of factorization in charmless nonleptonic B decays [2–6], we would like to pursue this framework for charmless Λ_b decays. It is customarily argued that final state interactions (FSIs) are expected to play a minor role in charmless hadronic b decays due to the large energy release in these decay processes. Furthermore, in the nonleptonic decays of heavy mesons, the W exchange contribution is known to be negligible in comparison to the factorizable one due to helicity and color suppression. But the W exchange in baryon decays is not subject to helicity and color suppression and can be as important as a factorizable one. The experimental measurement of the decay modes $\Lambda_c \rightarrow \Sigma^0 \pi^+$, $\Sigma^+ \pi^-$, and $\Lambda_c \rightarrow \Xi^0 K^+$, which do not receive any factorizable contributions, indicates that W exchange indeed plays an essential role in charmed baryon decays. Nevertheless, the W -exchange contribution is expected to be less important in the nonleptonic decays of bottom baryons as shown by the following arguments: the W -exchange contribution to the total decay width of heavy baryon relative to the spectator diagram is of the order of $P = 32\pi^2 |\psi_{Qq}(0)|^2 / m_Q^3$ [14]. Therefore, although W exchange plays a dramatic role in the charmed baryon case, it becomes negligible in bottom baryon decays. So we have not taken into account the W -exchange contribution in our analysis. The renormalization scheme and scale problems with the factorization approach for matrix elements can be circumvented by employing scale- and scheme-independent effective Wilson coefficients. The form factors at maximum recoil have been calculated using the nonrelativistic quark model [15] and the nearest pole dominance has been used to extrapolate them to the required q^2 point.

The paper is organized as follows. The kinematics of hyperon decays is presented in Sec. II. In Sec. III we discuss the effective Hamiltonian together with the quark level matrix elements and the numerical values of the Wilson coefficients. Using the factorization ansatz we evaluate the matrix elements in the nonrelativistic quark model in Sec. IV. Section V contains our results and discussion.

II. KINEMATICS OF HYPERON DECAYS

In this section we present the kinematics of nonleptonic hyperon decays. The most general Lorentz-invariant amplitude for the decay $\Lambda_b \rightarrow B_f P$ (where P is a pseudoscalar meson) can be written as

$$\mathcal{M}(\Lambda_b \rightarrow B_f P) = i \bar{u}_f(p_f) (A + B \gamma_5) u_{\Lambda_b}(p_i), \quad (2)$$

where u_f and u_{Λ_b} are the Dirac spinors for B_f and Λ_b baryons; A and B are parity-violating S -wave and parity-conserving P -wave amplitudes, respectively. The corresponding decay rate (Γ) and up-down asymmetry parameter are given as [16,17]

$$\Gamma = \frac{p_c}{8\pi} \left\{ \frac{(m_i + m_f)^2 - m_P^2}{m_i^2} |A|^2 + \frac{(m_i - m_f)^2 - m_P^2}{m_i^2} |B|^2 \right\},$$

$$\alpha = - \frac{2\kappa \text{Re}(A^* B)}{|A|^2 + \kappa^2 |B|^2}, \quad (3)$$

where m_i , m_f , and m_P are the masses of the initial baryon, final baryon, and pseudoscalar meson respectively, p_c is the c.m. momentum, and $\kappa = p_c / (E_f + m_f) = \sqrt{(E_f - m_f)(E_f + m_f)}$.

For the $\Lambda_b \rightarrow B_f V$ (where V is the vector meson) decay mode, the general form for the amplitude is given as

$$\mathcal{M}(\Lambda_b \rightarrow B_f V) = \bar{u}_f(p_f) \epsilon^{*\mu} [A_1 \gamma_\mu \gamma_5 + A_2 (p_f)_\mu \gamma_5 + B_1 \gamma_\mu + B_2 (p_f)_\mu] u_{\Lambda_b}(p_i), \quad (4)$$

where ϵ^μ is the polarization vector of the emitted vector meson. The corresponding decay rate and asymmetry parameter are given as [17]

$$\Gamma = \frac{p_c}{8\pi} \frac{E_f + m_f}{m_i} \left\{ 2(|S|^2 + |P_2|^2) + \frac{E_V^2}{m_V^2} (|S + D|^2 + |P_1|^2) \right\},$$

$$\alpha = \frac{4m_V^2 \text{Re}[(S^* P_2) + 2E_V^2 \text{Re}[(S + D)^* P_1]]}{2m_V^2 (|S|^2 + |P_2|^2) + E_V^2 (|S + D|^2 + |P_1|^2)}, \quad (5)$$

with

$$S = -A_1,$$

$$D = - \frac{p_c^2}{E_V (E_f + m_f)} (A_1 - m_i A_2),$$

$$P_1 = - \frac{p_c}{E_V} \left(\frac{m_i + m_f}{E_f + m_f} B_1 + m_i B_2 \right),$$

$$P_2 = \frac{p_c}{E_f + m_f} B_1. \quad (6)$$

III. EFFECTIVE HAMILTONIAN

The effective Hamiltonian \mathcal{H}_{eff} for the hadronic charmless Λ_b decays is given as

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{uq}^* [c_1(\mu) O_1^\mu(\mu) + c_2(\mu) O_2^\mu(\mu)] - V_{tb} V_{tq}^* \sum_{i=3}^{10} c_i(\mu) O_i(\mu) \right\} + \text{H.c.}, \quad (7)$$

where $q = d, s$ and $c_i(\mu)$ are the Wilson coefficients evaluated at the renormalization scale μ . The operators O_{1-10} are given as

$$\begin{aligned}
O_1^u &= (\bar{u}b)_{V-A}(\bar{q}u)_{V-A}, & O_2^u &= (\bar{u}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \\
O_{3(5)} &= (\bar{q}b)_{V-A} \sum_{q'} (\bar{q}'q')_{V-A(V+A)}, \\
O_{4(6)} &= (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A(V+A)}, \\
O_{7(9)} &= \frac{3}{2} (\bar{q}b)_{V-A} \sum_{q'} e_{q'} (\bar{q}'q')_{V+A(V-A)}, \\
O_{8(10)} &= \frac{3}{2} (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A(V-A)}, \tag{8}
\end{aligned}$$

where $O_{1,2}$ are the tree-level current-current operators, O_{3-6} are the QCD, and O_{7-10} are the electroweak penguin operators. $(\bar{q}_1 q_2)_{(V\pm A)}$ denote the usual $(V\pm A)$ currents. The sum over q' runs over the quark fields that are active at the scale $\mu = O(m_b)$, i.e., $(q' \in u, d, s, c, b)$. The Wilson coefficients depend (in general) in the renormalization scheme and the scale μ at which they are evaluated. In the next to leading order their values obtained in the naive dimensional regularization (NDR) scheme at $\mu = m_b(m_b)$ as [18]

$$\begin{aligned}
c_1 &= 1.082, & c_2 &= -0.185, & c_3 &= 0.014, & c_4 &= -0.035, \\
c_5 &= 0.009, & c_6 &= -0.041, & c_7 &= -0.002\alpha, & c_8 &= 0.054\alpha, \\
c_9 &= -1.292\alpha, & c_{10} &= 0.263\alpha. \tag{9}
\end{aligned}$$

However, the physical matrix elements $\langle B_f M | \mathcal{H}_{eff} | \Lambda_b \rangle$ are obviously independent of both the scheme and scale. Hence the dependence in the Wilson coefficients must be canceled by the corresponding scheme and scale dependence of the matrix elements of the operators. However, in the factorization approximation, the hadronic matrix elements are written in terms of form factors and decay constants which are scheme and scale independent. So to achieve cancellation the various one loop corrections are absorbed into the effective Wilson coefficients c_i^{eff} , which are scheme and scale independent: i.e.,

$$\langle q\bar{u}u | \mathcal{H}_{eff} | b \rangle = \sum_{i,j} c_i^{eff}(\mu) \langle q\bar{u}u | O_j | b \rangle^{tree}. \tag{10}$$

The effective Wilson coefficients $c_i^{eff}(\mu)$ may be expressed as [2-6]

$$\begin{aligned}
c_1^{eff}|_{\mu=m_b} &= c_1(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma^{(0)T} \ln \frac{m_b}{\mu} + \hat{r}^T \right)_{1i} c_i(\mu), \tag{11} \\
c_2^{eff}|_{\mu=m_b} &= c_2(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma^{(0)T} \ln \frac{m_b}{\mu} + \hat{r}^T \right)_{2i} c_i(\mu),
\end{aligned}$$

$$\begin{aligned}
c_3^{eff}|_{\mu=m_b} &= c_3(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma^{(0)T} \ln \frac{m_b}{\mu} + \hat{r}^T \right)_{3i} c_i(\mu) \\
&\quad - \frac{\alpha_s}{24\pi} (C_t + C_p + C_g), \\
c_4^{eff}|_{\mu=m_b} &= c_4(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma^{(0)T} \ln \frac{m_b}{\mu} + \hat{r}^T \right)_{4i} c_i(\mu) \\
&\quad + \frac{\alpha_s}{8\pi} (C_t + C_p + C_g), \\
c_5^{eff}|_{\mu=m_b} &= c_5(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma^{(0)T} \ln \frac{m_b}{\mu} + \hat{r}^T \right)_{5i} c_i(\mu) \\
&\quad - \frac{\alpha_s}{24\pi} (C_t + C_p + C_g), \\
c_6^{eff}|_{\mu=m_b} &= c_6(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma^{(0)T} \ln \frac{m_b}{\mu} + \hat{r}^T \right)_{6i} c_i(\mu) \\
&\quad + \frac{\alpha_s}{8\pi} (C_t + C_p + C_g), \\
c_7^{eff}|_{\mu=m_b} &= c_7(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma^{(0)T} \ln \frac{m_b}{\mu} + \hat{r}^T \right)_{7i} c_i(\mu) + \frac{\alpha}{8\pi} C_e, \\
c_8^{eff}|_{\mu=m_b} &= c_8(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma^{(0)T} \ln \frac{m_b}{\mu} + \hat{r}^T \right)_{8i} c_i(\mu), \\
c_9^{eff}|_{\mu=m_b} &= c_9(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma^{(0)T} \ln \frac{m_b}{\mu} + \hat{r}^T \right)_{9i} c_i(\mu) + \frac{\alpha}{8\pi} C_e, \\
c_{10}^{eff}|_{\mu=m_b} &= c_{10}(\mu) + \frac{\alpha_s}{4\pi} \left(\gamma^{(0)T} \ln \frac{m_b}{\mu} + \hat{r}^T \right)_{10i} c_i(\mu),
\end{aligned}$$

where \hat{r}^T and $\gamma^{(0)T}$ are the transpose of the matrices \hat{r} and $\gamma^{(0)}$ arise from the vertex corrections to the operators O_1-O_{10} derived in [13], which are explicitly given in Ref. [6]

The quantities C_t , C_p , and C_g arise from the penguin-type diagrams of the operators $O_{1,2}$, the penguin-type diagrams of the operators O_3-O_6 , and the tree level diagrams of the dipole operator O_g , respectively, which are given in the NDR scheme [after modified minimal subtraction scheme ($\overline{\text{MS}}$) renormalization] by

$$\begin{aligned}
C_t &= - \left(\frac{\lambda_u}{\lambda_t} \tilde{G}(m_u) + \frac{\lambda_c}{\lambda_t} \tilde{G}(m_c) \right) c_1, \tag{12} \\
C_p &= [\tilde{G}(m_q) + \tilde{G}(m_b)] c_3 \\
&\quad + \sum_{i=u,d,s,c,b} \tilde{G}(m_i) (c_4 + c_6),
\end{aligned}$$

$$C_g = -\frac{2m_b}{\sqrt{\langle k^2 \rangle}} c_g^{eff}, \quad c_g^{eff} = -1.043,$$

$$C_e = -\frac{8}{9} \left(\frac{\lambda_u}{\lambda_t} \tilde{G}(m_u) + \frac{\lambda_c}{\lambda_t} \tilde{G}(m_c) \right) (c_1 + 3c_2),$$

$$\tilde{G}(m_q) = \frac{2}{3} - G(m_q, k, \mu),$$

$$G(m, k, \mu) = -4 \int_0^1 dx x(1-x) \ln \left(\frac{m^2 - k^2 x(1-x)}{\mu^2} \right). \quad (13)$$

It should be noted that the quantities C_t , C_p , and C_g depend on the CKM matrix elements, the quark masses, the scale μ and k^2 , the momentum transferred by the virtual particles appearing in the penguin diagrams. In the factorization approximation there is no model-independent way to keep track of the k^2 dependence; the actual value of k^2 is model dependent. From the simple kinematics of charmless nonleptonic B decays [19] one expects k^2 to be typically in the range

$$\frac{m_b^2}{4} \leq k^2 \leq \frac{m_b^2}{2}. \quad (14)$$

Since the branching ratios depend crucially on the parameter k^2 , here we would like to take a specific value for it from the above-mentioned range. Here we will use for the two body penguin induced decays $\Lambda_b \rightarrow B_f M$ as done for the charmless $B \rightarrow PP$ decays [20]. Assuming that in the rest frame of the Λ_b baryon the spectator diquarks both in the initial and final baryons have negligible momentum and the momentum is shared equally between the two quarks of the emitted meson, the average momentum transfer for $b \rightarrow qu\bar{u}$ transitions [$q=d$ for $\Lambda_b \rightarrow p(\pi/\rho)$ and $q=s$ for $\Lambda_b \rightarrow p(K/K^*)$ and $\Lambda(\pi/\rho)$ transitions] is given as

$$\langle k^2 \rangle = m_b^2 + m_q^2 - 2m_b E_q. \quad (15)$$

The energy E_q of the q quark in the final meson is determined from

$$E_q + \sqrt{E_q^2 - m_q^2 + m_u^2} + \sqrt{4(E_q^2 - m_q^2) + m_u^2} = m_b, \quad (16)$$

where m_b , m_q , and m_u denote the masses of the decaying b quark, daughter q quark, and the u quark created as a $u\bar{u}$ pair from the virtual gluon, photon, or Z particle in the penguin loop.

For numerical calculation we have taken the CKM matrix elements expressed in terms of the Wolfenstein parameters with the values $A=0.815$, $\lambda = \sin \theta_c = 0.2205$, $\rho = 0.175$, and $\eta = 0.37$ [6]. Using the mass renormalization equations with three loop β function, the values of the current quark masses are evaluated at various energy scales in Ref. [21]. Since the energy released in the decay mode $\Lambda_b \rightarrow p \pi^-$ is of the order of m_b , we take the current quark mass values at the scale $\mu \sim m_b$ from [21] as $m_u(m_b) = 3.2$ MeV, $m_d(m_b)$

TABLE I. Numerical values of the effective Wilson coefficients c_i^{eff} for $b \rightarrow s$ and $b \rightarrow d$ transitions evaluated at $k^2/m_b^2 = 0.499$ for $b \rightarrow s$ and at $k^2/m_b^2 = 0.5$ for the $b \rightarrow d$ processes.

	$b \rightarrow s$	$b \rightarrow d$
c_1^{eff}	1.168	1.168
c_2^{eff}	-0.365	-0.365
c_3^{eff}	0.0225 + i0.0043	0.0224 + i0.0038
c_4^{eff}	-(0.0467 + i0.0129)	-(0.0454 + i0.0115)
c_5^{eff}	0.0133 + i0.0043	0.0131 + i0.0038
c_6^{eff}	-(0.0481 + i0.0129)	-(0.0475 + i0.0115)
c_7^{eff}/α	-(0.0299 + i0.0356)	-(0.0294 + i0.0329)
c_8^{eff}/α	0.055	0.055
c_9^{eff}/α	-(1.4268 + i0.0356)	-(1.426 + i0.0329)
c_{10}^{eff}/α	0.48	0.48

= 6.4 MeV, $m_s(m_b) = 90$ MeV, $m_c(m_b) = 0.95$ GeV, and $m_b(m_b) = 4.34$ GeV. Thus we obtain $k^2/m_b^2 = 0.5$ for $b \rightarrow du\bar{u}$ transitions and $k^2/m_b^2 = 0.499$ for $b \rightarrow su\bar{u}$ transitions. Using these values of k^2 the estimated values of the effective renormalization scheme and scale independent Wilson coefficients for $b \rightarrow d$ and $b \rightarrow s$ transitions are given in Table I.

IV. EVALUATION OF THE MATRIX ELEMENTS

After obtaining the effective Wilson coefficients now we want to calculate the matrix element $\langle B_f M | O_i | \Lambda_b \rangle$ where O_i are the four quark current operators listed in Eq. (8), using the factorization approximation. In this approximation, the hadronic matrix elements of the four quark operators $(\bar{u}b)_{(V-A)}(\bar{q}u)_{(V-A)}$ is split into the product of two matrix elements, $\langle M | (\bar{q}u)_{(V-A)} | 0 \rangle$ and $\langle B_f | (\bar{u}b)_{(V-A)} | \Lambda_b \rangle$, where Fierz transformation has been used so that the flavor quantum numbers of the currents match those of the hadrons. Since Fierz rearranging yields operators which are in the color singlet-singlet and octet-octet forms, this procedure results, in general, in matrix elements which have the right flavor quantum numbers but involve both singlet-singlet and octet-octet current operators. However, there is no experimental information available for the octet-octet part. So in the factorization approximation, one discards the color octet-octet piece and compensates this by treating N_c , the number of colors, as a free parameter, and its value is extracted from the data of two body nonleptonic decays.

The matrix elements of the $(V-A)(V+A)$ operators, i.e., (O_6 and O_8), can be calculated as follows. After Fierz ordering and factorization they contribute as [22]

$$\begin{aligned} \langle B_f M | O_6 | \Lambda_b \rangle &= -2 \sum_{q'} \langle M | \bar{q}(1 + \gamma_5)q' | 0 \rangle \\ &\quad \times \langle B_f | \bar{q}'(1 - \gamma_5)b | \Lambda_b \rangle. \end{aligned} \quad (17)$$

Using the Dirac equation the matrix element can be rewritten as

TABLE II. Values of the form factors at zero momentum transfer evaluated using the nonrelativistic quark model

Decay process	$f_1(0)$	$m_i f_2(0)$	$m_i f_3(0)$	$g_1(0)$	$m_i g_2(0)$	$m_i g_3(0)$
$\Lambda_b \rightarrow p$	0.043	-0.022	-0.009	0.092	-0.02	-0.047
$\Lambda_b \rightarrow \Lambda$	0.061	-0.025	-0.008	0.107	-0.014	-0.043

$$\langle B_f M | O_6 | \Lambda_b \rangle = -[R_1 \langle B_f | V_\mu | \Lambda_b \rangle - R_2 \langle B_f | A_\mu | \Lambda_b \rangle] \langle M | (V-A)_\mu | 0 \rangle, \quad (18)$$

with

$$R_1 = \frac{2m_M^2}{(m_b - m_u)(m_q + m_u)}, \quad R_2 = \frac{2m_M^2}{(m_b + m_u)(m_q + m_u)}, \quad (19)$$

where the quark masses are the current quark masses. The same relation works for O_8 .

Thus under the factorization approximation the baryon decay amplitude is governed by a decay constant and baryonic transition form factors. The general expression for the baryon transition is given as

$$\begin{aligned} \langle B_f(p_f) | V_\mu - A_\mu | \Lambda_b(p_i) \rangle \\ = \bar{u}_{B_f}(p_f) \{ f_1(q^2) \gamma_\mu + i f_2(q^2) \sigma_{\mu\nu} q^\nu + f_3(q^2) q_\mu \\ - [g_1(q^2) \gamma_\mu + i g_2(q^2) \sigma_{\mu\nu} q^\nu + g_3(q^2) q_\mu] \gamma_5 \} u_{\Lambda_b}(p_i), \end{aligned} \quad (20)$$

where $q = p_i - p_f$. In order to evaluate the form factors at maximum momentum transfer, we have employed the non-relativistic quark model [15], where they are given as

$$\begin{aligned} f_1(q_m^2)/N_{fi} &= 1 - \frac{\Delta m}{2m_i} + \frac{\Delta m}{4m_i m_q} \left(1 - \frac{\Lambda_b}{2m_f} \right) (m_i + m_f - \eta \Delta m) \\ &\quad - \frac{\Delta m}{8m_i m_f m_Q} \bar{\Lambda} (m_i + m_f - \eta \Delta m), \\ f_3(q_m^2)/N_{fi} &= \frac{1}{2m_i} - \frac{1}{4m_i m_f} (m_i + m_f - \eta \Delta m) \\ &\quad - \frac{\bar{\Lambda}}{8m_i m_f m_Q} [(m_i + m_f) \eta + \Delta m], \\ g_1(q_m^2)/N_{fi} &= \eta + \frac{\Delta m \bar{\Lambda}}{4} \left(\frac{1}{m_i m_q} - \frac{1}{m_f m_Q} \right) \eta, \\ g_3(q_m^2)/N_{fi} &= -\frac{\bar{\Lambda}}{4} \left(\frac{1}{m_i m_q} - \frac{1}{m_f m_Q} \right) \eta, \end{aligned} \quad (21)$$

with $\bar{\Lambda} = m_f - m_q$, $\Delta m = m_i - m_f$, $q_m^2 = \Delta m^2$, $\eta = 1$, and m_Q and m_q are the constituent quark masses of the interacting quarks of initial and final baryons with values m_u

$= 338$ MeV, $m_d = 322$ MeV, $m_s = 510$ MeV, and $m_b = 5$ GeV. N_{fi} is the flavor factor:

$$N_{fi} = \text{flavor spin} \langle p | b_u^\dagger b_b | \Lambda_b \rangle_{\text{flavor spin}}, \quad (22)$$

which is equal to $1/\sqrt{2}$ for $\Lambda_b \rightarrow p$ and $1/\sqrt{3}$ for $\Lambda_b \rightarrow \Lambda$ transitions [17]. Since the calculation of the q^2 dependence of the form factors is beyond the scope of the nonrelativistic quark model, we will follow the conventional practice to assume a pole dominance for the form factor q^2 behavior as

$$f(q^2) = \frac{f(0)}{(1 - q^2/m_V^2)^2}, \quad g(q^2) = \frac{g(0)}{(1 - q^2/m_A^2)^2}, \quad (23)$$

where m_V (m_A) is the pole mass of the vector (axial vector) meson with the same quantum number as the current under consideration. The pole masses are taken as $m_V = 5.32$ (5.42) GeV and $m_A = 5.71$ (5.86) GeV for $b \rightarrow d$ ($b \rightarrow s$) transitions [17]. Assuming a dipole q^2 behavior for form factors and taking the masses of the particles from Ref. [9], the obtained values of the form factors at zero momentum transfer are given in Table II.

The matrix element $\langle M | (V-A)_\mu | 0 \rangle$ is related to the decay constants of the charged pseudoscalar and vector mesons f_P and f_V as

$$\langle 0 | A_\mu | P(q) \rangle = i f_P q_\mu, \quad \langle 0 | A_\mu | V(\epsilon, q) \rangle = f_V m_V \epsilon_\mu. \quad (24)$$

The decay constants for the neutral mesons (i.e., π^0 and ρ^0) are taken to be $1/\sqrt{2}$ times that of the corresponding charged mesons. Thus we obtain the transition amplitudes for various $\Lambda_b \rightarrow B_f P$ decay modes as given below.

A. $\Lambda_b \rightarrow B_f P$ transitions

1. $\Lambda_b \rightarrow p \pi^-$

Since in this case the final state has isospin $I_f = 3/2, 1/2$ we have $\Delta I = 3/2$ and $1/2$. From the flavor-flow topologies for $b \rightarrow d u \bar{u}$ transitions, we find that the isospin decomposition of the effective Hamiltonian is as follows: the tree diagrams have $\Delta I = 3/2, 1/2$, the QCD penguins $\Delta I = 1/2$, and the electroweak penguins $\Delta I = 3/2, 1/2$ components. Hence both tree and penguin diagrams (QCD as well as the electroweak penguin diagrams) contribute to this channel and hence the amplitude is given as

$\mathcal{M}(\Lambda_b \rightarrow p \pi^-)$

$$\begin{aligned}
&= i \frac{G_F}{\sqrt{2}} f_\pi \bar{u}_p(p_f) \{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \\
&\quad \times [a_4 + a_{10} + (a_6 + a_8) R_1] \} [f_1(m_\pi^2)(m_i - m_f) \\
&\quad + f_3(m_\pi^2) m_\pi^2] + \{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* [a_4 + a_{10} \\
&\quad + (a_6 + a_8) R_2] \} [g_1(m_\pi^2)(m_i + m_f) \\
&\quad - g_3(m_\pi^2) m_\pi^2] \gamma_5 u_{\Lambda_b}(p_i). \quad (25)
\end{aligned}$$

2. $\Lambda_b \rightarrow p K^-$

It can be seen from the flavor-flow topologies for $b \rightarrow su\bar{u}$ transitions that the effective Hamiltonian has isospin components such as the tree diagram with $\Delta I=1,0$, QCD penguin diagrams with $\Delta I=0$, and electroweak penguin diagrams with $\Delta I=1,0$ components. Since the final state (pK) has isospin 1 and 0, we have $\Delta I=1,0$ for this process. Thus we find that tree and QCD as well as the electroweak penguin diagrams will contribute to this channel and we obtain the amplitude as

$$\begin{aligned}
\mathcal{M}(\Lambda_b \rightarrow p K^-) &= i \frac{G_F}{\sqrt{2}} f_K \bar{u}_p(p_f) \{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* \\
&\quad \times [a_4 + a_{10} + (a_6 + a_8) R_1] \} [f_1(m_K^2) \\
&\quad \times (m_i - m_f) + f_3(m_K^2) m_K^2] + \{ V_{ub} V_{us}^* a_1 \\
&\quad - V_{tb} V_{ts}^* [a_4 + a_{10} + (a_6 + a_8) R_2] \} \\
&\quad \times [g_1(m_K^2)(m_i + m_f) \\
&\quad - g_3(m_K^2) m_K^2] \gamma_5 u_{\Lambda_b}(p_i). \quad (26)
\end{aligned}$$

3. $\Lambda_b \rightarrow \Lambda \pi^0$

For $\Lambda_b \rightarrow \Lambda \pi^0$ we have only $\Delta I=1$ and from the flavor-flow diagrams for $b \rightarrow su\bar{u}$ processes, we find that only the tree and electroweak penguin diagrams will contribute to this channel:

$\mathcal{M}(\Lambda_b \rightarrow \Lambda \pi^0)$

$$\begin{aligned}
&= i \frac{G_F}{2} f_\pi \bar{u}_\Lambda(p_f) \left[V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \left(\frac{3}{2} (a_9 - a_7) \right) \right] \\
&\quad \times \{ [f_1(m_\pi^2)(m_i - m_f) + f_3(m_\pi^2) m_\pi^2] \\
&\quad + [g_1(m_\pi^2)(m_i + m_f) - g_3(m_\pi^2) m_\pi^2] \gamma_5 \} u_{\Lambda_b}(p_i). \quad (27)
\end{aligned}$$

B. $\Lambda_b \rightarrow B_f V$ transitions

Here we obtain the transition amplitudes for $\Lambda_b \rightarrow B_f V$ decay channels. As seen from the flavor-flow diagrams for $\Lambda_b \rightarrow B_f P$ processes, in this case also the $\Lambda_b \rightarrow p \rho$ and $p K^*$

receive contributions from tree as well as QCD and electroweak penguins diagrams whereas $\Lambda_b \rightarrow \Lambda \rho$ has only tree and electroweak penguin contributions. Thus we obtain the corresponding transition amplitudes as follows.

1. $\Lambda_b \rightarrow p \rho^-$

$\mathcal{M}(\Lambda_b \rightarrow p \rho^-)$

$$\begin{aligned}
&= \frac{G_F}{\sqrt{2}} f_\rho m_\rho \epsilon^{*\mu} \bar{u}_p(p_f) \\
&\quad \times \{ [V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* (a_4 + a_{10})] [f_1(m_\rho^2) - f_2(m_\rho^2) \\
&\quad \times (m_i + m_f)] \gamma_\mu + 2f_2(m_\rho^2)(p_f)_\mu - \{ [g_1(m_\rho^2) + g_2(m_\rho^2) \\
&\quad \times (m_i - m_f)] \gamma_\mu + 2g_2(m_\rho^2)(p_f)_\mu \} \gamma_5 \} u_{\Lambda_b}(p_i). \quad (28)
\end{aligned}$$

2. $\Lambda_b \rightarrow p K^{*-}$

$\mathcal{M}(\Lambda_b \rightarrow p K^{*-})$

$$\begin{aligned}
&= \frac{G_F}{\sqrt{2}} f_{K^*} m_{K^*} \epsilon^{*\mu} \bar{u}_p(p_f) \{ [V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* \\
&\quad \times (a_4 + a_{10})] [f_1(m_{K^*}^2) - f_2(m_{K^*}^2)(m_i + m_f)] \gamma_\mu \\
&\quad + 2f_2(m_{K^*}^2)(p_f)_\mu - \{ [g_1(m_{K^*}^2) + g_2(m_{K^*}^2) \\
&\quad \times (m_i - m_f)] \gamma_\mu + 2g_2(m_{K^*}^2)(p_f)_\mu \} \gamma_5 \} u_{\Lambda_b}(p_i). \quad (29)
\end{aligned}$$

3. $\Lambda_b \rightarrow \Lambda \rho^0$

$\mathcal{M}(\Lambda_b \rightarrow \Lambda \rho^0)$

$$\begin{aligned}
&= \frac{G_F}{2} f_\rho m_\rho \epsilon^{*\mu} \bar{u}_\Lambda(p_f) \\
&\quad \times \left\{ \left[V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \left(\frac{3}{2} (a_7 + a_9) \right) \right] [f_1(m_\rho^2) \right. \\
&\quad - f_2(m_\rho^2)(m_i + m_f)] \gamma_\mu + 2f_2(m_\rho^2)(p_f)_\mu \\
&\quad - \{ [g_1(m_\rho^2) + g_2(m_\rho^2)(m_i - m_f)] \gamma_\mu \\
&\quad \left. + 2g_2(m_\rho^2)(p_f)_\mu \} \gamma_5 \right\} u_{\Lambda_b}(p_i). \quad (30)
\end{aligned}$$

The coefficients a_1, a_2, \dots, a_{10} are combinations of the effective Wilson coefficients given as

$$\begin{aligned}
a_{2i-1} &= c_{2i-1}^{eff} + \frac{1}{(N_c^{eff})} c_{2i}^{eff}, \quad a_{2i} = c_{2i}^{eff} + \frac{1}{(N_c^{eff})} c_{2i-1}^{eff}, \\
i &= 1, 2, \dots, 5, \quad (31)
\end{aligned}$$

TABLE III. Branching ratios for various charmless $\Lambda_b \rightarrow B_f M$ decay modes.

Decay processes	$N_c^{eff}=2$	$N_c^{eff}=3$	$N_c^{eff}=\infty$	Expt. [9]
$\Lambda_b \rightarrow p \pi^-$	8.52×10^{-7}	9.29×10^{-7}	11.57×10^{-7}	$< 5 \times 10^{-5}$
$\Lambda_b \rightarrow p K^-$	1.38×10^{-6}	1.54×10^{-6}	1.87×10^{-6}	$< 5 \times 10^{-5}$
$\Lambda_b \rightarrow \Lambda \pi^0$	1.2×10^{-8}	1.58×10^{-8}	3.22×10^{-8}	-
$\Lambda_b \rightarrow p \rho^-$	1.22×10^{-6}	1.38×10^{-6}	1.55×10^{-6}	-
$\Lambda_b \rightarrow p K^{*-}$	2.99×10^{-7}	2.71×10^{-7}	4.075×10^{-7}	-
$\Lambda_b \rightarrow \Lambda \rho^0$	1.93×10^{-8}	2.52×10^{-8}	5.1×10^{-8}	-

where N_c^{eff} is the effective number of colors. The terms in Eq. (31) are proportional to $\xi = 1/N_c^{eff}$ originating from Fierz rearranging of the operators O_i to produce quark currents to match the quark content of the hadrons in the initial and final states after adopting the factorization approximation. This well-known procedure results in general in matrix elements with the right flavor quantum number but involves both color singlet-singlet and color octet-octet operators. In the naive factorization approximation one discards the color octet-octet operators. This amounts to having $N_c^{eff}=3$. To compensate for these neglected octet-octet and other nonfactorizing contributions one treats $\xi = 1/N_c^{eff}$ in Eq. (31) as a phenomenological free parameter. To maintain the predictive power, it is assumed that ξ is universal (i.e., process independent) for classes of decays sharing similar kinematics. This treatment is known as the ‘‘generalized factorization hypothesis.’’ In the literature, it has often been assumed that the same effective parameter N_c^{eff} can be used to account for nonfactorizable contributions to the matrix elements of all the operators in the effective weak Hamiltonian [2–4] or that two parameters $N_c^{eff}(LL)$ and $N_c^{eff}(LR)$, referring to the operators with structure $(V-A)(V-A)$ and $(V-A)(V+A)$, respectively, would suffice to account for these decays [5,6,23]. A recent analysis of $B \rightarrow D \pi$ data gives $N_c^{eff} \sim 2$ [23]. Analyzing a number of measured two body charmless nonleptonic B decays Ali and Greub [2] have obtained the range as $2 \leq N_c^{eff} \leq \infty$. On the other hand, Mannel and Roberts [24] have used $N_c^{eff} = \infty$ to study the nonleptonic decays of Λ_b baryons. So in order to have an idea about the magnitude of the branching ratios, here we have taken three sets of values, i.e., 2, 3, and ∞ , for the effective number of colors. In theory ξ can be obtained by calculating the octet-octet and other nonfactorizing contributions and can in principle be different for all operators. It was also recently shown in the QCD factorization approach by Beneke *et al.* [25] that ξ is in general operator and channel dependent. Since we have taken N_c^{eff} to be universal (i.e., operator and channel independent), there is the likelihood of some uncertainties in our predicted values.

It should be noted from Table I that the dominant coefficients are a_1 and a_2 for current-current amplitudes, a_4 and a_6 for QCD penguin induced amplitudes, and a_9 for electroweak penguin induced amplitudes. Furthermore, it can also be seen that the coefficients a_1 , a_4 , a_6 , and a_9 are in general N_c^{eff} stable, whereas the others depend strongly on it. Therefore for charmless b decays whose amplitudes depend dominantly on N_c^{eff} stable coefficients, their decay rates can

be reliably predicted within the factorization approach even in the absence of information on nonfactorizable effects.

C. Classification of the factorized amplitudes

Applying the effective Hamiltonian (7), the factorizable decay amplitudes for $\Lambda_b \rightarrow B_f M$ decay processes obtained within the generalized factorization approach are given in Eqs. (25)–(30). In general the two body charmless B meson decays are classified into six classes: class-I decay modes dominated by the external W emission characterized by the parameter a_1 ; class-II decay modes dominated by the color suppressed internal W emission characterized by the parameter a_2 ; class-III decays involving both external and internal W emissions described by $a_1 + r a_2$; class-IV decays dominated by the QCD penguin parameter $a_4 + R a_6$; class-V modes, those whose amplitudes are governed by the effective coefficients a_3 , a_5 , a_7 , and a_9 ; class-VI modes involving the interference of a_{even} and a_{odd} .

Assuming the same classification for charmless Λ_b decays we now find the classes for the decay processes in which we are interested.

I. $\Lambda_b \rightarrow p(\pi/\rho)$

These decays proceed at the tree level through $b \rightarrow u \bar{u} d$ and at the loop level via $b \rightarrow d$ penguin diagrams. Since in terms of the Wolfenstein parametrization

$$V_{ub} V_{ud}^* \approx A \lambda^3 (\rho - i \eta), \quad V_{tb} V_{td}^* \approx A \lambda^3 (1 - \rho + i \eta) \quad (32)$$

are of the same order of magnitude, it is clear that these decays are tree dominated as the penguin contributions are suppressed by the smallness of the penguin coefficients. Hence these decay modes belong to the class-I category.

TABLE IV. Asymmetry parameter (α) for various charmless $\Lambda_b \rightarrow B_f M$ decay modes.

Decay processes	$N_c^{eff}=2$	$N_c^{eff}=3$	$N_c^{eff}=\infty$
$\Lambda_b \rightarrow p \pi^-$	-0.77	-0.77	-0.77
$\Lambda_b \rightarrow p K^-$	-0.77	-0.77	-0.77
$\Lambda_b \rightarrow \Lambda \pi^0$	-0.89	-0.89	-0.89
$\Lambda_b \rightarrow p \rho^-$	-0.71	-0.71	-0.71
$\Lambda_b \rightarrow p K^{*-}$	-0.68	-0.68	-0.68
$\Lambda_b \rightarrow \Lambda \rho^0$	-0.78	-0.78	-0.78

2. $\Lambda_b \rightarrow p(K/K^*)$

These decays proceed at the tree level through $b \rightarrow u\bar{u}s$ and via $b \rightarrow s$ penguin diagrams. In this case,

$$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta), \quad V_{tb}V_{ts}^* = -A\lambda^2, \quad (33)$$

the magnitude of $V_{tb}V_{ts}^*$ being approximately (10^2) times larger than that of $V_{ub}V_{us}^*$. Hence these processes are dominated by the QCD penguin coefficients and belong to class IV.

3. $\Lambda_b \rightarrow \Lambda(\pi/\rho)$

These decays proceed at the tree level through internal W emission $b \rightarrow u\bar{u}s$ and via $b \rightarrow s$ electroweak penguin diagrams. Since the magnitude of $V_{tb}V_{ts}^*$ is larger than $V_{ub}V_{us}^*$, these decays are dominated by electroweak penguin diagrams and belong to class V. Since the electroweak coefficients are smaller than those of tree and QCD penguin coefficients, the branching ratios for these type transitions are in general smaller than the other decay modes that we have considered.

V. RESULTS AND DISCUSSIONS

After obtaining the transition amplitudes for various decay processes we now proceed to estimate their branching ratios and asymmetry parameters. Comparing the evaluated transition amplitudes for $\Lambda_b \rightarrow B_f M$ processes [Eqs. (25)–(30)] with the corresponding generalized amplitudes given in Eqs. (2),(4) one can easily determine the coefficients A , B , A_1 , A_2 , B_1 , and B_2 . Hence the branching ratios and asymmetry parameters are estimated with Eqs. (3),(5),(6). Using the various pseudoscalar and vector meson decay constants (in MeV) such as $f_\pi = 130.7$, $f_K = 159.8$, $f_K^* = 221$, and $f_\rho = 216$, the estimated branching ratios and asymmetry parameters are presented in Tables III and IV, respectively, for three different sets of the effective number of colors. It is seen that the branching ratios are maximum for $N_c^{eff} = \infty$; however, α is stable for all three sets. The estimated branching ratios for $\Lambda_b \rightarrow p\pi$ and pK for all three sets of N_c^{eff} lie

below the present experimental upper limit $BR(\Lambda_b \rightarrow p\pi/pK) < 5 \times 10^{-5}$ [9]. It should also be noted that the decay modes $\Lambda_b \rightarrow \Lambda(\pi/\rho)$ have the smallest branching ratios in comparison to the others. This is so because these decay modes receive contributions from CKM as well as color suppressed tree and electroweak penguin diagrams and moreover they are dominated by the latter. It is naively believed that in charmless b decays the contributions from the electroweak penguin diagrams are negligible compared to QCD penguin diagrams because of the smallness of electroweak Wilson coefficients. Thus the estimated branching ratios for $\Lambda_b \rightarrow \Lambda(\pi/\rho)$ are one order smaller than the $\Lambda_b \rightarrow p(\pi/\rho), p(K/K^*)$ transitions.

To summarize, using the next-to-leading order QCD corrected effective Hamiltonian, we have obtained the branching ratios and asymmetry parameters for charmless hadronic Λ_b decays, within the framework of a generalized factorization. The nonfactorizable contributions are parametrized in terms of the effective number of colors, N_c^{eff} . So in addition to the naive factorization approach ($N_c^{eff} = 3$), here we have taken two more values for N_c^{eff} , i.e., $N_c^{eff} = 2$ and ∞ . The baryonic form factors at maximum momentum transfer (q_m^2) are evaluated using the nonrelativistic quark model and the extrapolation of the form factors from q_m^2 to the required q^2 value is done by assuming pole dominance. The obtained branching ratios for $\Lambda_b \rightarrow p\pi$, pK processes lie within the present experimental upper limit. Though the branching ratios for these modes are small, they could be accessible at future hadron colliders with large b production. Furthermore, with large data on Λ_b baryons expected in the near future, these decay channels will serve as a testing ground to look for CP violation in and beyond standard model.

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