Schwarzschild black hole in the dilatonic domain wall

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In the dilatonic domain wall model, we study the Schwarzschild black hole as a solution to the Kaluza-Klein zero mode effective action which is equivalent to the Brans-Dicke (BD) model with a potential. This can describe the large Randall-Sundrum (RS) black hole whose horizon is to be the intersection of the black cigar with the brane. The black cigar located far from the five-dimensional anti–de Sitter (AdS$_5$) horizon is known to be stable, but any explicit calculation for stability of the RS black hole at $z=0$ has not yet been performed. Here its stability is investigated against the z-independent perturbations composed of odd-even parities of graviton ($h_{\mu\nu}$) and BD scalar ($h_{44}=2\varphi$). It seems that the RS black hole is classically unstable because it has a potential instability at a wavelength $\lambda > 1/(2k)$. However, this is not allowed inside an AdS$_5$ box of the size $1/(2k)$. Thus the RS black hole becomes stable. The RS black hole can be considered as a stable remnant at $z=0$ of the black cigar.

I. INTRODUCTION

Recently there has been much interest in the Randall-Sundrum brane world [1–3]. A key idea of this model is that our universe may be a brane embedded in the higher dimensional space. A concrete model is a single three brane embedded in the five-dimensional anti–de Sitter space (AdS$_5$), which acts like a box of the size $\sim 1/k$. Randall and Sundrum (RS) have shown that a longitudinal part ($h_{\mu\nu}$) of the metric fluctuations satisfies the Schrödinger-like equation with an attractive delta function. As a result, the massless Kaluza-Klein (KK) modes which describe the localized gravity on the brane were found. Furthermore, the massive KK modes lead to corrections to the Newtonian potential such as $V(r) = G_N(m_1 m_2)/r[1+(1/k^2 r^2)]$.

However, we would like to point out that this has been done in the four-dimensional (4D) Minkowski space with the RS gauge.[1] It seems that this gauge is so restrictive. In order to have a well-defined theory on the brane, one has to include nonzero transverse parts of $h_{4\mu}$ and h_{44} at the beginning. Ivanov and Volovich discussed along this direction by choosing the 5D de Donder gauge[2] [4,5]. Also authors in Ref. [6] studied the propagation of the metric including nonzero transverse parts. It turned out that there are no massless scalar and vector propagations in the RS background. This implies that the RS gauge is a good choice for describing the brane world. Furthermore, it is shown that the RS Minkowski spacetime is stable only under the RS gauge.

If the Minkowski metric on the brane is replaced by any 4D Ricci-flat one, for example the Schwarzschild metric, the 5D metric still satisfies the Einstein equation with a negative cosmological constant. In this process we obtain a black

string solution in an AdS$_5$ [7].[3] This is given by $\bar{G}_{MN}^{BS} = H^{-2}(z)[\bar{g}_{\mu\nu}^S,1]$ with the Schwarzschild metric $\bar{g}_{\mu\nu}^S = \mathrm{diag}[-(1-2M/r),(1-2M/r)^{-1},r^2,r^2\sin^2\theta]$. It is shown that this black string is unstable near the AdS$_5$ horizon of $z=\infty$ but it is stable far from the horizon. This is a result of the Gregory-Laflamme instability [8], which states that the black string has a tendency to fragment near the AdS$_5$ horizon [9]. A stable object left behind would resemble a black cigar, although we do not know its explicit metric. Hence we have the RS black hole picture on the brane located at $z=0$. The horizon of the black hole on the brane will be determined from the intersection of the black cigar with the brane. Here we assume that this is large as $r_{EH} = 2M > 1/k$. In this case an observer on the brane perceives exactly the Schwarzschild solution, without any correction arising from the extra dimension [10]. In other words, any massive KK modes are not excited in these circumstances. Then the zero modes of the bulk graviton can describe this situation very well. Hence the zero mode approach becomes a powerful technique in the study of black holes on the brane.

On the other hand, Youm showed that the RS solution can be found in the dilatonic domain wall [11,12]. In order to study physics on the brane, we need its effective action. It is known that zero mode effective action takes a form of the Brans-Dicke model with a potential.

In this paper, we study stability of the large RS black hole with z-independent perturbations such as $h_{\mu\nu}(x),h_{44}(x)$. We find that these are massless graviton and scalar modes propagating in the RS black hole background. Here we do not require the 4D TTF gauge which is useful for the RS Minkowski space. Instead we choose the Regge-Wheeler (RW) gauge for our study of spherically symmetric background.

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[1]In fact, this gauge for h_{MN} is composed of Gaussian-normal (GN) gauge ($h_{44}=h_{4\mu}=0$) and 4D transverse, trace-free (TTF) gauge ($\partial^\mu h_{\mu\nu}=0$, $h^\mu{}_\mu=0$).

[2]This corresponds to the 5D TTF gauge ($\partial_M h^{MN}=0, h^M{}_M=0$).

[3]Precisely, this is not a 5D black hole solution to the RS model. At present, nobody knows that.

II. RANDALL-SUNDRUM SOLUTION AND BRANS-DICKE-TYPE MODEL

We start with the 5D bulk action and the 4D domain wall action as [12]

$$S = S_{bulk} + S_{DW} \tag{1}$$

with

$$S_{bulk} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-G} \left[R_5 - \frac{4}{3} \partial_M D \partial^M D - e^{-2aD}\Lambda \right], \tag{2}$$

$$S_{DW} = -\sigma_{DW} \int d^4x \sqrt{-\gamma} e^{-aD}, \tag{3}$$

where σ_{DW} is the tension of the domain wall and γ is the determinant of the induced metric $\gamma_{\mu\nu} = \partial_\mu X^M \partial_\nu X^N G_{MN}$ for the domain wall. Here $M,N = 0,1,2,3,4(x^4 = z)$ and $\mu,\nu = 0,1,2,3(x^\mu = x)$. ''$D$'' denotes the dilaton. We are interested in the second RS solution with[4]

$$\bar{G}_{MN} = H^{-2}(z)\eta_{MN}, \quad \bar{D} = 0, \quad \Lambda = -12k^2,$$

$$\sigma_{DW} = 6k/\kappa_5^2, \quad a = 0 \tag{4}$$

with $H = k|z| + 1$ and $\eta_{MN} = \text{diag}[-++++]$. Here overbar ($^-$) means the background value. In this paper we follow the Misner-Thorne-Wheeler (MTW) conventions [13].

In order to obtain a 4D effective action, we introduce the metric $\hat{G}_{MN}$ in $G_{MN} = H^{-2}(z)\hat{G}_{MN}$ which satisfies $\partial_z \hat{G}_{MN} = 0$. Explicitly, the line element is given by

$$dS_5^2 = G_{MN} dx^M dx^N = H^{-2}\hat{G}_{MN} dx^M dx^N$$

$$= H^{-2}[g_{\mu\nu}(x)dx^\mu dx^\nu + \Phi^2(x)dz^2]. \tag{5}$$

Off-diagonal elements are not allowed because if they exist, they violate the Z_2-symmetry argument.[5] At this stage we wish to remind the reader that $\Phi(x)$ has nothing to do with the radion which is necessary for stabilizing the distance between two branes in the first RS model [1]. This is because here we consider the second RS model [2]. Substituting Eq. (5) with $D = 0$ into Eq. (1) and integrating it over z lead to the Brans-Dicke (BD)-type model with a potential [12]

$$S_{RS} = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left[\Phi R + 6k^2 \left(\Phi + \frac{1}{\Phi} - 2 \right) \right] \tag{6}$$

with $\kappa_4^2 = k\kappa_5^2$. This is our key action which is suitable for the study of physics defined on the brane. Also it can be derived from the RS model directly. It is emphasized that the

first term (ΦR) is the BD term with $\omega = 0$ [14]. The BD model with $\omega = 0$ corresponds to the massless Kaluza-Klein model with $g_{\mu\nu}$, $g_{44}(\sim\Phi^2)$, and $g_{\mu4}(\sim A_\mu) = 0$.[6] In this sense, we wish to call Φ the BD scalar. The second term arises from the fact that the 5D spacetime is an AdS$_5$ with a negative cosmological constant Λ and a domain wall located at $z = 0$. Equivalently, this means that the RS solution gives us a nonfactorizable geometry with $H = k|z| + 1$. Hence this term accounts for the feature of the RS-type solution and it plays the role of an effective potential. From now on we wish to separate the pure BD model $[\mathcal{L}_{BD} = \Phi R]$ from the RS model $[\mathcal{L}_{RS} = \Phi R + 6k^2(\Phi + 1/\Phi - 2)]$ for comparison.

From Eq. (6) we derive the equations of motion ($\delta_\Phi S_{RS} = 0$, $\delta_{g_{\mu\nu}} S_{RS} = 0$)

$$R + 6k^2 \left(1 - \frac{1}{\Phi^2} \right) = 0, \tag{7}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{\Phi} \left\{ \nabla_\mu \nabla_\nu \Phi - g_{\mu\nu}\Box\Phi \right.$$

$$\left. + 3k^2 \left(\Phi + \frac{1}{\Phi} - 2 \right) g_{\mu\nu} \right\}. \tag{8}$$

Contracting Eq. (8) with $g^{\mu\nu}$ and using Eq. (7) leads to the RS scalar equation

$$\Box\Phi + 2k^2 \left(-\Phi - \frac{3}{\Phi} + 4 \right) = 0. \tag{9}$$

Also from Eq. (8), its contraction form, and Eq. (9) one finds the other Einstein equation,

$$R_{\mu\nu} = \frac{1}{\Phi} \left\{ \nabla_\mu \nabla_\nu \Phi + 2k^2(-\Phi + 1)g_{\mu\nu} \right\}. \tag{10}$$

Here one finds a solution which satisfies all of Eqs. (7)–(10) simultaneously as

$$\bar{\Phi} = 1, \quad \bar{R} = 0, \quad \bar{R}_{\mu\nu} = 0. \tag{11}$$

This means that the Ricci-flat condition of $\bar{R}_{\mu\nu} = 0$ with $\bar{\Phi} = 1$ describes the 4D vacuum configuration correctly.

As an example, we choose the Minkowski spacetime

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu}, \quad \bar{\Phi} = 1. \tag{12}$$

To study the propagations on this background, we introduce the perturbation around the background (12) as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \Phi = \bar{\Phi} + \varphi \quad (g_{44} = \bar{g}_{44} + h_{44}). \tag{13}$$

Then the linearized equations to Eqs. (9) and (10) are found to be

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[4]With this ansatz, the dilatonic action Eq. (1) reduces to the second RS model exactly [2].

[5]One may introduce off-diagonal term of $2A_\mu(x)dx^\mu dz$. But this is not invariant under the reflection of $z \to -z$ [1]. Thus we must have $A_\mu(x) = 0$.

[6]This model is based on the action of $S_{KK} = (1/\kappa_5^2) \int d^5x \sqrt{-G}R_5$ with a factorizable geometry of $H = 1$.

$$\partial^2 \varphi + 4k^2 \varphi = 0, \tag{14}$$

$$\delta R_{\mu\nu}(h) - \partial_\mu \partial_\nu \varphi + 2k^2 \varphi \eta_{\mu\nu} = 0 \tag{15}$$

with

$$\delta R_{\mu\nu} = -\frac{1}{2}[\partial^2 h_{\mu\nu} + \partial_\nu \partial_\mu h^\rho{}_\rho - \partial^\rho \partial_\mu h_{\nu\rho} - \partial^\rho \partial_\nu h_{\mu\rho}]. \tag{16}$$

Under the 4D TTF gauge, one finds that $\delta R_{\mu\nu} = -\frac{1}{2}\partial^2 h_{\mu\nu}$ and $\eta^{\mu\nu}\delta R_{\mu\nu} = -\frac{1}{2}\partial^2 h^\mu{}_\mu = 0$. Contracting Eq. (15) with $\eta^{\mu\nu}$ leads to the other equation for φ

$$\partial^2 \varphi - 8k^2 \varphi = 0. \tag{17}$$

Equation (14) allows a tachyonic solution because it has a negative potential term of $-4k^2$. Fortunately we resolve this problem. We have two different Eqs. (14) and (17) for the same field of φ. Hence we require $\varphi = 0$ for consistency. This observation agrees with Refs. [6,12]. Then Eq. (15) reduces to an equation for the massless graviton without a matter source on the brane

$$\partial^2 h_{\mu\nu} = 0. \tag{18}$$

III. SCHWARZSCHILD BLACK HOLE SOLUTIONS

Introducing a spherically symmetric spacetime, one obtains the Schwarzschild black hole with $\bar{\Phi} = 1$ in the domain wall approach [7] as

$$\bar{g}_{\mu\nu} = \mathrm{diag}[-e^\nu, e^{-\nu}, r^2, r^2 \sin^2 \theta], \tag{19}$$

$$\bar{R}_{trtr} = 2\frac{M}{r^3}, \quad \bar{R}_{t\theta t\theta} = -\frac{(r-2M)M}{r^2},$$

$$\bar{R}_{t\phi t\phi} = \sin^2 \theta \bar{R}_{t\theta t\theta},$$

$$\bar{R}_{r\theta r\theta} = \frac{M}{r-2M}, \quad \bar{R}_{r\phi r\phi} = \sin^2 \theta \bar{R}_{r\theta r\theta},$$

$$\bar{R}_{\theta\phi\theta\phi} = -2Mr \sin^2 \theta,$$

$$\bar{R}_{\mu\nu} = 0, \quad \bar{R} = 0$$

with

$$e^\nu = 1 - \frac{2M}{r}. \tag{20}$$

Here $M = G_N M_4 = \kappa_4^2 M_4 / 8\pi = G_5 k M_4$ with G_N (4D Newtonian constant) and G_5 (5D Newtonian constant). M_4 is the mass of a large black hole with $M_4 > 1/(2k^2 G_5)$. The BD and RS black hole solutions are permitted because the Schwarzschild solution comes from the Ricci-flat condition. The BD (RS) black holes denote the Schwarzschild black hole with $\mathcal{L}_{\mathrm{BD}}$ ($\mathcal{L}_{\mathrm{RS}}$), respectively. The BD black hole is

introduced for reference. To study these black holes specifically, we introduce the perturbation

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad \Phi = \bar{\Phi} + \varphi. \tag{21}$$

Then the linearized equations to Eqs. (9) and (10) are found as

$$\bar{\Box}\varphi + 4k^2 \varphi = 0, \tag{22}$$

$$\delta R_{\mu\nu}(h) - \bar{\nabla}_\nu \bar{\nabla}_\mu \varphi + 2k^2 \varphi \bar{g}_{\mu\nu} = 0 \tag{23}$$

with the Lichnerowicz operator $\delta R_{\mu\nu}(h)$ [15,16],

$$\delta R_{\mu\nu}(h) = -\frac{1}{2}[\bar{\Box}h_{\mu\nu} + \bar{\nabla}_\nu \bar{\nabla}_\mu h^\rho{}_\rho - \bar{\nabla}^\rho \bar{\nabla}_\mu h_{\nu\rho} - \bar{\nabla}^\rho \bar{\nabla}_\nu h_{\mu\rho}]. \tag{24}$$

We note that Eq. (23) is not a diagonalized form to obtain eigenmodes. If one introduces $\hat{h}_{\rho\nu} = h_{\rho\nu} + \varphi \bar{g}_{\rho\nu}$, then this leads to

$$\delta R_{\mu\nu}(\hat{h}) = 0. \tag{25}$$

This is the perturbed equation of pure 4D gravity for $\hat{h}_{\mu\nu}$.[7] A way to analyze Eqs. (22) and (23) is known [19,20]. For instance, it is possible if one uses the RW gauge instead of the 4D TTF gauge. The perturbation $\hat{h}_{\mu\nu}$ falls into two distinct classes — odd and even parities with $(-1)^{l+1}$ and $(-1)^l$, respectively. l denotes an angular quantum number on S^2: $\bar{L}^2 Y_{lm}(\theta,\phi) = -l(l+1)Y_{lm}(\theta,\phi)$. Among ten components in the axisymmetric perturbation, one can always choose six components by taking into account the general coordinate transformations: $x^{\mu\prime} = x^\mu + \epsilon\xi^\mu$ [21,22]. This is a choice of the RW gauge. And this is obvious here because we consider the propagation of gravitons on the brane. In the RW gauge we assign two components (h_0, h_1) for odd parity

$$\hat{h}^{\mathrm{odd}}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & h_0(r) \\ 0 & 0 & 0 & h_1(r) \\ 0 & 0 & 0 & 0 \\ h_0(r) & h_1(r) & 0 & 0 \end{pmatrix} e^{-i\omega t} \sin\theta \frac{dP_l(\theta)}{d\theta} \tag{26}$$

with Legendre polynomial $P_l(\theta)$. For even parity, we have $\hat{h}^{even}_{\mu\nu} = h^{even}_{\mu\nu} + \varphi \bar{g}_{\mu\nu}$, where $h^{even}_{\mu\nu}$ is composed of four components (H_0, H_1, H_2, K) as

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[7]Its stability analysis was performed 30 years ago [17,18].

$$h_{\mu\nu}^{\text{even}}=\begin{pmatrix} H_0\left(1-\dfrac{2M}{r}\right) & H_1 & 0 & 0 \\[2mm] H_1 & H_2\left(1-\dfrac{2M}{r}\right)^{-1} & 0 & 0 \\[2mm] 0 & 0 & r^2K(r) & 0 \\[2mm] 0 & 0 & 0 & r^2\sin^2\theta K(r) \end{pmatrix}e^{-i\omega t}P_l(\theta). \tag{27}$$

These two cases are never mixed and thus they provide two degrees of freedom which is necessary for describing a massless spin-two particle.

A. RS scalar perturbation

Let us first analyze the RS scalar perturbation using Eq. (22). Considering $[\varphi \propto \psi(r)/r]Y_{lm}(\theta,\phi)e^{-i\omega t}$ and the background (19), one finds the Schrödinger-type equation

$$\frac{d^2\psi}{dr^{*2}}+(\omega^2-V_\psi^{\text{RS}})\psi=0, \tag{28}$$

where the RS scalar potential is given by

$$V_\psi^{\text{RS}}(r)=\left(1-\frac{2M}{r}\right)\left\{\frac{l(l+1)}{r^2}+\frac{2M}{r^3}-4k^2\right\} \tag{29}$$

with the tortoise coordinate $r^*=r+2M\ln[(r/2M)-1]$. From the analysis in Ref. [20], we find that for $\omega=i\alpha$, $0<\alpha<2k$, the scalar perturbation has an exponentially growing mode of $e^{\alpha t}$. Therefore this system may be classically unstable. In other words, "$-4k^2$" in the potential induces an instability of the asymptotically flat space of $r\to\infty$. Thus we call it a potential instability. This may imply that the RS black hole solution is classically unstable. However, it is not true. Any exponentially growing mode is not allowed for the RS spherically symmetric background. An important point to remember is that an AdS$_5$ acts like a box of size[8] $\sim(2k)^{-1}$ [7], so it may allow an unstable perturbation of wavelength with $\lambda\leqslant(2k)^{-1}$ only [23]. However, for this case, we cannot find any consistent (unstable) solution [20]. In the unstable case of $0<\alpha=\lambda^{-1}<2k$, one finds a condition of $\lambda>(2k)^{-1}$ which is forbidden inside an AdS$_5$. The instability problem can be cured by considering the bulk spacetime. Hence we can include the RS scalar mode, as a physical field, which propagates in the RS black hole background.

B. Odd parity perturbation for $\hat{h}_{\mu\nu}$

Now we discuss the odd parity perturbation for $\hat{h}_{\mu\nu}$. From Eq. (23), we have three equations:

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[8]Although a conventional length scale of an AdS$_5$ is determined by $\int d^5x\sqrt{-G}R_5$ in Eq. (2) as $\int dzH^{-3}=1/k$, we here choose a size of the AdS$_5$ box as $1/(2k)$ for a definite calculation. This comes from: $-\Lambda\int d^5x\sqrt{-G}\to\int dzH^{-5}=1/(2k)$.

$$\delta R_{03}=\delta R_{13}=\delta R_{23}=0. \tag{30}$$

Here we obtain the Regge-Wheeler equation using $Q\equiv(h_1/r)[1-(2M/r)]$

$$\frac{d^2Q}{dr^{*2}}+[\omega^2-V_{\text{RW}}(r)]Q=0, \tag{31}$$

where the Regge-Wheeler potential is given by

$$V_{\text{RW}}(r)=\left(1-\frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2}-\frac{6M}{r^3}\right). \tag{32}$$

Further h_0 is not an independent mode, it can be expressed in terms of Q as $h_0=(i/\omega)(d/dr^*)(rQ)$. We note that the BD black hole [19] takes the same potential as in Eq. (32) for the odd parity perturbation. This case has already been analyzed by Vishveshwara in Ref. [17] and an allowed solution is a scattering state

$$Q_\infty=e^{-i\omega r^*}+A^-e^{i\omega r^*}\quad(r^*\to\infty),$$

$$Q_{2M}=B^-e^{-i\omega r^*}\quad(r^*\to-\infty). \tag{33}$$

C. Even parity perturbation for $\hat{h}_{\mu\nu}$

In this case, from the remaining seven equations, we have the Zerilli equation in Ref. [17]

$$\frac{d^2\psi_Z}{dr^{*2}}+[\omega^2-V_Z(r)]\psi_Z=0, \tag{34}$$

where the Zerilli potential is given by

$$V_Z(r)=\left(1-\frac{2M}{r}\right)$$
$$\times\left\{\frac{2\lambda^2(\lambda+1)r^3+6\lambda^2Mr^2+18\lambda M^2r+18M^3}{r^3(\lambda r+3M)^2}\right\} \tag{35}$$

with $\lambda=(l-1)(l+2)/2$. Here $\psi_Z(r)$ is a gauge invariant combination of $H_0,H_1,H_2,K,\psi/r$ [19,21,22]. At this stage

we would like to comment that the BD equation $[\delta R_{\mu\nu}(h) - \nabla_\nu\nabla_\mu\varphi = 0]$ in Ref. [19] takes the same equation as in Eq. (34). Also it is easily shown that an allowed solution is a plane wave like Eq. (33). This fact can be easily read off from the shape of potentials V_{RW} and V_Z. Because these all belong to positive potential barrier for $l \geq 2$, there exist scattering states only. In other words, there are no bound state solutions. This means that one cannot find any exponentially growing mode in the graviton sector, even if k^2 term is involved in the RS Eq. (23).

IV. DISCUSSIONS

We investigate the zero mode sector of the 5D dilatonic domain wall solution. This sector is very useful for describing the RS black hole on the brane. Assuming a spherically symmetric spacetime, one has a large black hole on the brane. We perform the analysis of stability to see whether or not the RS black hole truly exists. It is well known that the BD black hole is stable. Here one finds an exponentially growing mode for the RS black hole because of the negative nature of its potential. If there is an exponentially growing mode, its black hole is classically unstable. From the analysis of φ in the RS Minkowski spacetime, we have $\varphi = 0$ under the 4D TTF gauge. On the other hand, we cannot have $\varphi = 0$ in the spherically symmetric black hole spacetime with the RW gauge. Fortunately, we find that this instability is not

allowed inside an AdS_5 whose size is $(2k)^{-1}$. Hence the instability problem is cured. Actually BD scalar as well as gravitons can propagate in the black hole spacetime. Also this implies that the large RS black hole is stable.

For the metric perturbations, we do not worry about their stability even for considering $\varphi \neq 0$. Choosing the RW gauge, the graviton sector including φ leads to the well-known two classes of odd and even parities. Since this sector always has positive potential barriers for any l with $l \geq 2$ [18], there are no exponentially growing modes. Also this can be confirmed from the other side. If we introduce a new tensor $\hat{h}_{\mu\nu}$, Eq. (23) reduces to Eq. (25). This is nothing but the perturbed equation for pure 4D gravity, which was proved to be stable 30 years ago.

In conclusion, the large black hole in the dilaton domain wall is stable. This can represent a stable form on the brane for the RS black cigar whose metric is not known up to now. Finally we comment that the RS black hole can be described by massless gravitons and a scalar mode with smaller wavelength than the size of an AdS_5 box.

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