Soft double-diffractive Higgs boson production at hadron colliders

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We evaluate the nonperturbative contribution to the double-diffractive production of the Higgs boson, which arises due to the QCD scale anomaly if the mass of the Higgs boson, M_H, is smaller than the mass of the top quark, M_T, $M_H < M_T$. The cross section appears to be larger than expected from perturbative calculations; we find $\sigma_H = 0.019-0.14$ pb at Fermilab Tevatron energies, and $\sigma_H = 0.01-0.27$ pb at the energy of the CERN LHC.

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I. INTRODUCTION

In this paper we suggest a new mechanism for ''soft'' double-diffractive production of the Higgs boson. We consider three reactions

$$p+p \to p+[\mathrm{LRG}]+H+[\mathrm{LRG}]+p, \tag{1}$$

$$p+p \to X_1+[\mathrm{LRG}]+H+[\mathrm{LRG}]+X_2, \tag{2}$$

$$p+p \to p+[\mathrm{LRG}]+H+[\mathrm{LRG}]+X_2, \tag{3}$$

where LRG denotes the large rapidity gap between produced particles and X corresponds to a system of hadrons with masses much smaller than the total energy. These reactions have such a clean signature for experimental searches [see Fig. 1, where the lego plot is shown for the reaction of Eq. (1)] that they have been the subject of continuing theoretical studies during this decade (see Refs. [1–7]).

The main idea behind all calculations, starting from the Bialas-Landshoff paper [2], is to describe the reactions of Eq. (1) and Eq. (2) as double Pomeron (DP) Higgs boson production (see Fig. 2). In Fig. 2, the Pomerons are the so-called ''soft'' Pomerons for which one uses the phenomenological Donnachie-Landshoff form (see Ref. [8]), while the vertex γ can be calculated in perturbative QCD (PQCD).

We can demonstrate the problems and uncertainties of such a kind of approach by considering the simplest PQCD diagram for double-Pomeron Higgs boson production (DPHP) [see Fig. 3(a)]. This diagram leads to the amplitude[1] [2,6]

$$M(qq \to qHq) = \frac{2}{9} 2 g_H \int \frac{d^2 Q_\perp}{Q_\perp^2 Q_{1,\perp}^2 Q_{2,\perp}^2} 4 \alpha_S(Q_\perp^2) \times (\vec{Q}_{1,\perp} \cdot \vec{Q}_{2,\perp}), \tag{4}$$

where g_H is the Higgs coupling that has been evaluated in perturbative QCD [10]. For the reaction of Eq. (1), $|t_1| = |\vec{Q}_\perp - \vec{Q}_{1,\perp}| \approx |t_2| = |\vec{Q}_\perp - \vec{Q}_{2,\perp}| \approx 2/B_{el}$ and, therefore,

$$M(q+q \to q+H+q) \propto \int \frac{d^2 Q_\perp}{Q_\perp^4}. \tag{5}$$

Equation (5) has an infrared divergence which is regularized by the size of the colliding hadrons. In other words, one can see that already the simplest diagrams show that the DP Higgs boson production is, in a sense, a ''soft'' process. Taking into account the emission of extra gluons denoted in Fig. 3(b) as Pomeron builders, we recover the exchange of the ''soft'' Pomerons.

Nevertheless, the emission vertex for the Higgs boson can still be calculated in PQCD since the typical distances inside the quark triangle in Fig. 3(a) are rather short, $\propto 1/M_T$,

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[1] In Ref. [2] all calculations have been performed in detail but the numerical factor in front was corrected in Ref. [6]. At high energy the Weizsäcker-Williams approximation is used which leads to a simple form [9] of the propagators for the t-channel gluons in Fig. 3(a):

$$G_{\mu\nu} = \frac{1}{q_{i,\perp}^2} \frac{2 q_{i,\perp,\mu} q_{i,\perp,\nu}}{q_{i,+} q_{i,-}} + O\left(\frac{1}{s}\right).$$

This form of the propagator allows us to sum over gluon polarizations and to obtain a simple formula of Eq. (4).

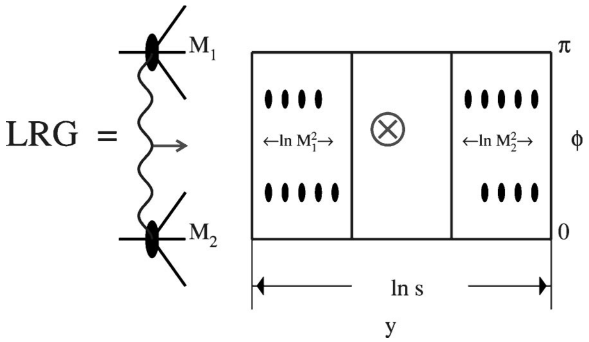

FIG. 1. Lego plot for the double Pomeron Higgs boson production process.

where M_T is the mass of the t quark. The coupling g_H has been evaluated in Ref. [10] and is given by

$$g_H^2 = \sqrt{2} G_F \alpha_S^2 (M_H^2) N^2 / 9\pi^2, \tag{6}$$

where N is a function of the ratio M_T/M_H which was calculated in Refs. [10,4].

In this paper we consider an alternative approach to DPHP, in which we estimate the value of the cross section from nonperturbative QCD. In Sec. II we review a nonperturbative method suggested by Shifman, Vainshtein, and Zakharov [11] for the evaluation of the coupling of the Higgs boson to hadrons; it is valid if the mass of the Higgs boson is smaller than the mass of the top quark. In Sec. III we develop a method of obtaining the DPHP cross section using the approach of Ref. [11]. The problem of the survival of large rapidity gaps (LRGs) will be discussed in Sec. IV. We conclude in Sec. V with a discussion of our results and of the uncertainties inherent in our approach.

II. COUPLING OF THE HIGGS BOSON TO HADRONS IN NONPERTURBATIVE QCD

To evaluate the nonperturbative coupling of the Higgs boson to hadrons, we need to have a closer look at the properties of the energy-momentum tensor in QCD. The trace of this tensor is given by

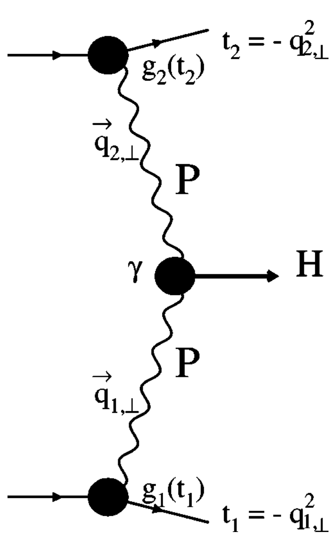

FIG. 2. Double-Pomeron Higgs boson production process.

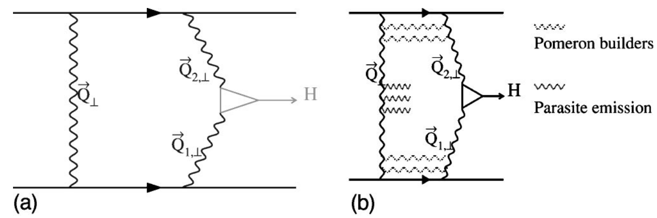

FIG. 3. Double Pomeron Higgs boson production in the Born approximation [Fig. 3(a)] and in leading log approximation [Fig. 3(b)] of PQCD.

$$\Theta_\alpha^\alpha = \frac{\beta(g)}{2g} G^{\alpha\beta a} G_{\alpha\beta}^a + \sum_{l=u,d,s} m_l (1+\gamma_{m_l}) \bar{q}_l q_l$$

$$+ \sum_{h=c,b,t} m_h (1+\gamma_{m_h}) \bar{Q}_h Q_h, \tag{7}$$

where γ_m are the anomalous dimensions; in the following we will assume that the current quark masses are redefined as $(1+\gamma_m)m$. The appearance of the scalar gluon operator in Eq. (7) is the consequence of a scale anomaly [12,13]. The QCD beta function can be written as

$$\beta(g) = -b \frac{g^3}{16\pi^2} + \cdots, \quad b = 9 - \frac{2}{3} n_h, \tag{8}$$

where n_h is the number of heavy flavors $(c, b, \ldots)$. Since there is no valence heavy quarks inside light hadrons, at scales $Q^2 < 4m_h^2$ one expects a decoupling of the heavy flavors. This decoupling was consistently treated in the framework of the heavy-quark expansion [11]; to order $1/m_h$, only the triangle graph with external gluon lines contributes. Explicit calculation shows [11] that the heavy-quark terms transform in the piece of the anomalous gluonic part of Θ_α^α:

$$\sum_h m_h \bar{Q}_h Q_h \rightarrow -\frac{2}{3} n_h \frac{g^2}{32\pi^2} G^{\alpha\beta a} G_{\alpha\beta}^a + \cdots. \tag{9}$$

It is immediately clear, from Eqs. (9), (7), and (8), that the heavy-quark terms indeed cancel the part of the anomalous gluonic term associated with heavy flavors, so that the matrix element of the energy-momentum tensor can be rewritten in the form

$$\Theta_\alpha^\alpha = \frac{\tilde{\beta}(g)}{2g} G^{\alpha\beta a} G_{\alpha\beta}^a + \sum_{l=u,d,s} m_l \bar{q}_l q_l, \tag{10}$$

where heavy quarks do not appear at all; the beta function in Eq. (10) includes the contributions of light flavors only:

$$\tilde{\beta}(g) = -9 \frac{g^3}{16\pi^2} + \cdots. \tag{11}$$

Because the mass of the Higgs boson, M_H, is presumably large, its coupling to hadrons involves knowledge of hadronic matrix elements at the scale $Q^2 \sim M_H^2$, at which the heavy quarks in general are not expected to decouple. However, if the Higgs boson mass M_H is smaller than the mass of the top quark, M_T, one can still perform an expansion in the ratio M_H/M_T; we expect this to be a reasonable procedure if $M_H \leqslant 100$ GeV. In this case, one finds

$$M_T \overline{t} t \rightarrow -\frac{2}{3}\frac{g^2}{32\pi^2} G^{\alpha\beta a} G^a_{\alpha\beta} + \cdots. \tag{12}$$

Since the mass of the hadron is defined as the forward matrix element of the energy-momentum tensor, the expression (12) leads to the following Yukawa vertex for the coupling of a Higgs boson to the hadron:

$$2^{1/4} G_F^{1/2} H \phi_h^2 \langle h | M_T \overline{t} t | h \rangle = 2^{1/4} G_F^{1/2} H \phi_h^2 \frac{2M^2}{27}; \tag{13}$$

this relation is valid in the chiral limit of massless light quarks [see Eq. (10)]; M_T is the mass of the heavy quark and M is the hadron mass. We put the number of light quarks, $N_F = 3$, and the number of colors, $N_c = 3$; ϕ_h, and H are hadron and Higgs operators. Note that, as a consequence of scale anomaly, Eq. (13) does not have an explicit dependence on the coupling α_s.

III. ESTIMATES FOR DOUBLE-POMERON HIGGS BOSON PRODUCTION CROSS SECTIONS

A. General formulas for double-Pomeron Higgs boson production

The amplitude for Higgs boson production in the Pomeron approach is given by (see, for example, Refs. [2,14])

$$M(h+h \rightarrow h+H+h) = g_1(t_1) g_2(t_2) \gamma(t_1,t_2) \eta_+(t_2) \eta_+(t_1) \times \left(\frac{s}{s_2}\right)^{\alpha_P(t_2)} \left(\frac{s}{s_1}\right)^{\alpha_P(t_1)}, \tag{14}$$

where $s_1 = (P_1 + q_1)^2$ and $s_2 = (P_2 + q_1)^2$ ($P_{1,2}$ are momenta of incoming hadrons); $\eta_+(t_i)$ is a signature factor, which for the Pomeron is

$$\eta_+(t_i) = i + \tan^{-1}\left(\frac{\pi \alpha_P(t_i)}{2}\right), \tag{15}$$

where $\alpha_P(t)$ is the Pomeron trajectory, $\alpha_P(t) = 1 + \Delta_P + \alpha'_P t$, with $\Delta_P \approx 0.08$ [8]; all other notations are evident from Fig. 2.

The cross section for DPHP in the central rapidity region ($y_H = 0$, where y_h is the rapidity of the produced Higgs boson) can be written as

$$\left.\frac{d\sigma}{dy_H dt_1 dt_2}\right|_{y_H=0} = \frac{1}{2s} |M(h+h \rightarrow h+H+h)|^2 \times \prod_{i=1,2} \frac{d^3 P'_i}{(2\pi)^3 2P'_{i,0}} \frac{d^2 p_{H,\perp}}{2(2\pi)^3} \times (2\pi)^4 \delta^{(4)}(P_1 + P_2 - P'_1 - P'_2 - p_H) \tag{16}$$

where P'_i are momenta of recoil hadrons, while p_H is the momentum of the produced Higgs boson.

Performing all integrations and recalling that $s_1 \cdot s_2 = M_H^2 \cdot s$ we obtain

$$\left.\frac{d\sigma}{dy_H dt_1 dt_2}\right|_{y_H=0} = \frac{2 g_1^2(t_1) g_2^2(t_2) \gamma^2(t_1,t_2)}{\pi (16\pi)^2} \left(\frac{s}{M_H^2}\right)^{2\Delta_P} \times e^{\alpha'_P \ln(s/M_H^2)[t_1+t_2]}. \tag{17}$$

We will assume that $\gamma(t_1,t_2)$ is a smooth function of t_1 and t_2 in comparison with $g_1(t_1)$ and $g_2(t_2)$. Indeed, the t dependence of g_i is related to the quark distribution inside the hadron while the t dependence of γ is determined by the mean transverse of the gluon inside the Pomeron. The typical scale for this momentum is $1/\alpha_P \approx 4$ GeV^2 which is much larger than the typical momentum of a quark in a hadron (≈ 0.1 GeV^2).

Using this assumption together with the simplest Gaussian parametrization for the vertex, $g_i(t_i) = g_i(0)\exp(-R_0^2|t_i|)$, we obtain

$$\left.\frac{d\sigma}{dy_H}\right|_{y_H=0} = \frac{8 g_1^2(0) g_2^2(0)}{\pi[16\pi B_{el}(s/M_H^2)]^2} \gamma^2(t_1=0,t_2=0) \left(\frac{s}{M_H^2}\right)^{2\Delta_P}. \tag{18}$$

Recalling now the well-known relation between the total and elastic cross sections for one Pomeron exchange, namely,

$$R_{el}(s) = \frac{\sigma_{el}(s)}{\sigma_{tot}(s)} = \frac{g_1(0) g_2(0)}{16\pi B_{el}(s)} \left(\frac{s}{s_0}\right)^{\Delta_P}, \tag{19}$$

where $B_{el} = 4R_0^2 + 2\alpha'_P \ln s$, one can derive

$$\left.\frac{d\sigma}{dy_H}\right|_{y_H=0} = \gamma^2(t_1=0,t_2=0) \times \frac{8}{\pi} R_{el}^2 \left(\frac{s}{M_H^2} s_0\right). \tag{20}$$

There is only one unknown factor in Eq. (20), namely, $\gamma^2(t_1=0,t_2=0)$. In the next subsection we present estimates for this factor using the nonperturbative approach that has been discussed in the Sec. II.

B. Production vertex $\gamma(t_1=0,t_2=0)$

Our estimate of $\gamma(t_1=0,t_2=0)$ consists of two steps: (1) For positive values of $t_1 = t_2 = m^2_{glueball}$ we can obtain $\gamma(t_1 = m^2_{glueball}, t_2 = m^2_{glueball})$ from Eq. (13); (2) using Eq. (14)

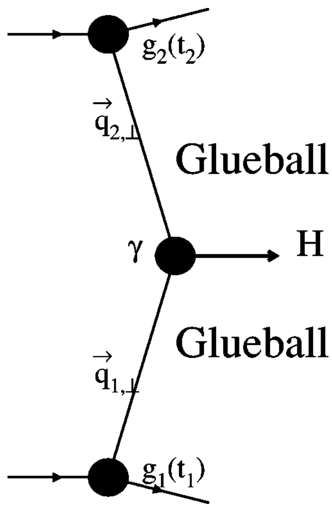

FIG. 4. Higgs boson emission from the glueball.

we can make an analytic continuation to the region $t_1 < 0$ and $t_2 < 0$, which corresponds to the scattering process.

We will assume that there exists a tensor 2^{++} glueball which lies on the Pomeron trajectory, namely, that its mass satisfies the following relation:

$$\alpha_P(t = m^2_{glueball}) = 1 + \Delta + \alpha'_P(0) m^2_{glueball} = 2. \tag{21}$$

There is no undisputed experimental evidence for such a meson but lattice calculations give for its mass $m_{glueball} = 2.4$ GeV [15]. This mass is a little bit higher than can be expected from Eq. (21) with the experimental $\alpha'_P(0) = 0.25\ \mathrm{GeV}^{-2}$ [8]. On the other hand, it is possible to describe experimental data using a smaller value of $\alpha'_P(0) \approx 0.17\ \mathrm{GeV}^{-2}$ which is needed to satisfy Eq. (21) with $m_{glueball} = 2.4$ GeV, assuming the presence of substantial shadowing corrections [16].

For the diagram in Fig. 4 the vertex $\gamma_{glueball}$ can be easily evaluated from Eq. (13); it is equal to

$$\gamma_{glueball} = 2^{1/4} G_F^{1/2} \frac{2 m^2_{glueball}}{27}. \tag{22}$$

One can see that Eq. (14) leads to the contribution described by Fig. 4. Indeed, for $t_i \to m^2_{glueball}$,

$$\eta_+(t_i) \to \frac{2}{\pi \alpha'_P (m^2_{glueball} - t_i)}. \tag{23}$$

(A more detailed discussion of the analytic properties of Reggeon exchange can be found in Ref. [17].) Using Eq. (23) and comparing Eq. (14) with the diagram of Fig. 4, we conclude that

$$\gamma(t_1 = m^2_{glueball}, t_2 = m^2_{glueball}) = \frac{\pi}{2} \alpha'_P(0) \gamma_{glueball}. \tag{24}$$

The Reggeon approach cannot tell us anything about the relation between $\gamma(t_1 = m^2_{glueball}, t_2 = m^2_{glueball}$ and $\gamma(t_1 = 0, t_2 = 0)$. The only thing that we can claim is that the signature factor takes into account the steepest part of the t behavior. Therefore, in the next subsection we will assume that

$$\gamma(t_1 = m^2_{glueball}, t_2 = m^2_{glueball}) = \gamma(t_1 = 0, t_2 = 0); \tag{25}$$

this is an extreme assumption which can be used to obtain an upper bound on the cross section. Uncertainties related to this and other assumptions we make will be discussed in detail in Sec. III D and in the summary, Sec. V.

Using Eq. (14), Eq. (17), and Eq. (22) we can calculate the width of Higgs boson decay into two glueballs, which is equal to

$$\Gamma(H \to 2\ \ \text{glueballs}(2^{++})) = \frac{\gamma^2_{glueball}}{32 M_H \pi} \left(\frac{M_H^2}{2 m^2_{glueball}} \right)^2 \approx 100\text{–}200\ \ \text{KeV} \ll \Gamma(H \to \text{hadrons}). \tag{26}$$

It should be stressed that no extra factors $M_H^2 / 2 m^2_{glueball}$ will appear for the width of the decay into glueballs with higher spins, due to the analytical continuation given by Eq. (14) and Eq. (17).[2]

C. Magnitude of the cross section

Using Eq. (22), Eq. (24), and Eq. (25) we can rewrite Eq. (20) in the simple form

$$\left.\frac{d\sigma}{dy_H}\right|_{y_H = 0} = 2\pi (\alpha'_P m^2_{glueball})^2 \frac{4\sqrt{2} G_F}{27^2} R^2 \left(\frac{s}{M_H^2} s_0 \right). \tag{27}$$

For $M_H = 100$ GeV, the factor $S/M_H^2 s_0$ is equal to 400 GeV^2 for $s_0 = 1\ \ \mathrm{GeV}^2$. Therefore, we can take $R_{el} \simeq 0.175$ (see Fig. 5) for Fermilab Tevatron energies. Equation (27) leads to

$$\left.\frac{d\sigma}{dy_H}\right|_{y_H = 0} (M_h = 100\ \ \text{GeV}, \quad \sqrt{s} = 1800\ \ \text{GeV}) = 6.4\ \ \text{pb}. \tag{28}$$

This is a very large number, especially if we recall that the total inclusive cross section for Higgs boson production in perturbation theory is on the order of 1 pb [18]. However, this estimate does not yet contain the suppression due to the (small) probability of the rapidity gap survival, which will be discussed in Sec. IV, where we present our final results. Since R_{el} grows with energy, $R_{el} \propto s^{\Delta}$, we expect that the cross section at the CERN Large Hadron Collider LHC energy is approximately 2 times larger than the one in Eq. (28).

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[2]We would like to thank V. Khoze and M.Ryskin for drawing our attention to the danger that a partial width of the decay in two glueballs could be large due to longitudinal polarization of the produced glueballs.

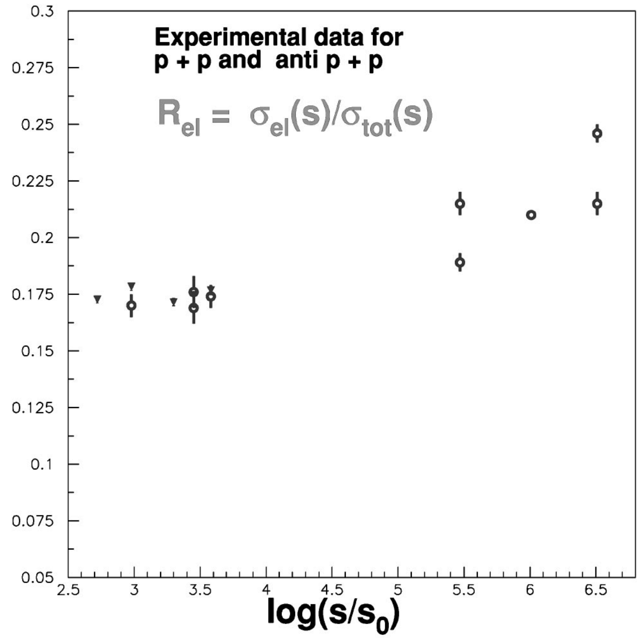

FIG. 5. Experimental data for the ratio $R_{el}(s)=\sigma_{el}(s)/\sigma_{tot}(s)$. Here $s_0=1$ GeV^2 and the logarithm is taken on base 10.

D. Uncertainties of our estimates

(1) Let us start with the value of R_{el}. We took it from the experimental data, but we nevertheless have two uncertainties associated with it. First, Eq. (19) is written for one Pomeron exchange while in experimental data at $\sqrt{s}\approx 20$ GeV we have about 30% contamination from the secondary Reggeons [8]. If we try to extract the one Pomeron exchange from the data, it reduces the value of the cross section for DPHP by 1.7 times. Therefore, the value for the cross section can be about 3.8 pb rather than Eq. (28). The second uncertainty in evaluation of R_{el} is the value of s_0; even though $s_0=1$ GeV appears in all phenomenological approaches [8,16], we have no theoretical argument for the value of s_0. However, since the ratio R_{el} in Fig. 5 is a rather smooth function of energy, we do not expect that the uncertainty in the value of s_0 can introduce a large error.

(2) We can take into account also the reactions of Eq. (2) and Eq. (3). In Eq. (27) we would then have to substitute

$$R_{el}\to R_D=R_{el}+\frac{\sigma^{DD}(s)}{\sigma_{tot}}, \tag{29}$$

where σ^{DD} is the cross section of the double-diffraction dissociation. Unfortunately, we do not have conclusive data on this cross section. However, recent Collider Detector at Fermilab (CDF) measurements [19] show that this cross section could be rather large (about 4.7 mb at the Tevatron energy).

(3) The principal uncertainty, however, is associated with the continuation from $t=m^2_{glueball}$ to $t=0$. This is a question which at present can only be addressed in the framework of different models. For example, in the Veneziano model [20] instead of $\eta_+(t)$ [see Eq. (15)] a new factor appears, namely,

$$\eta^V_+(t_i)=\Gamma(2-\alpha_P(t_i))e^{i\pi\alpha_P(t_i)/2}. \tag{30}$$

Equation (30) does not give the factor of $\pi/2$ in Eq. (24) and, therefore, decreases the value of the cross section given by Eq. (27) by a factor of 2.5. We will return to the discussion of the analytic continuation in the summary section.

(4) As we have discussed in Sec. II, we can evaluate the value of $\gamma(t_1=m^2_{glueball},t_2=m^2_{glueball})=\gamma_{glueball}$ only if $M_H/M_T<1$. The accuracy of Eq. (22) is $O(M_H^2/M_T^2)$ and we thus believe that Eq. (22) gives a reasonable estimate of $\gamma_{glueball}$ for Higgs mesons with $M_H\leqslant 100$ GeV.

IV. SURVIVAL OF LARGE RAPIDITY GAPS

As has been discussed intensively during the past decade (see Refs. [1,21–30]), the cross section of Eq. (27) has to be multiplied by a factor S^2_{spect}, which is the survival probability of large rapidity gap processes. The ''experimental'' cross section is therefore given by

$$\left.\frac{d\sigma(pp\to ppH)}{dy}\right|_{y=0}=S^2_{spect}\left.\frac{d\sigma_P(pp\to ppH)}{dy}\right|_{y=0}. \tag{31}$$

Here, $d\sigma_P(pp\to ppH)/dy$ denotes the cross section calculated in Eq. (27). The factor S^2_{spect} has a very simple meaning—it is the probability of the absence of inelastic interactions of the spectators which could produce hadrons inside the LRG. We have rather poor theoretical control of the value of the survival probability; this fact reflects the lack of knowledge of the ''soft'' physics stemming from nonperturbative QCD. Different models exist (see, for example, Refs. [25–28] and references therein), leading to the values about $S^2_{spect}\approx 10^{-1}$–$10^{-3}$ at Tevatron energies. Therefore, at the moment we have to use additional experimental information to obtain an estimate for S^2_{spect}; we use a ''theoretical approach'' only to discuss rather qualitative properties of the survival probability. For double-Pomeron processes, this S^2_{spect} has been discussed in Ref. [31]. The result of this analysis is that the value of the survival probability for double-Pomeron production is almost the same as for ''hard'' dijet production with the LRG between them. Fortunately, the value of S^2_{spect} has been measured [32], and is equal to 0.07 for the highest Tevatron energy.

Multiplying Eq. (28) by $S^2_{spect}=0.07$ and taking into account suppression due to the factor of Eq. (30), we obtain

$$\left.\frac{d\sigma}{dy_H}\right|_{y_H=0}(M_h=100\ \text{GeV},\ \sqrt{s}=1800\ \text{GeV})=0.2\ \text{pb}. \tag{32}$$

This estimate is not our final result yet, since we still have to correct it by the additional suppression factor S^2_{par} which describes the probability of the absence of parasite gluon emission around the Higgs boson production vertex [see Fig. 4(b)] [6,7,33]. As was argued in Ref. [7],

$$S^2_{par}=e^{-\langle N_G(\Delta y=\ln(M_H^2/s_0))\rangle}, \tag{33}$$

with

$$\langle N_G(\Delta y=\ln(M_H^2/s_0))\rangle=\frac{N_{hadrons}(\Delta y=\ln(M_H^2/s_0))}{N_{hadrons}\ \ (\text{one minijet})}\approx 2-4. \tag{34}$$

It gives $S_{par}^2=0.14$–0.014.

The appearance of this factor can be illustrated by the following argument: one of the most important differences between the diagrams of Fig. 2 and of Fig. 4 is the fact that the Pomeron exchange is almost purely imaginary while the glueball exchange leads to the real amplitude. The imaginary amplitude describes the production of particles and the Pomeron is associated with the inelastic process with large multiplicity. Therefore, normally, in a large rapidity region $\Delta y=\ln(M_H^2/s_0)$ we expect to see a large number of produced particles while in Fig. 2 we require that only one Higgs boson be produced. Therefore, it seems reasonable to expect a suppression for double-diffractive Higgs boson production, and this suppression can be described by Eq. (33) and Eq. (34).

It is worthwhile mentioning that in our approach, strictly speaking, there is no survival probability due to parasite emission; a smallness assigned to the factor S_{par}^2 therefore has to be interpreted as a suppression due to analytical continuation from $q_i^2=m_{glueball}^2$ to $q_i^2=0$. If we use a Gaussian formula for such a form factor $e^{-r^2(m_{glueball}^2-q_i^2)}$, Eq. (33) and Eq. (34) mean that the value of $r^2=(0.5$–$1)/m_{glueball}^2$. We think that it is a reasonable estimate for possible suppression, which looks self-consistent both from the point of view of the analytical continuation and from the microscopic picture of gluon emisson.

Finally, for the Tevatron energy we expect

$$\left.\frac{d\sigma(pp\to ppH)}{dy}\right|_{y=0}(\sqrt{s}=1.8\ \ \text{TeV})=S_{spect}^2S_{par}^2\left.\frac{d\sigma_P(pp\to ppH)}{dy}\right|_{y=0}=(0.0038-0.028)\ \ \text{pb}. \tag{35}$$

Extrapolating to the LHC energy, we have two effects that work in different directions: the rise of the Pomeron contribution and the decrease of the S_{spect}^2 with energy. From Ref. [31] we expect that $S_{spect}^2(\sqrt{s}=14\ \text{TeV})/S_{spect}^2(\sqrt{s}=1.8\ \text{TeV})\approx 0.75$ while the rise of the Pomeron exchange leads to an extra factor of 2 in Eq. (35). Therefore, our final estimate for the LHC is

$$\left.\frac{d\sigma(pp\to ppH)}{dy}\right|_{y=0}(\sqrt{s}=14\ \ \text{TeV})=S_{spect}^2S_{par}^2\left.\frac{d\sigma_P(pp\to ppH)}{dy}\right|_{y=0}=(0.0013-0.040)\text{pb}. \tag{36}$$

Equations (35) and (36) give significantly larger (by about 5 times) cross sections than expected for double-diffractive production in PQCD [33]. However, Ref. [7] contains an estimate of the upper bound on double Pomeron Higgs boson production in PQCD obtained by choosing the largest possible value for S_{par}^2 [see Eq. (34)]. This upper bound appears to be about 7 times larger than the highest value in Eq. (35).

V. SUMMARY AND DISCUSSION

The approach suggested in this paper is based entirely on nonperturbative QCD. We believe that such an approach is logically justified for diffractive Higgs boson production since even PQCD calculations show that this is, to a large extent, a ''soft'' process [see Eq. (5) and the following discussion]. However, just because of this, we have to stress again that the accuracy of our calculation is not very good. We feel, however, that our results support the idea [7] that in the PQCD approach to diffractive Higgs boson production the running QCD coupling has to be taken at the ''soft'' scale $Q^2\sim 1\ \text{GeV}^2$. As was argued in Ref. [7], in the Brodsky-Lepage-Mackenzie (BLM) prescription [34] of taking into account the running QCD coupling one can insert quark bubbles only in the t-channel gluon lines in Fig. 3. Therefore, the running QCD coupling depends on the transverse momenta of these gluons, and they are determined by the ''soft'' scale.[3] However, this statement about the scale in the running coupling constant should be taken with great caution since, as is shown in Ref. [33], a soft scale of the running coupling constant appears only when parasite emissions of Fig. 3(b) are taken into account in PQCD. This ''soft'' scale is essential for both the running coupling constant and for the parasite emission [see Eq. (11) of Ref. [33]]. It is interesting to note that Eq. (13) indeed does not depend on the QCD coupling, demonstrating the nonperturbative, ''soft'' character of the discussed process.

We obtain quite large values for the cross section of the diffractive Higgs boson production—after integration over the Higgs rapidity y in Eq. (35) and Eq. (36) we get

$$\sigma(pp\to ppH)(\sqrt{s}=1.8\ \ \text{TeV})=0.019-0.14\ \ \text{pb}. \tag{37}$$

and

$$\sigma(pp\to ppH)(\sqrt{s}=14\ \ \text{TeV})=0.0095-0.26\ \ \text{pb}. \tag{38}$$

Comparing our estimates with the ones based on PQCD [2–7] we conclude that the lowest of our values of the cross section of double-Pomeron Higgs boson production is about the same as the highest one in the PQCD approach. However, both our approach and the PQCD one suffer from large uncertainties, stemming from the analytical continuation in

[3]In this soft regime, the dependence on the coupling constant in the Pomeron can disappear as a consequence of the scale anomaly [35].

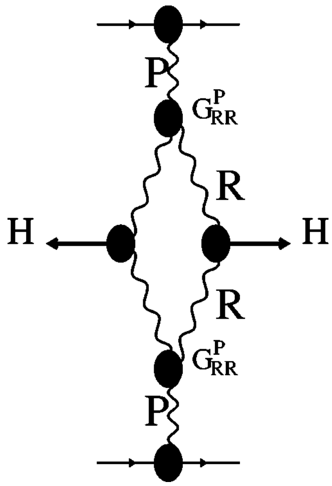

FIG. 6. Mueller diagram [36] for ''soft'' inclusive Higgs boson production.

our approach and from the survival probability of the rapidity gap and the absence of ''parasite emission'' S^2_{par} in PQCD.

Let us point out that Eq. (37) shows that the double-Pomeron Higgs boson production constitutes a substantial part of the total inclusive Higgs boson production. Moreover, our calculations lead to an additional contribution to the inclusive cross section which is shown in Fig. 6.

(Note that the triple-Pomeron interaction gives a very small contribution to the process in Fig. 6 due to the small real part in the Pomeron exchange [37].) Using the same approach as in derivation of Eq. (17) we obtain

$$\left.\frac{d\sigma_{incl}(pp\to H+X)}{dy}\right|_{y_H=0} = \gamma_R^2(t_1=0,t_2=0)\frac{2g_1(0)g_2(0)(G^P_{RR})^2}{\pi(16\pi B_R)^2}\frac{1}{\Delta_R^2}\left(\frac{s}{M_H^2}\right)^{\Delta_P}, \tag{39}$$

where $\Delta_R \approx 0.5$. As a first approximation we can take [see Eq. (24)]

$$\gamma_R(t_1=0,t_2=0)=\frac{\pi}{2}2^{1/4}G^{1/2}\frac{2\alpha_R' m_f^2}{27}, \tag{40}$$

where m_f is the mass of the f meson which is the first resonance on the secondary Reggeon trajectory, and $\alpha_R' m_f^2 = 1.5$. Substituting Eq. (40) into Eq. (39) we obtain

$$\left.\frac{d\sigma_{incl}(pp\to H+X)}{dy}\right|_{y_H=0} = \frac{(G^P_{RR})^2}{8B_R}\frac{B_R}{B_{el}}2^{1/2}G_F\frac{1}{3} \times R\left(\frac{s}{M_H^2}s_0\right). \tag{41}$$

Equation (41) gives

$$\left.\frac{d\sigma_{incl}(pp\to H+X)}{dy}\right|_{y_H=0} = \left(\frac{G^P_{RR}}{g}\right)^2 3.4\times 10^{-7} \ \text{mb}, \tag{42}$$

which does not yet contain the suppression arising from the analytical continuation. We take this suppression into account by a multiplying Eq. (42) by factor $S^2_{par}=0.14$–0.014. Unfortunately, we do not know the value for the ratio G^P_{RR}/g. In the triple-Pomeron parametrization of the cross section of diffractive dissociation in hadron reactions [38] this ratio changes from 1 to 0. For $G^P_{RR}/g=1$ we get for the ''soft'' inclusive cross section the value of 43–430 pb. On the other hand, taking the Field-Fox value [38] for this ratio we obtain a much smaller, but still very sizable value of 0.43–4.28 pb. It is thus clear that the evaluation of the ''soft'' contribution to the inclusive Higgs boson production is plagued by large uncertainties; however, it might be bigger than the PQCD one [18].

We hope that this paper will help to look at diffractive Higgs boson production from a different viewpoint, and will stimulate much needed further work. To our surprise, despite the very different nonperturbative method used here, our estimates for the double-diffractive production turn out to be not that far from the PQCD calculation [33] (the average is about 5 times larger). It adds some confidence to both approaches and gives us hope that one will be able to perform a reliable calculation in the nearest future.

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