Radion dynamics and electroweak physics

Csaba Csáki*
Theory Division T-8, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

Michael L. Graesser†
Department of Physics, University of California, Santa Cruz, California 95064

Graham D. Kribs‡
Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
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The dynamics of a stabilized radion in the Randall-Sundrum model with two branes is investigated, and the effects of the radion on electroweak precision observables are evaluated. The radius is assumed to be stabilized using a bulk scalar field as suggested by Goldberger and Wise. First the mass and the wave function of the radion is determined including the back reaction of the bulk stabilization field on the metric, giving a typical radion mass of the order of the weak scale. This is demonstrated by a perturbative computation of the radion wave function. A consequence of the background configuration for the scalar field is that after including the back reaction the Kaluza-Klein states of the bulk scalars couple directly to the standard model fields on the TeV brane. Some cosmological implications are discussed, and in particular it is found that the shift in the radion at late times is in agreement with the four-dimensional effective theory result. The effect of the radion on the oblique parameters is evaluated using an effective theory approach. In the absence of a curvature-scalar Higgs mixing operator, these corrections are small and give a negative contribution to *S*. In the presence of such a mixing operator, however, the corrections can be sizable due to the modified Higgs and radion couplings.

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I. INTRODUCTION

Extra dimensional theories where standard model fields are localized on a brane [1–8] have recently attracted a lot of attention, since such models have several distinct features from ordinary Kaluza-Klein (KK) theories. In particular, Randall and Sundrum [4] presented a simple model based on two branes and a single extra dimension, where the hierarchy problem could be solved due to the exponentially changing metric along the extra dimension. In order to obtain a phenomenologically acceptable model, the radion field (which corresponds to fluctuations in the distance of the two branes) has to get a mass, otherwise it would violate the equivalence principle [9], and also result in unconventional cosmological expansion equations [10,11]. The simplest mechanism for radius stabilization has been suggested by Goldberger and Wise [12], who employed an additional bulk scalar which has a bulk mass term and also couples to both branes (for another issues related to the radion potential, see Ref. [13]). Both the cosmology and collider phenomenology crucially depend on the mass and couplings of the radion. In particular, there is no radion moduli problem if the radion mass is *O* (TeV) and its couplings to standard model (SM) fields is *O* (TeV^{-1}). This is also the most favorable scenario for discovering the radion at a future collider.

In fact it was shown in Refs. [14] and [15], that the radion will have the above properties for the Goldberger-Wise scenario. However, the calculation of Refs. [14] and [15] were using a naïve ansatz for the radion field which ignores both the radion wave function and the back reaction of the stabilizing scalar field on the metric. The validity of this approximation has recently been questioned [16].

Therefore in this paper we analyze the coupled radion-scalar system in detail from the 5D point of view. We derive the coupled differential equations governing the dynamics of the system, and find the mass eigenvalues for some limiting cases. Because of the coupling between the radion and the bulk scalar, we find that there will be a single KK tower describing the system, with the metric perturbations nonvanishing for every KK mode. This implies that the standard model fields localized on the TeV brane will couple to every KK mode from the bulk scalar, and this could provide a means to directly probe the stabilizing physics.

Using the coupled equations for the radion-scalar system, we analyze the late-time behavior of the radion in an expanding universe, and find that the troubling 55 component of Einstein's equation just determines the shift of the radion. This shift completely agrees with the shift obtained in Ref. [14] using the 4D effective theory.

Given that we have established that the radion mass is *O* (TeV) and that its couplings to SM particles is *O* (TeV^{-1}), it is reasonable to consider its effects on SM phenomenology. Some direct collider signatures for the radion and loop corrections have been discussed in Refs. [17–23]. In the second half of this paper the effects of the radion on the oblique

*Email address: csaki@lanl.gov
†Email address: graesser@scipp.ucsc.edu
‡Present address: Department of Physics, University of Wisconsin, Madison, WI 53706. Email address: kribs@pheno.physics.wisc.edu

parameters[1] are calculated using an effective theory approach similar to Ref. [25]. Since in the RS model the radion is the only new state well below the TeV scale, a low-energy effective theory including only the radion and SM fields is used. The effects of other heavy modes are accounted for by including nonrenormalizable operators at the cutoff scale. In the absence of a curvature-scalar Higgs mixing operator, the corrections from the radion are small, but give a negative contribution to S. In the presence of such a mixing operator the corrections could be sizable due to the modified radion and Higgs couplings.

This paper is organized as follows: in Sec. II we review the Randall-Sundrum model and radius stabilization by bulk scalar fields. We also summarize the explicit example of Ref. [26] which we will be using for our explicit computation of the radion mass and couplings to SM fields. In Sec. III we present our ansatz for the coupled metric and scalar fluctuations based on the analysis of Refs. [27] and [28] of the radion without a stabilizing potential. We will derive a single ordinary differential equation, whose eigenmodes will yield the KK modes for the radion-scalar system. In Sec. IV we analyze the generic properties of this equation. In the general case we find that the system is not described by a Hermitian Schrödinger operator. However, we identify a convenient limit, in which the differential operator is in fact Hermitian, and the eigenfunctions are manifestly orthogonal. In Sec. V we analyze the eigenfunctions in this limit, and find the approximate masses for the KK tower. In this analysis, the back reaction of the metric is neglected, which results in the lightest mode still being massless. The effect of the back reaction on the lightest mode is taken into account in Sec. VI, where we find the mass of the radion to be of the order (but slightly lighter) than the weak scale. In Sec. VII we discuss the couplings of the radion and the KK tower to SM fields on the brane. We find that the radion coupling *exactly* agrees with the results in Refs. [14] and [15], while the couplings of the other KK modes of the scalar field are suppressed by the mass of the given mode, and is proportional to the backreaction of the metric due to the scalar background. In Sec. VIII we demonstrate that in an expanding universe the shift in the radion at late times agrees with the 4D effective theory result obtained in Ref. [14]. Having established the mass and coupling of the radion, we write an effective Lagrangian in Sec. IX without any specific mechanism of radius stabilization and neglecting the contributions of the KK modes. In Sec. X we add a curvature-scalar Higgs coupling to the effective Lagrangian, and discuss how the couplings are modified. Then, in Sec. XI we calculate the Feynman rules in a general gauge. These allow us to compute the oblique parameters via one-loop vacuum polarization diagrams with radions in the loop in Sec. XII. The radion correction is log divergent (unlike the Higgs boson), and so we also write the nonrenormalizable operators at the cutoff scale that provide the necessary counterterms. The size of the new contributions are shown for various cases in several figures in the numerical results part of Sec. XII. We also estimate limits on the radion mass as a function of the cutoff scale in Sec. XIII. Finally, we conclude in Sec. XIV.

[1]Loop effects for theories with large extra dimensions have been analyzed in Ref. [24].

II. REVIEW OF THE RANDALL-SUNDRUM MODEL AND THE GOLDBERGER-WISE MECHANISM

Randall and Sundrum presented a very interesting proposal for solving the hierarchy problem [4]. By introducing a fifth dimension where the bulk geometry is anti–de Sitter, a large hierarchy between the Planck scale and the TeV scale is obtained with only a mild fine-tuning. Two branes are introduced, located at the boundaries of the anti–de Sitter space. By tuning the bulk cosmological constant $\Lambda \equiv -6k^2/\kappa^2$, the tensions V_P and V_T on the Planck and TeV branes, respectively, such that $V_P = -V_T = 6k/\kappa^2$ (where κ^2 is the 5D Newton constant related to the 5D Planck mass by $\kappa^2 = 1/2M^3$) one obtains a 4D Poincaré invariant solution. The metric is then

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \tag{2.1}$$

where the Planck brane and TeV branes are located at $y=0$ and $y=r_0$. For a moderate choice of $kr_0 \sim O(50)$, a large hierarchy between the Planck scale and the weak scale is generated.

Since this solution is obtained for any value of r_0, some mechanism is required to fix $r_0 \sim 50/k$ as opposed to some other value of r_0. This must also be done without introducing any large fine-tuning. Further, small shifts in the separation between the two branes do not change the energy, and so are described in an effective theory by the fluctuations of a massless particle, the "radion." This particle couples like a Brans-Dicke scalar and must be massive to recover ordinary 4D Einstein gravity [9,14].

One way to achieve these requirements is to introduce a bulk scalar field ϕ that has a bulk potential $V(\phi)$ [12]. To stabilize the brane distance, potentials $\lambda_{P,T}(\phi)$ on the Planck and TeV branes, respectively, are also included. The competition between the brane and bulk Lagrangians generates a vacuum expectation value (VEV) for ϕ, which results in a 4D vacuum energy that depends on r_0. For a simple choice of polynomial potentials a large hierarchy is then easily obtained with a mild fine-tuning [12], and the resulting mass for the radion is O (TeV) [14,15].

The phenomenology of the radion depends on the strength of its coupling to the brane fields. Using the following naïve ansatz to describe the radion $b(x)$,

$$ds^2 = e^{-2k|y|b(x)}ds_4^2 - b(x)^2 dy^2, \tag{2.2}$$

Refs. [14] and [15] computed the normalization of the radion kinetic term to be

$$\frac{3}{4}e^{-2kr_0}\frac{kr_0^2}{\kappa^2}(\partial b)^2. \tag{2.3}$$

Fields living on the TeV brane couple to the radion through the induced metric, with an interaction

$$\frac{kr_0}{2}b(x)\mathrm{Tr}T_{\mu\nu}=\frac{r(x)}{\sqrt{6}\Lambda_W}\mathrm{Tr}T_{\mu\nu}, \tag{2.4}$$

where r is the canonically normalized radion, $\Lambda_W = M_{Pl}e^{-kr_0} \sim O$ (TeV),

$$M_{Pl}^2=(1-e^{-2kr_0})/(k\kappa^2)\sim 1/(k\kappa^2),$$

and $T_{\mu\nu}$ is the physical energy-momentum tensor of the TeV brane fields. It is then clear that the radion couples as $\sim 1/$TeV to the standard model fields. Obtaining an acceptable phenomenology then requires that the radion mass is O (TeV), which is easily satisfied by the Goldberger-Wise mechanism.

The phenomenology of the radion is then crucially dependent on the normalization of the kinetic term. In fact, in the computation leading to the O (TeV^2) prefactor in Eq. (2.3) there is a cancellation between two terms of $O(M_{Pl}^2)$. The origin of this cancellation remains somewhat mysterious, and the absence of this cancellation would clearly lead to different predictions. In Ref. [16] it was pointed out that there are additional contributions to the radion kinetic term not included in Refs. [14] and [15]. In particular, the profile of the stabilizing field depends on r_0, and so a small change in $r_0 \to b(x)$ distorts the background field. It was found that this results in an $O(M_{Pl}^2)$ correction to the radion kinetic term, thereby drastically changing the phenomenology of the radion.

We review the resolution of this issue in the first part of this paper. Some of the results presented in Secs. III–VI are already contained in the work by Tanaka and Montes [29], even though the results of this paper were obtained independently of Ref. [29]. We explicitly determine the wave function of the radion when there is a stabilizing mechanism. We find that the radion mass is typically O (TeV). In the limit that the backreaction of the stabilizing fields on the metric is small, we find that the correction of the stabilizing field to the radion kinetic term is subdominant to the gravitational contribution. We also find that once the stabilizing field has a nonzero VEV, the Kaluza-Klein (KK) tower couples directly to the brane world fields, with 1/TeV normalization, and amplitude depending on the size of the backreaction.

The action we consider is[2]

$$-M^3\int d^5x\sqrt{g}\mathcal{R}+\int d^5x\sqrt{g}\left(\frac{1}{2}\nabla\phi\nabla\phi-V(\phi)\right)$$
$$-\int d^4x\sqrt{g_4}\lambda_P(\phi)-\int d^4x\sqrt{g_4}\lambda_T(\phi), \tag{2.5}$$

where g_4 is the induced metric on the branes. The background VEV for ϕ and background metric that preserve 4D Lorentz invariance is

[2]The action is integrated over the circle rather than the line segment.

$$\phi(x,y)=\phi_0(y), \tag{2.6}$$

$$ds^2=e^{-2A}\eta_{\mu\nu}dx^\mu dx^\nu-dy^2. \tag{2.7}$$

The Einstein equations are then

$$R_{ab}=\kappa^2\tilde{T}_{ab}=\kappa^2\left(T_{ab}-\frac{1}{3}g_{ab}g^{cd}T_{cd}\right), \tag{2.8}$$

with $\kappa^2=1/(2M^3)$. For this background the scalar and metric field equations are

$$4A'^2-A''=-\frac{2\kappa^2}{3}V(\phi_0)-\frac{\kappa^2}{3}\sum_i \lambda_i(\phi_0)\delta(y-y_i), \tag{2.9}$$

$$A'^2=\frac{\kappa^2\phi_0'^2}{12}-\frac{\kappa^2}{6}V(\phi_0), \tag{2.10}$$

$$\phi_0''=4A'\phi_0'+\frac{\partial V(\phi_0)}{\partial\phi}+\sum_i\frac{\partial\lambda_i(\phi_0)}{\partial\phi}\delta(y-y_i). \tag{2.11}$$

Here primes denote $\partial/\partial y$, and we reserve ∂_μ to denote derivative with respect to the comoving 4D space-time coordinates x^μ. The boundary equations for A and ϕ_0 are obtained by matching the singular terms in the above equations. This gives

$$[A']|_i=\frac{\kappa^2}{3}\lambda_i(\phi_0), \tag{2.12}$$

$$[\phi_0']|_i=\frac{\partial\lambda_i(\phi_0)}{\partial\phi}. \tag{2.13}$$

For analytical solutions we use an approach presented in Refs. [26] and [30]. A particular class of potentials V is considered which can be written in the form

$$V(\phi)=\frac{1}{8}\left(\frac{\partial W(\phi)}{\partial\phi}\right)^2-\frac{\kappa^2}{6}W(\phi)^2. \tag{2.14}$$

Then a solution to the following first order equations,

$$\phi_0'=\frac{1}{2}\frac{\partial W}{\partial\phi},\quad A'=\frac{\kappa^2}{6}W(\phi_0), \tag{2.15}$$

automatically solves both the Einstein and scalar field equations, once the appropriate boundary conditions are solved. The virtue of this method is that for simple choices of W it is possible to also solve for the back reaction of ϕ_0 on the metric. This will be important for us, since we find that only after including the back reaction of the stabilizing field does one find that the radion acquires a mass.

In particular, to obtain some analytic results the following superpotential [26] will be used:

$$W(\phi)=\frac{6k}{\kappa^2}-u\phi^2 \tag{2.16}$$

with brane potentials

$$\lambda(\phi)_{\pm}=\pm W(\phi_{\pm})\pm W'(\phi_{\pm})(\phi-\phi_{\pm})+\gamma_{\pm}^{2}(\phi-\phi_{\pm})^{2}. \tag{2.17}$$

Here $+/-$ refer to Planck/TeV brane. The solution is [26]

$$\phi_0(y)=\phi_P e^{-uy}, \tag{2.18}$$

$$A(y)=ky+\frac{\kappa^2\phi_P^2}{12}e^{-2uy}. \tag{2.19}$$

The separation distance r_0 is then fixed by matching ϕ_0 at 0 and r_0 to ϕ_P and ϕ_T which gives $ur_0=\ln\phi_P/\phi_T$. So the quantity

$$e^{-ur_0}=\frac{\phi_T}{\phi_P} \tag{2.20}$$

is not a (hierarchically) small number, since both ϕ_P and ϕ_T are $O(M_{Pl}^{3/2})$. This combination will appear later in the expression for the radion mass. Also for future reference, since the back reaction corresponds to the second term in A, the limit of a small back reaction is $\kappa^2\phi_P^2,\ \kappa^2\phi_T^2\ll 1$, and $u>0$, but with $\phi_P/\phi_T=$const, so that u is kept constant.

III. COUPLED FIELD EQUATIONS

When $\phi_0=0$ there is always a static solution independent of the value of r_0.[3] The small fluctuations in the relative position between the two branes then describe a massless particle ("the radion"), and its wave function is [27] $G(x,y)=2F(x,y)=2ke^{2ky}R(x)$ and where $\Box R=0$. Since the coupling of the radion to the standard model fields is $\sim$1/TeV [14,15], obtaining an acceptable phenomenology requires that this radion acquires a mass.

We therefore consider the spectrum of perturbations about the above background which stabilizes the interbrane separation. A general ansatz to describe the spin-0 fluctuations is

$$\phi(x,y)=\phi_0(y)+\varphi(x,y), \tag{3.1}$$

$$ds^2=e^{-2A-2F(x,y)}\eta_{\mu\nu}dx^{\mu}dx^{\nu}-[1+G(x,y)]^2dy^2. \tag{3.2}$$

In order to describe all gravitational excitations of the model, one would need to add also the degrees of freedom in the graviton, by replacing $\eta_{\mu\nu}\rightarrow\eta_{\mu\nu}+h_{\mu\nu}^{TT}$. One can show that the Einstein equations with this replacement will have the radion and the graviton decoupled. This metric ansatz (3.2) [together with the two Eqs. (3.12) and $G=2F$ which we will shortly derive] fixes our gauge choice. One can show, that the effect of the remaining gauge transformations that preserve the form of Eqs. (3.2) and (3.12) just amount to a 4D gauge transformation on the graviton field $h_{\mu\nu}$ and can be used to impose a convenient 4D gauge for the graviton. In the following we will only concentrate on the radion field. Using this ansatz the Einstein and scalar field equations are linearized to obtain some coupled equations for F, G, and φ. The linearized Einstein equations are

$$\delta R_{ab}=\kappa^2\delta\tilde{T}_{ab}. \tag{3.3}$$

Inspecting the $\delta R_{\mu\nu}$ equation one immediately concludes that $G=2F$. For

$$\delta R_{\mu\nu}=\cdots+2\partial_{\mu}\partial_{\nu}F-\partial_{\mu}\partial_{\nu}G+\ldots, \tag{3.4}$$

where the ellipses all contain terms$\sim\eta_{\mu\nu}$. Since to linear order in the perturbations the sources $\delta\tilde{T}_{\mu\nu}$ are also all $\sim\eta_{\mu\nu}$, the $\partial_{\mu}\partial_{\nu}$ term in $\delta R_{\mu\nu}$ term must vanish. This gives $G=2F+c$. However, in the limit $F\rightarrow 0$, or $G\rightarrow 0$ we should recover the background solution, so $c=0$. In what follows we set $G=2F$. Then the coupled field equations are

$$\delta R_{\mu\nu}=\eta_{\mu\nu}\Box F+e^{-2A}\eta_{\mu\nu}\times(-F''+10A'F'+6A''F-24A'^2F), \tag{3.5}$$

$$\delta R_{\mu 5}=3\partial_{\mu}F'-6A'\partial_{\mu}F, \tag{3.6}$$

$$\delta R_{55}=2e^{2A}\Box F+4F''-16A'F'. \tag{3.7}$$

The source terms are

$$\delta\tilde{T}_{\mu\nu}=-\frac{2}{3}e^{-2A}\eta_{\mu\nu}[V'(\phi_0)\varphi-2V(\phi_0)F]-\frac{1}{3}e^{-2A}\eta_{\mu\nu}\sum_i\left(\frac{\partial\lambda_i(\phi_0)}{\partial\phi}\varphi-4\lambda_i(\phi_0)F\right)\delta(y-y_i), \tag{3.8}$$

$$\delta\tilde{T}_{\mu 5}=\phi_0'\partial_{\mu}\varphi, \tag{3.9}$$

$$\delta\tilde{T}_{55}=2\phi_0'\varphi'+\frac{2}{3}V'(\phi_0)\varphi+\frac{8}{3}V(\phi_0)F+\frac{4}{3}\sum_i\left(\frac{\partial\lambda_i(\phi_0)}{\partial\phi}\varphi+2\lambda_i(\phi_0)F\right)\delta(y-y_i). \tag{3.10}$$

The linearized scalar field equation is

$$e^{2A}\Box\varphi-\varphi''+4A'\varphi'+\frac{\partial^2V}{\partial\phi^2}(\phi_0)\varphi=-\sum_i\left(\frac{\partial^2\lambda_i(\phi_0)}{\partial\phi^2}\varphi+2\frac{\partial\lambda_i(\phi_0)}{\partial\phi}F\right)\delta(y-y_i)-6\phi_0'F'-4\frac{\partial V}{\partial\phi}F. \tag{3.11}$$

Notice that the $R_{\mu 5}$ may be integrated immediately to obtain

$$\phi_0'\varphi=\frac{3}{\kappa^2}(F'-2A'F). \tag{3.12}$$

[3] After two fine-tunings which are independent of r_0. But only one fine-tune remains after radius stabilization [12,14].

An integration constant $k(y)$ has been sent to zero since we require that the fluctuations F and φ are also localized in x. This Eq. (3.12) together with the metric ansatz (3.2) fixes our gauge choice. One can show that the effect of the remaining gauge transformations that preserve the form of Eqs. (3.2) and (3.12) just amount to a 4D gauge transformation on the graviton field $h_{\mu\nu}$ and can be used to impose a convenient 4D gauge for the graviton.

These equations must be supplemented by the boundary conditions for F and φ on the two branes. These are obtained by identifying the singular terms in above equations. *A priori* the Einstein equations give two boundary conditions for each wall. It is, however, straightforward to show that one of them is trivially satisfied once A satisfies the jump Eq. (2.12). The two remaining boundary equations are

$$[F']=\frac{2\kappa^2}{3}\lambda_i(\phi_0)F+\frac{\kappa^2}{3}\frac{\partial\lambda_i}{\partial\phi}(\phi_0)\varphi, \qquad (3.13)$$

$$[\varphi']|_i=\frac{\partial^2\lambda_i}{\partial\phi^2}(\phi_0)\varphi+2\frac{\partial\lambda_i}{\partial\phi}F. \qquad (3.14)$$

Upon using the jump equations for the background the first equation is seen to be equivalent to Eq. (3.12) and so provides no new constraints. Then only the second boundary condition must be implemented. A convenient limit will at times be considered in this paper. The second boundary condition simplifies in the limit of a stiff boundary potential. Namely, if $\partial^2\lambda_i/\partial\phi^2 \gg 1$ then the second boundary condition is just $\varphi|_i=0$. Then in this limit the first boundary condition is just

$$(F'-2A'F)|_i=0. \qquad (3.15)$$

A single equation for F is obtained as follows. One considers the combination $e^{2A}\delta R_{\mu\nu}+\delta R_{55}$ *in the bulk*. The point of this combination is to eliminate terms of the form $V(\phi_0)\varphi$. This leaves a bulk equation for F and φ' only:

$$e^{2A}\Box F+F''-2A'F'=\frac{2\kappa^2}{3}\phi_0'\varphi'. \qquad (3.16)$$

One then eliminates φ' in favor of F using Eq. (3.12). This gives

$$F''-2A'F'-4A''F-2\frac{\phi_0''}{\phi_0'}F'+4A'\frac{\phi_0''}{\phi_0'}F=e^{2A}\Box F, \qquad (3.17)$$

to be solved in the bulk. This is the principle equation that will be studied and solved below. We note in passing that each eigenmode $\Box F_n=-m_n^2F_n$ to this equation has two integration constants and one mass eigenvalue. One constant corresponds to the overall normalization. The remaining integration constant is fixed by the boundary condition at the Planck brane, and the mass is determined by the boundary condition on the TeV brane. In the stiff potential approximation we use the boundary condition given by Eq. (3.15).

It is then possible to show that a solution F to the above equation automatically implies that the φ equation and the remaining Einstein equation are satisfied. In particular, starting with Eq. (3.16), one uses the derivative of Eq. (3.12) to eliminate F''. The resulting equation, call it E, is then differentiated and the combination $0=E'-2A'E$ is constructed. Using the background field equations and Eq. (3.12) one arrives at the φ equation. Finally, the $\delta R_{\mu\nu}$ equation is obtained from the δR_{55} equation after substituting for φ'.

IV. GENERAL PROPERTIES OF THE EQUATION

First we show that the single ordinary differential equation for $F(y)$ given in Eq. (3.17) can always be brought into the Schrödinger form. For this we first transform the equations into the coordinate system where the background metric is conformally flat. This is achieved by the change of variables $dze^{-A(z)}=dy$, where $A(z)=A[y(z)]$. In these coordinates the equation simplifies to

$$F''-3A'F'-4A''F-2\frac{\phi_0''}{\phi_0'}F'+4\frac{\phi_0''}{\phi_0'}A'F=-m^2F. \qquad (4.1)$$

After the rescaling of the field F by $F=e^{3/2A}\phi_0'\tilde{F}$ we obtain the Schrödinger-like equation

$$-\tilde{F}''+\left[\frac{9}{4}A'^2+\frac{5}{2}A''-A'\frac{\phi_0''}{\phi_0'}+2\left(\frac{\phi_0''}{\phi_0'}\right)^2-\frac{\phi_0'''}{\phi_0'}\right]\tilde{F}=m^2\tilde{F}. \qquad (4.2)$$

However, this by itself does not guarantee Hermiticity of the differential operator in Eq. (4.2). The reason is that this operator is defined only on a finite strip, and therefore in addition to writing the equation in a Schrödinger form one also has to ensure that one has Hermitian boundary conditions for F. For the differential operator in Eq. (4.2) to actually be Hermitian on the finite strip between the two branes, one also has to require that for any two functions F_1, F_2 on the strip $F_1'(0)F_2(0)-F_1(0)F_2'(0)-F_1'(z_b)F_2(z_b)+F_1(z_b)F_2'(z_b)=0$, where 0 and z_b denote the positions of the branes in the conformally flat z coordinates. Once this condition is satisfied, it is automatically guaranteed by the usual theorems that all eigenvalues m_n^2 are real, that the eigenfunctions are orthogonal to each other and that they form a complete set. The actual boundary conditions that F has to satisfy can be derived from the general boundary condition given in Eq. (3.14). In the particular model considered in this paper the boundary condition in the y coordinates is given by

$$\pm\varphi'=\gamma_\pm^2\varphi\pm 2u\phi_\pm F. \qquad (4.3)$$

In the special limit when $\gamma_\pm\to\infty$ this boundary condition reduces to $\varphi=0$ on the two boundaries, which together with the constraint Eq. (3.12) between φ and F just implies

$$(F'-2A'F)|_i=0 \qquad (4.4)$$

at the two branes. Upon transforming to the Schrödinger basis and z coordinates the boundary condition will be replaced by

$$\tilde{F}'=\tilde{F}\left(\frac{1}{2}A'-\frac{\phi_0''}{\phi_0'}\right) \tag{4.5}$$

at the branes. This boundary condition clearly satisfies the Hermiticity properties and thus will ensure the appearance of only real mass eigenvalues of the coupled system. This will also be the case that we will analyze in full detail in the following sections. As for the general case, when γ_i is finite, the boundary condition will not be Hermitian. This can be easily seen from the fact that the general boundary condition involves φ' at the branes, which should be expressed from Eq. (3.12) in terms of F'', F', and F at the brane. The appearance of F'' in the boundary condition will generically ruin the Hermiticity of the operator. Nevertheless, one may eliminate F'' in favor of the eigenvalue, and one can in principle solve for F. The non-Hermiticity by itself, however, does not mean that the eigenvalues are not real. In fact, since ϕ is a real scalar and F a component of the metric tensor, both of these functions have to be real to start with, which guarantees at least the appearance of only real eigenvalues. While for the model studied here (see Sec. VI) the radion is not tachyonic, it is unclear whether for a general potential this remains true. However, the orthogonality of the solutions is not guaranteed by anything, and will likely be violated in general for the non-Hermitian boundary conditions. It would be interesting to understand the physics behind the nonorthogonality of these solutions in more detail.

V. APPROXIMATE SOLUTION FOR THE KK TOWER

We have seen that the coupled radion-scalar system leads to a single ordinary second order differential equation. From now on we will always assume that we can use the limit $\gamma_i \to\infty$, and be able to use the Hermitian boundary conditions (4.4). In the following we will present an approximate solution to these equations. For this, we will first neglect the back reaction of the nonvanishing scalar background on the metric. This will lead us to a simple Bessel-type equation, which will give a very good approximation for the masses of the KK tower of the fields. However, surprisingly, in this approximation the radion [which we identify as the lowest lying solution of Eq. (3.17)] remains massless. Therefore, after presenting this approximation, we will give a perturbative analysis for the effect of the back reaction of the metric on the radion mass. We will find that as expected, the radion mass will be of order TeV, but somewhat lighter just as predicted in Refs. [14] and [15].

To find the actual wave functions and masses for the radion-scalar system, we will use the particular model put forward by de Wolfe *et al.* [26] and summarized in Sec. II. First we neglect the back reaction of the scalar field background on the metric, which seems to be a good approximation as long as $\kappa\phi_{P,T}\ll 1$. In this case the equation for the radion field F reduces in the Schrödinger frame to

$$-F''+\frac{\alpha(\alpha+1)k^2}{(kz+1)^2}F=m^2F, \tag{5.1}$$

where α is given by

$$\alpha=-\frac{3}{2}-\frac{u}{k}. \tag{5.2}$$

In these coordinates the boundary conditions at the brane simplify to

$$F'+\frac{\alpha k}{kz+1}F=0 \tag{5.3}$$

at the locations of the branes at $z=0$ and $z_b\equiv(1/k)(e^{kr_0}-1)$. The solutions of these equations are given by linear combinations of the Bessel functions $J_{\alpha+1/2}$ and Neumann functions $N_{\alpha+1/2}$:

$$F_n(z)=a_n\left(z+\frac{1}{k}\right)^{1/2}N_{\alpha+1/2}\left[m_n\left(z+\frac{1}{k}\right)\right]+b_n\left(z+\frac{1}{k}\right)^{1/2}J_{\alpha+1/2}\left[m_n\left(z+\frac{1}{k}\right)\right]. \tag{5.4}$$

The mass eigenvalues m_n can be determined from the boundary condition (5.3). Using the relation for Bessel functions

$$Z_n'(x)=Z_{n-1}(x)-\frac{n}{x}Z_n(x) \tag{5.5}$$

the boundary conditions at the two branes simply reduce to

$$a_nN_{\alpha-1/2}\left(\frac{m_n}{k}\right)+b_nJ_{\alpha-1/2}\left(\frac{m_n}{k}\right)=0,$$
$$a_nN_{\alpha-1/2}\left(\frac{m_ne^{kr_0}}{k}\right)+b_nJ_{\alpha-1/2}\left(\frac{m_ne^{kr_0}}{k}\right)=0, \tag{5.6}$$

which yields the simple equation

$$b(m_n)=J_{\alpha-1/2}\left(\frac{m_n}{k}\right)\frac{N_{\alpha-1/2}\left(\frac{m_ne^{kr_0}}{k}\right)}{N_{\alpha-1/2}\left(\frac{m_n}{k}\right)}-J_{\alpha-1/2}\left(\frac{m_ne^{kr_0}}{k}\right)=0, \tag{5.7}$$

which can be used to determine the mass eigenvalues m_n. This can be done numerically. In Fig. 1 we show the lowest mass eigenvalues for $\alpha=-2.5$, which corresponds to the somewhat unrealistic value $u/k=1$. In Fig. 2 we show the dependence of the first nonvanishing mass eigenvalue on the value of $\alpha=-3/2-u/k$. One can easily see from Eq. (5.7), that $m=0$ is always a solution to Eq. (5.7), therefore in the approximation we are using the radion is still massless. For the higher states of the KK tower it is a good approximation to use the mass eigenvalues obtained from Eq. (5.7), because the masses are of the order (and even larger) than the TeV scale, thus in the limit of small back reaction that we are considering throughout the paper these masses will be only slightly modified. The radion (which appeared as the zero mode above), however, needs special treatment, since the shift in the mass (which is usually negligible for the higher KK modes) coming from the back reaction of the metric

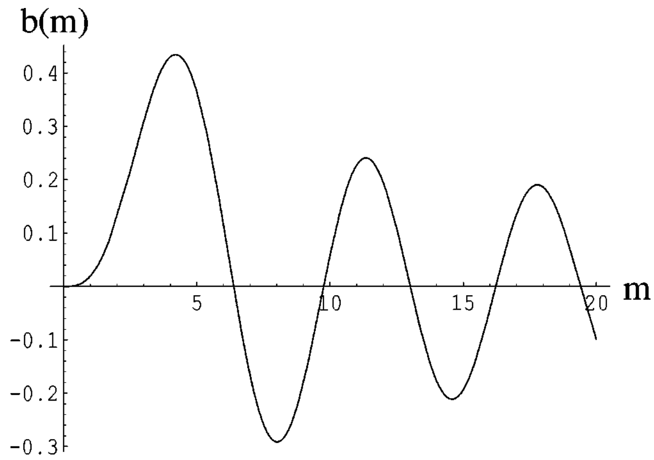


FIG. 1. The lowest mass eigenvalues for the coupled radion-scalar system for $u/k=1$ are given by the zeroes of the function $b(m)$ defined in Eq. (5.7). On this plot m is given in units ke^{-kr_0}, therefore the mass spacings are given by the TeV scale. Note that in the approximation leading to this equation the lowest lying state is still massless.

background due to the scalar field is the leading order contribution to the mass for the radion. Below we will estimate the size of the radion mass in perturbation theory.

VI. RADION MASS

In the previous section we have seen what the approximate wave functions and masses are for the KK tower of the coupled radion-scalar system. In this approximation of neglecting the back reaction, however, we have still found a vanishing radion mass. This is in fact easy to show for a general stabilizing potential. From Eq. (3.17), $F=e^{2A}$ is always a solution with zero mass if A'' is neglected in the bulk. Thus the radion mass is always proportional to the back reaction of the metric independently of the details of the potential of the stabilizing scalar field. In the following, we will show how the back reaction generates a nonvanishing mass for the radion field. For this, we start with the equation describing the radion wave function in the y coordinates:

$$F''-2A'F'-4A''F+2uF'-4uA'F+m^2e^{2A}F=0, \tag{6.1}$$

where $A(y)$ is given in Eq. (2.19). The appropriate boundary condition is $F'-2A'F=0$ at the branes. In the special limit $\gamma_\pm\to\infty$ the other boundary condition is $\varphi=0$. Thus we will treat the back reaction as a perturbation, and look for the solution in terms of a perturbative series in $l\equiv\kappa\phi_P/\sqrt{2}$. Then we write the solution as

$$F_0=e^{2k|y|}[1+l^2f_0(y)],\quad m_r^2=l^2\tilde{m}^2,$$

$$A(y)=k|y|+\frac{l^2}{6}e^{-2u|y|}. \tag{6.2}$$

Expanding the solution as above and keeping only the leading terms in l^2 we obtain the equation

$$f_0''+2(k+u)f_0'=-\tilde{m}^2e^{2k|y|}-\frac{4}{3}(k-u)ue^{-2u|y|} \tag{6.3}$$

along with the boundary conditions

$$f_0'+\frac{2}{3}ue^{-2u|y|}=0 \tag{6.4}$$

at the location of the branes. One can easily find the most general solution for f from the equation in the bulk, which is given by

$$f_0'(y)=Ce^{-2(k+u)|y|}-\frac{\tilde{m}^2}{2(2k+u)}e^{2k|y|}-\frac{2(k-u)u}{3k}e^{-2u|y|}, \tag{6.5}$$

where the integration constant C along with the radion mass $\tilde{m}$ is determined by the boundary conditions at the brane. This way we obtain the radion mass to be

$$m^2_{\text{radion}}=\frac{4l^2(2k+u)u^2}{3k}e^{-2(u+k)r_0}, \tag{6.6}$$

where r_0 denotes the location of the brane. Note that this result is very similar to the answer obtained from the effective theory computation using the naïve ansatz [14,15], except for the important difference in the power of u/k. The exact result obtained here scales as $(u/k)^2$, whereas the effective theory result would scale as[4] $(u/k)^{3/2}$. It would be very interesting to understand the origin of this different scaling. For this model to give the correct value of the weak scale without reintroducing a large fine-tuning again one needs $u/k\approx\frac{1}{37}$, thus the radion mass turns out to be somewhat lighter than the TeV scale. It is suppressed by the factor $l(u/k)e^{-ur_0}$ compared to the TeV scale. Thus in this approximation $m_{\text{radion}}\sim l/40$ TeV, which could be at least in the range of a few GeV's. Of course, we need to emphasize that

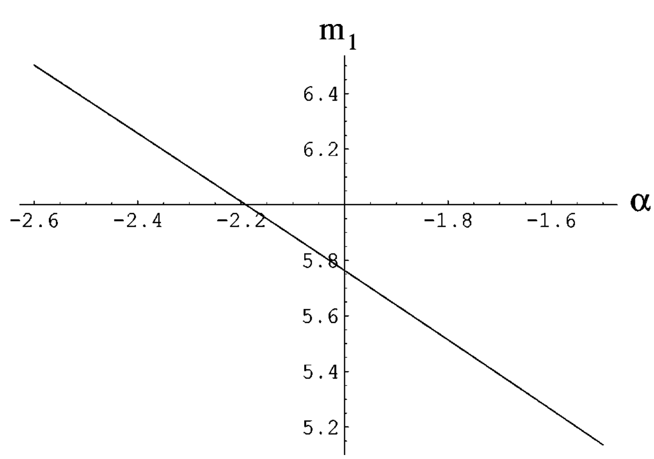


FIG. 2. The dependence of the mass of the first KK mode on α. Here m_1 is again given in units ke^{-kr_0} and is therefore of the order of the TeV scale.

[4]We thank Jim Cline for these observations.

l is not necessarily small for the stabilization mechanism to work, we took this limit only for calculational convenience.

VII. COUPLING TO SM FIELDS

In this section the coupling of the radion and KK tower of ϕ to the TeV brane are obtained. In particular we demonstrate that the bulk scalar field gives a small correction to the radion kinetic term, and thus the kinetic terms obtained from the Einstein-Hilbert part of the action dominate, justifying the results obtained using the naïve ansatz [14,15].

In the previous section it was seen that by including the backreaction an $\mathcal{O}$ (TeV) mass for the radion is obtained. The wave function is then

$$F_0(x,y)=e^{2k|y|}[1+l^2 f_0(y)]R(x), \tag{7.1}$$

where $f_0(y)$ is given by the integral of Eq. (6.5). Since the radion mass is $\mathcal{O}$ (TeV), and by assumption $l^2 \ll 1$, we see by inspection that the back reaction induces a small correction to the unperturbed wave function. So for the purposes of determining the coupling of the radion to the TeV brane it is sufficient to include only the unperturbed wave function, namely $F(x,y)=e^{2k|y|}R(x)$. Then a straightforward calculation gives

$$\begin{aligned}-M^3\int dy\sqrt{g}\mathcal{R}&\supset 6M^3(\partial R)^2\int e^{-2A}e^{4k|y|}\\&=\frac{6M^3}{k}(e^{2kr_0}-1)(\partial R)^2.\end{aligned}\tag{7.2}$$

So the normalized radion $r(x)$ is $R(x)=r(x)e^{-kr_0}/\sqrt{6}M_{Pl}$, since $M^3/k=M_{Pl}^2/2$. This implies a coupling to the TeV brane fields which is

$$\begin{aligned}&R(x)e^{2kr_0}[1+\mathcal{O}(l^2)]\mathrm{Tr}T_{\mu\nu}\\&\quad=\frac{r(x)}{\sqrt{6}M_{Pl}e^{-kr_0}}\mathrm{Tr}T_{\mu\nu}[1+\mathcal{O}(l^2)],\end{aligned}\tag{7.3}$$

where the left-hand side of this equation is a consequence of the fact that the induced metric on the TeV brane is $e^{-2A(r_0)}[1-e^{2kr_0}R(x)\eta_{\mu\nu}]$. The coupling obtained this way agrees precisely with Eq. (2.4). This is perhaps surprising, since the latter computation used an ansatz which did not satisfy the equations of motion. This makes us suspect that results which depend only on the leading unperturbed form of the radion wave function will be correctly captured by the naïve ansatz.

Now we address the issue that was originally raised by Ref. [16]. Is the radion kinetic term dominated by the kinetic term of the bulk scalar field or the bulk gravity action, and in particular is the former hierarchically larger? To answer this, we need the change in φ caused by a fluctuation in the radion. Since $\varphi=0$ when the back reaction is not included, we must include the leading back reaction correction to the radion wave function, given by the integral of Eq. (6.5). From Eq. (3.12) we compute that the change in φ to $\mathcal{O}(l^2)$ due to the radion is

$$\begin{aligned}\varphi&=\frac{3}{\kappa^2\phi_0'}(F'-2A'F)=\frac{3l^2}{\kappa^2\phi_0'}\left(F_0'+\frac{2u}{3}e^{-2u|y|}\right)\\&=\frac{3l^2R(x)}{\kappa^2\phi_0'}e^{2k|y|}f_3(y),\end{aligned}\tag{7.4}$$

where

$$\begin{aligned}f_3(y)&\equiv f_0'(y)+\frac{2}{3}ue^{-2u|y|}\\&=Ce^{-2(k+u)|y|}-\frac{\tilde{m}^2}{2(2k+u)}e^{2k|y|}+\frac{2u^2}{3k}e^{-2u|y|}.\end{aligned}\tag{7.5}$$

This fluctuation in ϕ then contributes to the radion kinetic term at $O(l^2)$ an amount

$$\begin{aligned}&\int dye^{-4A}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\\&\quad=\frac{9l^2}{2\kappa^2u^2}(\partial R)^2\int dye^{2(k-u)|y|}f_3(y)^2.\end{aligned}\tag{7.6}$$

From Eq. (7.2) the unnormalized contribution from bulk gravity to the kinetic term is $\sim e^{2kr_0}$. So we only need to consider those contributions from ϕ which are comparable or larger to this. Recalling that $m^2\sim e^{-2kr_0}$, it is seen that the largest terms in Eq. (7.6) are at best e^{2kr_0}. Explicitly performing the integral one finds that it is

$$\begin{aligned}\delta\mathcal{L}=2l^2\frac{u^2}{\kappa^2k^2}(\partial R)^2e^{2kr_0-6ur_0}&\left(\frac{1}{3k-u}+\frac{1}{k-3u}-\frac{1}{k-u}\right)\\&\times[1+O(l^2)].\end{aligned}\tag{7.7}$$

This is typically $\sim l^2u^2e^{2kr_0}M^3/k^3$, which is smaller than Eq. (7.2) since we assuming that the back reaction is small, $l\ll 1$, and also that $u \ll k$ to obtain a realistic hierarchy. So the radion kinetic term is dominated by the contribution from the bulk gravity, and receives a small correction from the stabilizing bulk scalar field.

In Sec. IV it was found that for the simple boundary conditions $\varphi=0$ (corresponding to the limit $\partial^2\lambda_\pm/\partial\phi^2\gg 1$) a self-adjoint equation for F was obtained. The general solution to this is

$$F(x,y)=\sum_n \alpha_n F_n(x,y), \tag{7.8}$$

where F_n is a mass eigenstate, and the $\alpha_n's$ are some numbers. We expect that F includes the massive radion, but where did all the other states come from? It is helpful to reconsider what happens when the back reaction is neglected. In this limit the KK tower in F completely disappears and only the (massless) radion remains. This may be observed from Eq. (3.12), since neglecting the back reaction corresponds to $\kappa^2\phi_0\ll 1$, and this amounts to setting $F'-2A'F=0$. The only solution for F in this case is the radion zero mode $F=e^{2k|y|}$. Once the back reaction is included, however, the fluctuating modes in ϕ and F are correlated through Eq. (3.12). In particular, a general fluctuation φ induces a change in F. The sum over KK states appearing above is then just the decomposition of F into these KK eigenstates. It is then expected that the coefficients α_n for the nonradion states to be suppressed by the back reaction.

The preceding remarks imply that the TeV brane fields, which couple to the induced metric F, also directly couple to the KK tower, by an amount suppressed by the back reaction.[5] Since $F\sim\phi_0'\varphi$ is already suppressed by the back reaction, therefore in order to compute the induced metric to lowest order in the back reaction we can use the zeroth order wave functions for φ. The normalized KK fields are given by

$$\varphi_n(x,z)=\frac{\psi_n(x)}{N_n}(kz+1)^2 J_{2+u/k}(m_n z+1/k)[1+l^2 f_n(y)]. \tag{7.9}$$

Here $\psi_n(x)$ are the normalized 4D fields satisfying $\Box\psi_n=-m_n^2\psi_n$. The orthogonality of these solutions when the back reaction vanishes $(l=0)$ follows from the boundary condition $\varphi_n=0$ and the properties of the Bessel functions. Also $\psi_n(x)$ are the normalized 4D fields satisfying $\Box\psi_n=-m_n^2\psi_n$. The normalization constant is

$$N_n=\frac{1}{\sqrt{k}}e^{kr_0}J_{3+u/k}\left(\frac{m_n}{k}e^{kr_0}\right). \tag{7.10}$$

As discussed previously, the lowest order masses m_n are determined by $J_{2+u/k}(e^{2kr_0}m_n/k)=0$ and are real since the operator equation with these boundary conditions is self-adjoint. The coupling of these fields to the TeV brane is given by

$$F(x,y=r_0)\mathrm{Tr}T_{\mu\nu}, \tag{7.11}$$

where F is the solution of Eq. (3.12) for the solutions φ_n given above. One finds the coupling

$$\frac{l}{m_n}\lambda_s x_n^u\psi_n(x)\mathrm{Tr}T_{\mu\nu}. \tag{7.12}$$

The model-dependent couplings that appear are

$$\lambda_S=\frac{\sqrt{2}}{3}u\kappa\sqrt{k}e^{-ur_0}\sim\mathcal{O}\left(\frac{u}{k}\right) \tag{7.13}$$

and

$$x_n^u=\frac{J_{1+u/k}\left(\frac{m_n}{k}e^{kr_0}\right)}{J_{3+u/k}\left(\frac{m_n}{k}e^{kr_0}\right)} \tag{7.14}$$

is a numerical constant of $O(1)$. While the inclusion of the back reaction leads to a TeV suppressed coupling for the KK modes, the size decreases rather rapidly due to the $1/m_n\sim1/\mathrm{TeV}$ suppression, as may be observed from inspecting Fig. 1.

The coupling discussed here implies that the KK modes of ϕ can be directly produced at future colliders, and they also decay directly to standard model fields. This may be puzzling at first, since the stabilizing potential may have a global discrete symmetry, such as Z_2, which would naïvely imply that some of these KK modes are stable. The background VEV for ϕ explicitly breaks this symmetry, however, and this allows for all the KK modes to decay into the brane world fields.

The direct coupling of the KK modes from the stabilizing fields may have interesting implications for search strategies and current limits on the Randall-Sundrum framework. In particular, it may be important to *not* neglect the stabilizing potential when discussing these issues. However, when the back reaction is small, the size of their couplings is suppressed by $u/k\sim\frac{1}{40}$ compared to the that of the radion. Therefore, in what follows, we neglect these states in the loop computations.

VIII. COSMOLOGICAL IMPLICATIONS

The subject of brane cosmology has recently attracted much interest [10,11,14,31–37]. Most of this was due to the realization that the expansion of a brane universe could be significantly different from the ordinary Friedmann-Robertson-Walker (FRW) cosmology [10,11]. However, it did not take very long to realize that this is simply due to the fact that a generic brane model (like the one presented in Ref. [10]) cannot give the ordinary cosmological evolution, since gravity is in general manifestly higher dimensional. This means that in these models the 4D effective theory is usually not described by ordinary Einstein gravity, but generically a complicated scalar-tensor theory of gravity. However, observations show that our Universe is described by Einstein's theory of relativity to a high precision, therefore one has to require from the outset that a brane model reproduces ordinary Einstein gravity, at least at long enough distances. Once this is achieved, the cosmological expansion will be automatically described by the ordinary Friedmann-Robertson-Walker (FRW) Universe, which simply follows from the fact that the effective theory is ordinary Einstein gravity. Thus one can see that the issue of unconventional cosmologies is nothing else but the issue of whether one recovers 4D gravity. This issue manifests itself in the case of the Randall-Sundrum two-brane model due to the presence of the radion field. Without a stabilizing potential, the radion field will be massless, and yield additional long range forces, and also contribute to the expansion of the Universe, yield-

[5]The KK modes of the scalar field do not mix with the KK modes of the graviton. The reason is that the only way they could mix is by a coupling of the 5D trace of the metric to the scalar KK modes. However, the graviton is traceless, and the trace is basically identified with the radion, therefore no additional graviton-scalar mixing could be introduced.

ing an unconventional cosmology, which is presumably excluded by the requirement for a successful nucleosynthesis [11]. Thus the radion field has to obtain a mass. Once it is massive, gravity on both branes will be ordinary 4D gravity, and thus the cosmology will be conventional below temperatures comparable to the radion mass. This has been explained in great detail in Ref. [14], and also in Ref. [31]. In Ref. [14] a simplified calculation for the cosmological expansion has been presented, where the wave function of the radion has been neglected, and also the effects of the stabilizing scalar field were included by adding a five dimensional potential for the radion field $V(b)$. Assuming that that the potential $V(b)$ is very steep, it was shown from a perturbative solution of the bulk equations that the ordinary FRW Universe is recovered. It was also argued that the 55 component of the Einstein equation, which in the absence of a stabilizing potential usually leads to the unconventional expansion equations, will only determine the shift in the radion field due to matter on the wall, and does not result in unconventional cosmologies once the radius is stabilized. Below we demonstrate, that the results of Ref. [14] which were neglecting the radion wave function, and also did not include the fluctuations of the scalar field at the brane remain valid in the more precise framework of radion stabilization explained in the previous sections. In particular, we will show that the result obtained for the shift in the radion field due to matter on the walls in Eq. (4.15) of Ref. [14] is exactly reproduced in the full calculation.

To compute G_{55} we use the ansatz

$$ds^2 = n(t,y)^2dt^2 - a(t,y)^2d^2x - b(t,y)^2dy^2 \tag{8.1}$$

for which

$$G_{55} = 3\left\{\frac{a'}{a}\left(\frac{a'}{a}+\frac{n'}{n}\right) - \frac{b^2}{n^2}\left[\frac{\dot a}{a}\left(\frac{\dot a}{a}-\frac{\dot n}{n}\right)-\frac{\ddot a}{a}\right]\right\}. \tag{8.2}$$

The jump equations for a and n on the TeV brane imply [10]

$$\frac{[a']}{a} = -\frac{\kappa^2}{3}[\lambda_-(\phi)+\rho]b,$$

$$\frac{[n']}{n} = \frac{\kappa^2}{3}[-\lambda_-(\phi)+3p+2\rho]b. \tag{8.3}$$

Here ρ and p are the bare energy matter density and pressure on the TeV brane, which are related to the physically measured quantities on the TeV brane by $\rho_0 = \rho e^{-4A_0}$, etc., where e^{-A_0} is the scale factor on the TeV brane. Then averaging the G_{55} equation about the TeV brane and linearizing to $\mathcal{O}(\rho, F, \varphi)$ gives

$$\langle G_{55}\rangle = \kappa^4\frac{\lambda_-^2(\phi_0)}{6} - 3e^{2A}\left[\left(\frac{\dot a}{a}\right)^2+\frac{\ddot a}{a}\right] - \frac{\kappa^4\lambda_-(\phi_0)}{12}(3p-\rho) + \frac{\kappa^4\lambda_-(\phi_0)}{3}\frac{\partial\lambda_-(\phi_0)}{\partial\phi}\varphi + \frac{2\kappa^4\lambda_-^2(\phi_0)}{3}F. \tag{8.4}$$

Terms with $\dot n$ are higher order in ρ and are dropped, and $b = 1+2F$ has been used. For late-time cosmology in the presence of radion stabilization it is reasonable to use the FRW equation

$$\left(\frac{\dot a_0}{a_0}\right)^2 + \frac{\ddot a_0}{a_0} = -\frac{1}{6M_{Pl}^2}(3p_* - \rho_* + 3p_0 - \rho_0). \tag{8.5}$$

Note that this includes a contribution from matter (p_*) on the Planck brane. Also implicit in the use of this equation is the assumption that the time variation of the radion is negligible, which is justified *a posteriori*. Then using the relation $\kappa^4\lambda_-(\phi_0) = -6(1-e^{-2A_0})/M_{Pl}^2$ gives

$$\langle G_{55}\rangle = \frac{\kappa^4\lambda_-^2(\phi_0)}{6} + \frac{e^{4A_0}}{2M_{Pl}^2}[3p_0-\rho_0+e^{-2A_0}(3p_*-\rho_*)] + \frac{\kappa^4\lambda_-(\phi_0)}{3}\frac{\partial\lambda_-(\phi_0)}{\partial\phi}\varphi + \frac{2\kappa^4\lambda_-^2(\phi_0)}{3}F. \tag{8.6}$$

The G_{55} equation is

$$G_{55} = \kappa^2 T_{55} = \kappa^2\left[\frac{1}{2}\phi'^2 - g_{55}\left(\frac{1}{2}(\nabla\phi)^2 - V(\phi)\right)\right]. \tag{8.7}$$

Then the averaging of T_{55} and linearizing using Eq. (3.2) gives

$$\kappa^2\langle T_{55}\rangle = \kappa^2\left(\frac{1}{2}\phi_0'^2 - V(\phi_0)\right) + \kappa^2\left(\phi_0'\varphi' - 4FV - \frac{\partial V}{\partial\phi}\varphi\right) \tag{8.8}$$

with all quantities are evaluated on the TeV brane. Using the background bulk Eq. (2.10) and the jump Eq. (2.12) the leading terms are seen to cancel. Then after using the background equations, the jump equations for the background fields, and some algebra gives

$$\frac{e^{4A_0}}{2M_{Pl}^2}[3p_0-\rho_0+e^{-2A_0}(3p_*-\rho_*)] = \kappa^2\left(\phi_0'\varphi' - 2\phi_0'^2F - \frac{\partial V}{\partial\phi}\varphi - 4A'\phi_0'\varphi\right). \tag{8.9}$$

Using Eq. (3.16) to eliminate φ', Eq. (3.17) to eliminate F'' in favor of the mass eigenvalue, and Eq. (2.11) to eliminate ϕ_0'' finally gives

$$\frac{e^{4A_0}}{2M_{Pl}^2}[3p_0-\rho_0+e^{-2A_0}(3p_*-\rho_*)] = -3e^{2A_0}m_r^2F. \tag{8.10}$$

But the shift in the distance between the two branes is obtained by integrating the line element, which gives $\delta r_0 = R(e^{2kr_0}-1)/k$. Then since $F = Re^{2kr_0}$, one obtains

$$\frac{\delta r_0}{r_0} = \frac{1}{6kr_0}\frac{(1-e^{-2A_0})}{m_r^2M_{Pl}^2e^{-2A_0}}[\rho_0-3p_0+e^{-2A_0}(\rho_*-3\rho_*)] \tag{8.11}$$

which is *precisely* the result found in Ref. [14] obtained by using a 4D effective theory.[6] This is perhaps not surprising, since for constant radion field the naïve ansatz and the full metric including the wave function of the radion are equivalent up to a coordinate transformation. So in an adiabatic approximation the leading order result using the naïve ansatz should agree with that obtained from using the correct radion wave function, if the fluctuations in the scalar field are ignored. It is less clear why the full answer including the contribution from the scalar field turns out to be exactly equal to the calculation using the naïve ansatz. Note that matter on the Planck brane causes a smaller shift in the radion compared to an equal amount of matter on the TeV brane. This is because the radion wave function is peaked at the TeV brane, and it couples more weakly to the Planck brane relative to the TeV brane by precisely the amount e^{-2A_0}. Thus one finds the very general result that in the presence of matter on the branes and a stabilizing mechanism, the G_{55} equation determines the shift in the radion.

IX. EFFECTIVE 4D LAGRANGIAN

In the previous sections we have argued that in the presence of a stabilizing potential the linear couplings of the radion and bulk scalars is given by

$$\frac{1}{2}(\partial r)^2-\frac{1}{2}m^2r^2+\sum_n \frac{1}{2}[(\partial\psi_n)^2-m_n^2\psi_n^2]+DH^\dagger DH$$
$$+\left(\frac{r(x)}{\sqrt{6}\Lambda}+\sum_n \alpha_n\frac{\psi_n(x)}{\Lambda_n}\right)\mathrm{Tr}T_{\mu\nu}+\xi H^\dagger H\mathcal{R}-V(H). \tag{9.1}$$

The masses appearing here are O (TeV), and their particular value depends on the details of the stabilizing mechanism. The scale $\Lambda=e^{-kr_0}M_{Pl}$ in the Randall-Sundrum model, but here we have left it general. The other scales are $\Lambda_n\sim m_n$, and the α_n are also model dependent, and vanish in the limit of small back reaction. In the remaining sections we restrict ourselves to the above Lagrangian, and do not commit ourselves to any specific mechanism of radius stabilization. For the electroweak analysis we neglect the contributions of the KK modes from ϕ.

Note that in the above Lagrangian we have also included a curvature Higgs scalar operator $H^\dagger H\mathcal{R}$. The presence of this operator leads to interesting signals for discovering the Higgs and radion at future colliders [21]. In particular, the branching fractions of the Higgs and radion to gg and $\overline{b}b$ can be substantial different from that of the SM Higgs.

As discussed in Ref. [21], the presence of the conformal term $H^\dagger HR$ leads to both kinetic and mass mixing between the neutral Higgs and radion. Below we summarize the relevant formulas for mixing and couplings. The interested reader is referred to the next section for details.

One finds that the ''gauge'' h and r are related to the mass eigenstates h_m and r_m by

$$h=\left(\cos\theta-\frac{6\xi\gamma}{Z}\sin\theta\right)h_m+\left(\sin\theta+\frac{6\xi\gamma}{Z}\cos\theta\right)r_m, \tag{9.2}$$

$$r=\cos\theta\frac{r_m}{Z}-\sin\theta\frac{h_m}{Z}, \tag{9.3}$$

where

$$\tan 2\theta=12\xi\gamma Z\frac{m_h^2}{m_r^2-m_h^2-6\xi\gamma^2(1-12\xi)}, \tag{9.4}$$

$$\gamma=\frac{v}{\sqrt{6}\Lambda},\quad Z^2=1+6\xi\gamma(1-6\xi), \tag{9.5}$$

where $v\approx 246\,\mathrm{GeV}$ is the electroweak VEV. Requiring that the quantity Z^2 be positive (in order to avoid ghostlike states) places an upper bound on the value of ξ, for a given γ. Physically this requirement comes from maintaining positive definite kinetic terms for h and ϕ.

In this basis, the couplings of the physical radion and Higgs appropriate for tree-level studies are

$$-\left[\left(\cos\theta-(6\xi-1)\frac{\gamma\sin\theta}{Z}\right)h_m\right.$$
$$\left.+\left(\sin\theta+(6\xi-1)\frac{\gamma\cos\theta}{Z}\right)r_m\right]\mathrm{Tr}T_{\mu\nu}. \tag{9.6}$$

In the $\xi\to 0$ limit one recovers

$$-(h-\gamma r)\mathrm{Tr}T_{\mu\nu}, \tag{9.7}$$

obtained in Refs. [14] and [15]. Note that $\mathrm{Tr}T_{\mu\nu}$ includes SM Higgs contributions.

X. CURVATURE-SCALAR MIXING

In this section the effects of introducing a curvature-scalar interaction are reviewed. The discussion parallels Ref. [21], however, some of the resulting formulas are slightly different because here terms of $\mathcal{O}(\gamma^2)$ and $\mathcal{O}(\gamma^2\xi^2)$ are kept.

We begin with the couplings of the radion and Higgs to the SM fields before electroweak symmetry breaking. The induced metric on the TeV wall is

$$g_{\mu\nu}^{ind}(x)=e^{-2A(r_0)-2e^{2kr_0R(x)}}g_{\mu\nu}(x), \tag{10.1}$$

where the warp factor includes the back reaction, although its inclusion is not necessary for our purposes. The canonically normalized radion r is

$$R(x)=e^{-kr_0}\frac{r(x)}{\sqrt{6}M_{Pl}}. \tag{10.2}$$

[6]A translation dictionary between two different notations is required: $kr_0=m_0b_0/2$.

So we express the induced metric expanded about a Minkowski metric as

$$g_{\mu\nu}^{ind}(x)=e^{-2A(r_0)-2r(x)\gamma/v}\eta_{\mu\nu}\equiv e^{-2A(r_0)}\Omega^2(r)\,\eta_{\mu\nu} \tag{10.3}$$

with

$$\gamma=\frac{v}{\sqrt{6}\Lambda},\quad \Lambda=M_{Pl}e^{-kr_0}. \tag{10.4}$$

The four dimensional effective action we consider is

$$\begin{aligned}S_{\rm TeV}=&\int d^4x\sqrt{g_{ind}}[g_{ind}^{\mu\nu}D_\mu H^\dagger D_\nu H-V(H)]\\&+\int d^4x\sqrt{g}\frac{1}{2}[(\nabla r)^2-m_r^2r^2]\\&+\int d^4x\sqrt{g_{ind}}\xi R(g_{ind})H^\dagger H+S_{\rm SM}.\end{aligned} \tag{10.5}$$

To canonically normalize the Higgs and other SM fields, we perform the field-independent redefinition

$$H\to e^{A(r_0)}H,\quad \psi\to e^{3A(r_0)/2}\psi. \tag{10.6}$$

In this basis the Higgs-radion potential is

$$V(H,r)=\Omega^4(r)V(H). \tag{10.7}$$

Note that V also includes the effective 4D cosmological constant, which we assume to vanish. Clearly this potential has a minimum at the same location as $V(H)$, so that the electroweak symmetry breaking (EWSB) vacuum is $r=0$ and $H^0=v/\sqrt{2}$.

We consider the presence of the curvature mixing term

$$\mathcal{L}_\xi=\sqrt{g_{ind}}\xi\mathcal{R}(g_{ind})H^\dagger H. \tag{10.8}$$

Our choice of signs for ξ is such that the Higgs potential receives a positive mass-squared correction in a de Sitter phase when ξ is positive. Since this is a renormalizable interaction, there is no reason for it not to be present, or to be suppressed. What makes this operator important in this case is that $\mathcal{R}$ contains the induced metric, rather than just the ordinary 4D metric. In particular,

$$\mathcal{R}[\Omega^2(r)\,\eta_{\mu\nu}]=-6\Omega^{-2}[\Box\ln\Omega+(\nabla\ln\Omega)^2]. \tag{10.9}$$

So the curvature-scalar interaction is

$$\mathcal{L}_\xi=-6\xi\Omega^2[\Box\ln\Omega+(\nabla\ln\Omega)^2]H^\dagger H. \tag{10.10}$$

To see the effect of the curvature scalar interaction we expand $H^0=(v+h)/\sqrt{2}$ and $\Omega(r)=1-\gamma r/v+\ldots$. We need to only expand Ω to linear order since the derivative terms are already of $O(r)$. This gives at quadratic order

$$\mathcal{L}_\xi=6\xi\gamma h\Box r+3\xi\gamma^2(\partial r)^2, \tag{10.11}$$

where a total derivative has been dropped. The ξ terms clearly introduce kinetic mixing. The full radion-Higgs Lagrangian to be diagonalized is

$$\begin{aligned}\mathcal{L}=&-\frac{1}{2}h\Box h-\frac{1}{2}m_h^2h^2-\frac{1}{2}(1+6\xi\gamma^2)r\Box r-\frac{1}{2}m_r^2r^2\\&+6\xi\gamma h\Box r.\end{aligned} \tag{10.12}$$

The mass parameters m_r, m_h are the masses of the radion and Higgs, respectively, in the limit $\xi=0$. The kinetic terms are diagonalized by the shift $h=h'+6\xi\gamma r'/Z$, and $r=r'/Z$. Here

$$Z^2=1+6\xi\gamma^2(1-6\xi) \tag{10.13}$$

is the coefficient of the radion kinetic term after undoing the kinetic mixing, and is therefore required to be positive in order to keep the radion kinetic term positive definite. For a fixed cutoff Λ this restricts the size of the mixing parameter ξ. It must lie in the range

$$\frac{1}{12}\left(1-\sqrt{1+\frac{4}{\gamma^2}}\right)\leqslant\xi\leqslant\frac{1}{12}\left(1+\sqrt{1+\frac{4}{\gamma^2}}\right) \tag{10.14}$$

for nonzero values of γ. Otherwise one has a ghostlike radion field, which presumably signals an instability of the theory.

This rescaling diagonalizes the kinetic terms, but introduces mixing in the mass matrix. A final rotation $h'=\cos\theta h_m+\sin\theta r_m$ and $r'=\cos\theta r_m-\sin\theta h_m$ brings the Lagrangian to canonical form. With the above definition of the sign of the rotation, the rotation angle is

$$\tan 2\theta=12\xi\gamma Z\frac{m_h^2}{m_r^2-m_h^2(Z^2-36\xi^2\gamma^2)}. \tag{10.15}$$

We note that for moderate values of ξ and γ (i.e., $Z^2>36\xi^2\gamma^2$) the mixing angle $\tan 2\theta$ is negative when $m_h>m_r$. For small γ we can expand

$$\tan 2\theta=12\xi\gamma\frac{m_h^2}{m_r^2-m_h^2}+O(\gamma^2). \tag{10.16}$$

Putting everything together, the relation between the gauge and mass eigenstates is

$$h=\left(\cos\theta-\frac{6\xi\gamma}{Z}\sin\theta\right)h_m+\left(\sin\theta+\frac{6\xi\gamma}{Z}\cos\theta\right)r_m, \tag{10.17}$$

$$r=\cos\theta\frac{r_m}{Z}-\sin\theta\frac{h_m}{Z}. \tag{10.18}$$

The mass eigenvalues are easily obtained

$$m_{\pm}^2=\frac{1}{2Z^2}(m_r^2+(1+6\xi\gamma^2)m_h^2\pm\{[m_r^2-m_h^2(1+6\xi\gamma^2)]^2+144\gamma^2\xi^2m_r^2m_h^2\}^{1/2}). \tag{10.19}$$

The heavier state $(+)$ is identified with the state with the larger of (m_h^2,m_r^2).

XI. RADION COUPLINGS AND FEYNMAN RULES

In this section we derive the Feynman rules relevant to the computation of the oblique parameters *S, T, U*.

Before proceeding, we pause to ask whether higher-order couplings such as

$$\phi^2\mathrm{Tr}T_{\mu\nu} \tag{11.1}$$

also affect in particular the electroweak precision measurements *S*, *T*, and *U*. This operator could either be directly present, or generated from the above linear coupling due to a nontrivial kinetic term for the radion. Although this operator contributes at one loop to the gauge boson two point functions, it is easy to see that they do not contribute to *T*, since the m_V^2 contained in $\mathrm{Tr}T_{\mu\nu}$ is canceled by the $1/m_V^2$ appearing in the expression for *T*, nor to *S* or *U* since the contribution of this operator to the vacuum polarizations is momentum independent. Thus we need to only consider the linear coupling

$$\frac{\gamma r}{v}\mathrm{Tr}T. \tag{11.2}$$

This operator will have a contribution to the oblique corrections similar to that of the standard model Higgs boson. First we discuss the Feynman rules for the interactions from the $(\gamma r/v)\mathrm{Tr}T$ operator. The interaction Lagrangian term relevant for the gauge-boson propagator corrections is just given by

$$\mathcal{L}_{\mathrm{int}}=-\frac{\gamma}{v}r(2M_W^2W_\mu^+W^{\mu-}+M_Z^2Z_\mu Z^\mu). \tag{11.3}$$

In addition, to ensure gauge invariance of the results, one also has to examine the gauge fixing terms carefully. The gauge fixing Lagrangian for the *W* and the *Z* are given by

$$\mathcal{L}_{gf}=\sqrt{g}\left[-\frac{1}{\alpha}(-D_\mu W^{\mu+}+i\alpha M_W\Psi^+)(-D_\mu W^{\mu-}-i\alpha M_W\Psi^-)-\frac{1}{2\alpha}(-D_\mu Z^\mu+\alpha M_Z\Psi)^2\right], \tag{11.4}$$

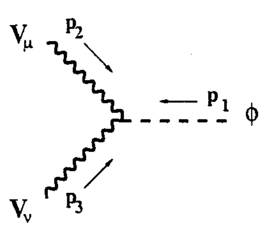
where the Ψ's are the would-be-Goldstone bosons, and α is the gauge fixing parameter in the R_α gauge. Note that since the gravitational background is nontrivial, we have replaced the ordinary derivatives by covariant derivatives. The background metric is given by $g_{\mu\nu}=\Omega^2(r)\eta_{\mu\nu}=e^{-2\gamma r/v}\eta_{\mu\nu}$, therefore the covariant derivative of a vector will take the form

$$D_\mu V^\mu=\Omega^{-2}\left(\partial_\mu V^\mu-\frac{2\gamma}{v}\partial_\mu rV^\mu\right). \tag{11.5}$$

Thus from the gauge fixing terms one also obtains three-point interaction vertices of the form

$$\mathcal{L}_{gf}=\frac{2\gamma}{v\alpha}(\partial_\mu Z^\mu)(\partial_\nu rZ^\nu)+\frac{2\gamma}{v\alpha}(\partial_\mu W^{\mu+})(\partial_\nu rW^{\nu-})+\frac{2\gamma}{v\alpha}(\partial_\mu W^{\mu-})(\partial_\nu rW^{\nu+}). \tag{11.6}$$

With these operators added, the Feynman rule for the three-point function is given by



$$\frac{-i2M_V^2\gamma}{v}\eta_{\mu\nu}-\frac{i2\gamma}{\alpha v}(p_{2\mu}p_{1\nu}+p_{3\nu}p_{1\mu}) \tag{11.7}$$

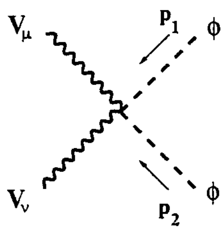
In addition to the cubic vertices evaluated above, there are also four-point couplings of the radion and the gauge bosons. These terms will not contribute to the oblique electroweak corrections, however, they will be important to obtain a gauge invariant answer for the gauge boson vacuum polarization diagrams. These terms arise from two different sources. The first source is the conformal coupling $-e^{-\gamma r/v}\mathrm{Tr}T$ to the trace of the energy momentum tensor. In the formalism of Ref. [14] this can be obtained by a very careful expansion of the interaction terms $|D_\mu H|^2+(1/\sqrt{6}\Lambda)\partial r(H^\dagger DH+\mathrm{H.c.})$. Either way one finds the additional operators

$$\frac{\gamma^2}{v^2}r^2\left(\frac{M_W^2}{2}W_\mu^+W^{\mu-}+M_Z^2Z_\mu Z^\mu\right). \tag{11.8}$$

The other source of quartic interaction terms are again the gauge fixing terms. One simply expands these to higher order to obtain the interaction terms

$$-\frac{2\gamma^2}{\alpha v^2}\partial_\mu r\partial_\nu r(Z^\mu Z^\nu+2W^{\mu+}W^{\nu-}). \tag{11.9}$$

The interaction terms involving the would-be Goldstone bosons just contribute a total derivative, and thus they can be omitted. The above operators give rise to the following Feynman rule for the four-point function:



$$\frac{i2M_V^2\gamma^2}{v^2}\eta_{\mu\nu}+\frac{i4\gamma^2}{\alpha v^2}(p_{1\mu}p_{2\nu}+p_{1\nu}p_{2\mu}) \tag{11.10}$$

The presence of the terms proportional to $1/\alpha$ in Eqs. (11.7) and (11.10) are in fact crucially important to obtain gauge invariant amplitudes, however, for calculations in the unitary gauge $\alpha\to\infty$ their effect vanishes.

This is, however, not the complete story for the Feynman rules. The reason is that the radion couples conformally to the metric and so is not the same as the Higgs. Thus, for example, one finds that at one loop the radion has the anomalous coupling

$$\frac{r}{\Lambda}b_G\frac{\alpha_G}{8\pi}G_{\mu\nu}G^{\mu\nu} \tag{11.11}$$

in addition to the usual momentum-dependent coupling obtained from one-loop diagrams with internal fermions. Here b_G is the beta function. This may be understood as due to the scaling anomaly together with r as a generator of scale transformations. Diagrammatically this result is obtained by preserving the conformal coupling of r when the theory is regulated. For dimensional regularization this means that the radion must couple conformally to the D-dimensional metric. Since the linear coupling of the radion is obtained from varying the induced metric, for loop computations the radion should couple instead to $\mathrm{Tr}_D T_{\mu\nu}$, but where now the trace is evaluated in D dimensions. This differs from the above coupling by some operators whose coefficient vanishes when $D\to 4$. Since this $\varepsilon=4-D$ dependence can be offset by poles appearing in the loops, the appearance of these additional operators can result in finite nonzero results in the $D\to 4$ limit. These operators will indeed have a nonvanishing contribution to the S and U parameter. Next we calculate the Feynman rules for these ''anomalous'' couplings. The interactions we should therefore study are

$$\mathcal{L}=-\frac{h}{v}\mathrm{Tr}_4 T+\frac{\gamma r}{v}\mathrm{Tr}_D T. \tag{11.12}$$

We point out that it is the original ''gauge'' radion r that has the conformal coupling to the metric, and consequently it is this field which appears in the above interactions. But a result of the curvature-scalar term interaction is to introduce mixing between the radion and Higgs, which implies that after transforming to the mass basis the physical Higgs boson h_m will have couplings similar to those above.

The radion will thus have the interaction terms ($D=4-\varepsilon$)

$$\begin{aligned}\mathcal{L}&=\gamma\frac{r}{v}\mathrm{Tr}_D T_{\mu\nu}\\&=\gamma\frac{r}{v}\mathrm{Tr}_4 T-\gamma\frac{\varepsilon}{4}\frac{r}{v}F_{\mu\nu}F^{\mu\nu}+\gamma\frac{\varepsilon}{2}\frac{r}{v}(2M_W^2W^+W^-\\&\quad+M_Z^2Z^2),\end{aligned} \tag{11.13}$$

where the first term is the one we have already discussed above. We will later add in the Higgs and mixing coefficients. The terms relevant to the gauge boson propagators from the last two terms are

$$\begin{aligned}\mathcal{L}_{\mathrm{int}}^{\mathrm{anom}}=&-\frac{\varepsilon\gamma}{2v}r[(\partial_\mu Z_\nu)^2-(\partial_\mu Z_\nu)(\partial_\nu Z_\mu)\\&+2(\partial_\mu W_\nu^+)(\partial^\mu W^{\nu-})-2(\partial_\mu W_\nu^+)(\partial^\nu W^{\mu-})\\&-2M_W^2W_\mu^+W^{\nu-}-M_Z^2Z_\mu Z^\nu].\end{aligned} \tag{11.14}$$

In D dimensions, one also has to modify the gauge fixing terms. The covariant derivative of a vector field will be modified to

$$D_\mu V^\mu=\Omega^{-2}\left(\partial_\mu V^\mu-\frac{(D-2)\gamma}{v}\partial_\mu r V^\mu\right). \tag{11.15}$$

In addition, the $\sqrt{g}$ factor in front of the gauge fixing terms have to be modified to Ω^D. Thus these interaction terms modify the Feynman rules for the interaction vertex given in Eq. (11.7) to

$$\frac{-2iM_V^2\gamma}{v}\left(1-\frac{\varepsilon}{2}\right)\eta_{\mu\nu}+\frac{i\gamma\varepsilon}{v}(p_2\cdot p_3\eta_{\mu\nu}-p_{2\nu}p_{3\mu})-\frac{i(2-\varepsilon)\gamma}{\alpha v}(p_{2\mu}p_{1\nu}+p_{3\nu}p_{1\mu})-\frac{i\gamma\varepsilon}{\alpha v}p_{2\mu}p_{3\nu}. \tag{11.16}$$

In addition, the four-point vertex in Eq. (11.10) is also modified by terms proportional to ε, which, however, do not contribute to a calculation in the unitary gauge.

We close this section by discussing how to take the mixing between the Higgs and the radion due to the possible presence of the curvature-scalar mixing operator into account. The interaction in the gauge basis is

$$\mathcal{L}=-\frac{h}{v}\mathrm{Tr}_4T+\frac{\gamma r}{v}\mathrm{Tr}_DT. \tag{11.17}$$

Using

$$h=ah_m+br_m, \tag{11.18}$$

$$r=ch_m+dr_m, \tag{11.19}$$

where the coefficients a, b, c, and d can be read off from Eqs. (10.17) and (10.18), then

$$\mathcal{L}=\left(-(a-\gamma c)\frac{h_m}{v}+(\gamma d-b)\frac{r_m}{v}\right)\mathrm{Tr}_4T+(ch_m+dr_m)\frac{\gamma}{v}\left(-\frac{\varepsilon}{4}F_{\mu\nu}F^{\mu\nu}+\frac{\varepsilon}{2}M_V^2V^2\right). \tag{11.20}$$

Thus the Feynman rules for the radion (and also for the Higgs boson) have to be modified such that the above mixing terms are taken properly into account, for example, for the mass eigenstate radion the Feynman rule will be

$$\frac{-2iM_V^2}{v}\left(\gamma d-b-\frac{\gamma d\varepsilon}{2}\right)\eta_{\mu\nu}+\frac{i\gamma d\varepsilon}{v}(p_2\cdot p_3\eta_{\mu\nu}-p_{2\nu}p_{3\mu})-\frac{i(2-\varepsilon)\gamma d}{\alpha v}(p_{2\mu}p_{1\nu}+p_{3\nu}p_{1\mu})-\frac{i\gamma\varepsilon d}{\alpha v}p_{2\mu}p_{3\nu}. \tag{11.21}$$

XII. ELECTROWEAK PRECISION MEASUREMENTS

In this section we consider the corrections of the Randall-Sundrum model to the oblique parameters. Our analysis also applies more generally to a model with a light scalar coupled conformally to the SM metric but with a typical coupling of $\mathcal{O}$ (TeV^{-1}).

Our approach is to use an effective theory with cutoff $\mathcal{O}(\Lambda)$, similar to the approach taken in Ref. [25]. Below this scale the only light fields are the radion and Higgs boson whose contributions we are going to calculate explicitly. In our approach the effect of any modes heavier than the cutoff are included by introducing higher dimension operators that directly contribute to the oblique parameters. This in principle includes the effects of the heavy spin-2 KK states, for example, which are typically heavier than the radion. A direct computation of the effect of the heavy spin-2 states using a momentum-dependent regulator has been presented in Ref. [18].

In the previous section the radion coupling to the gauge bosons was obtained and found to be similar to that of the Higgs boson. The contribution of the Higgs boson to the oblique parameters is by itself divergent, but these divergences are canceled by the contribution from the pseudo-Goldstone bosons (or the longitudinal states of the massive gauge bosons). Thus for the radion one expects a divergent contribution, but in contrast to the Higgs boson there is no additional source to cancel this. This is perhaps not surprising since the radion interactions are nonrenormalizable.

A set of operators that provide the necessary counterterms for the wave-function renormalization is

$$\mathcal{O}_X=\frac{g^2Z_X}{\Lambda^2}\left(H^\dagger H\mathrm{Tr}W_{\mu\nu}W^{\mu\nu}+\frac{1}{2}\tan^2\theta_WH^\dagger HB_{\mu\nu}B^{\mu\nu}+\tan\theta_WH^\dagger W_{\mu\nu}B^{\mu\nu}H\right), \tag{12.1}$$

where $W_{\mu\nu}=W^a\tau^a$, with the generators normalized to $\frac{1}{2}$. Note that the last operator is gauge invariant, since for a gauge transformation U, $W_{\mu\nu}\to UW_{\mu\nu}U^\dagger$. Setting the Higgs boson to its VEV in the above operator gives

$$\mathcal{O}_X\to\frac{g^2Z_Xv^2}{2\Lambda^2}\left(W^+_{\mu\nu}W^{-\mu\nu}+\frac{1}{2\cos^2\theta_W}Z_{\mu\nu}Z^{\mu\nu}\right). \tag{12.2}$$

In this model the radion does not contribute to $\gamma\gamma$ and γZ wave-function renormalization, and the absence of these counterterms uniquely fixes the coefficients in Eq. (12.1). The explicit computation of the ZZ and WW wave-function renormalizations demonstrates that the above operator has the correct relative factor between the two gauge bosons. This is perhaps nontrivial, since there is no additional degree of freedom to fix this relative factor. We also note that an identical operator to Eq. (12.2) is also required in technicolor theories in order to cancel the divergent contribution of pseudo-Goldstone bosons to the oblique parameters [40].

The operator which provides the counterterms for the mass renormalization is

$$\mathcal{O}_M=\frac{Z_M}{2\Lambda^2}[g'^2(D_\mu H^\dagger H)(H^\dagger D^\mu H)+g^2H^\dagger H(D_\mu H^\dagger D^\mu H)]. \tag{12.3}$$

The first operator that appears here violates the custodial symmetry, and in particular contributes to the Z but not the W mass. After electroweak symmetry breaking they together reduce to

$$\mathcal{O}_M \to \frac{Z_M}{\Lambda^2}\left(M_W^4 W_\mu^+ W^{-\mu} + \frac{M_Z^4}{2} Z^\mu Z_\mu \right). \tag{12.4}$$

Note that it is m_V^4 which appears, so we explicitly see that this operator contributes to the ρ parameter. In this case it is trivial to obtain the correct mass renormalization from Eq. (12.3), since here there are two coefficients to be determined from only two constraints. So in addition to the usual standard model renormalizations, these wave function and mass counter terms are also required to renormalize the model.

The model with the radion represents an effective theory valid for $E \lesssim \Lambda$. The dimension-six operators discussed above are obtained by integrating some unknown degrees of freedom in the full theory, and in the effective theory they appear with some unknown coefficients $Z_i(\Lambda)$. These for example could include the effects of integrating out the heavy spin-2 KK modes. Since the divergences for which the two above operators act as counter terms arise at one loop order, they are proportional to $\gamma^2/(16\pi^2)$. Thus it is reasonable to expect that the finite part of the operator is also of the same order, and in order to match the form of the explicitly calculated one-loop corrections we will write the (finite) coefficients as

$$Z_M = \frac{\gamma^2\Lambda^2}{16\pi^2 v^2} a_M, \quad Z_X = \frac{\gamma^2\Lambda^2}{16\pi^2 v^2} a_X, \tag{12.5}$$

where we expect that the dimensionless parameters a_M and a_X are at most of order 1. Note that since $\gamma^2 \sim \Lambda^{-2}$, in this parameterization the dimension-six operators are still suppressed by Λ^2.

These coefficients parameterize the unknown physics integrated out at the scale $E \sim \Lambda$. Since, however, the radion at one loop contributes to the anomalous dimension of these operators, when comparing to the experimental results evaluated at the Z mass large logarithms of $O(\ln \Lambda/M_Z)$ appear from this anomalous scaling and this effect should be included. Following Wilson, a one-loop Wilsonian renormalization group equation is obtained for the operator coefficients. In the leading logarithm approximation the value of these coefficients at the weak scale is determined to be

$$Z_i(M_Z) = Z_i(\Lambda) + \frac{\beta_i}{16\pi^2} \ln \frac{\Lambda^2}{M_Z^2}, \tag{12.6}$$

with the β_i determined from the coefficient of the $\ln \mu$ term (or equivalently, from the $1/\varepsilon$ poles) in an explicit one-loop computation. To compute the oblique parameters, one then adds the contribution of these renormalized operators to the finite parts of the one-loop diagrams. In the leading logarithm approximation this amounts to simply replacing the $1/\varepsilon$ poles in the gauge-boson self-energies with $\ln \Lambda/M_Z$.

To compute the oblique parameters one uses the Feynman rules in the previous section to compute the two Feynman diagrams that contribute to the vacuum polarizations. Both diagrams have one internal radion, and one uses the three point function and the other uses the four-point function. Our convention for the sign of the vacuum polarizations Π_{VV} is that

$\mathbf{V}_\mu$  $\mathbf{V}_\nu$

$$= i\Pi_{VV}^{\mu\nu}(p^2) = i\eta^{\mu\nu}\Pi_{VV}(p^2) + ip^\mu p^\nu \tilde{\Pi}_{VV}(p^2) \tag{12.7}$$

and only the first term is computed. The generic form of the radion contribution is

$$\Pi_{VV}(p^2) = \Pi_{VV}^S(p^2) + \Pi_{VV}^A(p^2). \tag{12.8}$$

Here "A" denotes the anomalous contribution due to the conformal coupling of the radion, and "S" denotes the standard contribution which is also similar to the Higgs contribution (when $\xi = 0$). The anomalous couplings are discussed in the previous section. By an appropriate rescaling of the coupling we can also use Π^S for the Higgs boson. When $\xi \neq 0$, the results given below can also be used to compute the oblique parameter after an appropriate redefinition of the couplings and masses. The modification to the expression for the oblique parameters is summarized in Eq. (12.21).

Inspecting the Feynman diagrams for the vacuum polarizations one finds that the quadratic divergences cancel between the two diagrams, leaving only a logarithmic divergence. Therefore this justifies the use of dimensional regularization. An explicit computation of the Feynman diagrams in unitary gauge and for vanishing curvature scalar parameter ξ gives

$$\Pi^S_{VV}(0) = -\frac{\gamma^2}{16\pi^2}\frac{m_V^4}{v^2}\left(\frac{6}{\varepsilon}+\frac{5}{2}-\frac{m_r^2}{2m_V^2}\right.$$
$$\left.+3\frac{m_V^2\ln m_V^2/\mu^2 - m_r^2\ln m_r^2/\mu^2}{m_r^2-m_V^2}\right), \quad (12.9)$$

$$\Pi^S_{VV}(m_V^2) = \frac{\gamma^2}{16\pi^2}\frac{m_V^4}{v^2}\left(-\frac{20}{3\varepsilon}-\frac{2}{3}\frac{m_r^2}{m_V^2}+\frac{1}{3}\frac{m_r^4}{m_V^4}+\frac{10}{9}\right)$$
$$+\frac{\gamma^2}{16\pi^2}\frac{m_V^4}{v^2}\left(4-\frac{4}{3}\frac{m_r^2}{m_V^2}+\frac{m_r^4}{m_V^4}\right)$$
$$\times\int_0^1 dx\ \ln[x^2m_V^2+(1-x)m_r^2]/\mu^2$$
$$+\frac{\gamma^2}{16\pi^2}\frac{m_V^4}{v^2}\left[-\frac{m_r^2}{m_V^2}\left(\frac{m_r^2}{3m_V^2}-1\right)\ln\frac{m_r^2}{\mu^2}\right.$$
$$\left.+\frac{1}{3}\left(\frac{m_r^2}{m_V^2}-2\right)\ln\frac{m_V^2}{\mu^2}\right], \quad (12.10)$$

where to avoid confusion with defining too many γ's, the renormalization scale μ appearing includes the usual factors of 4π and Euler's constant γ_E. An analytic expression for the Feynman parameter integral may be obtained, but it is not very illuminating. A powerful check on these expressions is gauge invariance. We have explicitly checked that in the general R_α gauge, the gauge parameter α cancels from the expression and reproduces the above results. We note that the divergences appearing here have the form as given by the operators in Eqs. (12.1) and (12.3). For $\Pi(0)_{VV}\sim m_V^4$ as required by Eq. (12.3). The difference between $\Pi(m_V^2)$ and $\Pi(0)$ gives the divergence proportional to p^2, but in the above equations p^2 has already been set to m_V^2. With this in mind, by inspection the coefficient of p^2 is $\sim m_V^2$, as required by Eq. (12.1). So the set of operators given by Eqs. (12.1) and (12.3) provide an appropriate set of counterterms. We renormalize using the modified numerical subtraction scheme (MS). Then using the Wilsonian approach outlined above, in the leading logarithm approximation we just replace $1/\varepsilon$ with $\ln \Lambda/M_Z$, that is,

$$\frac{2}{\varepsilon}-\gamma_E+\ln 4\pi \rightarrow \ln\frac{\Lambda^2}{M_Z^2}, \quad (12.11)$$

and set $\mu = M_Z$.

The anomalous contribution is finite and is

$$\Pi^A_{VV}(p^2) = \frac{\gamma^2}{16\pi^2}\frac{m_V^4}{v^2}\left(-\frac{10}{3}\frac{p^2}{m_V^2}+6-2\frac{m_r^2}{m_V^2}\right). \quad (12.12)$$

We note that the m_r^2 term does not contribute to the oblique parameters since it is p^2-independent and also $\sim m_V^2$.

The Particle Data Group (PDG) convention for the oblique parameters that we use here is

$$T = \frac{1}{\alpha}\left(\frac{\Pi_{WW}(0)}{M_W^2}-\frac{\Pi_{ZZ}(0)}{M_Z^2}\right), \quad (12.13)$$

$$S = \frac{4\sin^2\theta_W\cos^2\theta_W}{\alpha}\left(\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2}-\frac{\Pi_{ZZ}(0)}{M_Z^2}\right), \quad (12.14)$$

$$S+U = \frac{4\sin^2\theta_W}{\alpha}\left(\frac{\Pi_{WW}(M_W^2)}{M_W^2}-\frac{\Pi_{WW}(0)}{M_W^2}\right). \quad (12.15)$$

More generally one must also include the $\Pi_{Z\gamma}$ and $\Pi_{\gamma\gamma}$ self-energies. They have been dropped here since they do not receive contributions from either the Higgs boson or the radion.

Using the above expressions one can evaluate the contribution of the radion to S and T in the limit of a large radion mass. One obtains, for $\xi=0$,

$$S = \frac{\gamma^2}{\pi}\left(a_X-\frac{1}{12}\ln\frac{\Lambda^2}{m_r^2}-\frac{5}{72}\right), \quad (12.16)$$

$$T = \frac{3\gamma^2}{16\pi\cos^2\theta_W}\left(-\frac{a_M}{3}+\ln\frac{\Lambda^2}{m_r^2}+\frac{5}{6}\right). \quad (12.17)$$

Inspecting the above expressions for the Π's one finds that there is no divergent contribution to U, and this is consistent with the fact that $\mathcal{O}_X$ and $\mathcal{O}_M$ provide no counterterms for U.

It is interesting that the radion contribution to S is negative and to T is positive. This is easy to understand by comparing this result to the contribution of the Higgs in the SM. In fact, for $\gamma=1$, $m_r=m_h$, and $Z_i(\Lambda)=0$, the radion result is identical to the (logarithmic) contribution of the SM physical Higgs. But there the total correction to S and T is finite, so that for the Higgs the Λ dependence in the above expression is canceled by loops of W's and Z, leaving a $\ln m_h/M_Z$ dependence. Then one obtains the usual positive (negative) correction to $S(T)$. For the radion though the large logarithms are present, and since $\Lambda > m_r$, the radion contribution to S is negative, and to T it is positive. Recalling that $\gamma^2 = v^2/6\Lambda^2$, the size of the above correction is only significant for small values of Λ.

We conclude with a comment on the decoupling behavior of the radion. Inspecting the above expressions we see that for large m_r the radion contribution scales as

$$\frac{1}{\Lambda^2}\ln\frac{\Lambda_C}{m_r}. \quad (12.18)$$

For the purposes of this paragraph we distinguish the cutoff of the effective theory Λ_C from the mass scale $\Lambda = M_{Pl}e^{-kr_0}$ appearing in the radion coupling. In our analysis we have approximated $\Lambda_C\sim\Lambda$. If the radion mass is much larger then Λ, then the cutoff of the effective theory is the much higher scale $E\sim m_r$ or larger. Then it is more natural to express all higher dimension operators as suppressed by m_r^{-1} or Λ_C^{-1}. But then the coupling of the radion to the gauge bosons contains the very large coefficient m_r/Λ. It is then inappropriate to use the one-loop approximation. So to remain within the validity of the approximations used here one must also increase Λ for large m_r. Then the radion contribution decouples.

For large radion mass it is conceivable that the coupling of the radion to TrT decreases. This cannot be seen in our computations here because we have made the approximation of using only the zero mode wave function to determine the coupling, but have included the back reaction perturbatively to compute the radion mass. For very large radion mass these approximations are invalid, and then one must exactly solve the equations. It is then conceivable that for large radion mass the radion wave function on the TeV brane decreases in such a manner that the coupling to TrT remains natural, i.e., $O(m_r^{-1})$.

Following standard practice we define a reference model in which one computes the oblique parameters within the standard model, which means for some specific value for the Higgs mass. Since the curvature scalar operator mixes the radion and Higgs, the two physical scalars are some mixture of the gauge Higgs boson and radion. This mixture is not a unitary rotation due to the kinetic mixing between the states. The couplings of the ''Higgs boson'' in this case is somewhat different than in the standard model, and for the purposes of computing the oblique parameters it is easiest to think of this as a new model, rather than as a perturbation to the standard model. So in computing the oblique parameters the standard model Higgs contribution (for the reference Higgs mass) should be subtracted out, and the contribution of the physical states in this model added back in. That is,

$$X - X_{SM}^{\rm ref} = X_{\rm new}(m_h, m_r, \Lambda, \xi) - X_H^{\rm ref}(m_h = m_h^{\rm ref}). \tag{12.19}$$

As mentioned, $X_H^{\rm ref}(m_h = m_h^{\rm ref})$ is the contribution from only the Higgs, with mass $m_h^{\rm ref}$ and with standard model couplings, and it is independent of Λ, m_r, and the curvature-mixing parameter ξ. The quantity $X_{\rm SM}^{\rm ref}$ is the full SM contribution, with the Higgs set at the reference mass $m_h^{\rm ref}$. The new physics contribution contains two pieces,

$$X_{\rm new} = X_R + X_H, \tag{12.20}$$

which describe the contribution of the physical radion (X_R), and the physical Higgs (X_H). In the limit of no curvature-scalar mixing, this last contribution is just that of a standard model Higgs with mass m_h. For a general curvature scalar mixing one just needs to include the effect of the mixing coefficients a, b, c, and d given in the previous section. Then

$$\begin{aligned} X_{\rm new} = & \left(\cos\theta - \frac{\gamma}{Z}(6\xi - 1)\sin\theta \right)^2 X(m_h^{\rm phys}, \gamma = 1) \\ & + \left(\sin\theta + \frac{\gamma}{Z}(6\xi - 1)\cos\theta \right)^2 X(m_r^{\rm phys}, \gamma = 1) \\ & - \frac{\gamma^2}{Z^2}(6\xi - 1) X^A. \end{aligned} \tag{12.21}$$

Here X^A is the anomalous contribution, obtained from the vacuum polarization (12.12), but dropping the m_r^2 term since this does not contribute to any of the oblique parameters. The other X's are obtained from using the vacuum polarizations (12.9) and (12.10), and inserting the physical mass of the appropriate state.

We note that, for example, the full anomalous contribution to S from the above formula is

$$S_{\rm new}^A = \frac{5\gamma^2}{6Z^2\pi}(6\xi - 1) \tag{12.22}$$

and is negligible for reasonable values of γ and ξ. The anomalous contribution to T is even smaller.

Numerical results

The ''new'' contribution X is constrained to lie within the measured values (extracted assuming $m_h^{\rm SM\,ref} = 100$ GeV) [38]

$$S_{\rm meas} = -0.07 \pm 0.11, \tag{12.23}$$

$$T_{\rm meas} = -0.10 \pm 0.14, \tag{12.24}$$

$$U_{\rm meas} = 0.11 \pm 0.15 \tag{12.25}$$

(the errors are for one sigma). X can be easily calculated once m_h, m_r, Λ, and ξ are specified. Notice that since the current best-fit values for the electroweak parameters are nonzero, the new contributions can be more weakly or more strongly constrained depending on whether they add destructively or constructively with the Higgs boson, respectively. As a first example, we show the contribution to S and T in Fig. 3 as a function of $m_h = m_r$ (the ''gauge'' masses), fixing $\Lambda = 1$ TeV. Each contour corresponds to a different value of ξ, and the contours end when a physical mass exceeds the cutoff. The unshaded region corresponds to the 1σ allowed region. Notice that T is a strong constraint on small (gauge) masses, while S is a strong constraint for large masses. Also, the the case with $\xi = 0$ is nearly identical to the ordinary SM Higgs contribution, since the radion contribution that can be separated out is strongly suppressed by the coupling γ^2. In Fig. 4 we show the contribution to S and T as a function of m_r with $m_h = 300$ GeV. Notice that the contributions are nearly independent of m_r for small curvature scalar mixing.

The above results illustrate a general trend that with small or absent curvature-scalar mixing, the bound on the Higgs boson mass is not significantly affected by the presence of the radion. This is not true, however, if we allow ξ to take larger values. It is easiest to first illustrate that radion physics with large curvature scalar mixing can significantly relax the bound on the Higgs mass, by scanning through the parameter space (choosing $m_h = m_r$ for simplicity) for values that satisfy one or two sigma limits on the electroweak parameters. We find sets of parameters that are not minor perturbations on the SM limit allow the physical masses of the Higgs and radion to be several hundred GeV, and perhaps even TeV scale. In Fig. 5 we show the range of physical masses and the range of ξ as a function of the cutoff scale. In general there is

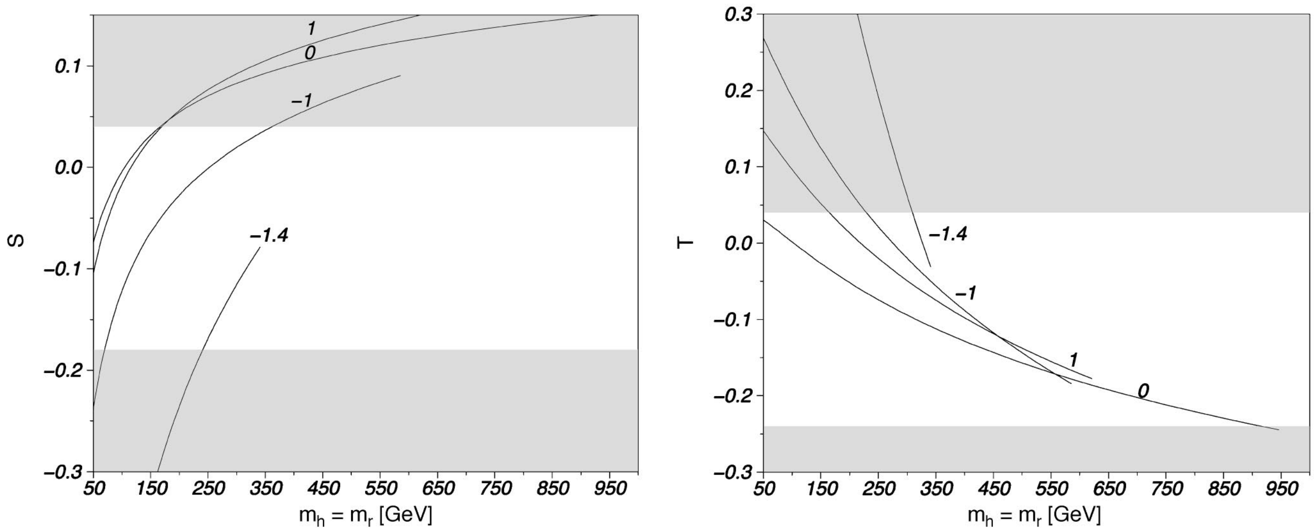


FIG. 3. The contributions to S, T as a function of the ''gauge'' masses $m_h = m_r$. Each line is a contour for a fixed curvature scalar mixing ξ. The cutoff scale was chosen to be 1 TeV ($\gamma = 0.1$). The shaded regions are excluded by the PDG measurements to one sigma.

not a unique mapping between the figures, however the ''shark fin'' structure for the one sigma region in Fig. 5(a) does correspond to the ''inverse fin'' in Fig. 5(b). Notice that at two sigma the physical Higgs boson mass can be much larger than the SM bound throughout the parameter space shown, and even at one sigma there exists a narrow range of large, negative curvature-scalar mixing where the physical Higgs mass could be of order a TeV. The latter result arises from a cancellation between the physical Higgs and radion contributions with the SM reference contribution. This can be seen in the limit of a large radion and Higgs boson mass. Since the dependence on the masses is only logarithmic, we can approximate the masses as being equal. Then, for example,

$$S_{\rm new} = \frac{1}{\pi}\left(\frac{1}{12}\ln\frac{m^2}{M_z^2} - \frac{5}{72}\right) - (6\xi - 1)^2 \frac{\gamma^2}{Z^2\pi}\left(\frac{1}{12}\ln\frac{\Lambda^2}{m^2} + \frac{5}{72}\right). \tag{12.26}$$

The first contribution is just the usual correction from the Higgs boson. But the second correction can be potentially large and negative due to the dependence on the ξ parameter. It should be emphasized that the large correction is due to the (nonunitary) kinetic mixing between the radion and Higgs boson, or equivalently, due to the nonstandard couplings of the radion and Higgs boson in the mass basis. Hence, while this region is provocative, it nonetheless requires fine-tuning.

These results have assumed that the contribution from the nonrenormalizable counterterms is small, meaning a_X and

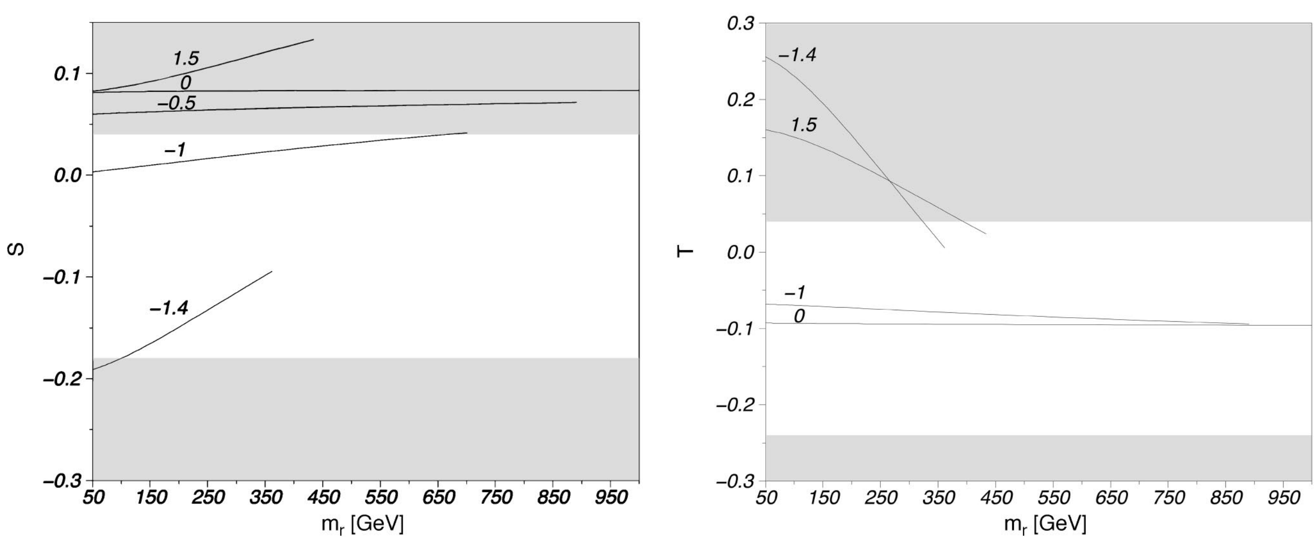


FIG. 4. Same as Fig. 3 except that m_h is fixed to 300 GeV. Notice that the contributions to S and T are nearly independent of the radion mass if the curvature mixing is small since the radion contribution is suppressed by $\sim\gamma$.

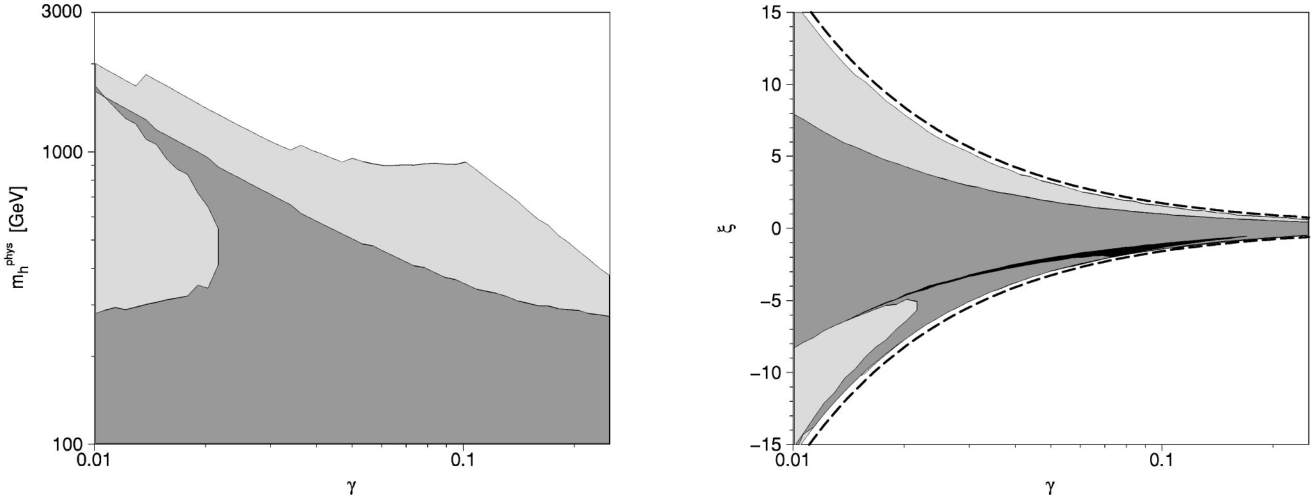


FIG. 5. The allowed region of $m_h^{\rm phys}$ and ξ as a function of the inverse of the cutoff scale $\gamma = v/\Lambda$ by requiring *S*, *T*, *U* do not exceed the one sigma (dark region) or two sigma (light region) measurements from the PDG. The dashed lines correspond to the theoretical bound requiring the kinetic term is non-negative [see Eq. (9.14)]. The black sliver corresponds to the region where $m_h^{\rm phys} \approx 300$ GeV.

a_M are less than order 1. For larger coefficients the allowed regions of parameter space, albeit only at moderately large γ. In Fig. 6 we show the shift in the contours, for the two sigma region, resulting from taking $a_X = \pm 10$. (a_M was also taken to be 10, but the effect on the contours was negligible.)

XIII. LIMITS ON RADION MASS

As we found in Sec. VI, the mass of the radion is expected to be significantly below the the cutoff scale, placing it in a region that can be directly probed by experiments. The previous section has shown that the radion couples much like a Higgs boson, and in the limit $\xi \to 0$, the tree-level couplings of the radion are simply scaled by γ. Let us first consider the bounds in this case.

In the SM, the current bound on the Higgs boson mass comes primarily from the CERN e^+e^- collider LEP processes $e^+e^- \to Z^* \to Zh$, with the value $m_h^{\rm SM} \lesssim 108$ GeV [39]. For the radion, an exactly analogous production process occurs $e^+e^- \to Z^* \to Zr$, except that the *ZZr* coupling has a factor of γ. To a good approximation, we can therefore estimate the production cross section of radions at LEP by simply scaling the Higgs cross section by γ^2.

The decay of the radion is somewhat more complicated, however. As we discussed in Sec. XI, the radion couples directly to gauge bosons through the conformal anomaly. Although this coupling is one-loop suppressed, it competes with Yukawa suppressed interactions and, for the case *rgg*, can be comparable or even dominate [21,22]. In the radion mass range well below the $t\bar{t}$ threshold, the ratio of the two largest widths can be expressed as

$$\frac{\Gamma(r \to gg)}{\Gamma(r \to b\bar{b})} = \frac{\alpha_s^2 c_3^2}{12\pi^2 \beta^3} \left(\frac{m_r}{m_b} \right)^2, \qquad (13.1)$$

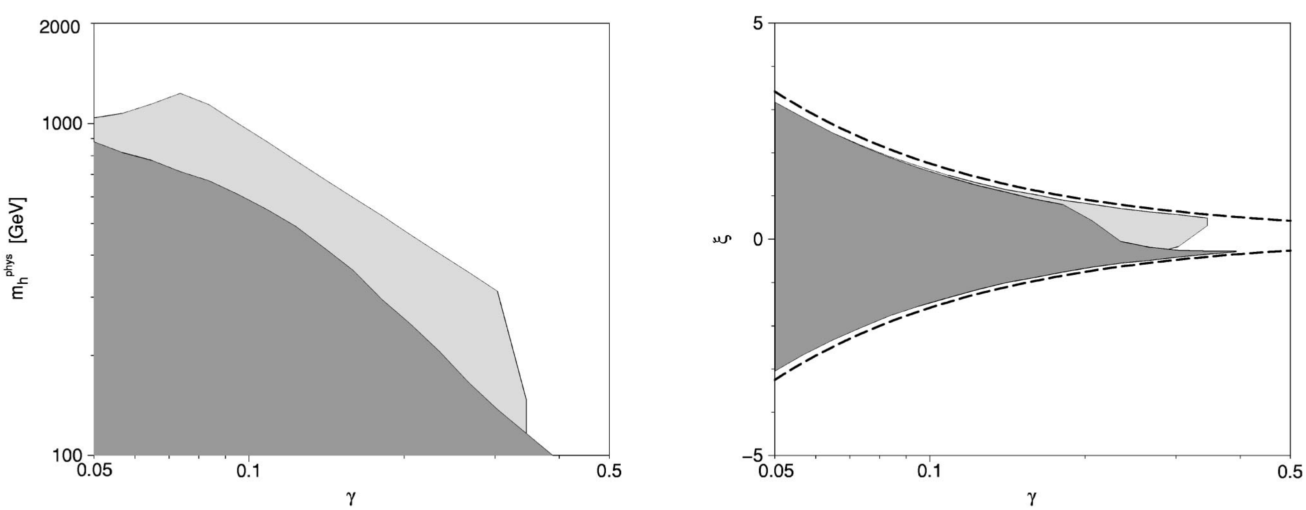


FIG. 6. The shift in the two sigma contours shown in Fig. 5 resulting from taking the coefficients of the nonrenormalizable operators to be $a_X = 10$ (dark region) and $a_X = -10$ (light region).

where $\beta^2 = 1 - 4m_b^2/m_r^2$ and $c_3 \approx \frac{23}{3}$ is roughly the one-loop QCD β-function coefficient (approximately including the smaller contribution from the one-loop triangle diagram with top quarks). Notice that the coupling γ^2 cancels in this ratio. This can be written in the suggestive form

$$\frac{\Gamma(r \to gg)}{\Gamma(r \to b\overline{b})} \approx 1/\beta^3 \left(\frac{m_r}{12 m_b} \right)^2. \tag{13.2}$$

Hence $r \to gg$ dominates for the region $12m_b \lesssim m_r \lesssim 2M_W$. The search strategy for the radion is therefore significantly different from the Higgs in this mass window, namely searching for a pair of gluon jets instead of a pair of b jets. Similarly, the radion has a different production cross section at hadron colliders via gluon fusion, proportional to the conformal anomaly enhanced width into gluons but suppressed by the usual γ^2 [21].

Determining an accurate bound on the radion mass in the region that can be probed by LEP requires a detailed analysis of detecting a two gluon plus Z signal. We will not attempt this here. Instead, the expected bound on the radion mass can be roughly estimated as a function of the coupling if we assume that some number of production events N at LEP could not have escaped detection (or be lumped into SM backgrounds). Near the kinematical limit, the best bound will always come from the highest energy data. For lower mass radions, a lower center-of-mass energy results in a slightly higher cross section.[7] Since the bound for a given radion mass is limited only by luminosity, we can combine the multitude of LEP runs at various energies by weighting by the integrated luminosity accumulated. The bound is then simply

$$\gamma^2 < \frac{N}{\Sigma \sigma_{\sqrt{s}}(e^+e^- \to Zh; m_h = m_r) \times \int \mathcal{L}_{\sqrt{s}}}, \tag{13.3}$$

where the sum is over the various recent LEP runs with center-of-mass energy $\sqrt{s}$. N encodes all of the detailed analyses of backgrounds, signal efficiencies, etc., and is in general not independent of energy or radion mass. In Fig. 7 we simply show the bound obtained if $N=20$ or 100 (the integrated luminosity for each energy was also summed) corresponding to producing 20 or 100 events summed over all four LEP experiments. These numbers were chosen since searches for Higgs bosons typically need a few to tens of events (per detector) for a statistically significant signal-to-background ratio. Notice that no bound on the radion mass is expected from the recent LEP runs once γ is less than about 0.1.

One could also search for light radions, $m_r \lesssim 60$ GeV, at LEP I via the decay $Z \to f\overline{f} + r$, through the same coupling discussed above. However, the expected bound obtained by using this procedure is no better than that found above for m_r larger than about 10 GeV.

[7] For instance, $\sigma_{\sqrt{s}=189\ \mathrm{GeV}}(e^+e^- \to rZ)/\sigma_{\sqrt{s}=202\ \mathrm{GeV}}(e^+e^- \to rZ) \sim 1.3$ for small m_r.

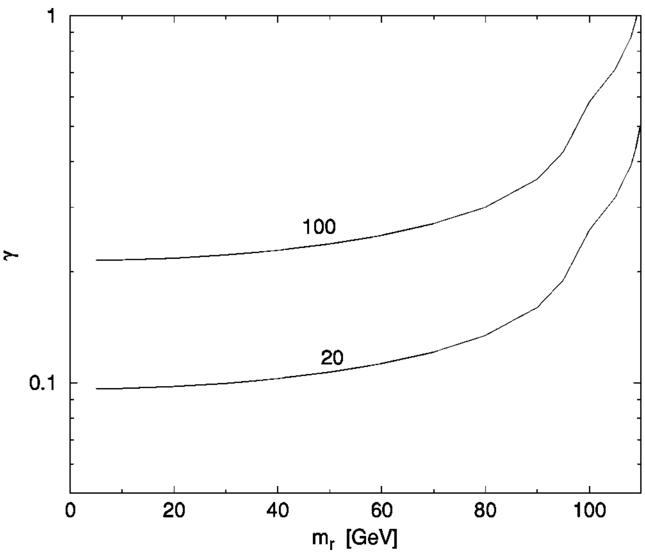


FIG. 7. The bound on the radion mass as a function of γ is shown, assuming the signal could be extracted from background once the radion production cross section times integrated luminosity exceeds $N=20$ or 100 events at LEP (summed over the four experiments).

We have not attempted to estimate a bound on γ for a radion mass less than about 10 GeV. We really do not expect the radion to be several orders of magnitude below the cutoff scale, and so at the outset it seems this mass region is unnatural. But, the presence of several low energy production processes (and rare decays) could be important, so a considerably more careful analysis than what we have attempted here is needed.

When curvature scalar mixing is included, the coupling ZZr is modified as shown in Eq. (11.20). The above analysis can be translated into this more general case, but now the coupling is not simply γ but a function of the curvature-scalar mixing as well. In addition, the SM Higgs couplings are also modified, and so its production and decay are also affected. In particular, the production cross section could be either enhanced or suppressed. (This is similar to what happens in two Higgs doublet models, such as the MSSM.) An interesting signal for RS with curvature-scalar mixing could be observing a nonstandard cross section or decay rate for a SM-like Higgs boson.

XIV. CONCLUSIONS

In this paper we have analyzed the coupled radion-scalar system in detail, including the back reaction of the bulk stabilizing scalar on the metric. We derived the coupled differential equations governing the dynamics of the system, and found the mass eigenvalues for some limiting cases. We find that due to the coupling between the radion and the bulk scalar, there will be a single KK tower describing the system, with the metric perturbations nonvanishing for every KK mode. This implies that the standard model fields localized on the TeV brane will couple to every KK mode from the bulk scalar, and this could provide a means to directly probe the stabilizing physics. We also found that in an expanding universe the shift in the radion at late times completely agrees with the effective theory result of Ref. [14].

We also calculated the contributions of the radion to the oblique parameters using an effective theory approach. Since the radion is the only new state well below the TeV scale, we argued that a low-energy effective theory including only the radion and SM fields is sufficient, as long as appropriate nonrenormalizable counterterms at the cutoff scale are added. In the absence of a curvature-scalar Higgs mixing operator, the size of the contribution to the oblique parameters due to the radion is small. In the presence of such a mixing operator, the corrections can be much larger due to the modified radion and Higgs couplings. In particular, including only the mixed radion and Higgs fields as ''new physics,'' we calculated the range of curvature-scalar mixing for a given cutoff scale that allows the physical Higgs boson mass to be up to of order the cutoff scale, while S_{new} and T_{new} were within the experimental limits. However, the parameters must be increasingly fine-tuned to achieve a Higgs boson mass that exceeds a few hundred GeV.

Note added. After Secs. III–VI were completed, we were informed that many of the results of these sections are also contained in Ref. [29]. We thank Riccardo Rattazzi for pointing us to this reference.

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[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *ibid.* **436**, 257 (1998).

[2] I. Antoniadis, Phys. Lett. B **246**, 377 (1990); J. D. Lykken, Phys. Rev. D **54**, 3693 (1996); R. Sundrum, *ibid.* **59**, 085009 (1999); K. R. Dienes, E. Dudas, and T. Gherghetta, Phys. Lett. B **436**, 55 (1998); G. Shiu and S. H. Tye, Phys. Rev. D **58**, 106007 (1998); Z. Kakushadze and S. H. Tye, Nucl. Phys. **B548**, 180 (1999).

[3] P. Horava and E. Witten, Nucl. Phys. **B475**, 94 (1996); **B460**, 506 (1996).

[4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).

[5] M. Gogberashvili, Europhys. Lett. **49**, 396 (2000); Mod. Phys. Lett. A **14**, 2025 (1999); hep-ph/9908347.

[6] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, and N. Kaloper, Phys. Rev. Lett. **84**, 586 (2000); J. Lykken and L. Randall, J. High Energy Phys. **06**, 014 (2000); C. Csáki and Y. Shirman, Phys. Rev. D **61**, 024008 (2000); A. E. Nelson, *ibid.* (to be published), hep-th/9909001.

[7] N. Kaloper, Phys. Rev. D **60**, 123506 (1999); T. Nihei, Phys. Lett. B **465**, 81 (1999); H. B. Kim and H. D. Kim, Phys. Rev. D **61**, 064003 (2000).

[8] A. G. Cohen and D. B. Kaplan, Phys. Lett. B **470**, 52 (1999); C. P. Burgess, L. E. Ibanez, and F. Quevedo, *ibid.* **447**, 257 (1999); A. Chodos and E. Poppitz, *ibid.* **471**, 119 (1999); T. Gherghetta and M. Shaposhnikov, Phys. Rev. Lett. **85**, 240 (2000).

[9] T. Shiromizu, K. Maeda, and M. Sasaki, Phys. Rev. D **62**, 024012 (2000); J. Garriga and T. Tanaka, Phys. Rev. Lett. **84**, 2778 (2000); S. B. Giddings, E. Katz, and L. Randall, J. High Energy Phys. **03**, 023 (2000).

[10] P. Binetruy, C. Deffayet, and D. Langlois, Nucl. Phys. **B565**, 269 (2000); P. Binetruy, C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. B **477**, 285 (2000).

[11] C. Csáki, M. Graesser, C. Kolda, and J. Terning, Phys. Lett. B **462**, 34 (1999); J. M. Cline, C. Grojean, and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999); D. J. Chung and K. Freese, Phys. Rev. D **61**, 023511 (2000).

[12] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999).

[13] J. Garriga, O. Pujolas, and T. Tanaka, hep-th/0004109; U. Gunther and A. Zhuk, hep-ph/0006283; D. J. Toms, hep-th/0005189; W. D. Goldberger and I. Z. Rothstein, Phys. Lett. B **491**, 339 (2000).

[14] C. Csáki, M. Graesser, L. Randall, and J. Terning, Phys. Rev. D **62**, 045015 (2000).

[15] W. D. Goldberger and M. B. Wise, Phys. Lett. B **475**, 275 (2000).

[16] J. M. Cline and H. Firouzjahi, hep-ph/0006037; J. M. Cline (private communication).

[17] H. Davoudiasl, J. L. Hewett, and T. G. Rizzo, Phys. Rev. Lett. **84**, 2080 (2000).

[18] H. Davoudiasl, J. L. Hewett, and T. G. Rizzo, Phys. Rev. D (to be published), hep-ph/0006041.

[19] H. Davoudiasl, J. L. Hewett, and T. G. Rizzo, hep-ph/0006097.

[20] U. Mahanta and S. Rakshit, Phys. Lett. B **480**, 176 (2000); P. Das, S. Raychaudhuri, and S. Sarkar, J. High Energy Phys. **07**, 050 (2000); U. Mahanta and A. Datta, Phys. Lett. B **483**, 196 (2000); U. Mahanta, hep-ph/0004128; hep-ph/0008042; D. K. Ghosh and S. Raychaudhuri, hep-ph/0007354.

[21] G. F. Giudice, R. Rattazzi, and J. D. Wells, hep-ph/0002178.

[22] S. B. Bae, P. Ko, H. S. Lee, and J. Lee, Phys. Lett. B **487**, 299 (2000).

[23] J. E. Kim, B. Kyae, and J. D. Park, hep-ph/0007008.

[24] G. F. Giudice, R. Rattazzi, and J. D. Wells, Nucl. Phys. **B544**, 3 (1999); T. Han, J. D. Lykken, and R. Zhang, Phys. Rev. D **59**, 105006 (1999); M. L. Graesser, *ibid.* **61**, 074019 (2000); P. Nath and M. Yamaguchi, *ibid.* **60**, 116006 (1999); M. Hayakawa, hep-ph/9904445; T. G. Rizzo and J. D. Wells, Phys.

Rev. D **61**, 016007 (2000); T. Han, D. Marfatia, and R. Zhang, *ibid.* **62**, 125018 (2000).
[25] S. Dawson and G. Valencia, Phys. Rev. D **53**, 1721 (1996); S. Alam, S. Dawson, and R. Szalapski, *ibid.* **57**, 1577 (1998); B. A. Dobrescu and J. Terning, Phys. Lett. B **416**, 129 (1998); L. Hall and C. Kolda, *ibid.* **459**, 213 (1999); J. A. Bagger, A. F. Falk, and M. Swartz, Phys. Rev. Lett. **84**, 1385 (2000); R. Barbieri and A. Strumia, Phys. Lett. B **462**, 144 (1999); hep-ph/0007265.
[26] O. DeWolfe, D. Z. Freedman, S. S. Gubser, and A. Karch, Phys. Rev. D **62**, 046008 (2000).
[27] C. Charmousis, R. Gregory, and V. A. Rubakov, Phys. Rev. D **62**, 067505 (2000).
[28] L. Pilo, R. Rattazzi, and A. Zaffaroni, J. High Energy Phys. **07**, 056 (2000).
[29] T. Tanaka and X. Montes, Nucl. Phys. **B582**, 259 (2000).
[30] K. Behrndt and M. Cvetic, Phys. Lett. B **475**, 253 (2000); A. Chamblin and G. W. Gibbons, Phys. Rev. Lett. **84**, 1090 (2000); K. Skenderis and P. K. Townsend, Phys. Lett. B **468**, 46 (1999); M. Gremm, *ibid.* **478**, 434 (2000); C. Csáki, J. Erlich, T. J. Hollowood, and Y. Shirman, Nucl. Phys. **B581**, 309 (2000).
[31] P. Kanti, I. I. Kogan, K. A. Olive, and M. Pospelov, Phys. Lett. B **468**, 31 (1999); Phys. Rev. D **61**, 106004 (2000); P. Kanti, K. A. Olive, and M. Pospelov, Phys. Lett. B **481**, 386 (2000); Phys. Rev. D **62**, 126004 (2000).
[32] H. B. Kim, Phys. Lett. B **478**, 285 (2000); J. M. Cline and H. Firouzjahi, hep-ph/0005235; P. Bowcock, C. Charmousis, and R. Gregory, hep-th/0007177; J. E. Kim and B. Kyae, Phys. Lett. B **486**, 165 (2000); H. Stoica, S. H. Tye, and I. Wasserman, *ibid.* **482**, 205 (2000); C. Barcelo and M. Visser, *ibid.* **482**, 183 (2000); R. N. Mohapatra, A. Perez-Lorenzana, and C. A. de Sousa Pires, hep-ph/0003328;
[33] S. W. Hawking, T. Hertog, and H. S. Reall, Phys. Rev. D **62**, 043501 (2000); H. Kodama, A. Ishibashi, and O. Seto, *ibid.* **62**, 064022 (2000); D. Langlois, *ibid.* **62**, 126012 (2000); C. van de Bruck, M. Dorca, R. H. Brandenberger, and A. Lukas, *ibid.* **62**, 123515 (2000).
[34] D. Ida, J. High Energy Phys. **09**, 014 (2000); J. Garriga and M. Sasaki, Phys. Rev. D **62**, 043523 (2000); H. Collins and B. Holdom, *ibid.* **62**, 105009 (2000); M. Brandle, A. Lukas, and B. A. Ovrut, *ibid.* **63**, 026003 (2001); J. Lesgourgues, S. Pastor, M. Peloso, and L. Sorbo, Phys. Lett. B **489**, 411 (2000); C. Park and S. Sin, *ibid.* **485**, 239 (2000); C. J. Hogan, Phys. Rev. Lett. **85**, 2044 (2000); Y. V. Shtanov, hep-th/0005193; V. Barger, T. Han, T. Li, J. D. Lykken, and D. Marfatia, Phys. Lett. B **488**, 97 (2000); L. Anchordoqui and K. Olsen, hep-th/0008102; K. Enqvist, E. Keski-Vakkuri, and S. Rasanen, hep-th/0007254.
[35] R. Gregory, V. A. Rubakov, and S. M. Sibiryakov, Phys. Rev. Lett. **84**, 5928 (2000); Phys. Lett. B **489**, 203 (2000); C. Csáki, J. Erlich, and T. J. Hollowood, Phys. Rev. Lett. **84**, 5932 (2000); Phys. Lett. B **481**, 107 (2000); G. Dvali, G. Gabadadze, and M. Porrati, *ibid.* **484**, 112 (2000); **484**, 129 (2000); C. Csáki, J. Erlich, T. J. Hollowood, and J. Terning, hep-th/0003076; A. Gorsky and K. Selivanov, Phys. Lett. B **485**, 271 (2000).
[36] I. I. Kogan, S. Mouslopoulos, A. Papazoglou, G. G. Ross, and J. Santiago, Nucl. Phys. **B584**, 313 (2000); hep-th/0006030; I. I. Kogan and G. G. Ross, Phys. Lett. B **485**, 255 (2000); S. Mouslopoulos and A. Papazoglou, J. High Energy Phys. **11**, 018 (2000); D. Choudhury, D. P. Jatkar, U. Mahanta, and S. Sur, *ibid.* **09**, 021 (2000).
[37] S. S. Gubser, Phys. Rev. D (to be published), hep-th/9912001; J. E. Kim, B. Kaye, and H. M. Lee, *ibid.* **62**, 045013 (2000); N. Kaloper, Phys. Lett. B **474**, 269 (2000); M. Sasaki, T. Shiromizu, and K. Maeda, Phys. Rev. D **62**, 024008 (2000); M. G. Ivanov and I. V. Volovich, hep-th/9912242; K. Uzawa, Y. Morisawa, and S. Mukohyama, Phys. Rev. D **62**, 064011 (2000); L. Mersini, hep-ph/0001017; R. Dick and D. Mikulovicz, Phys. Lett. B **476**, 363 (2000); T. Chiba, Phys. Rev. D **62**, 021502 (2000); K. Koyama and J. Soda, Phys. Lett. B **483**, 432 (2000); S. Nojiri, S. D. Odintsov, and S. Zerbini, Phys. Rev. D **62**, 064006 (2000); P. F. Gonzalez-Diaz, Phys. Lett. B **481**, 353 (2000); C. Grojean, Phys. Lett. B **479**, 273 (2000); M. Chaichian and A. B. Kobakhidze, *ibid.* **488**, 117 (2000); S. Ichinose, hep-th/0003275; A. Masiero, M. Peloso, L. Sorbo, and R. Tabbash, Phys. Rev. D **62**, 063515 (2000); J. E. Kim, B. Kyae, and H. M. Lee, Nucl. Phys. **B582**, 296 (2000); C. Barcelo and M. Visser, *ibid.* **B584**, 415 (2000); Phys. Rev. D **63**, 024004 (2001); L. Anchordoqui, C. Nunez, and K. Olsen, J. High Energy Phys. **10**, 050 (2000).
[38] Particle Physics Data Group, D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
[39] ''Searches for Higgs bosons: preliminary combined results using LEP data collected at energies up to 202 GeV,'' The LEP Collaborations, CERN EP/2000-055.
[40] M. Golden and L. Randall, Nucl. Phys. **B361**, 3 (1991); B. Holdom and J. Terning, Phys. Lett. B **247**, 88 (1990); M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990).