

Leptogenesis and neutrino parameters

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We calculate the baryonic asymmetry of the universe in the baryogenesis-via-leptogenesis framework, assuming first a quark-lepton symmetry and then a charged-neutral lepton symmetry. We match the results with the experimentally favored range. In the first case all the oscillation solutions to the solar neutrino problem, except the large mixing matter solution, can lead to the allowed range, but with fine-tuning of the parameters. In the second case the general result is quite similar. Some related theoretical hints are discussed.

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I. INTRODUCTION

Strong indications for nonzero neutrino mass and mixing come from solar and atmospheric neutrino experiments. In fact, if interpreted in terms of neutrino oscillations, such experiments, together with the tritium beta decay end point, imply small neutrino masses [1].

In the minimal standard model (MSM) the neutrino is massless because there are no right-handed neutrino singlets and there is no Higgs scalar triplet. The simplest way to get a mass for the neutrino field is by adding the right-handed state ν_R , the analogue of the quark state u_R in the leptonic sector, in which case it becomes possible to build both a Dirac mass term $m_\nu \bar{\nu}_L \nu_R$ and a Majorana mass term $(1/2)m_R \bar{\nu}_L^c \nu_R$ for the right-handed neutrino. The Dirac mass m_ν is expected to be of the same order of magnitude of the quark or charged lepton masses, while the Majorana mass m_R is not constrained. A popular mechanism to obtain a very small neutrino mass is the seesaw mechanism [2], where the right-handed neutrino mass is very large and as a consequence a very light left-handed Majorana neutrino appears, with a mass $m_L \approx m_\nu^2/m_R$.

The MSM plus the right-handed neutrino (which we would like to call SM) is also a minimal scenario to produce a baryonic asymmetry in the universe, according to the Fukugita-Yanagida baryogenesis-via-leptogenesis mechanism [3,4]. In this framework the out-of-equilibrium decays of right-handed neutrinos generate a leptonic asymmetry which is partially transformed into a baryonic asymmetry by electroweak sphaleron processes [5].

The baryonic asymmetry depends on both the Dirac and the right-handed Majorana neutrino mass matrices. Therefore, assuming a quark-lepton symmetry or a charged-neutral lepton symmetry, we should be able to determine the value of the baryonic asymmetry, and to match it with the experimental bounds coming from nucleosynthesis in the standard big bang theory. This is the main subject of the present paper, already discussed by several authors [6–9]. However, our approach is quite different, more general and direct. We scan over the neutrino parameter space, using several forms

for the Dirac mass matrices, and taking into account the vacuum and matter solutions to the solar neutrino problem. A graphical representation of the results is given, from which one can eventually infer approximate bounds on neutrino parameters. Both the nonsupersymmetric (SM) and the supersymmetric (SSM) cases are considered.

Section II is about neutrino oscillation data, from which one may obtain light neutrino masses and mixings. Section III deals with the quark-lepton symmetry, which allows one to get the Dirac and heavy neutrino mass matrices. In Sec. IV, after a short collection of the relevant formulas of the baryogenesis-via-leptogenesis mechanism, the calculation of the baryonic asymmetry is carried out, based on the content of Secs. II and III. In Sec. V the same calculation is done assuming a charged-neutral lepton symmetry. Finally, in Sec. VI, we give our conclusions and a brief discussion.

II. NEUTRINO PARAMETERS

From the phenomenological point of view the baryonic asymmetry also depends on which solution for solar neutrinos is taken into account. Therefore, in this section we summarize the neutrino oscillation data that we will use in our analysis. For atmospheric neutrinos the best fit is [10]

$$\Delta m_a^2 = 3.5 \times 10^{-3} \text{ eV}^2$$

$$\sin^2 2\theta_a = 1.0,$$

that is maximal mixing. For solar neutrinos we have three matter [Mikheyev-Smirnov-Wolfenstein (MSW)] solutions [11]: the small mixing angle (SMA)

$$\Delta m_s^2 = 5.4 \times 10^{-6} \text{ eV}^2$$

$$\sin^2 2\theta_s = 0.006,$$

the large mixing angle (LMA)

$$\Delta m_s^2 = 1.8 \times 10^{-5} \text{ eV}^2$$

$$\sin^2 2\theta_s = 0.76,$$

and the low- Δm^2 (LOW) solution

$$\Delta m_s^2 = 7.9 \times 10^{-8} \text{ eV}^2$$

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$$\sin^2 2\theta_s = 0.96.$$

Moreover, we also have the vacuum oscillation (VO) solution [11]

$$\Delta m_s^2 = 8.0 \times 10^{-11} \text{ eV}^2$$

$$\sin^2 2\theta_s = 0.75.$$

The latest day-night and spectral data favor the LMA and LOW solutions, but do not exclude the others [12]. Further information on neutrino oscillations comes from the CHOOZ experiment [13] which gives $\sin \theta_c \leq 0.16$ for $\Delta m_c^2 > 1 \times 10^{-3} \text{ eV}^2$.

Therefore, neutrinos do have masses and mixings, and a unitary matrix $U_{\alpha i}$ ($\alpha = e, \mu, \tau; i = 1, 2, 3$) relates the mass eigenstates ν_i to the weak eigenstates ν_α :

$$\nu_{\alpha L} = \sum_i U_{\alpha i} \nu_{iL}.$$

It is clear that $\Delta m_s^2 \ll \Delta m_a^2$. According to Ref. [14] we assume

$$\Delta m_s^2 = m_2^2 - m_1^2, \quad \Delta m_a^2 = m_3^2 - m_1^2$$

where the numbering corresponds to the family index. Moreover, we work with the hierarchical spectrum of light neutrinos, $m_1 \ll m_2 \ll m_3$. Then, $m_3^2 \approx \Delta m_a^2$, $m_2^2 \approx \Delta m_s^2$, and for m_1 we take $10^{-4} m_2 < m_1 < 10^{-1} m_2$.

The mixing matrix U [the Maki-Nakagawa-Sakata (MNS) matrix [15]] can be written as the standard parametrization of the CKM matrix (including one phase δ') times a diagonal phase matrix $D = \text{diag}(e^{i\varphi_1}, e^{i\varphi_2}, 1)$ [16,17]. Hence, it depends on three angles and three phases. From neutrino oscillation data we can determine the three angles [14,16]. For $|U_{e3}|$, related to the result of the CHOOZ experiment, we use the bound

$$|U_{e3}| \leq 0.2,$$

while U_{e2} and $U_{\mu 3}$ are obtained from the best fits of atmospheric and solar neutrinos. Then we are left with five free neutrino parameters: $|U_{e3}|$, $\delta = \arg(U_{e3})$, m_1 , φ_1 , φ_2 . Choosing values for the free parameters leads to a complete determination of light masses and the mixing matrix U . These will be used in the following section, together with the quark-lepton symmetry, to obtain the heavy neutrino mass matrix.

III. SEESAW MECHANISM WITH QUARK-LEPTON SYMMETRY

The Lagrangian for the relevant lepton sector is (for simplicity we do not write the 1/2 factor in the Majorana terms)

$$\mathcal{L} = \bar{e}_L M_e e_R + \bar{\nu}_L M_\nu \nu_R + g \bar{\nu}_L e_L W + \bar{\nu}_L^c M_R \nu_R \quad (1)$$

where M_e is the mass matrix of charged leptons, M_ν is the mass matrix of Dirac neutrinos, and M_R the mass matrix of right-handed Majorana neutrinos. The effective Lagrangian of the seesaw mechanism is

$$\mathcal{L}_{ss} = \bar{e}_L M_e e_R + \bar{\nu}_L M_L \nu_R^c + g \bar{\nu}_L e_L W + \bar{\nu}_L^c M_R \nu_R \quad (2)$$

with the light neutrino mass matrix M_L given by

$$M_L = -M_\nu M_R^{-1} M_\nu^T. \quad (3)$$

Setting

$$U_L M_L U_L^T = D_L, \quad U_{eL} M_e U_{eR}^\dagger = D_e, \quad (4)$$

where D_L , D_e are diagonal matrices, we obtain the MNS matrix as

$$U = U_L U_{eL}^\dagger. \quad (5)$$

Inverting Eq. (3) we get the heavy neutrino mass matrix

$$M_R = -M_\nu^T M_L^{-1} M_\nu. \quad (6)$$

Now, assuming a quark-lepton symmetry, we take the pair of Hermitian matrices

$$M_\nu = \frac{m_\tau}{m_b} \begin{pmatrix} 0 & 0 & \sqrt{m_u m_t} \\ 0 & m_c & 0 \\ \sqrt{m_u m_t} & 0 & m_t \end{pmatrix}, \quad (7)$$

$$M_e = \frac{m_\tau}{m_b} \begin{pmatrix} 0 & \sqrt{m_d m_s} e^{i\alpha} & 0 \\ \sqrt{m_d m_s} e^{-i\alpha} & -3m_s & \sqrt{m_d m_b} \\ 0 & \sqrt{m_d m_b} & m_b \end{pmatrix}, \quad (8)$$

with one phase $\alpha = \pi/2$ in M_{e12} . These lepton mass matrices are obtained in the following way. We take the five texture zero model for the quark mass matrices M_u and M_d from Ref. [18], which studies the phenomenologically viable textures. The quark mass matrices are related to the lepton mass matrices by an approximate running factor m_b/m_τ from the high scale where the quark-lepton symmetry should hold [19], and in addition a factor -3 is included in M_{e22} in order to have a good relation between charged lepton and down quark masses [20]. For five texture zeros the matrices M_u , M_d lead to the simple meaningful relations

$$V_{us} \approx \sqrt{\frac{m_d}{m_s}}, \quad V_{cb} \approx \sqrt{\frac{m_d}{m_b}}, \quad V_{ub} \approx \sqrt{\frac{m_u}{m_t}}. \quad (9)$$

In this way, by means of Eqs. (4)–(8), we can calculate M_R and then its eigenvalues M_1, M_2, M_3 [21]. The quark-lepton symmetry is usually obtained within unified theories such as $SU(5)$ and mostly $SO(10)$, where quarks and leptons belong to the same multiplets. In particular, the factor -3 in M_{e22} is due to suitable Yukawa couplings of these multiplets with the **45** [in $SU(5)$] or **126** [in $SO(10)$] Higgs representations. However, here we can also regard the quark-lepton symmetry as a phenomenological feature.

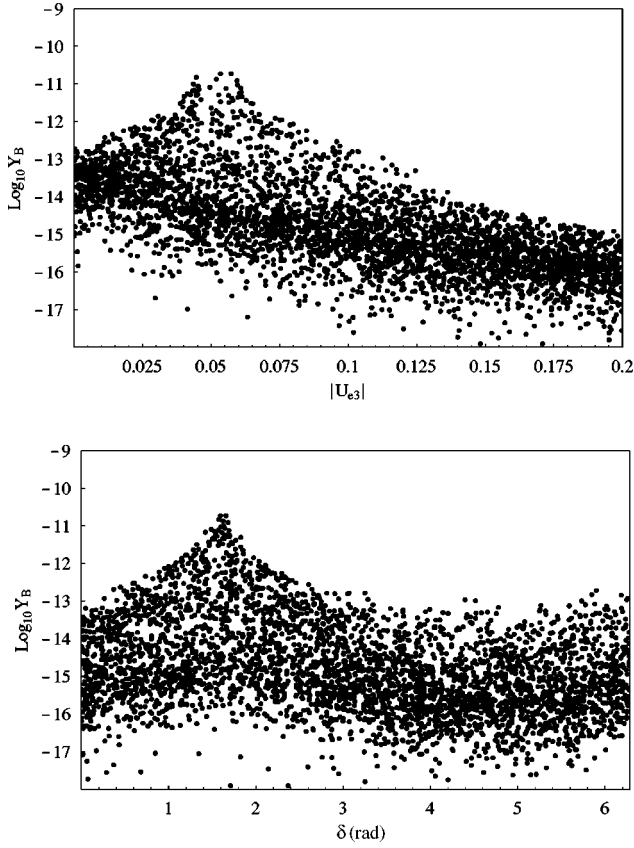


FIG. 1. The baryonic asymmetry Y_B vs $|U_{e3}|$ and δ for SMA, quark-lepton symmetry, five texture zeros.

IV. THE BARYONIC ASYMMETRY

A baryonic asymmetry can be generated from a leptonic asymmetry [3]. In order to study this baryogenesis-via-leptogenesis mechanism we diagonalize M_e :

$$\mathcal{L}' = \bar{e}_L D_e e_R + \bar{\nu}_L M'_\nu \nu_R + g \bar{\nu}_L e_L W + \bar{\nu}_L^c M_R \nu_R,$$

where

$$M'_\nu = U_{eL} M_\nu, \quad (10)$$

and also M_R by means of $U_R M_R U_R^T = D_R$:

$$\mathcal{L}'' = \bar{e}_L D_e e_R + \bar{\nu}_L M''_\nu \nu_R + g \bar{\nu}_L e_L W + \bar{\nu}_L^c D_R \nu_R$$

where

$$M''_\nu = M'_\nu U_R^T \equiv M_D. \quad (11)$$

Because of the electroweak sphaleron effect, the baryonic asymmetry Y_B is related to the leptonic asymmetry Y_L by [22]

$$Y_B = a Y_{B-L} = \frac{a}{a-1} Y_L \quad (12)$$

with

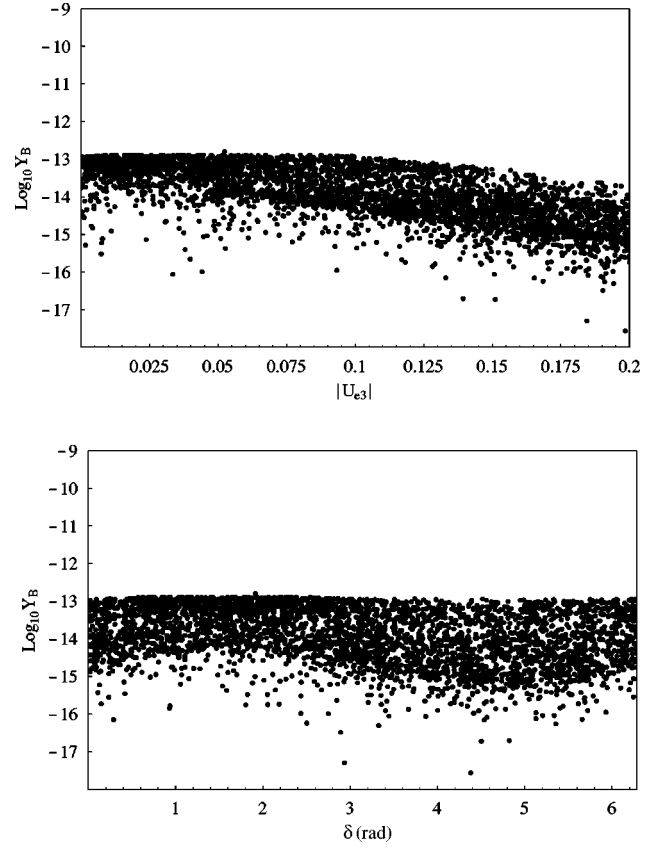


FIG. 2. The baryonic asymmetry Y_B vs $|U_{e3}|$ and δ for LMA, quark-lepton symmetry, five texture zeros.

$$a = \frac{8N_f + 4N_H}{22N_f + 13N_H},$$

where N_f is the number of families (three) and N_H the number of Higgs doublets (one in the SM and two in the SSM; $a \approx 1/3$ in both cases). Remember that

$$Y_B = \frac{n_B - n_{\bar{B}}}{7.04 n_\gamma}$$

where $n_{B,\bar{B},\gamma}$ are number densities. The leptonic asymmetry can be written as [4]

$$Y_L = d \frac{\epsilon_1}{g^*} \quad (13)$$

where, in the SM, the CP -violating asymmetry ϵ_1 is given by [23,24]

$$\epsilon_1 = \frac{1}{8\pi v^2 (M_D^\dagger M_D)_{11}} \sum_{j=2,3} \text{Im}[(M_D^\dagger M_D)_{j1}]^2 f\left(\frac{M_j^2}{M_1^2}\right), \quad (14)$$

with

$$f(x) = \sqrt{x} \left[1 - (1+x) \ln \frac{1+x}{x} - \frac{1}{x-1} \right],$$

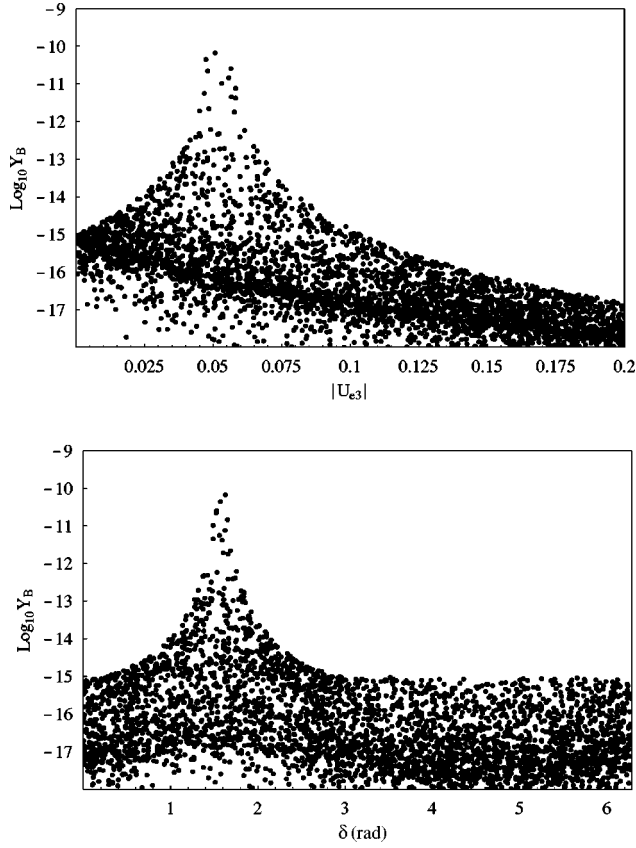


FIG. 3. The baryonic asymmetry Y_B vs $|U_{e3}|$ and δ for VO, quark-lepton symmetry, five texture zeros.

$g^*(\text{SM}) = 106.75$; v is the vacuum expectation value (VEV) of the SM Higgs doublet. In the SSM, $v \rightarrow v \sin \beta$, $f(x) \rightarrow g(x)$, $g^*(\text{SSM}) = 228.75$,

$$g(x) = -\sqrt{x} \left[\ln \frac{1+x}{x} + \frac{2}{x-1} \right],$$

and a factor 4 is included in ϵ_1 [23], due to more decay channels. For a hierarchical spectrum of heavy neutrinos $f \approx -3M_1/2M_j$, $g \approx -3M_1/M_j \approx 2f$, with a very good accuracy. Equation (14) arises from the interference between the tree level and one loop decay amplitudes of the lightest heavy neutrino, and includes vertex and self-energy corrections. The latter may be dominant if M_1 and M_j are nearly equal, so that an enhancement of the asymmetry may occur.

A good approximation for d , the dilution factor, is inferred from Refs. [25–27]:

$$d = (0.1 k)^{1/2} \exp[-(4/3)(0.1 k)^{1/4}] \quad (15)$$

for $k \geq 10^6$,

$$d = 0.24/k(\ln k)^{3/5} \quad (16)$$

for $10 \leq k \leq 10^6$, and

$$d = 1/2k, \quad d = 1 \quad (17)$$

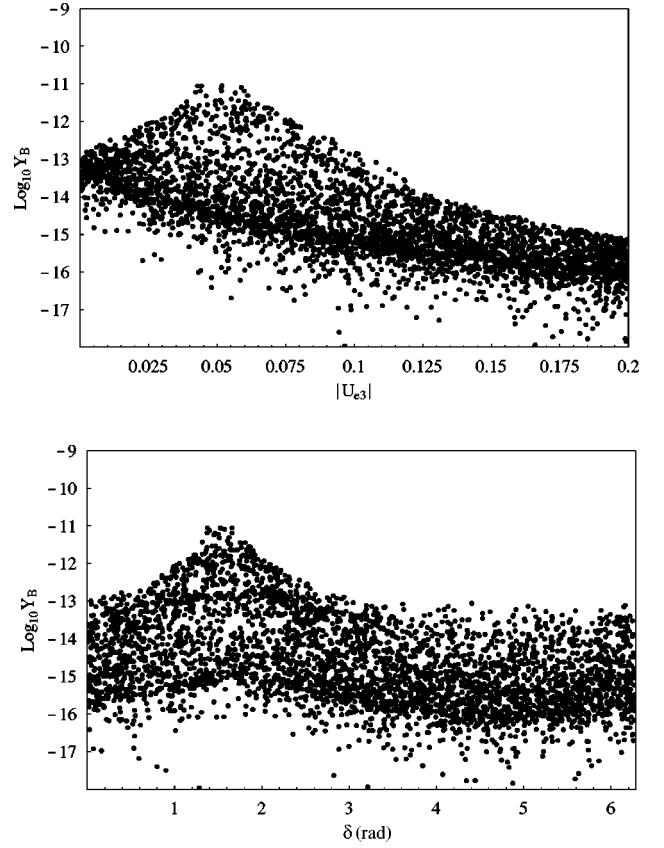


FIG. 4. The baryonic asymmetry Y_B vs $|U_{e3}|$ and δ for LOW, quark-lepton symmetry, five texture zeros.

for $1 \leq k \leq 10$, $0 \leq k \leq 1$, respectively, where the parameter k is

$$k = \frac{M_P}{1.7v^2 32\pi\sqrt{g^*}} \frac{(M_D^\dagger M_D)_{11}}{M_1}, \quad (18)$$

and M_P is the Planck mass. In the SSM the critical value 10^6 for k is lowered, but in our calculation k remains always much smaller. The presence of the dilution factor in Eq. (13) takes into account the washout effect produced by inverse decay and lepton number violating scattering.

We make a random extraction of the free neutrino parameters for a total of 8000 points and we plot Y_B versus such parameters. As expected, about 4000 points give a negative Y_B . Only $|U_{e3}|$ and δ show a major effect and the results for the five texture zero model are presented in Figs. 1–4, according to the four different solar neutrino solutions. Changing the other parameters, in particular U_{e2} and m_2 , within the allowed experimental limits, does not affect the general result. Since the favored range for the baryonic asymmetry is [28]

$$Y_B = (1.7 - 8.9) \times 10^{-11},$$

one can look at the region of Y_B between 10^{-11} and 10^{-10} . The SMA, VO and LOW solutions can produce the required amount of baryonic asymmetry, but with fine-tuning of the parameters. Notice that we plot $\text{Log}_{10} Y_B$, which is negative.

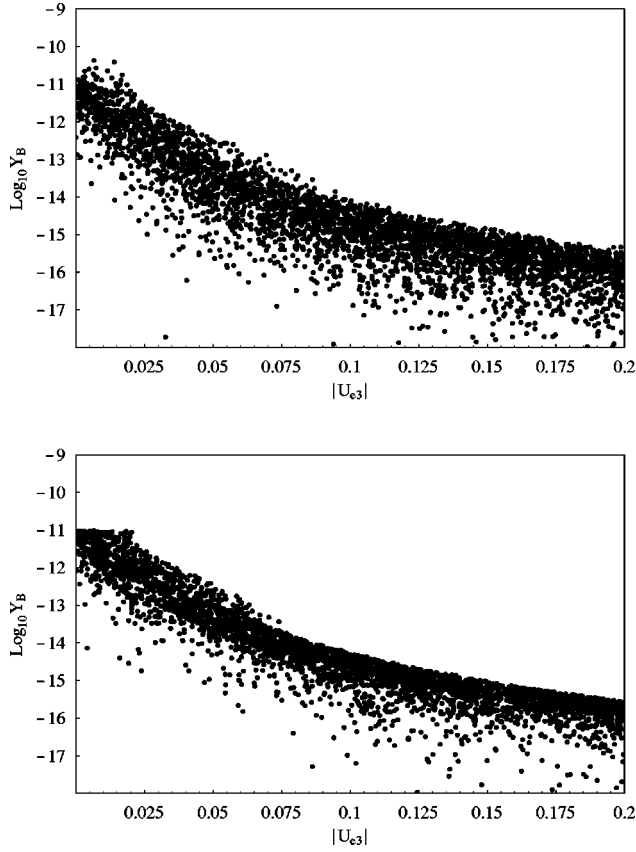


FIG. 5. The baryonic asymmetry Y_B vs $|U_{e3}|$ for SMA and LOW, quark-lepton symmetry, four texture zeros.

However, the trend is clear. For example, we find the phase δ tuned around $\pi - \alpha$, which corresponds to $\delta' = \alpha$, $\sin \theta_{e3} < 0$. Moreover, according to Ref. [9], we find an enhancement of the asymmetry for $|U_{e3}| \approx V_{D12} U_{\mu 3}$, where $V_D = U_{\nu L} U_{eL}^\dagger$ is the mixing matrix in the Dirac sector (the analogue of $V_{CKM} = V_u V_d^\dagger$). Since $V_{D12} \approx (1/3) \sqrt{m_d/m_s} \approx 0.07$, we get the maximum of Y_B around $|U_{e3}| \approx 0.07 \times 0.7 \approx 0.05$. In a similar way one can explain why the SMA solution shows an enhancement, contrary to the LMA solution. In fact, the further condition is $U_{e2} \approx V_{D12} U_{\mu 2}$, which is compatible with the SMA but not with the LMA. The two conditions correspond to the decoupling of M_1 from m_3 and m_2 , respectively. Note that in Ref. [9] $V_{D12} \approx 0.21$ because there the factor -3 in our M_{e22} is absent. In this case one has a different enhancement value for U_{e2} , $|U_{e3}|$, so that the SMA is also excluded. The presence of the factor -3 allows the SMA to be reliable for leptogenesis, for the matrix texture (7),(8).

In the supersymmetric case the calculated baryonic asymmetry is increased by a factor of nearly 6. In fact, going from the SM to the SSM, there is a factor 4 due to ϵ_1 , a factor $1/\sqrt{2}$ due to g^* (for $d \sim 1/k$) and a factor 2 due to $g(x)$: $4 \times (1/\sqrt{2}) \times 2 \approx 6$. Hence, in the present context, the SSM works better for leptogenesis, with respect to the SM.

To test the dependence of Y_B on matrix texture, let us consider a second pair of Hermitian matrices [18], with four texture zeros, which is formed by the same matrix M_e as in

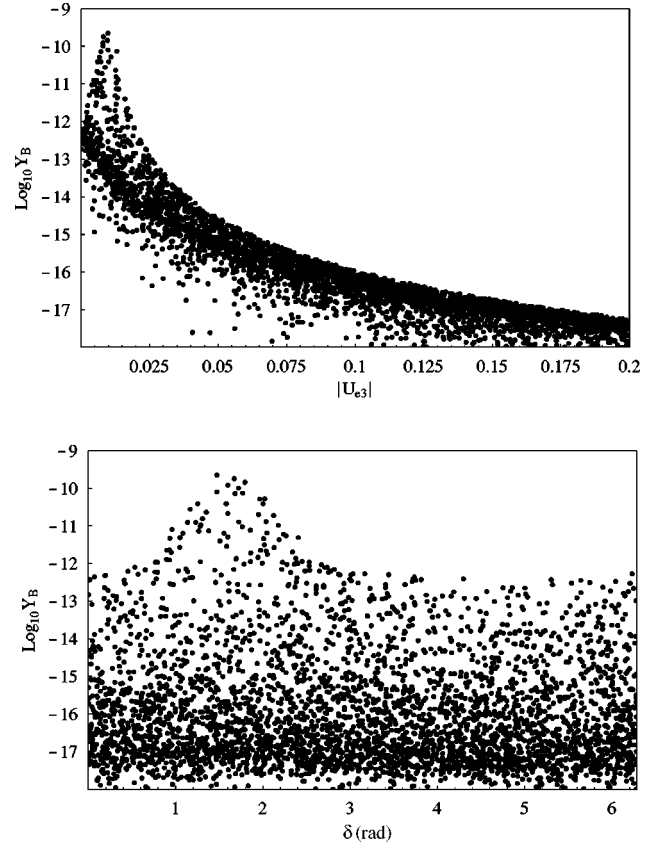


FIG. 6. The baryonic asymmetry Y_B vs $|U_{e3}|$ and δ for VO, quark-lepton symmetry, four texture zeros.

Eq. (8), but with a further phase $\pi/2$ in M_{e23} , and

$$M_\nu = \frac{m_\tau}{m_b} \begin{pmatrix} 0 & \sqrt{m_u m_c} & 0 \\ \sqrt{m_u m_c} & m_c & \sqrt{m_u m_t} \\ 0 & \sqrt{m_u m_t} & m_t \end{pmatrix}. \quad (19)$$

Note that this form for M_ν has entries 1-2 and 2-3 filled in, with respect to matrix (7), while $M_{\nu 13} = 0$. Useful results are in Figs. 5, 6, for the SMA, LOW and VO solutions, while the LMA gives too small asymmetry as in the foregoing case. For SMA, LOW to work one has $|U_{e3}| \approx 0.01$, and for VO also $\delta \approx \pi/2$.

V. THE CASE OF A CHARGED-NEUTRAL LEPTON SYMMETRY

If there is a charged-neutral lepton symmetry, the Dirac neutrino mass matrix is related to the charged lepton mass matrix rather than to the up quark mass matrix: $M_\nu \sim M_e$, with

$$M_e = \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & 0 \\ \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_e m_\tau} \\ 0 & \sqrt{m_e m_\tau} & m_\tau \end{pmatrix}, \quad (20)$$

for example (four texture zeros). This form for M_e is obtained by analogy to M_d in Ref. [18]. There are some theo-

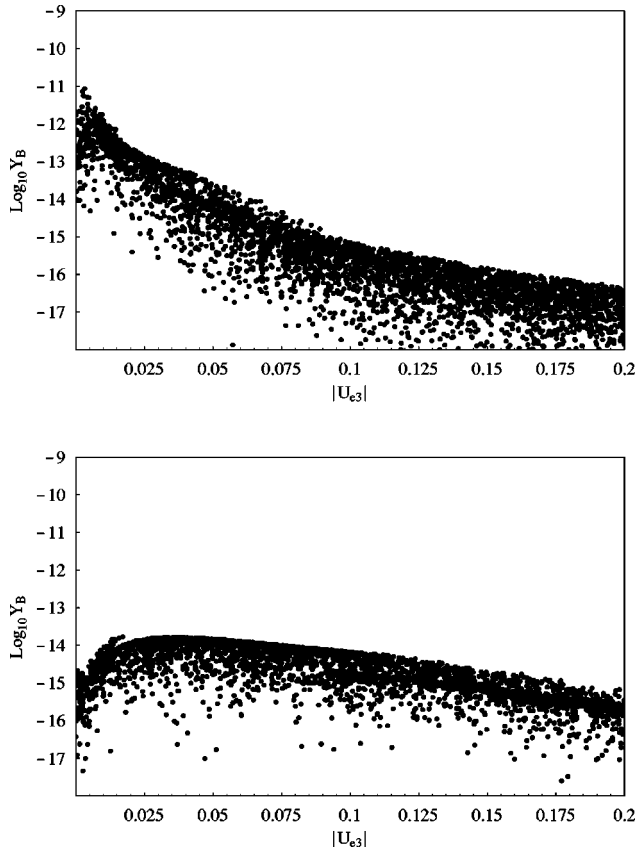


FIG. 7. The baryonic asymmetry Y_B vs $|U_{e3}|$ for SMA and LMA, charged-neutral lepton symmetry, four texture zeros.

retical (left-right) models [29] with a charged-neutral lepton symmetry, along with an up-down symmetry. However, again, we can also assume it as a phenomenological hypothesis. The position of phases is somewhat arbitrary. We put phases $\pi/2$ in M_{e12} , M_{e23} in order to have a mixing in the Dirac sector similar to the previous case. The value of the baryonic asymmetry Y_B is quite analogous to the case of a quark-lepton symmetry, see Figs. 7, 8. However, the maximum level of asymmetry is now reached for $|U_{e3}| \approx 0$.

Also for this case, we have checked the dependence on matrix texture by using the Hermitian matrices formed by six texture zeros

$$M_\nu \sim M_e \sim \begin{pmatrix} 0 & 0 & \sqrt{m_e m_\tau} \\ 0 & m_\mu & 0 \\ \sqrt{m_e m_\tau} & 0 & m_\tau \end{pmatrix},$$

with a single phase $\pi/2$ in M_{e13} , but we have found no relevant difference with the asymmetry generated by matrix (20).

VI. CONCLUSION AND DISCUSSION

The baryonic asymmetry Y_B has been calculated using a random extraction for five of the nine neutrino parameters (three light masses; three angles and three phases in the mixing matrix) and assuming quark-lepton symmetry or

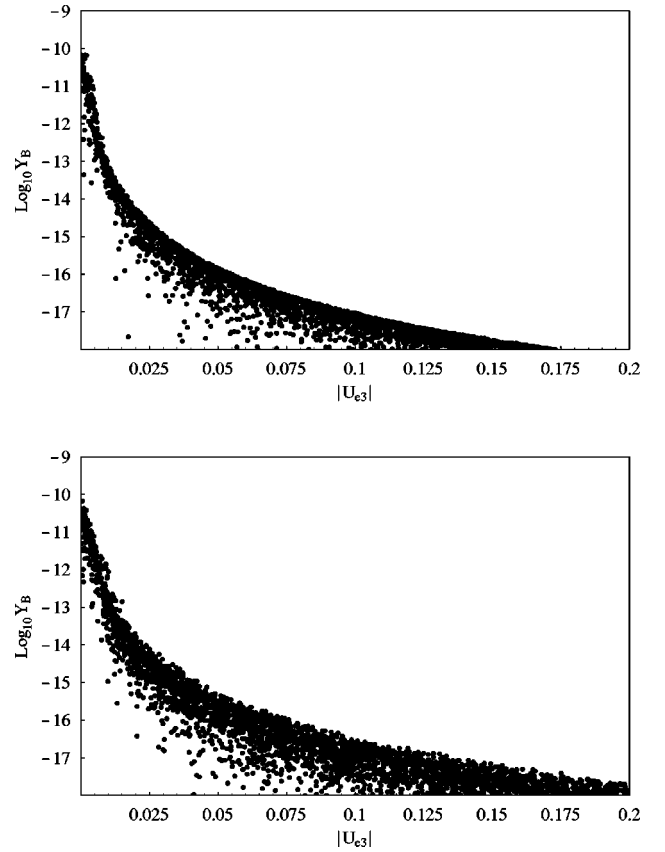


FIG. 8. The baryonic asymmetry Y_B vs $|U_{e3}|$ for VO and LOW, charged-neutral lepton symmetry, four texture zeros.

charged-neutral lepton symmetry for the Dirac mass matrices. Other parameters have been checked. As a result, for quark-lepton symmetry, we find that the SMA, VO, and LOW solutions for solar neutrinos are able to generate enough asymmetry, especially in the supersymmetric case, but with fine tuning and selected values of the parameters $|U_{e3}|$ and δ . For charged-neutral lepton symmetry the general results are similar.

Let us discuss some related theoretical issues. Unified theories such as $SO(10)$, or left-right models such as $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, naturally contain heavy Majorana neutrinos, generated at the unification or left-right scale [30], but also contain other particles, for example additional gauge bosons. Usually these particles are much heavier than the lightest heavy Majorana neutrino, so that they are sufficiently decoupled from the leptogenesis process, as confirmed in Ref. [31]. In this way, the idea of baryogenesis through leptogenesis may be attractive also within unified or left-right models. The VO solution with quark-lepton symmetry gives the scale of M_R around the Planck mass, while the SMA and LMA solutions give the scale of M_R near the unification scale (10^{16} GeV) [21]. Also the LOW solution may be consistent with the unification scale. Thus, it is hard to reconcile the VO solution with quark-lepton symmetry, in the context of unified theories, whereas for the SMA and perhaps the LOW solutions this is possible. We point out that if the LMA is the right solution to the solar neutrino problem, then the framework used in this

paper does not work for leptogenesis. If the VO solution is right, then a good amount of leptogenesis can be obtained, but with fine tuning and outside normal unified models. The SMA and LOW solutions may be consistent with both unified theories and leptogenesis bounds, for selected values of the complex parameter U_{e3} .

In the case of charged-neutral lepton symmetry the VO solution gives M_R around the unification scale, while the SMA and LMA solutions give M_R near the intermediate (left-right) scale (10^{12} GeV) [21]. The LOW solution lies between the two. Hence, the VO solution may be consistent with both the unification scale and leptogenesis, and the SMA with both the intermediate scale and leptogenesis.

Since the quark-lepton symmetry is natural in unified models, while the charged-neutral (up-down) symmetry is natural in left-right models, in the present context the preferred solution for solar neutrinos could be the SMA. However, neutrino data slightly favor the LMA solution [32]. A possible alternative for the baryogenesis-via-leptogenesis mechanism to work more extensively is by means of horizontal symmetries [33].

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