U(1) gauge field of the Kaluza-Klein theory in the presence of branes

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We investigate the zero mode dimensional reduction of Kaluza-Klein unification in the presence of a single brane in the infinite extra dimension. We treat the brane as fixed, not a dynamical object, and do not require orbifold symmetry. It seems that, contrary to the standard Kaluza-Klein models, the four-dimensional (4D) effective action is no longer invariant under U(1) gauge transformations due to the explicit breaking of isometries in the extra dimension by the brane. Surprisingly, however, the linearized perturbation analysis around the RS vacuum shows that the Kaluza-Klein gauge field does possess U(1) gauge symmetry at the linear level. In addition, the graviscalar also behaves differently from the 4D point of view. Some physical implications of our results are also discussed.

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I. INTRODUCTION

In the standard five-dimensional Kaluza-Klein (KK) approach where the full spacetime manifold is factorized as $M_4 \otimes S^1$, the five-dimensional gravitation theory which has reparametrization invariance on S^1 can be interpreted as a gauge theory of the Virasoro group from the four-dimensional point of view (see Ref. [1], and references therein). After the geometric spontaneous symmetry breaking of the Virasoro invariance, the excitations of the 5D gravitational fields are split into 4D massless gravitational fields, massless gauge fields, massless scalar field, and an infinite tower of massive spin-2 fields [1,2]. In particular, the U(1) gauge symmetry of the vector fields in the 4D effective action is originated from the translational isometries in the extra dimension.

Recently, there have been lots of interest in the phenomenon of localization of gravity proposed by Randall and Sundrum (RS) [3,4] (for previous relevant work see references therein). RS [4] assumed a single positive tension 3-brane and a negative bulk cosmological constant in the five-dimensional spacetime. There have been developed a large number of brane world models afterwards [5,6]. The introduction of branes usually gives rise to the ''warping'' of the extra dimensions, resulting in nonfactorizable spacetime manifolds. More importantly, the presence of branes breaks the translational isometries in extra dimensions. Therefore, it would be very interesting to see how the conventional Kaluza-Klein picture changes in the brane world scenarios.

Dobado and Maroto [7] recently have incorporated the effect of the presence of the brane in the Kaluza-Klein reduction by introducing Goldstone bosons (GB) fields. The GB fields parametrize the excitations of the brane [8] and so the GB correspond to the spontaneous symmetry breaking of the translational isometries of the compactified extra dimensions by the brane. It has been shown that in the dimensional reduction of the KK unifications a sort of Higgs mechanism related to the GB [9] gives mass to the KK graviphotons. (See also the appendix of Ref. [10] although it has the opposite sign for the mass term.)

On the other hand, there are other approaches to confine standard model particles on the brane by allowing the fields to live in the bulk spacetime. For example, bulk gauge bosons have been considered in Refs. [11,12]. It is important to derive the zero mode effective action because its zero modes (massless modes) correspond to the standard model particles localized on the brane.

In this paper, contrary to the approach mentioned above where the brane is treated as a dynamical object, we have treated the brane as fixed, and investigate the zero mode dimensional reduction of the Kaluza-Klein unifications. The brane world model considered in this paper is the RS one with a single 3-brane in the infinite fifth dimension [4]. As expected, the breaking of isometries in the extra dimension by the brane makes the 4D effective action not being invariant under U(1) gauge transformations. Interestingly, however, the analysis of the linearized equation around the RS background shows that the KK gauge field does possess the U(1) gauge symmetry at the linear level.

In Sec. II, we carry out the dimensional reduction of the KK unifications in the presence of a single brane with some ansatz for the zero mode excitations. In Sec. III, the linearized perturbations of the 4D effective action obtained are analyzed around the RS vacuum solution. Some physical implications of our results are discussed in Sec. IV.

II. KALUZA-KLEIN REDUCTION

The RS model with a single brane can be described by the following action in 5D spacetimes [4,5,13]:

$$I=\int d^4x \int_{-\infty}^{\infty} dz \frac{\sqrt{-\hat{g}}}{16\pi G_5}(\hat{R}-2\Lambda)-\int d^4x \sqrt{-\hat{g}_B}\sigma. \quad (1)$$

Here G_5 is the 5D Newton's constant, Λ the bulk cosmological constant of 5D spacetime, $\hat{g}_B$ the determinant of the induced metric describing the brane, and σ the tension of the brane. We assume that the value of σ is fine-tuned such that $\Lambda=-6k^2(<0)$ with $k=4\pi G_5\sigma/3$. Let us introduce the domain-wall metric in the following form:

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$$
\begin{aligned}
ds^2 &= \hat{g}_{MN}dx^M dx^N \\
&= H^{-2}(z)g_{MN}dx^M dx^N \\
&= H^{-2}(z)[\gamma_{\mu\nu}dx^\mu dx^\nu + \phi^2(dz-\kappa A_\mu dx^\mu)^2]. \qquad (2)
\end{aligned}
$$

Here $H=k|z|+1$, $\phi^2=g_{55}$, and $\kappa A_\mu=-g_{5\mu}/g_{55}$. The standard Kaluza-Klein decomposition of the metric is given by

$$
(g_{MN})=\begin{pmatrix} \gamma_{\mu\nu}+\kappa^2\phi^2 A_\mu A_\nu & -\kappa\phi^2 A_\mu \\ -\kappa\phi^2 A_\nu & \phi^2 \end{pmatrix},
$$

$$
(g^{MN})=\begin{pmatrix} \gamma^{\mu\nu} & \kappa A^\mu \\ \kappa A^\nu & \phi^{-2}(1+\kappa^2\phi^2 A\cdot A) \end{pmatrix} \qquad (3)
$$

with $A^\mu=\gamma^{\mu\nu}A_\nu$ and $A\cdot A=A_\mu A^\mu$. Here κ is the gauge coupling constant.

Under the specific class of coordinate transformations such as

$$
x^\mu \rightarrow \tilde{x}^\mu=\tilde{x}^\mu(x), \quad z\rightarrow \tilde{z}=z+f(x), \qquad (4)
$$

we obtain

$$
\tilde{\gamma}_{\mu\nu}=\frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu}\gamma_{\alpha\beta}, \quad \tilde{A}_\mu=\frac{\partial x^\alpha}{\partial \tilde{x}^\mu}A_\alpha+\kappa^{-1}\frac{\partial f}{\partial \tilde{x}^\mu},
$$

$$
\tilde{\phi}(\tilde{x},\tilde{z})=\phi(x,z) \qquad (5)
$$

according to

$$
\tilde{g}_{MN}=\frac{\partial x^P}{\partial \tilde{x}^M}\frac{\partial x^Q}{\partial \tilde{x}^N}g_{PQ}.
$$

We see that $\gamma_{\mu\nu}$ transforms similar to a four-dimensional metric tensor, and ϕ a scalar field under diffeomorphisms in Eq. (4). However, we point out that the 5D diffeomorphisms are split into the 4D diffeomorphisms plus the gauge transformations for the field A_μ.

In this paper, we are mainly interested in the zero mode effective action. In general, it is a nontrivial problem to determine what the ''zero mode'' is if the full spacetime is not factorizable. As an ansatz for the zero mode, we assume that $\gamma_{\mu\nu}$, A_μ, and ϕ are functions of x coordinates only, i.e., no z-coordinate dependence. If one requires the Z_2 (e.g., R/Z_2) orbifold symmetry in the brane world model, there will be no vector zero mode fluctuations. It follows because in the presence of Z_2 orbifold symmetry in Eq. (2) the vector gauge field A_μ should satisfy $A_\mu(x,-z)=-A_\mu(x,z)$ and so $A_\mu(x)=0$. In what follows, we consider general cases without having the orbifold symmetry in the theory.

The above assumption comes from the crucial observation that the graviton zero mode $h_{\mu\nu}$ in $\gamma_{\mu\nu}=\eta_{\mu\nu}+\kappa h_{\mu\nu}$ depends only on ''x'' even if one starts from $h_{\mu\nu}(x,z)=H^{3/2}\psi(z)\hat{h}_{\mu\nu}(x)$ in the RS approach [4] where the transverse fluctuations are fixed (e.g., $h_{5\mu}=h_{55}=0$). For the zero mode solution with $m^2=0$, we have $\psi^0(z)=c_h H^{-3/2}$, thus we find $h^0_{\mu\nu}(x,z)=c_h\hat{h}_{\mu\nu}(x)$ with a constant c_h. Other examples are the form of the zero modes for the bulk spin-0 and spin-1 fields on the RS background [14]. For the spin-0 field $\Phi(x,z)=H^{3/2}\chi(z)\hat{\phi}(x)$, we have $\chi=c_\Phi H^{-3/2}$ for the zero mode and hence its localized zero mode is given by $\Phi^0(x,z)=c_\Phi\hat{\phi}(x)$. In the case of the spin-1 field $V_\mu(x,z)=H^{3/2}\sigma(z)v_\mu(x)$, one finds $\sigma=c_V H^{-3/2}$ for the zero mode and hence its zero mode is given by $V^0_\mu(x,z)=c_V v_\mu(x)$. From the observations mentioned above, we may propose that the zero modes are constants with respect to ''z.'' Furthermore we stress that for $h_{\mu5}\neq 0, h_{55}\neq 0$, it may be not a correct way to obtain the zero modes from the linearized equations. This is because their forms are too complicated to analyze the zero modes [13]. Even if we choose the gauge fixing, it is hard to obtain the consistent zero mode solutions. Hence the integration of Eq. (1) over z could be a good starting point to obtain the zero modes.

Note first that

$$
\sqrt{-\hat{g}}=H^{-5}\phi\sqrt{-\gamma}, \qquad (6)
$$

$$
\sqrt{-\hat{g}_B}=H^{-4}(z=0)\sqrt{-\gamma}\sqrt{|\delta^\mu_\nu+\kappa^2\phi^2 A^\mu A_\nu|}. \qquad (7)
$$

Using $\hat{g}_{MN}=H^{-2}g_{MN}$, one has

$$
\hat{R}=H^2\left[R(g)+8\frac{\nabla_P\nabla^P H}{H}-20\frac{\nabla_P H\nabla^P H}{H^2}\right]. \qquad (8)
$$

Since

$$
\begin{aligned}
\nabla_P\nabla^P H &= H''(\phi^{-2}+\kappa^2 A\cdot A)+\kappa H' \\
&\quad\times\left(\phi^{-1}A^\mu\partial_\mu\phi+\frac{1}{2}A^\mu\gamma^{\alpha\beta}\partial_\mu\gamma_{\alpha\beta}+\partial_\mu A^\mu\right),
\end{aligned} \qquad (9)
$$

we have

$$
\begin{aligned}
& 8\frac{\nabla_P\nabla^P H}{H}-20\frac{\nabla_P H\nabla^P H}{H^2} \\
&= \left[8\frac{H''}{H}-20\left(\frac{H'}{H}\right)^2\right](\phi^{-2}+\kappa^2 A\cdot A) \\
&\quad +8\kappa\frac{H'}{H}\left(\phi^{-1}A^\mu\partial_\mu\phi+\frac{1}{2}A^\mu\gamma^{\alpha\beta}\partial_\mu\gamma_{\alpha\beta}+\partial_\mu A^\mu\right),
\end{aligned} \qquad (10)
$$

where the prime denotes differentiation with respect to z.

Then, the five-dimensional action Eq. (1) is given by

$$I=\frac{1}{16\pi G_5}\int d^4x\sqrt{-\gamma}\phi\bigg[R(g)\int dzH^{-3}+(\phi^{-2}+\kappa^2A\cdot A)\int dzH^{-3}\bigg(8\frac{H''}{H}-20\frac{H'^2}{H^2}\bigg)+8\kappa\bigg(\phi^{-1}A^\mu\partial_\mu\phi+\frac{1}{2}A^\mu\gamma^{\alpha\beta}\partial_\mu\gamma_{\alpha\beta}+\partial_\mu A^\mu\bigg)\int dz\frac{H'}{H^4}-2\Lambda\int dzH^{-5}\bigg]-\int d^4x\sqrt{-\gamma}\sqrt{|\delta^\mu_\nu+\kappa^2\phi^2A^\mu A_\nu|}\sigma. \tag{11}$$

It should be pointed out that the 5D Ricci scalar curvature constructed from g_{MN} $R(g)$, is independent of z coordinate since the metric elements g_{MN} are functions of x^μ only. Using $H'=k\theta(z)$, $H''=2k\delta(z)$, $\int_{-\infty}^{\infty}dzH^{-3}=1/k$, and $\int_{-\infty}^{\infty}dzH^{-5}=1/2k$ for the RS model with a single brane, one gets the 4D effective action

$$I_{KK}=\frac{1}{16\pi G_5}\frac{1}{k}\int d^4x\sqrt{-\gamma}[\phi R(g)-\phi\Lambda+6k^2(\phi^{-1}+\kappa^2\phi A\cdot A)-16\pi G_5k\sigma\sqrt{|\delta^\mu_\nu+\kappa^2\phi^2A^\mu A_\nu|}] \tag{12}$$

$$=\frac{1}{16\pi G_4}\int d^4x\sqrt{-\gamma}[\phi R(g)+6k^2(\phi^{-1}+\phi-2\sqrt{|\delta^\mu_\nu+\kappa^2\phi^2A^\mu A_\nu|}+\kappa^2\phi A\cdot A)]. \tag{13}$$

Here the 4D Newton's constant is defined as $G_4=G_5k$. Notice from Eq. (12) that it reproduces the ordinary KK reduction in the presence of the cosmological constant as the brane at $z=0$ disappears (i.e., σ, $k\to 0$). The 5D scalar curvature $R(g)$ is related to the 4D Ricci scalar curvature constructed from $\gamma_{\mu\nu}$, $R(\gamma)$, as

$$\int d^4x\sqrt{-\gamma}\phi R(g)=\int d^4x\sqrt{-\gamma}\phi\bigg[R(\gamma)-\frac{\kappa^2}{4}\phi^2F^2\bigg]+\oint[\cdots]. \tag{14}$$

The last term is the surface term. The field strength is defined as $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and $F^2=F_{\mu\nu}F^{\mu\nu}$. Using Eq. (14), one finally obtains

$$I_{KK}=\frac{1}{16\pi G_4}\int d^4x\sqrt{-\gamma}\bigg[\phi R(\gamma)-\frac{\kappa^2}{4}\phi^3F^2+6k^2(\phi^{-1}+\phi-2\sqrt{|\delta^\mu_\nu+\kappa^2\phi^2A^\mu A_\nu|}+\kappa^2\phi A\cdot A)\bigg], \tag{15}$$

where we have omitted the surface terms.

We observe that the zero mode gravitational degrees of freedom in the 5D spacetime are split into the 4D gravitational fields $\gamma_{\mu\nu}$, a vector field A_μ, and a graviscalar field ϕ as usual. However, the properties of the vector field and the scalar field are very different from those in the conventional KK reduction. The first two terms in this effective action are the same form as in the ordinary dimensional reduction of the Kaluza-Klein unifications, and they have the U(1) gauge symmetry. The difference from the conventional KK reduction is only the last term which is proportional to the brane tension squared. If one started from the KK metric decomposition with $A_\mu=0$ and $\phi=1$ in Eq. (3), this ''potential'' term would disappear and one obtains the ordinary Einstein gravity on the brane with zero effective cosmological constant as well known. As can be easily seen in Eq. (12), this happens because of the fine tuning between the brane tension σ and the 5D bulk cosmological constant Λ.

The appearance of the nonlinear term (e.g., $\sqrt{|\delta^\mu_\nu+\kappa^2\phi^2A^\mu A_\nu|}$) as well as the squared term in A_μ shows not only that the 4D effective action no longer has the gauge symmetry, but also that KK photons are not massless. This arises from the presence of the brane ($k\neq 0$) in the five-dimensional spacetime. Because Eq. (15) contains a nonlinear term which generates a lot of terms, a truncated form of the effective action may be useful to understand the dynamics of the fields easily. If one expands the full action up to the order of κ^2, one has

$$I^T_{\rm KK}\simeq\frac{1}{16\pi G_4}\int d^4x\sqrt{-\gamma}\bigg[\phi R(\gamma)-\frac{\kappa^2}{4}\phi^3F^2+6k^2(\phi^{-1}+\phi-2+\kappa^2\phi(1-\phi)A\cdot A)\bigg]. \tag{16}$$

Here we used $\sqrt{|\delta^\mu_\nu+\kappa^2\phi^2A^\mu A_\nu|}\simeq 1+\frac{1}{2}\kappa^2\phi^2A\cdot A$.

In order to explicitly see how the dynamical aspect of the ϕ field comes out, let us conformally transform the metric as

$$\gamma_{\mu\nu}\to\overline{\gamma}_{\mu\nu}=\phi\gamma_{\mu\nu}. \tag{17}$$

Then, the zero-mode effective action Eq. (15) is written by

$$I_{\rm KK}=\frac{1}{16\pi G_4}\int d^4x\sqrt{-\overline{\gamma}}\bigg[R(\overline{\gamma})-\frac{\kappa^2}{4}\phi^3F^2-\frac{3}{2}\phi^{-2}\overline{\nabla}^\mu\phi\overline{\nabla}_\mu\phi+6k^2\phi^{-2}(\phi^{-1}+\phi-2\sqrt{|\delta^\mu_\nu+\kappa^2\phi^3A^\mu A_\nu|}+\kappa^2\phi^2A\cdot A)\bigg] \tag{18}$$

up to the surface terms. Here $F^2=\overline{\gamma}^{\mu\nu}\overline{\gamma}^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}$, $A^\mu=\overline{\gamma}^{\mu\alpha}A_\alpha$, and $A\cdot A=\overline{\gamma}^{\mu\nu}A_\mu A_\nu$. The truncated effective action in this Einstein frame is also given by

$$I^T_{\mathrm{KK}} \simeq \frac{1}{16\pi G_4}\int d^4x\sqrt{-\bar{\gamma}}\Bigg\{R(\bar{\gamma}) - \frac{\kappa^2}{4}\phi^3F^2 - \frac{3}{2}\phi^{-2}\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi + 6k^2[\phi^{-2}(\phi^{-1}+\phi-2) + \kappa^2(1-\phi)A\cdot A]\Bigg\}. \tag{19}$$

As shall be shown below, the form of these actions above also suggests that the metric tensor which is responsible for the 4D Einstein gravitation is not the $\gamma_{\mu\nu}$, but indeed the $\bar{\gamma}_{\mu\nu}$.

Now the field equations derived from the effective action (19) are given by

$$R_{\mu\nu} = \frac{\kappa^2}{2}\phi^3\left(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}\bar{\gamma}_{\mu\nu}F^2\right) - 6k^2\kappa^2(1-\phi)A_\mu A_\nu + \frac{3}{2}\phi^{-2}[\bar{\nabla}_\mu\phi\bar{\nabla}_\nu\phi - 2k^2\bar{\gamma}_{\mu\nu}(\phi^{-1}+\phi-2)], \tag{20}$$

$$\bar{\nabla}^\mu F_{\mu\nu} + 12k^2\kappa^2\phi^{-3}(1-\phi)A_\nu = -3\phi^{-1}\bar{\nabla}^\mu\phi F_{\mu\nu}, \tag{21}$$

$$\bar{\nabla}^\mu\bar{\nabla}_\mu\phi - \phi^{-1}\bar{\nabla}^\mu\phi\bar{\nabla}_\mu\phi + 2k^2[\phi^{-1}(4-\phi-3\phi^{-1}) - \kappa^2\phi^2A\cdot A] = \frac{\kappa^2}{4}F^2. \tag{22}$$

Here we used the truncated effective action for simplicity. The essential properties of solutions do not change.

It is well known in the RS model that if the flat metric on the brane is replaced by any Ricci-flat 4D metric then the 5D Einstein equations with a negative cosmological constant are still satisfied [15–17]. Now note that $A_\mu = 0$ and $\phi = 1$ satisfies Eqs. (21) and (22). In this case, Eq. (20) becomes $R_{\mu\nu} = 0$. Thus any 4D Ricci-flat metric $\bar{\gamma}_{\mu\nu}$ is a solution. Therefore, although based on the assumption for the zero mode which is z-coordinate independent, our results reproduce this well known property in the RS model. Such somewhat general agreement may indicates the validity of our ansatz for the zero mode. One can see that $\bar{\gamma}_{\mu\nu} = \eta_{\mu\nu}$ corresponds to the RS solution [4]. Since the metric for the 4D Schwartzschild black hole is Ricci-flat, the Schwartzschild black hole can be embedded in the brane world as well known. That is, assuming the spherically symmetric background $\hat{g}_{\mathrm{MN}} = H^{-2}(z)(\bar{\gamma}^S_{\mu\nu}, \phi^2)$ with $\bar{\gamma}^S_{\mu\nu} = \mathrm{diag}[1-2M/r, (1-2M/r)^{-1}, r^2, r^2\sin^2\theta]$ and $\phi = 1$, we obtain the black string solution in 5D anti–de Sitter (AdS_5) spacetime [16]. Since this black string solution is unstable near the AdS horizon, but stable far from it, it is likely to end up with a ‘‘black cigar’’ solution as conjectured in Ref. [16].

Secondly, it would be of interest to ask whether or not the Reissner-Nordström charged black hole can be embedded in the brane world. It is straightforward to see that the Reissner-Nordström black hole solution

$$\bar{\gamma}^{\mathrm{RN}}_{\mu\nu} = \mathrm{diag}[1-2M/r+Q'^2/r^2, (1-2M/r + Q'^2/r^2)^{-1}, r^2, r^2\sin^2\theta] \tag{23}$$

with $Q' = \kappa Q/2$, $F_{tr} = Q/r^2$, and $\phi = 1$, satisfies Eqs. (20) and (21). In this case we have $F_{tr} = -\partial_r A_0$ with $A_0 = Q/r$. However, it does not satisfy Eq. (22) (i.e., $-2k^2A\cdot A = F^2/4$). This is so because of the presence of both brane and ϕ. Thus, the 4D Reissner-Nordström black hole with $\phi = 1$ cannot be embedded in the brane world. In the absence of the brane (i.e., $k = 0$), there exist charged black hole solutions with nontrivial ϕ field. However, such a charged black hole is very different from the Reissner-Nordström black hole [18]. It does not seem that the presence of the brane changes this feature of the Kaluza-Klein theory much. On the other hand, however, if the scalar field were frozen somehow from the beginning in Eq. (3) (e.g., $\phi = 1$), there would be no equation like Eq. (22). Consequently, by observing Eqs. (20) and (21), one can easily find that the Reissner-Nordström black hole is a solution.

III. LINEARIZED PERTURBATION

In this section we consider the linearized perturbations around the RS vacuum solution $(\bar{\gamma}_{\mu\nu} = \eta_{\mu\nu}, A_\mu = 0, \phi = 1)$ for the dimensionally reduced effective action in Eq. (18). Actually, at the linear level, the truncated effective action in Eq. (19) is equivalent to the nontruncated one in Eq. (18). Let us introduce the perturbations around the RS solution

$$\gamma_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad A_\mu = 0 + a_\mu, \quad \phi = 1 + \kappa\varphi. \tag{24}$$

Consequently,

$$\bar{\gamma}_{\mu\nu} = \eta_{\mu\nu} + \kappa\bar{h}_{\mu\nu}, \quad \bar{h}_{\mu\nu} = h_{\mu\nu} + \varphi\eta_{\mu\nu}. \tag{25}$$

Then the bilinear action of Eq. (18) or (19) which governs the perturbative dynamics is given by

$$I^0_{\mathrm{KK}} = \frac{\kappa^2}{16\pi G_4}\int d^4x\Bigg\{-\frac{1}{4}[\partial^\mu\bar{h}^{\alpha\beta}\partial_\mu\bar{h}_{\alpha\beta} - \partial^\mu\bar{h}\partial_\mu\bar{h} + 2\partial^\mu\bar{h}_{\mu\nu}\partial^\nu\bar{h} - 2\partial^\mu\bar{h}_{\mu\alpha}\partial^\nu\bar{h}^\alpha_\nu] - \frac{1}{4}(\partial_\mu a_\nu - \partial_\nu a_\mu)\times(\partial^\mu a^\nu - \partial^\nu a^\mu) - \frac{3}{2}\partial_\mu\varphi\partial^\mu\varphi + 6k^2\varphi^2\Bigg\}, \tag{26}$$

where $\bar{h} = \eta^{\mu\nu}\bar{h}_{\mu\nu} = h + 4\varphi$. Surprisingly, it turns out that the bilinear effective action is invariant under the U(1) gauge transformation. The U(1) gauge symmetry breaking term in Eq. (19) appears as higher order term than the squared order: i.e., $6k^2\kappa^2(1-\phi)A\cdot A \simeq -6k^2\kappa^3\varphi a^\mu a_\mu$.

In order to understand what physical states there are, let us analyze the field equations as below. From the action Eq. (26) we have the equations of motion

$$\Box\bar{h}_{\mu\nu} + \partial_\mu\partial_\nu\bar{h} - (\partial_\mu\partial^\alpha\bar{h}_{\alpha\nu} + \partial_\nu\partial^\alpha\bar{h}_{\alpha\mu}) - \eta_{\mu\nu}(\Box\bar{h} - \partial^\alpha\partial^\beta\bar{h}_{\alpha\beta}) = 0, \tag{27}$$

$$\Box a_\mu - \partial_\mu(\partial_\nu a^\nu) = 0, \tag{28}$$

$$\Box \varphi + 4k^2\varphi = 0. \tag{29}$$

By taking the trace of Eq. (27), we have

$$\Box \bar{h} - \partial^\alpha \partial^\beta \bar{h}_{\alpha\beta} = 0. \tag{30}$$

Hence Eq. (27) becomes

$$\Box \bar{h}_{\mu\nu} + \partial_\mu \partial_\nu \bar{h} - (\partial_\mu \partial^\alpha \bar{h}_{\alpha\nu} + \partial_\nu \partial^\alpha \bar{h}_{\alpha\mu}) = 0. \tag{31}$$

So far we have not chosen any gauge for $\bar{h}_{\mu\nu}$. Now let us choose the transverse (or harmonic) gauge in the five-dimensional spacetime. Since

$$g_{MN} = \eta_{MN} + \kappa \epsilon_{MN}, \quad (\epsilon_{MN}) = \begin{pmatrix} h_{\mu\nu} & -a_\mu \\ -a_\nu & 2\varphi \end{pmatrix}, \tag{32}$$

the five-dimensional harmonic gauge $\partial^M \epsilon_{MN} = \frac{1}{2}\partial_N \epsilon$ is equivalent to

$$\partial^\mu \bar{h}_{\mu\nu} = \frac{1}{2}\partial_\nu \bar{h}, \quad \partial_\mu a^\mu = 0. \tag{33}$$

That is, the 5D harmonic gauge is split into the harmonic gauge for the 4D gravitational field and the Lorenz gauge for the 4D KK gauge field. Using these gauge conditions above, Eqs. (31) and (28) become

$$\Box \bar{h}_{\mu\nu} = 0, \quad \Box a_\mu = 0, \tag{34}$$

respectively. Therefore, it proves that $\bar{h}_{\mu\nu}$ and a_μ indeed represent the massless spin-2 gravitons and the massless spin-1 graviphotons on the brane, respectively.

On the other hand, the spin-0 scalar field fluctuation φ in Eq. (29) appears to be massive. However, it has a tachyonic mass $m_\varphi^2 = -4k^2$ proportional to the brane tension squared. It seems to indicate that the scalar fluctuation of the 5D gravitational degrees of freedom corresponds to the unstable mode in the RS background.

IV. DISCUSSION

We have investigated the KK zero mode effective action in the presence of a single brane in the extra dimension. Although the four-dimensional gravitational modes behaves as usual, the vector and scalar modes behave quite differently. In the 4D effective action, it seems that the vector field A_μ does not possess the U(1) gauge symmetry and that KK photons are not massless any more. The scalar field ϕ is also no longer massless and couples to the vector field.

However, this is not all of the story. The linearized perturbation analysis around the RS background spacetime shows that the 5D massless gravitational degrees of freedom are split into spin-2, spin-1, and spin-0 modes as the standard KK model up to κ^2 order. We have observed that the massive propagation of the vector mode in the 4D effective action is not revealed in the linearized perturbation. In order to observe the effect of the brane $(k \neq 0)$, thus, one needs to study the one-loop correction rather than the linearized one. For example, we expect the relevant vertex correction $(k^2\kappa^3\varphi a^\mu a_\mu)$ from the last term of Eq. (19).

We also have observed that the spin-0 mode propagation has a tachyonic mass, indicating some instability of the RS background spacetime through the ''radion'' or ''modulus'' field. Presumably, it suggests that some stabilization mechanism for the 55-metric component is necessary in order to have the stable RS background spacetime with a single brane as in the case of the two branes. On the other hand, if one requires the R/Z_2 orbifold symmetry in the brane world model, there will be no vector zero mode propagations as mentioned above. Thus, the R/Z_2 orbifold symmetry with the ''modulus'' field stabilization establishes the usual localization of gravity on the brane in the RS model [4].

In deriving the U(1) Maxwell term from the 5D RS brane model, we use the conventional Kaluza-Klein approach. Apparently, we find a nonlinear term as well as $A\cdot A$. This arises from a sort of brane-Higgs effect: Here the isometry of extra dimension was broken spontaneously by the presence of the brane. Hence we expect that the gauge field becomes massive. However, we have found that the massive propagation of the KK gauge field does not reveal at the linear level. Fortunately, instead we find the massless vector propagation. What will happen if we take a z-coordinate dependent or other form of ansatz for the ''zero modes''? We still expect there should be nonlinear or mass terms in the reduced effective action due to the broken isometry in the extra dimension. However, in order to answer whether or not the gauge field becomes massless when linearized, some further work in detail is needed. There were other attempts to achieve the 4D U(1) symmetry from the 5D U(1) bulk gauge fields [11,19].

On the other hand, the RS solution can be extended to accommodate the Schwarzschild black hole solution on the brane as a zero mode solution. This is possible because the RS solution is Ricci flat. Hence the Ricci-flat Schwarzschild solution can be embedded into the brane world by introducing the spherically symmetric spacetime. Now it is very important to check whether or not the RS brane world allows to have the Reissner-Nordström black hole on the brane. As is shown above, the Reissner-Nordström black hole cannot be embedded in the brane world, because this case of $A_0 \neq 0$ cannot be a solution to the effective action of Eq. (15) including the nonlinear term and $A\cdot A$. To obtain this black hole on the brane, it seems to be necessary to introduce some U(1) bulk gauge field in the five-dimensional spacetime whose dynamics is localized on the brane [20].

We have observed that the naive propagations of the scalar field gives rise to the tachyonic mass proportional to the tension of the brane. It may induce the instability of the RS vacuum. In the linearized gravity, by using the residual gauge freedom, one can also impose the traceless gauge in a source-free region in addition to the harmonic gauge. It follows mainly because ''$\Box$Trace'' vanishes in a *source-free* region. What will happen if such traceless gauge is imposed in our linearized analysis? The five-dimensional trace is $\epsilon = \eta^{MN}\epsilon_{MN} = h + 2\varphi = \bar{h} - 2\varphi$. Here $\bar{h} = h + 4\varphi$ is used. By combining the trace of Eq. (34) and Eq. (29), we have

$$\Box_{(5)}\epsilon=\Box\bar{h}-2\Box\varphi=8k^2\varphi, \tag{35}$$

where $\Box_{(5)}=\Box+\partial_5^2$. Thus, imposing the five-dimensional traceless gauge (i.e., $\epsilon=0$) directly results in $\varphi=0$, that is, no graviscalar fluctuation. Since $h=\epsilon-2\varphi$ and $\bar{h}=\epsilon+2\varphi$, it also means the four-dimensional traceless gauge (i.e., $h=\bar{h}=0$). In other words, we notice that the existence of the tachyonic graviscalar fluctuation is mutually inconsistent with imposing the traceless gauge condition. Therefore, the resolution of the instability of the RS background spacetime due to the graviscalar transforms to whether or not one can impose the traceless gauge. In the linearized gravity, the trace of metric fluctuations can be set to be zero by using remaining gauge freedom provided that there is no matter source in the region in consideration [21]. In our analysis, however, since the graviscalar field φ plays as a matter source in the trace equation above, it does not seem to be plausible imposing such gauge condition in the first place. Presumably such traceless gauge condition can be imposed on $\bar{h}$ $(\Box\bar{h}=0)$, but not on h $(\Box h=16k^2\varphi)$.

As mentioned above, another possible caveat of our result is that the tachyonic graviscalar fluctuation is merely an artifact of the ansatz for the zero mode we used in this paper. For instance, instead of $H(z)=k|z|+1$ in Eq. (2), let us assume $H(x,z)=k|z|\phi(x)+1$ for the zero mode. This ansatz is analogous to the form used in Ref. [22] for the case of RS model with two branes. Then the 4D effective action is given by

$$\begin{aligned}
I_{\mathrm{KK}}=&\frac{1}{16\pi G_4}\int d^4x\sqrt{-\gamma}\Big[R(\gamma)-\frac{\kappa^2}{4}\phi^2F^2\\
&+2\phi^{-2}\gamma^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+6k^2\big(2-2\sqrt{|\delta^\mu_\nu+\kappa^2\phi^2A^\mu A_\nu|}\\
&+\kappa^2\phi^2A\cdot A\big)\Big]\\
\simeq&\frac{1}{16\pi G_4}\int d^4x\sqrt{-\gamma}\Big[R(\gamma)-\frac{\kappa^2}{4}\phi^2F^2\\
&+2\phi^{-2}\gamma^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\Big].
\end{aligned} \tag{36}$$

Here we see that the graviscalar fluctuation as well as that of the gravivector becomes massless in the linearized perturbations. Therefore, in order to clarify the issues discussed above, further investigation is required on what the correct form of the ansatz is for the zero mode in the brane world scenarios.

Finally, it will be interesting to extend our study to various types of brane world models [6] as well as to the RS model with two positive and negative tension branes [3] and see how the U(1) gauge field behaves in the 4D effective action. It will be also worth investigating how graviphotons and graviscalar particles interact with the 5D bulk standard model particles [11,12,23] in the presence of branes.

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[1] Y.M. Cho and S.W. Zoh, Phys. Rev. D **46**, 3483 (1992).
[2] A. Salam and J. Strathdee,''On Kaluza-Klein Theory,'' ICTP Report No. IC/81-211; S. Randjbar-Daemi, A. Salam, and J. Strathdee, Nucl. Phys. **B214**, 491 (1983).
[3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[5] C. Csaki, J. Erlich, T.J. Hollowood, and Y. Shirman, Nucl. Phys. **B581**, 309 (2000).
[6] R. Gregory, V.A. Rubakov, and S.M. Sibiryakov, Phys. Rev. Lett. **84**, 5928 (2000); I.I. Kogan and G.G. Ross, Phys. Lett. B **485**, 255 (2000); J. Lykken, R.C. Myers, and J. Wang, J. High Energy Phys. **09**, 009 (2000).
[7] A. Dobado and A.L. Maroto, Nucl. Phys. **B592**, 203 (2000).
[8] M. Bando, T. Kugo, T. Noguchi, and K. Yoshioka, Phys. Rev. Lett. **83**, 3601 (1999); J. Hisano and N. Okada, Phys. Rev. D **61**, 106003 (2000).
[9] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D **59**, 086004 (1999); G. Dvali and M. Schifman, in L.B. Okun Festschrift, edited by V. Telegdi and K. Winter, Phys. Rep. **320**, 107 (1999).
[10] G. Dvali, I.I. Kogan, and M. Schifman, Phys. Rev. D **62**, 106001 (2000).
[11] H. Davoudiasl, J.L. Hewett, and T.G. Rizzo, Phys. Lett. B **473**, 43 (2000); A. Pomarol, *ibid.* **486**, 153 (2000); T. Gherghetta and A. Pomarol, Nucl. Phys. **B586**, 141 (2000); N. Kaloper, E. Silverstein, and L. Susskind, hep-th/0006192.
[12] P. Dimopoulos, K. Farakos, A. Kehagias, and G. Koutsoumbas, hep-th/0007079.
[13] G. Kang and Y.S. Myung, Phys. Lett. B **483**, 235 (2000); Y.S. Myung, G. Kang, and H.W. Lee, *ibid.* **478**, 294 (2000); Y.S. Myung and G. Kang, *ibid.* **497**, 296 (2001).
[14] B. Bajc and G. Gabadadze, Phys. Lett. B **474**, 282 (2000).
[15] R. Emparan, C.V. Johnson, and R.C. Myers, Phys. Rev. D **60**, 104001 (1999).
[16] A. Chamblin, S.W. Hawking, and H.S. Reall, Phys. Rev. D **61**, 065007 (2000); R. Gregory, Class. Quantum Grav. **17**, L125 (2000).
[17] Y.S. Myung and H.W. Lee, Phys. Rev. D **63**, 064034 (2001).
[18] J.H. Horne and G.T. Horowitz, Phys. Rev. D **46**, 1340 (1992); R.G. Cai and Y.S. Myung, *ibid.* **56**, 3466 (1997).

[19] R. Gaitan, Mod. Phys. Lett. A **14**, 1383 (1999).
[20] A. Chamblin, H. Real, H. Shinkai, and T. Shiromizu, Phys. Rev. D **63**, 064015 (2001).
[21] C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).
[22] W.D. Goldberger and M.B. Wise, Phys. Lett. B **475**, 275 (2000).
[23] D.A. Demir, Phys. Lett. B **495**, 357 (2000).