# Gravity on a brane in infinite-volume extra space

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We generalize the mechanism proposed in a previous paper and show that a four-dimensional relativistic tensor theory of gravitation can be obtained on a delta-function brane in flat infinite-volume extra space. In particular, we demonstrate that the induced Ricci scalar gives rise to Einstein's gravity on a delta-function type brane if the number of space-time dimensions is bigger than five. The bulk space exhibits the phenomenon of infrared transparency. That is to say, the bulk can be probed by gravitons with vanishing four-dimensional momentum square, while it is unaccessible to higher modes. This provides an attractive framework for solving the cosmological constant problem.

## I. INTRODUCTION AND SUMMARY

The size of compact extra dimensions could be as big as a millimeter if there is a brane in extra space on which standard model particles are localized [1]. Gravity, in this framework, spreads into the large compact extra dimensions, and that is why it is weak compared to other interactions.

Yet another possibility is to maintain a brane-world hypothesis without actually compactifying extra space, but rather giving it some nonzero curvature [2]. As a result, the invariant volume of the extra space is still finite and is determined by the bulk cosmological constant [2].

The aim of the present work is to look for models in which the extra dimensions are uncompactified and, moreover, they are flat at infinity. In this case, the invariant volume of extra space is truly infinite. The matter and gauge fields can be localized on such a brane using the field-theoretic mechanisms of Refs. [3,4] and [5] respectively, or the string theory mechanism for localization of matter and gauge fields on a D-brane [6]. The immediate challenge, however, is to obtain four-dimensional gravity on the brane world volume where the standard model particles live.

In Ref. [7] the mechanism was proposed by which four-dimensional gravity can be obtained on a brane in 5-dimensional flat space-time. The mechanism is based on the observation that the localized matter fields on a brane (which couple to bulk gravitons) can generate the localized four-dimensional world volume kinetic term for gravitons. That is to say, four-dimensional gravity is "pulled over" (or induced) from the bulk gravity to the brane world volume by the matter fields confined to a brane.

It was shown in Ref. [7] that the four-dimensional Newton law is recovered on a brane. On the other hand, the relativistic effects of this theory differ from those of Einstein's gravity. In fact, the induced gravity on a brane in 5-dimensional Minkowski space is tensor-scalar gravity [7].

In this work we would like to pursue a more general strategy and study the mechanism of Ref. [7] on a brane in space-time with the number of dimensions bigger than five. The

main motivation to go beyond 5-dimensions comes from the well known fact that in 5-dimensional space there is no *static (non-inflating)* and *stationary* domain wall solution with a nonzero positive tension [8–10] which could be embedded in a flat and infinite transverse space. In contrast, in dimensions bigger than five, such nonzero tension solutions with a non-inflating world volume can be found.

In the present work we will show that using the mechanism of Ref. [7] one can obtain the four-dimensional laws of *Einstein's gravity* on a *delta-function type* brane world-volume if $D > 5$. This difference between five-dimensional and $D > 5$-dimensional theories can be traced back to the fact that in the former case the transverse to the brane space is one-dimensional and the transverse Green's functions are finite at the origin, while in the latter case these functions diverge at zero.

The physical picture which we obtain can be summarized as follows. There is a flat non-zero tension 3-brane embedded in $D > 5$ space-time which is also flat at infinity. Thus, the invariant volume of the extra space is infinite. The localized matter on the brane couples to the bulk gravity. As a result of this coupling the four-dimensional kinetic term for gravitons is generated via quantum loops on the world volume. These "four-dimensional" gravitons are nothing but part of bulk gravitons. Furthermore, the induced four-dimensional kinetic term gives rise to four-dimensional laws of gravity on a brane world volume.

As a next step we would like to address a more pragmatic question: The infinite-volume theories—what are they good for? Certainly they have an independent academic interest. Furthermore, they could lead to the modification of the laws of gravity at ultra-large distances [11–15]. Most importantly, as it was pointed out in Refs. [14,15], these theories give a new, yet unexplored way of thinking about the cosmological constant problem. This will be discussed in details in Sec. VI. Here, we just briefly reiterate the arguments of [14,15]. One could start with a supersymmetric theory in high-dimensions, let us say with superstring theory, M-theory or some of the low-energy supergravity truncations. If the brane on which we live is a non-Bogomol'nyi-Prasad-Sommerfeld (BPS) brane, then it can be used to break all the supersymmetries on the world volume theory [16] (stability of such a brane can in principle be warranted by topological reasons,

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or equivalently by giving some conserved charges to the brane). Thus, classically the brane world volume is not supersymmetric, while the bulk is supersymmetric. The question is whether the bulk supersymmetry could be preserved in the full quantum theory. The answer is positive and the key point here is that the bulk has an infinite volume. Because of this, transmission of supersymmetry (SUSY) breaking from the brane world volume to the bulk vanishes as inverse volume. Thus, one has SUSY in the whole high-dimensional theory without Fermi-Bose degeneracy in 4D theory.[1] Furthermore, imposing the condition that the bulk preserves the $R$ symmetry along with local SUSY, we obtain that the bulk cosmological constant is zero to all orders in full quantum theory. How about the 4D cosmological constant which we should supposedly be observing? There are two parts to the answer to this question. The first one is that the 4D cosmological constant which is produced by the brane world volume theory can be re-absorbed by rescaling of the brane tension. Such a brane, in $D > 5$ will or will not inflate (in $D = 5$ it inflates with necessity [8–10]). The reason why it might not inflate in the present case is that the bulk supersymmetry and the induced term might not allow it to do so. Seemingly alternative, but in fact an equivalent way of thinking about this is as follows: At extremely low energies, the theory at hand is not four-dimensional. Rather it is higher-dimensional due to the presence of the infinite-volume bulk (in this sense the physics is inverted upside down compared to the theories with compact of warped extra dimensions, where at lower energies the theory becomes more and more four-dimensional). Therefore, what one ''sees'' at extremely low energies, is not the 4D cosmological constant but rather the higher-dimensional cosmological constant which is zero due to bulk SUSY and $R$ symmetry. We will discuss these issues in details in Sec. VI.

## II. GRAVITY ON A BRANE IN EXTRA MINKOWSKI SPACE

Let us suppose that the bulk action has the following general form:

$$S_{\text{bulk}} = \int d^{4+N} X \sqrt{|G|} \mathcal{L}(G_{AB}, \mathcal{R}_{ABCD}, \Phi), \quad (1)$$

where the capital Latin indices run over $D = (4+N)$-dimensional space-time. $G_{AB}$ denotes the metric of D-dimensional space-time, $\mathcal{R}_{ABCD}$ is the D-dimensional Riemann tensor, and $\Phi$ collectively denotes all other bulk fields. Suppose that there is a 3-brane embedded in this space. The surface term which is necessary for the correct variational procedure for the action will be implied in the bulk action in all the discussions below. We simplify further our presentation by considering the whole picture in the effective field theory framework. Our goal will be to find the

properties of the system which are independent of these simplifications. Let us suppose that the 3-brane is localized in the extra space so that it asymptotes to a flat space at infinity. We split the coordinates in D-dimensions as follows:

$$X^A = (x^\mu, y^m), \quad (2)$$

where Greek indices run over four-dimensional brane world volume, $\mu = 0,1,2,3$, and small Latin indices over the space transverse to a brane, $m,n,i,j = 4,5,\ldots,4+N$.

The Dirac-Nambu-Goto action for a brane takes the form

$$S_{\text{3-brane}} = -T \int d^4 x \sqrt{|\det \bar{g}|}, \quad (3)$$

where $T$ stands for the brane tension and $\bar{g}_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B G_{AB}$ denotes the induced metric on a brane. In the most part of this work, unless stated otherwise, we treat a brane as a *delta-function type* singular source which is located at a point $y_m = 0$ in the extra space. Moreover, for the moment we neglect its fluctuations. Therefore, the induced metric can be written as follows:

$$\bar{g}_{\mu\nu}(x) = G_{\mu\nu}(x, y_n = 0). \quad (4)$$

In general, there could be localized matter fields on the brane world volume. These can be taken into account by writing the following action for the brane:

$$\bar{S}_{\text{3-brane}}^{\text{matter}} = S_{\text{3-brane}} + \int d^4 x \sqrt{|\det \bar{g}|} \tilde{\mathcal{L}}(\phi), \quad (5)$$

where $\phi$ denotes collectively all the localized fields for which the four-dimensional Lagrangian density is $\tilde{\mathcal{L}}$.

Note that in the classical theory, which we are discussing so far, the 4D Ricci scalar on the brane world volume is not present. Thus, the localized particles separated at a distance $r$ on a brane interact via the $(4+N)$-dimensional gravitational force-law, that is $F \sim 1/r^{2+N}$. This holds as long as the classical theory is concerned. However, in full quantum theory the 4D Ricci scalar will be generated (along with other terms) on a brane world volume. This is due to quantum loops of the matter fields which are *localized* on a brane world volume [7] (see also Appendix A). As a result, the following world volume terms should be included into the consideration in the full quantum theory:

$$S_{\text{ind}} = \bar{M}^2 \int d^4 x \sqrt{|\det \bar{g}|} [\bar{\Lambda} + \bar{R}(x) + \mathcal{O}(\bar{R}^2)], \quad (6)$$

where $\bar{M}$ is some parameter which depends on details of the world volume model [7] (see also Appendix A). For phenomenological reasons $\bar{M}$ should be of the order of the 4D Planck scale:

$$\bar{M} \sim M_{\text{Pl}} \simeq 10^{19} \text{GeV}. \quad (7)$$

In Sec. V we will discuss how this big scale could be generated.

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[1]This is close in spirit to the $(2+1)$-dimensional example of [17] and $(3+1)$-dimensional example of [18], although the mechanism and the properties are very different.

After the brane is included in the theory translation invariance in the directions transverse to the brane is broken. On the other hand, four-dimensional reparametrization invariance ("gauge invariance") along the brane world-volume coordinates is preserved in the theory. This invariance requires that the induced terms in Eq. (6) preserve this 4D gauge symmetry. Possible operators with derivatives with respect to the transverse coordinates are not induced on the world volume of a delta-function type brane since the localized particles on a brane have zero transverse momenta. This can also be seen by explicit calculations of loop diagrams in gauge invariant regularization schemes.

$\bar{\Lambda}$ in Eq. (6) is an induced four-dimensional cosmological constant. The role of this term is that it renormalizes the brane tension. Furthermore, $\bar{R}(x)$ is the four-dimensional Ricci scalar which is constructed out of the induced metric $\bar{g}_{\mu\nu}(x)$ defined in Eq. (4).

All terms in Eq. (6) are consistent with the symmetries of the theory in which conformal invariance and SUSY are broken in the brane world volume. Any realistic brane-world model should possess these properties.[2] Moreover, the terms in Eq. (6) are relevant operators of the four-dimensional world volume theory which will be induced on the brane even if they were not present at the first place [7]. Since these terms are unavoidable in any realistic brane world scenario, we should study the physical consequences of term.

The total brane world volume action takes the form

$$S_W = -T' \int d^4x \sqrt{|\bar{g}|} + \bar{M}^2 \int d^4x \sqrt{|\bar{g}|} [\bar{R}(x) + \mathcal{O}(\bar{R}^2) + \cdots], \quad (8)$$

where $\bar{g} \equiv \det \bar{g}$, and $T' \equiv T - \bar{\Lambda}\bar{M}^2$ is the renormalized brane tension which absorbs the induced four-dimensional cosmological constant. Dots in this expression stand for other possible world volume matter fields and interactions which we will omit below for simplicity. The field theory on a brane world volume is an effective field theory with a cutoff.[3] In the effective field theory framework the higher derivative terms appearing in Eq. (8) are suppressed by higher powers of $\bar{M}$ and can be neglected in the leading approximation.

### III. FOUR-DIMENSIONAL GRAVITY ON A BRANE

In this section we study the laws of gravity on a brane with the world volume action given by Eq. (8). We adopt a simplest setup in which the gravitational part of the bulk Lagrangian contains only the Einstein term while the gravity on the world volume is given by Eq. (8):

$$M^{2+N} \int d^{4+N}X \sqrt{|G|} \mathcal{R}_{(4+N)} + \bar{M}^2 \int d^4x \sqrt{|\bar{g}|}\bar{R}. \quad (9)$$

Here, $M$ denotes the Planck constant of the bulk theory. For the simplicity of calculations we will temporarily neglect the brane tension term, i.e., we put $T' = 0$. This is not essential as long as one is dealing with a theory in dimensions higher than 5. In this case, there can exist brane solutions with *static* world volume which have a *non-zero* tension. Therefore, non-zero $T$ can consistently be restored back (see Secs. V and VI). However, the case $D = 5$ is special. As is well known [8–10], any stationary nonzero *positive* tension brane inflates in 5D. Hence, it is not feasible in 5D to go from the theory with a zero-tension brane to a viable model where the brane tension has a positive finite value. Moreover, it was claimed in Ref. [19] that any constant-curvature bulk in 5D cannot control the brane cosmological constant. We will discuss these issues in more details in Sec. VI. Before that, we assume that the number of space-time dimensions is greater than 6 (although, as we will show below, some of our results will also be applicable for the 6-dimensional case.).

#### A. Newtonian gravity on a brane

First we study Newtonian potential between two localized masses on a brane. For this we can drop temporarily the tensor structure in the graviton propagator. Effectively, this is equivalent to the exchange of a massless scalar mode in the world volume theory.[4] Thus, we define the prototype Lagrangian for this scalar:

$$M^{2+N} \int d^{4+N}X (\partial_A \Phi(x,y))^2 + \bar{M}^2 \int d^4x (\partial_\mu \Phi(x,y=0))^2. \quad (10)$$

Here, the first and the second terms are respectively counterparts of the bulk Ricci scalar $\mathcal{R}_{(4+N)}$ and the induced 4D Ricci scalar $\bar{R}$ in Eq. (9).[5]

We are looking for the distance dependence of interactions in a 4D world volume theory. For this we should find the corresponding retarded Green's function and calculate the potential. The equation for the Green's function takes the form

$$(M^{2+N}\partial_A \partial^A + \bar{M}^2 \delta^{(N)}(y_m)\partial_\mu \partial^\mu)G_R(x,y_m;0,0)$$
$$= -\delta^{(4)}(x)\delta^{(N)}(y_m), \quad (11)$$

where $G_R(x,y_m;0,0) = 0$ for $x_0 < 0$.

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[2]Note that the induced 4D cosmological term $\bar{\Lambda}$ is not generated if the brane world volume theory is supersymmetric. However, the other terms in Eq. (6) will still be induced in a non-conformal supersymmetric theory.

[3]This statement will be elucidated in details in Sec. V.

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[4]Even for warped backgrounds equations for scalars are similar to those of gravitons [20].

[5]Note that the scalar model considered here is just used as a demonstration for more complicated model of gravity. If a scalar field theory is considered independently by its own, then on a brane with a finite width one expects additional induced terms containing derivatives with respect to $y$. However, these terms are not generated for scalars on the delta function type brane considered in this work since all the particles in the matter loops which induce world volume terms have exactly zero transverse momenta.

The potential at a distance $r$ along the brane world volume is determined as follows:

$$V(r) = \int G_R(t, \vec{x}, y_m = 0; 0, 0, 0) dt, \qquad (12)$$

where $r \equiv \sqrt{x_1^2 + x_2^2 + x_3^2}$. To find a solution of Eq. (11) let us turn to Fourier-transformed quantities with respect to the world volume four-coordinates $x_\mu$:

$$G_R(x, y_m; 0, 0) \equiv \int \frac{d^4 p}{(2\pi)^4} e^{ipx} \tilde{G}_R(p, y_m). \qquad (13)$$

In Euclidean space equation (11) takes the form

$$(M^{2+N}(p^2 - \Delta_N) + \bar{M}^2 p^2 \delta^{(N)}(y_m)) \tilde{G}_R(p, y_m) = \delta^{(N)}(y_m). \qquad (14)$$

Here $p^2$ denotes the square of an Euclidean four-momentum, $p^2 = p_4^2 + p_1^2 + p_2^2 + p_3^2$, and $\Delta_N$ stands for the Laplacian of the $N$-dimensional transverse space.

We look for the solution of Eq. (14) in the following form:

$$\tilde{G}_R(p, y_m) = D(p, y_m) B(p), \qquad (15)$$

where $D(p, y_m)$ is defined as follows:

$$(p^2 - \Delta_N) D(p, y_m) = \delta^{(N)}(y_m). \qquad (16)$$

$B(p)$ is some function to be determined.[6] Using this decomposition one finds

$$\tilde{G}_R(p, y_m) = \frac{D(p, y_m)}{M^{2+N} + \bar{M}^2 p^2 D(p, 0)}. \qquad (17)$$

For the case $D > 5$ the function $D(p, y_m)|_{y_m=0}$ diverges.[7] Therefore, the expression for $\tilde{G}_R(p, y_m)$ has a jump at $y_m = 0$. In the brane world volume, i.e., for $y_m = 0$, we find

$$\tilde{G}_R(p, y_m = 0)| = \frac{1}{\bar{M}^2 p^2}, \qquad (18)$$

which is nothing but the Green's function for a four-dimensional theory.

Therefore, we conclude that the static potential between a couple of point-like sources on a brane in $D > 5$ scales with distance between them as a four-dimensional potential:

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[6]There are some subtleties related to the fact that seemingly one has a product of two singular functions in Eqs. (14) and (15). To avoid this, one should work with the regularized expression: $\tilde{G}_R(p, y_m) = \lim_{\epsilon_m \to 0} D(p, y_m + \epsilon_m) B_\epsilon(p)$. A careful analysis with this regularization shows that the formal derivation with singular functions presented below is valid.

[7]This is another reason why the case $D = 5$ is exceptional; here $D(p, y_m)|_{y_m=0}$ is finite; see [7] and Appendix B.

$$V(r, y_m = 0) = \frac{1}{8\pi \bar{M}^2} \frac{1}{r}. \qquad (19)$$

In the approximation of a delta-function type brane, which we adopt in the present work, this behavior is expected to hold all the way up to infinite distances. However, for a finite thickness brane, the four-dimensional Newton law might be changed at very large (presumably of the order of the present Hubble size) distances, see discussions in Sec. IV.

Outside of the brane, i.e., for $y_n \neq 0$, there are two, physically distinct cases to consider. These differ by the value of the four-momentum square: $p^2 = 0$, and $p^2 \neq 0$. We will discuss them in turn.

(i) *Vanishing four-momentum square*, $p^2 = 0$

In this case the expression (14) reduces to the equation for Euclidean Green's function in the transverse $N$-dimensional space. In general, this Green's function is well known:

$$\tilde{G}_R(0, y \neq 0) \propto \lim_{p^2 \to 0} \frac{1}{M^{2+N}} \left( \frac{p}{|y|} \right)^{(N-2)/2} K_{(N-2)/2}(p|y|), \qquad (20)$$

where $|y| \equiv \sqrt{y_1^2 + y_2^2 + \cdots + y_N^2}$ and $K(p|y|)$ denotes the McDonald function. For $p^2 = 0$ this function scales as follows:

$$\sim \frac{1}{|y|^{N-2}}, \quad \text{for} \quad N > 2. \qquad (21)$$

For the $D = 6$ case ($N = 2$) this has a logarithmic singularity at $p^2 = 0$:

$$\sim \ln(p|y|)|_{p^2=0} \to \infty, \quad \text{for} \quad N = 2. \qquad (22)$$

Therefore, for $p^2 = 0$ there exists a solution in the form of the $N$-dimensional Green's function (20). This means that the $p^2 = 0$ mode provides interactions between world volume states and bulk states. This mode should be produced with a non-zero three-momentum along the brane world volume.

(ii) *Non-vanishing four-momentum square*, $p^2 \neq 0$

In this case physics is rather different. Outside of a brane, i.e., for $y_m \neq 0$, the Green's function vanishes:

$$\tilde{G}_R(p, y_m \neq 0)|_{p^2 \neq 0} = 0. \qquad (23)$$

In other words, the $p^2 \neq 0$ mode cannot give rise to any interactions of the localized world volume matter with the matter which is placed in the bulk. Therefore, the interaction between a particle localized on a brane and a particle placed in the bulk is completely determined by the $p^2 = 0$ solution and has the form given in Eq. (21).

## B. Tensor structure of the graviton propagator

We have established in the previous subsection that the 4D Newton law is reproduced on the brane. However, this is not enough. The relativistic theory of gravitation on the brane should be the Einstein tensor gravity plus possible higher derivative terms. In the minimal 5D setup studied in

Ref. [7] this is not the case: the relativistic model on a brane world volume is tensor-scalar gravity. The unwanted additional scalar there comes from the extra polarization of a bulk 5D graviton [7]. In this section we will show that in $D > 5$ the four-dimensional theory on a delta-function type brane is consistent 4D tensor gravity.

To show this we need to study the tensor structure of the graviton propagator. Let us introduce the metric fluctuations:

$$G_{AB} = \eta_{AB} + h_{AB}. \qquad (24)$$

We choose *harmonic gauge* in the bulk:

$$\partial^A h_{AB} = \frac{1}{2} \partial_B h_C^C. \qquad (25)$$

It can be checked that the choice

$$h_{\mu m} = 0, \quad m = 4, \dots, 4+N, \qquad (26)$$

is consistent with the equations of motion for the action (9) which is amended by a point-like source term located on a brane. Thus, the surviving components of $h_{AB}$ are $h_{\mu\nu}$ and $h_{mn}$. In this gauge the $\{mn\}$ components of Einstein's equations yield

$$(6-D)\partial_A \partial^A h_m^m = (D-4)\partial_A \partial^A h_\mu^\mu. \qquad (27)$$

Indices in all these equations are raised and lowered by a flat space metric tensor. Finally, we come to the $\{\mu\nu\}$ components of the Einstein equation. After some rearrangements these take the form

$$M^{2+N}\left( \partial_A \partial^A h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\partial_A \partial^A h_\alpha^\alpha - \frac{1}{2}\eta_{\mu\nu}\partial_A \partial^A h_n^n \right)$$

$$+ \bar{M}^2 \delta^{(N)}(y_m)\left( \partial_\alpha \partial^\alpha h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\partial_\beta \partial^\beta h_\alpha^\alpha \right.$$

$$\left. + \frac{1}{2}\eta_{\mu\nu}\partial_\beta \partial^\beta h_n^n - \partial_\mu \partial_\nu h_n^n \right) = T_{\mu\nu}(x)\,\delta^{(N)}(y_m). \qquad (28)$$

Here, the energy-momentum tensor for the localized on a brane source is denoted by $T_{\mu\nu}(x)\,\delta^{(N)}(y_m)$. As before, there are two groups of terms on the left-hand side (LHS) of the equation (placed in separate parenthesis). The first group originates from the bulk Ricci scalar $\mathcal{R}_{(4+N)}$ and the second one from the induced 4D Ricci scalar $\bar{R}$.

In order to determine the tensor structure of the graviton propagator it is convenient to bring this expression to the following form:

$$(M^{2+N}\partial_A \partial^A + \bar{M}^2 \delta^{(N)}(y_m)\partial_\alpha \partial^\alpha)h_{\mu\nu}$$

$$= \left\{ T_{\mu\nu}(x) - \frac{1}{2}\eta_{\mu\nu}T_\alpha^\alpha(x) \right\}\delta^{(N)}(y_m)$$

$$- M^{2+N}\frac{1}{2}\eta_{\mu\nu}\partial_A \partial^A h_n^n + \bar{M}^2 \delta^{(N)}(y_m)\partial_\mu \partial_\nu h_n^n. \qquad (29)$$

The tensor structure of the terms which contain $T_{\mu\nu}$ on the RHS of this equation is that of for-dimensional Einstein's gravity:

$$\left\{ T_{\mu\nu}(x) - \frac{1}{2}\eta_{\mu\nu}T_\alpha^\alpha(x) \right\}. \qquad (30)$$

However, as we see there are two additional term on the RHS of Eq. (29) [written in the second line of Eq. (29)]. Let us start with the second term. In the momentum space this term is proportional to the product $p_\mu p_\nu$. Therefore, its contribution vanishes when it is convoluted with a conserved energy-momentum tensor. In that respect, it is similar to gauge dependent terms occurring in graviton propagators. This term is harmless. The first term, $\eta_{\mu\nu}\partial_A \partial^A h_n^n$, however, cannot be removed by gauge transformations. Depending on dimensionality of space-time, this term might or might not give rise to additional contributions to 4D gravity. For instance, in accordance with Eq. (27), in the four-dimensional case ($D=4$) the expression $\eta_{\mu\nu}\partial_A \partial^A h_n^n$ vanishes. Therefore, we recover ordinary 4D Einstein's gravity. However, in $D = 5$, as was shown in [7], this extra term gives rise to the additional scalar exchange and, as a result, the 4D theory is tensor-scalar gravity. Our goal is to establish which of these possibilities is realized in higher dimensions.

As before, let us make the Fourier-transform with respect to the four world volume coordinates. Equation (29) can be rewritten in the following form:

$$[M^{2+N}(p^2 - \Delta_N) + \bar{M}^2 \delta^{(N)}(y_m)p^2]\tilde{h}_{\mu\nu}(p,y_m)\tilde{T}'^{\mu\nu}$$

$$= \left\{ \tilde{T}_{\mu\nu}\tilde{T}'^{\mu\nu} - \frac{1}{2}\tilde{T}_\alpha^\alpha \tilde{T}'^\beta_\beta \right\}\delta^{(N)}(y_m) - \frac{1}{2}\tilde{T}'^\alpha_\alpha(p^2 - \Delta_N)\tilde{h}_m^m. \qquad (31)$$

Here, the sign tilde denotes the Fourier-transformed quantities and $\tilde{T}'^{\mu\nu}$ is a conserved energy-momentum tensor in the momentum space. As in the previous subsection there are two different types of solutions to this equation. Let us start with the solution on the brane world volume, i.e., that with $y_n = 0$. Using Eqs. (27), (31) and the methods we used for the scalar case [the decompositions similar to Eq. (15)] we find that

$$\tilde{h}_{\mu\nu}(p,y_m)\tilde{T}'^{\mu\nu}\big|_{y=0} = \left[ \tilde{T}_{\mu\nu}\tilde{T}'^{\mu\nu} - \frac{1}{2}\tilde{T}_\alpha^\alpha \tilde{T}'^\beta_\beta \right]\frac{1}{\bar{M}^2 p^2}. \qquad (32)$$

This is a perfectly 4D solution. Therefore, we find that the distance dependence and the tensor structure of the graviton propagator on a delta-function type brane in $D>5$ is that of 4D Einstein's gravity.

Having this result obtained, let us investigate what happens outside of the brane, i.e., for $|y| \neq 0$. As with scalars in the previous subsection, there are two different physical cases to consider.

(i) *Vanishing four-momentum square*, $p^2 = 0$

After some algebra one finds the following solution:

$$\tilde{h}_{\mu\nu}(0,y_m)\tilde{T}'^{\mu\nu}|_{p^2=0} = \left[ \tilde{T}_{\mu\nu}\tilde{T}'^{\mu\nu} - \frac{1}{D-2}\tilde{T}^{\alpha}_{\alpha}\tilde{T}'^{\beta}_{\beta} \right] D(0,y). \tag{33}$$

The expression for $D(0,y)$ is given in Eq. (21). This is just a N-dimensional solution with a D-dimensional tensor structure. Furthermore, this solution gives rise to interactions (33) between the matter which is localized on the brane and the matter which is placed in the bulk. This mode should be produced with a nonzero three-momentum directed along the brane world volume.

(ii) *Non-vanishing four-momentum square*, $p^2 \neq 0$

As is expected, this case is similar to that with scalars. Here we find

$$\tilde{h}_{\mu\nu}(p,y_n)\tilde{T}'^{\mu\nu}|_{y \neq 0} = 0. \tag{34}$$

Thus, the $p^2 \neq 0$ mode cannot produce interactions between the brane world volume matter and the bulk matter.

Let us note that in the world volume theory there are remaining components of the high-dimensional metric, $h_{mn}$, which from the four-dimensional standpoint look as scalar particles with no mass gap. In the full quantum theory there is no symmetry which protects masses of these states against radiative corrections. Therefore, they are expected to acquire on the brane potentials (masses) of order the cutoff of the world volume model.

Summarizing this section we conclude that the four-dimensional gravity on the delta-function type brane with the induced 4D Ricci scalar (in $D>5$ case) is consistent tensor gravity with correct Newtonian potential and the correct Einstein tensor structure of the graviton propagator.

## IV. INFRARED TRANSPARENCY OF EXTRA SPACE

The bulk of this paper is devoted to the study of a delta-function type brane. A similar consideration for a "fat" brane, which turns out to be much more involved and rich, will be given elsewhere [21]. In the present and next sections, however, we will discuss some qualitative features which should emerge when "fat" branes are considered.

In higher-dimensional theories one usually expects that extra space can be probed in very high energy accelerator experiments or, in very short distance gravity measurements. This is certainly true when there is no induced kinetic terms on the brane. In the scenarios where the extra space is compact, or, alternatively, if it is warped as in [2] the invariant physical volume of the extra space is finite. In the former case this is true by definition, while in the case of warped but

non-compact spaces this is also true since there is a non-trivial warp-factor which makes the invariant volume finite $\int_{-\infty}^{+\infty} dy \sqrt{g} < \infty$ (although the proper distance in this case is infinite). Therefore, at very high energies (or equivalently short distances) the extra finite-volume space is probed.

The natural question to ask is whether the same phenomenon holds in infinite-volume theories discussed in the present work. We will argue below that physics of the infinite-volume theories is inverted upside down compared to the finite-volume theories mentioned above. In fact, we will argue that the extra bulk space might be probed only at extremely small (close to zero) energies, or equivalently, at ultra-large (close to the present Hubble size) distances.

To see this let us recall the results of the previous sections. There we found that the Green's function on a brane behaves precisely as a 4D solution. That is to say, the world volume laws of gravity are four-dimensional up to infinite distances. We also have found that the solution with zero four-dimensional momentum square, $p^2 = 0$, has the dependence on the bulk coordinates. On the other hand, any other solution with $p^2 \neq 0$ is zero in the bulk. Thus, we say that the bulk is transparent only for the $p^2 = 0$ mode. This phenomenon holds as long as the brane is a delta-function type source.

However, the brane at hand might have certain finite transverse width. This could be set by the size of the core of the brane if it appears as a smooth soliton in the bulk, or by the effective size of the transverse fluctuations of the brane. In any case, if the brane has a finite width, there could exist the modes with tiny but still nonzero $p^2$ which would be able to leak into the bulk. In this respect, one could define a certain critical momentum, let us call it $p_c$, below which the theory could become higher-dimensional. Above this momentum the world volume theory behaves as a 4D model. Higher we go in momenta farther we are from $(4+N)$-dimensional theory, and the world volume physics becomes more- and more four-dimensional. This is precisely opposite to what one obtains in theories with finite-volume extra dimensions.[8]

In terms of distances, we define the critical distance

$$r_c \equiv \frac{1}{p_c}.$$

Below this distance physics is four-dimensional. However, for distances bigger than $r_c$ gravity changes. On phenomenological grounds, $r_c$ should at least be of the average size of clusters of galaxies or so. The shorter distances we probe, physics becomes more and more four-dimensional.

Let us try to make these arguments a bit more quantitative. For this let us look at the plane wave solutions of equations of motion (for simplicity we drop the tensor structures again):

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[8]Note that due to the specific nature of the $D=5$ case this phenomenon takes place in 5D models even for a brane which has a zero width (a delta-function type brane) [11–14].

$$(M^{2+N}(p^2-\Delta_N)+\bar{M}^2 p^2 \delta^{(N)}(y_m))e^{ipx}f(y)=0. \quad (35)$$

There is only one plane-wave solution to this equation with $f(0)\neq 0$, that is the wave with $p^2=0, f(y)=$ const. The reason for such a behavior is that the term with $\delta^{(N)}(y_m)$ in Eq. (35) dominates over the first term if $p^2\neq 0$ and $D>5$. Thus, the resulting intrinsic 4D nature of the theory for $p^2\neq 0$. On the other hand, for $p^2=0$ the second term in Eq. (35) can be made small and physics is determined by the first term which naturally gives rise to D-dimensional results.[9]

Let us now suppose that the brane has a very small but finite width $B$. In this case, the $\delta^{(N)}(y_m)$ function in Eq. (35) should be replaced by some smooth spread function. As before, there are two competing terms in Eq. (35), one is D-dimensional and another one is 4-dimensional. Qualitatively, the effects of these terms in the vicinity of a brane are weighted respectively by the quantities

$$M^{2+N}\left(p^2-\frac{1}{B^2}\right) \quad \text{and} \quad \bar{M}^2 p^2 \frac{1}{B^N}. \quad (36)$$

At large momenta and $N>2$ the second term dominates since $B$ is very small. Therefore, the theory is 4-dimensional in that regime. However, as we discussed above, there is a critical value of the momentum, $p_c$, at which the two terms in Eq. (36) could become comparable. This value can be estimated:

$$p_c^2 \sim \frac{M^{2+N}B^{N-2}}{M^{2+N}B^N+\bar{M}^2}. \quad (37)$$

Assuming now that the brane width is of the order of inverse $\bar{M}$, $B\sim 1/\bar{M}$, and, furthermore, assuming that $\bar{M}\gg M$, we arrive at the following estimate for the critical momentum and distance:

$$r_c^2 = p_c^{-2} \sim \frac{\bar{M}^N}{M^{2+N}}. \quad (38)$$

A simple estimate with $M\simeq$ TeV and $\bar{M}=10^{19}$ GeV gives the following result for the $D=10$ case, $r_c\sim 10^{26}$ cm. For smaller numbers of dimensions the value of $r_c$ decreases. At the distances smaller that $r_c$ we will observe the four-dimensional world. However, at distances bigger then $r_c$ the laws of gravity would change.[10]

## V. COMMENTS ON THE HIERARCHY PROBLEM

In the brane world scenarios with large compact [1] or warped extra dimensions with two branes [2] the hierarchy problem is solved due to the finite size of the extra space.

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[9]This suppression is similar to the one found in Ref. [22].

[10]Note that these two regimes should be matched at the distance scale $r_c$. This is non-trivial task for gravitons, since the number of physical degrees of freedom for them can change. Thus, the matching for the case of a ''fat'' brane needs detailed investigations [21].

The natural question is whether the hierarchy problem could be solved if the extra space has an infinite volume? Let us note that the scale $M$ can in principle take any value between the 4D Planck scale, $10^{19}$ GeV, and the fundamental Planck scale of the framework [1], that is a few TeV. In theories with infinite-volume extra dimensions there is no phenomenological motivation for any particular value of $M$ in that domain. Thus, we keep $M$ as a free parameter. If $M$ is of order the 4D Planck scale, then there is no hierarchy between $M$ and $\bar{M}$ to explain. However, one should still elucidate how the Higgs boson mass term is stabilized.

On the other hand, if the scale $M$ is in a TeV region, then the big hierarchy between $M$ and $\bar{M}$ should somehow be justified. In fact, there are two issues to be addressed in this respect:

Given the high-dimensional fundamental scale $M$, how can we obtain the scale on the brane, $\bar{M}$, which is much bigger then $M$?

How does one stabilize the standard model Higgs boson mass against quadratically divergent radiative corrections?

The answer to the first question is based on the fact that in theories which allow for brane solutions, there is a nonperturbative mass scale which is inversely proportional to the coupling constant of the theory. If the coupling is very small then the nonperturbative scale could be much bigger than the fundamental scale of the model. Then, it is rather natural that the brane energy density (tension) is related to the fundamental scale of the world volume theory as follows:

$$T \propto \frac{M^4}{\lambda}, \quad (39)$$

where $\lambda$ is the coupling constant, or some other positive power of the coupling constant of the fundamental theory. Let us ask the following question: What would be the value of the induced Planck constant, $\bar{M}$, on the world volume of this brane. In this simplest setup this constant is determined by the mass of the heaviest world volume particle that could propagate in the loop which induces the 4D world volume Ricci scalar (see Appendix A). Generically, if one studies quantum fluctuations of the brane in the field theory context, one finds that there is a heaviest fluctuation of the brane with the mass determined by the *inverse brane width*. Suppose that for simplicity of arguments this scale is related to $M$ as follows (in general, the power of the coupling constant in the expression below can be different, however we would like to discuss a qualitative effect which is independent of this assumption):

$$\bar{M} \propto (T)^{1/4} \propto \frac{M}{\lambda^{1/4}}. \quad (40)$$

Thus, in the weak coupling approximation, when $\lambda \to 0$, we obtain that

$$\bar{M} \gg M. \quad (41)$$

In order for this hierarchy to be big, the coupling constant should be really tiny. Such a small coupling could not be a coupling of the world volume field theory which should have at least some resemblance to the standard model. In this respect, one needs two sectors in the theory, one of them makes the brane with a huge tension as described above, and another sector governs the world volume physics. In fact, the situation might be a bit more trickier. The relation (39) is exact for D-branes in string theory if one identified $M$ with the string scale and $\lambda$ with the string coupling constant. In this case, the string coupling constant can be expressed via an exponent of the vacuum expectation value (VEV) of a dilaton

$$\lambda = \exp\left(\frac{\langle\phi\rangle}{M}\right) \ll 1. \tag{42}$$

Here we assume a certain mechanism for the dilaton stabilization which guarantees that the corresponding string theory is in a weak coupling regime. One expects that the field-theory cutoff of the brane world volume theory to be defined by $M$, since above this scale the string theory description sets in. This is certainly true, and at scales above $M$ the higher stringy modes should be taken into account. However, this does not eliminate the fact that there exist some states which have masses that are much bigger that the string scale [23]. The simplest example would be D0-branes with mass $m_0 = M/\lambda$. The heavy states could also come from fluctuations of the brane itself. For instance, generically there is a localized massive mode on a solitonic brane the mass of which is determined by the inverse width of the brane. As we go below the energy scale determined by this mass, the heavy state induces an appropriate world volume Ricci scalar. In this particular case, one can say that the 4D Ricci scalar is induced after integrating out the width of the brane. This will take place regardless of the fact that other states (presumably perturbative string states) can be entering the problem at a lower scale $M$. Thus, one way or other, the constant in front of the induced 4D Ricci term on the brane could in principle be determined by this nonperturbative scale:

$$\bar{M} \propto M \exp\left(-\frac{\langle\phi\rangle}{4M}\right) \gg M. \tag{43}$$

As long as the dilaton VEV is not a logarithmic function of a scale parameter, this gives rise to an *exponential* hierarchy between the 4D scale $\bar{M}$ and the fundamental scale $M$. For instance, the huge hierarchy between $M \sim$ (a few TeV) and $\bar{M} \sim 10^{19}$ GeV could be explained by a dilaton VEV which is only a 1/100 part of the fundamental scale $M$. This latter relatively small hierarchy could in principle be obtained as a result of some world volume coupling constant suppression of the corresponding terms. Note that as we discussed above, the coupling in the world volume theory should not be determined by the string coupling constant since this latter is extremely small. Thus, one needs again two sector in the fundamental theory: One sector would create a brane with a big tension and another one should be responsible for inter-

actions in the brane world volume theory. Something similar to the ''little string theory'' could do the job of the second sector [24].

Yet another possibility is to have a huge number of particles running in the loops which generate the 4D Ricci scalar on the brane world volume.[11] In this case, $\bar{M}$ can be much bigger than $M$ due to the multiplicity of states which induce $\bar{R}$. This might be possible to realize in the constructions where the number of massive states increases polynomially or better yet exponentially.

Let us now turn to the issue of stabilization of the Higgs boson mass term. If there were SUSY on the brane, this would be the standard supersymmetric stabilization scenario.

If we deal with a non-BPS brane, then there is no supersymmetry on the world volume, and the Higgs boson mass is driven by quadratically divergent diagrams toward the cutoff of the world volume theory. If the cutoff of the low-energy field theory on a brane is $M \sim$ (a few TeV), then this would do the job of cutting the divergent diagrams and the Higgs boson mass would not need additional stabilization. On the other hand, if the cutoff is huge, then to avoid phenomenological difficulties, the Higgs boson field should be thought of as a composite field. The compositeness scale can be somewhere in a few TeV region. In particular, this composite Higgs could be coming from the properties of the bulk.

Summarizing these discussions we emphasize that there are possibilities for solving the hierarchy problem using the very generic feature of the brane physics: In the presence of (D)branes there emerges a nonperturbative scale in the theory which can be much bigger then the fundamental scale of a given model. The detailed study of this issue should be performed within the framework of concrete examples of branes and world volume field theories. This task is beyond the scope of the present work.

## VI. ON THE COSMOLOGICAL CONSTANT PROBLEM

The unnatural small value of the cosmological constant is a problem shared by any theory of gravity which at low energies flows to $D=4$ non-supersymmetric theory. Such is any high-dimensional model with broken supersymmetry and *finite volume* extra space.

Apart from a tree-level fine-tuning there is an important issue of stability against quantum corrections. The effective 4D cosmological constant $\Lambda$ is power-law sensitive to the cutoff of the theory since it gets renormalized by (at least quadratically) divergent loops.

In a minimalistic scenario in which the low-energy theory is standard model (SM) plus Einstein's gravity (GR), one may conventionally split the loop contributions (denote it by $\Delta\Lambda$) to the 4D cosmological constant in the following two parts:

$$\Delta\Lambda = \Delta\Lambda_{SM} + \Delta\Lambda_{GR}, \tag{44}$$

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[11]This possibility was independently mentioned to us by D. Fursaev.

where $\Delta\Lambda_{SM}$ parametrizes the pure SM contribution, whereas $\Delta\Lambda_{GR}$ includes graviton loops. Such a split may be useful in theories with large extra dimensions in which the SM particles are localized on a brane and gravity propagates in the bulk. In this case $\Delta\Lambda_{SM}$ can be understood as the renormalization of the brane tension by the SM loops. In such a setup $\Lambda$, in general, gets contributions from both the brane tension $T$ and the bulk cosmological constant $\Lambda_{Bulk}$, so that we can write

$$\Lambda = F(T, \Lambda_{Bulk}), \tag{45}$$

where $F$ is some model-dependent function. In particular, in the approximation of flat $N$ compact extra dimensions one finds [1]

$$\Lambda = T + \Lambda_{Bulk} V_{Extra}, \tag{46}$$

where $V_{Extra}$ is the volume of extra space. For non-flat spaces the form of $F$ may be more complicated. However, the net result is that in theories with finite $V_{Extra}$, the brane tension $T$ and $\Lambda_{Bulk}$ must conspire with an extraordinary accuracy in order to give $\Lambda \simeq 0$. One may try to make $F$ insensitive to a brane tension by introducing extra bulk degrees of freedom (perhaps coupled conformally to the brane fields) [25–27]. However, even if this is the case, $\Delta\Lambda_{GR}$ remains a big problem. One may expect naively that graviton loops are at least $1/M_{Pl}^2$-suppressed, due to bulk SUSY. However, this is not true as it can be seen from the following simple argument. The lowest scale at which we have to break supersymmetry on a brane in the conventional approach is $\sim$ TeV. The Fermi-Bose mass splitting induced in the bulk by KK modes is then

$$\Delta m^2 \sim \frac{(TeV)^4}{V_{Extra} M^{2+N}}. \tag{47}$$

Summing up one-loop contributions from all KK states lighter than $M$ and using the relation (47) we get

$$\Delta\Lambda_{GR} \sim (TeV)^4. \tag{48}$$

We cannot simply ignore this contribution, or attribute it to our pure knowledge of quantum gravity, since it appears at a scale lower than $M$ where graviton loop contributions can be evaluated in effective field theory. Quantum gravity theory sets in only above the scale $M$. While a miraculous cancellation in Eq. (48) *a priory* cannot be excluded, it would imply some form of *non-decoupling* of a very high energy physics for low energy observables. This possibility will be disregarded in the present discussion. To summarize, the finite volume theories face at least a potential problem of radiative instability of the bulk cosmological constant.

Infinite volume theories, on the other hand, provide a loop-hole from the above argument due to the fact that in these theories Eq. (47) is violated

$$\frac{\bar{M}^2}{M^{2+N}} \neq V_{Extra} = \infty. \tag{49}$$

As a result, even if supersymmetry is broken on a brane, it could still be preserved in the bulk. At all energy scales the theory remains to be a $(4+N)$-dimensional supersymmetric model (at least for $D > 6$). The reason for this is that a localized brane in $D > 6$ gives an asymptotically flat metric in transverse directions. On such a space there are infinitely many Killing spinors (any constant spinor is a Killing spinors). This fact plays a crucial role in exact cancellation of quantum gravity loops in renormalization of the bulk cosmological constant.

Given the fact that we have the (super)symmetry reason for vanishing of the bulk cosmological constant, it is time to ask what is the role of the brane tension for an observable $\Lambda$. We shall argue now that due to an infinite volume $\Lambda$ could vanish for arbitrary $T$, provided $D > 6$ ($D = 6$ case is also possible, but is somewhat subtle due to the conical structure in the transverse space).

Suppose there exists a 3-brane solution to the D-dimensional Einstein equation:

$$M^{2+N}\left(\mathcal{R}_{AB} - \frac{1}{2}g_{AB}\mathcal{R}\right) = T_{AB}^{Brane} + T_{AB}^{Other\ fields}, \tag{50}$$

where $T_{AB}^{Brane}$ is the brane energy-momentum tensor and $T_{AB}^{Other\ fields}$ stands for the energy-momentum tensor of other bulk and/or brane fields.

As long as $D \geq 6$ this equation can have a static solutions for a nonzero tension brane. That is to say, the four-dimensional world volume of the solution is flat. The line element for these solutions takes the form:

$$ds^2 = A(|y|)\bar{g}_{\mu\nu}(x)dx^\mu dx^\nu + B(|y|)\delta_{mn}dy^m dy^n, \tag{51}$$

where $A$ and $B$ are the warp-factors which go to constants at infinity. The statement that the solution has a flat four-dimensional world volume means that

$$\bar{g}_{\mu\nu}^{solution} = \eta_{\mu\nu}, \quad \bar{R}|_{solution} = 0. \tag{52}$$

Before proceeding further let us summarize the main ingredients of the framework:

(I) The bulk cosmological constant should vanish due to the bulk supersymmetry and $R$ symmetry [when the model is embedded in supergravity (SUGRA) framework];

(II) The brane world volume metric is flat, because in $D \geq 6$ spaces there are flat brane solutions for an *arbitrary* tension $T$;

(III) The induced curvature term $\bar{R}$ on a brane world volume guarantees that gravity is four-dimensional on a brane.

To fulfill (III) we have to introduce the four-dimensional Ricci term $\bar{R}$ on the world volume in Eq. (50). The question is whether the 3-brane solution discussed above still persists after the induced terms (6) are taken into account. We will argue that the answer is positive in the case at hand. Indeed, with the induced terms included as in Eq. (6) the Einstein equation of motion takes the form

$$M^{2+N}\left(\mathcal{R}_{AB} - \frac{1}{2}g_{AB}\mathcal{R}\right) + \bar{M}^2 \delta^{(N)}(y_m)A(0)$$
$$\times\left\{-\frac{1}{2}\bar{\Lambda}\bar{g}_{\mu\nu} + \left(\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R}\right)\right\}\delta^\mu_A \delta^\nu_B$$
$$= T^{\text{Brane}}_{AB} + T^{\text{Other fields}}_{AB}. \tag{53}$$

Let us assume that $A(0)$ is a nonzero finite quantity, $A(0) < \infty$. Since on the solution with a flat world volume $\bar{R} = 0$, then the induced terms in Eq. (53) (all except the cosmological constant) vanish on the solution. The induced cosmological constant, on the other hand, can be re-absorbed into the brane tension on the RHS of Eq. (53). This rescaling of the tension changes parameters of the solution but not the form of the solution itself (similar to the change in the Schwarzschild solution caused by the rescaling of the mass of the spherically symmetric body). Therefore, we conclude that [if $A(0) < \infty$] Eq. (53) also has a static solution which differs from the solution of Eq. (52) by the redefinition of the brane tension:

$$T \rightarrow T - \bar{M}^2 \bar{\Lambda} A(0). \tag{54}$$

Thus, we can obtain a model on a brane which is embedded in an infinite-volume extra space. This brane has zero 4D cosmological constant and the correct 4D relativistic world volume gravity.

This, however, is not the solution to the cosmological problem yet. To really solve the problem, one should at least address the following three vital issues:

Why the solution with the flat brane world volume is unique? In general there might be a number of other solutions with an inflating world volume. As was suggested in [14,15], it could be bulk SUSY which might pick a single solution with a flat world volume.

Why is the flat solution stable against phase transitions on a brane world volume, let us say against the QCD or electroweak phase transitions?

Does the matter localized on a non-zero tension brane in the infinite-volume extra space gives rise to conventional (at least at some distances) Freedman-Robertson-Walker cosmology?

A 5D model was found in Ref. [28] in which bulk supersymmetry does control the world volume cosmological constant on a brane. On the other hand, the model is very restrictive so that the world volume theory turns out to be conformal [29].

Whether these issues can be answered positively in the present $D > 5$ framework (along with the issue of the existence of different solutions) is the subject of an ongoing investigation and will be reported elsewhere.

## VII. DISCUSSIONS AND CONCLUSIONS

The main objective of the present work was to explore the possibility of generating a relativistic 4D theory of gravitation on a singular brane which is embedded in a flat infinite-volume extra space. Four-dimensional gravity is obtained on

a brane world volume due to the induced 4D Ricci scalar [7]. We showed that if the number of extra dimensions is two or bigger, then the induced gravity on a *delta-function* type brane is a four-dimensional relativistic tensor theory. In particular, the tensor structure of the graviton propagator in this case is equivalent to that of Einstein's gravity.

In the present framework the extra space exhibits the phenomenon of infrared transparency. That is to say, the bulk can only be probed by the signals with zero 4D momentum square.

This approach offers new opportunities to study the cosmological constant problem. In particular, the bulk cosmological constant in this framework can be controlled by the bulk symmetries, while the brane cosmological constant can be re-absorbed into the brane tension. The generalization for the case of a ''fat'' brane and the question whether this framework could lead to a unique brane solution with a flat 4D world volume will be discussed elsewhere.

## APPENDIX A

In this Appendix we summarize the mechanism by which expression (6) can emerge on a brane world volume [7]. It is instructive to consider two different possibilities. If the brane is ''rigid'' then quantum fluctuations of the matter fields which are localized on the brane can induce the term (6) via loop diagrams. On the other hand, if the brane is allowed to fluctuate then generically there is at least one massive state in the spectrum of fluctuations of this brane. This state could for instance correspond to the breathing mode of the transverse size of the brane. From the point of view of a world volume observer this state looks as a massive mode of the world volume theory. Therefore, this mode can also run in loops and produce the terms (6). Below, we present our discussion in terms of the states of the localized matter on the brane. However, we keep in mind that the very same consideration applies to the massive fluctuations of the brane itself. Having this said, we write the matter energy-momentum tensor as follows:

$$T_{AB} = \begin{pmatrix} T_{\mu\nu}(x)\delta^{(N)}(y_m) & 0 \\ 0 & 0 \end{pmatrix}. \tag{A1}$$

As a result, the interaction Lagrangian of localized matter with D-dimensional metric fluctuations $h_{AB}(x,y) \equiv G_{AB}(x,y) - \eta_{AB}$, reduces to the following expression:
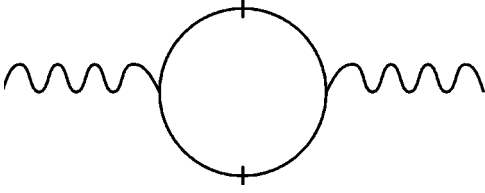
FIG. 1. The one-loop diagram which generates the 4D Ricci scalar $\bar{R}$. Wave lines denote gravitons; solid lines denote massive scalars or fermions. Vertical short lines on scalar or fermion propagators indicate that they are massive.

$$\mathcal{L}_{\text{int}} = \int d^N y h^{\mu\nu}(x, y_m) T_{\mu\nu}(x) \delta^{(N)}(y_m) = h^{\mu\nu}(x, 0) T_{\mu\nu}(x),$$
(A2)

where the 4D induced metric $\bar{g}_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}$ was defined earlier in Eq. (4). Due to this interaction, a 4D kinetic term can be generated for $\bar{g}_{\mu\nu}(x)$ in the full quantum theory. For instance, the diagram of Fig. 1 with massive scalars [30], or fermions [31,32] running in the loop would induce the following 4D term in the low-energy action:

$$\int d^4 x d^N y \, \delta^{(N)}(y_m) \sqrt{|g|} \bar{R}.$$
(A3)

The corresponding induced gravitational constant will be determined by a correlation function of the world-volume matter theory. The magnitude of this constant, as we have discussed in the text, depends on a world volume theory at hand and is vanishing in conformally invariant models, or nonzero if conformal invariance is broken (for detailed discussions see [31–33]). We assume that the world volume theory is not conformal and the second term in Eq. (9) is generated.

As we discussed, in general one induces on a brane the whole series in powers of the four-dimensional Ricci scalar $\bar{R}$. The very first term in this series is the induced 4D cosmological constant, $\bar{\Lambda} = \langle 0|T^\mu_\mu|0\rangle$. The higher-derivative terms can also be generated. We have to deal with these contributions separately. Let us start with the induced four-dimensional cosmological constant. As we discussed in Sec. VI, this just renormalizes the brane tension in $D > 5$, and does not change the world volume physics when the static brane solution exists.

Let us turn to the higher derivative terms. These are suppressed by higher powers of $\bar{M}$, thus their effects on 4D world volume gravity should be small. The only subtlety with these terms is that in certain cases they can give rise to ghosts in 4D theory. However, these ghosts are absent if the corresponding high-derivative terms come in certain combinations. For instance, it is known that in the second order in $\bar{R}$ the ghosts are absent if the $\bar{R}^2$ terms come in the Gauss-Bonnet combination [34]. We will be assuming that the bulk and world volume theories are consistent models with no

propagating ghosts or other unconventional states. Therefore, the resulting expression for induced Lagrangian on the brane is expected to be ghost free in such theories.[12]

## APPENDIX B

In this appendix we recall the properties of the 5D theory. We follow the discussions in [7]. At the end of the appendix we discuss some novel features.

The crucial difference from the theories in higher co-dimensions is that the Green's function in the transverse space in 5D is finite at the origin. In the notations of Eq. (16):

$$D(p, y) = \frac{1}{2p} \exp\{-p|y|\}.$$
(B1)

As a result, the expression for the retarded Green's function (17) takes the form [7]

$$\tilde{G}_R(p, y) = \frac{1}{2M^3 p + \bar{M}^2 p^2} \exp\{-p|y|\}.$$
(B2)

This gives rise to the potential which has 4D behavior at observable distances, but 5D behavior at ultra-large scales [7]. In this model, as we mentioned above, very low energy gravitons can leak into the bulk space even when the brane width is zero [12–14].

Let us now study the tensor structure of the graviton propagator [7]. The $(\mu\nu)$ components of the Einstein equation take the form

$$(M^3 \partial_A \partial^A + \bar{M}^2 \delta(y) \partial_\mu \partial^\mu) h_{\mu\nu}(x, y)$$
$$= \left\{ T_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} T^\alpha_\alpha \right\} \delta(y) + \bar{M}^2 \delta(y) \partial_\mu \partial_\nu h_5^5.$$
(B3)

This has a structure of a massive 4D graviton or, equivalently that of a massless 5D graviton. In this respect, it is instructive to rewrite this expression in the following form:

$$(M^3 \partial_A \partial^A + \bar{M}^2 \delta(y) \partial_\mu \partial^\mu) h_{\mu\nu}(x, y)$$
$$= \left\{ T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\alpha_\alpha \right\} \delta(y) - \frac{1}{2} M^3 \eta_{\mu\nu} \partial_A \partial^A h^\alpha_\alpha$$
$$+ \bar{M}^2 \delta(y) \partial_\mu \partial_\nu h_5^5.$$
(B4)

Here the tensor structure on the RHS is that of a 4D massless graviton. However, there is an additional contribution due to the trace part $h^\mu_\mu$ which is nonzero. Therefore, one is left with the theory of gravity which from the 4D point of view is

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[12]It might also happen that the bulk theory is ghost free, however ghosts emerge as artifacts of the truncation of the perturbative series in $\bar{R}$. In this case the perturbative approach with consistent subtraction schemes in each order of perturbation theory should be developed.

mediated by a graviton plus a scalar. As before, turning to the Fourier images in the Euclidean space we find

$$\tilde{h}_{\mu\nu}(p, y=0)\tilde{T}^{\mu\nu}(p) = \frac{\tilde{T}^{\mu\nu}\tilde{T}_{\mu\nu} - \frac{1}{3}\tilde{T}^{\mu}_{\mu}\tilde{T}^{\nu}_{\nu}}{\bar{M}^2 p^2 + 2M^3 p}. \qquad \text{(B5)}$$

Here the tilde sign denotes the Fourier-transformed quantities. Thus, the tensor structure of the graviton propagator in 4D world volume theory looks as follows:

$$\frac{1}{2}\eta^{\mu\alpha}\eta^{\nu\beta} + \frac{1}{2}\eta^{\mu\beta}\eta^{\nu\alpha} - \frac{1}{3}\eta^{\mu\nu}\eta^{\alpha\beta} + \mathcal{O}(p). \qquad \text{(B6)}$$

At short distances the potential scales as $1/r$ with the logarithmic corrections found in [7]. On the other hand, at large distances the $1/r^2$ behavior is recovered. The tensor structure of the propagator is that of 4D tensor-scalar gravity.

The presence of the extra polarization degrees of freedom is *not* acceptable from the phenomenological point of view [35,36]. The light bending by the Sun and the precession of the Mercury perihelion in this theory are incompatible with the existing data. The reason for this is that gravity in these models is mediated not only by two transverse polarizations of a 4D graviton, but also, by an additional polarization of a 5D-dimensional graviton. Thus, there is the excess of attraction in the theory. This extra attraction should be somehow removed in order to render the model compatible with the data.

A way to avoid the problem with the extra degrees of freedom in the world volume theory is to assume that the bulk gravity is in fact described by a topological theory which has no propagating degrees of freedom. As an example one could write in 5D the Chern-Simons term [37]. The terms in Eq. (6) will still be induced in the world volume theory as long as the brane is introduced (this is true for any number of dimensions). Therefore, the only propagating degrees of freedom will be those of 4D gravity on a brane world volume. These latter appear due to Eq. (6). In terms of equations this could be seen as follows: in a model with the bulk Chern-Simons term the first term in Eq. (28) is absent [37]. Thus, the tensor structure of the graviton propagator is determined by the second term in Eq. (28). Hence, the resulting tensor structure is that of four-dimensional gravity:

$$\frac{1}{2}\eta^{\mu\alpha}\eta^{\nu\beta} + \frac{1}{2}\eta^{\mu\beta}\eta^{\nu\alpha} - \frac{1}{2}\eta^{\mu\nu}\eta^{\alpha\beta}. \qquad \text{(B7)}$$

Let us note that higher dimensional topological gravity could in particular be obtained from certain compactifications of string theory [38].[13] The net result of this approach is that the world volume gravity on a brane is four-dimensional to all distances. That is to say, there is no crossover to the high-dimensional gravitational law at large distances. The theory in this case is intrinsically four-dimensional, that is to say no localized matter can escape from the brane to the bulk.

Finally one could incorporate scalars into the consideration in co-dimension-one theories [39,28,29].

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