Anisotropy dissipation in brane-world inflation

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We examine the behavior of an anisotropic brane-world in the presence of inflationary scalar fields. We show that, contrary to naive expectations, a large anisotropy does not adversely affect inflation. On the contrary, a large initial anisotropy introduces more damping into the scalar field equation of motion, resulting in greater inflation. The rapid decay of anisotropy in the brane-world significantly increases the class of initial conditions from which the observed universe could have originated. This generalizes a similar result in general relativity. A unique feature of Bianchi type I brane-world cosmology appears to be that for scalar fields with a large kinetic term the initial expansion of the Universe is quasi-isotropic. The Universe grows more anisotropic during an intermediate transient regime until anisotropy finally disappears during inflationary expansion.

I. INTRODUCTION

Observations of galaxies, quasistellar objects (QSO's) and the cosmic microwave background appear to indicate that we live in a Universe which is remarkably uniform on very large scales. Yet the homogeneity and isotropy of the universe is difficult to explain within the standard relativistic framework since, in the presence of matter, the class of solutions to the Einstein equations which evolve towards a Friedmann-Robertson-Walker (FRW) universe is essentially a set of measure zero [1]. The above statement is however only true for space-times containing ''normal'' matter satisfying energy conditions which ensure that (i) negative pressures can never grow so large as to dominate the energy density, $T_{00} \geq |T_{ij}|$, and (ii) the sum of the principle pressures of the fluid must be non-negative: $\Sigma_{i=1}^{3} T_{ii} \geq 0$. The inflationary scenario, based as it is on a form of matter which violates these energy conditions, radically alters the above picture. Indeed, as demonstrated in [2–7], a large class of spacetimes both homogenize and isotropize under the influence of an effective cosmological Λ term. Thus the inflationary scenario can successfully generate a homogeneous and isotropic FRW-like universe from initial conditions which, in the absence of Λ, would have resulted in a universe far removed from the one we live in today.

Recently there has been a great deal of interest in a cosmological scenario in which matter fields are confined to a 3-dimensional brane-world embedded in a higher dimensional ''bulk'' space [8]. This higher-dimensional cosmology generalizes the standard Kaluza-Klein picture by allowing the presence of large or even infinite non-compact extra dimensions. The issue of inflation on the brane was investigated in [9], where it was shown that on an FRW brane in 5-dimensional anti–de Sitter space, extra-dimensional effects are conducive to the advent of inflation (see also [10]). In this paper we address the kindred issue of anisotropic initial conditions. We demonstrate that even very large initial anisotropy cannot prevent brane-world inflation from occurring, thus generalizing a previous result in general relativity [4–7]. On the contrary, for a large class of initial conditions, the presence of an anisotropy actually *increases* the amount of inflation. Thus a scalar field dominated universe can eventually isotropise and inflate, even if its expansion was very anisotropic to begin with. A unique feature of brane cosmology is that the effective equation of state at high densities can become ultra stiff. Consequently matter can overwhelm shear for equations of state which are stiffer than dust, leading to quasi-isotropic early expansion of the Universe in such cases.

II. BRANE DYNAMICS

The 5-dimensional (bulk) field equations are

$$\widetilde{G}_{AB} = \widetilde{\kappa}^2 [-\widetilde{\Lambda}\widetilde{g}_{AB} + \delta(y)\{-\lambda g_{AB} + T_{AB}\}], \qquad (1)$$

where tildes denote the bulk generalization of standard general relativity quantities, and $\widetilde{\kappa}^2 = 8\pi/\widetilde{M}_p^3$, where $\widetilde{M}_p$ is the fundamental 5-dimensional Planck mass, which is typically much less than the effective Planck mass on the brane, $M_p = 1.2 \times 10^{19}$ GeV. The brane is given in Gaussian normal coordinates $x^A = (x^\mu, y)$ by $y = 0$, where $x^\mu = (t, x^i)$ are spacetime coordinates on the brane. The brane tension is λ, and $g_{AB} = \widetilde{g}_{AB} - n_A n_B$ is the induced metric on the brane, with n_A being the space-like unit normal to the brane. Standard-model matter fields confined to the brane make up the brane energy-momentum tensor T_{AB} (with $T_{AB}n^B = 0$). The bulk cosmological constant $\widetilde{\Lambda}$ is negative, and is the only 5-dimensional source of the gravitational field.

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The field equations induced on the brane are derived via an elegant geometric approach in [11], leading to new terms that carry bulk effects onto the brane:

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa^2 T_{\mu\nu} + \tilde{\kappa}^4 S_{\mu\nu} - \mathcal{E}_{\mu\nu}. \tag{2}$$

Here $\kappa^2 = 8\pi/M_p^2$ and

$$\lambda = 6\frac{\kappa^2}{\tilde{\kappa}^4}, \quad \Lambda = \frac{1}{2}\tilde{\kappa}^2(\tilde{\Lambda} + \frac{1}{6}\tilde{\kappa}^2\lambda^2). \tag{3}$$

We assume that $\tilde{\Lambda}$ is chosen so that $\Lambda = 0$. Extra-dimensional corrections to the Einstein equations on the brane are of two forms: firstly, the matter fields contribute local quadratic energy-momentum corrections via the tensor $S_{\mu\nu}$, and secondly, there are nonlocal effects from the free gravitational field in the bulk, transmitted via the projection $\mathcal{E}_{\mu\nu}$ of the bulk Weyl tensor. The matter corrections are given by

$$S_{\mu\nu} = \frac{1}{12} T_\alpha{}^\alpha T_{\mu\nu} - \frac{1}{4} T_{\mu\alpha} T^\alpha{}_\nu + \frac{1}{24} g_{\mu\nu}[3 T_{\alpha\beta} T^{\alpha\beta} - (T_\alpha{}^\alpha)^2],$$

and are significant at high energies, i.e., $\rho \gtrsim \lambda$. The projection of the bulk Weyl tensor is

$$\mathcal{E}_{AB} = \tilde{C}_{ACBD} n^C n^D,$$

which is symmetric and traceless and without components orthogonal to the brane, so that $\mathcal{E}_{AB} n^B = 0$ and $\mathcal{E}_{AB} \rightarrow \mathcal{E}_{\mu\nu} g_A{}^\mu g_B{}^\nu$ as $y \rightarrow 0$.

The Weyl tensor $\tilde{C}_{ABCD}$ represents the free, nonlocal gravitational field in the bulk, i.e., the part of the field that is not directly determined at each point by the energy-momentum tensor at that point. The local part of the bulk gravitational field is the Einstein tensor $\tilde{G}_{AB}$, which is determined locally via the bulk field equations (1). Thus $\mathcal{E}_{\mu\nu}$ transmits nonlocal gravitational degrees of freedom from the bulk to the brane, including tidal (or Coulomb), gravito-magnetic and transverse traceless (gravitational wave) effects.

If u^μ is the 4-velocity comoving with matter (which we assume is a perfect fluid or minimally-coupled scalar field), the nonlocal term has the form of a radiative energy-momentum tensor [12]:

$$\mathcal{E}_{\mu\nu} = \frac{-6}{\kappa^2\lambda}[\mathcal{U}(u_\mu u_\nu + \frac{1}{3} h_{\mu\nu}) + \mathcal{P}_{\mu\nu} + \mathcal{Q}_\mu u_\nu + \mathcal{Q}_\nu u_\mu],$$

where $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ projects into the comoving rest-space. Here

$$\mathcal{U} = -\frac{1}{6}\kappa^2\lambda \, \mathcal{E}_{\mu\nu} u^\mu u^\nu$$

is an effective nonlocal energy density on the brane (which need not be positive), arising from the free gravitational field in the bulk. It carries Coulomb-type effects from the bulk onto the brane. There is an effective nonlocal anisotropic stress

$$\mathcal{P}_{\mu\nu} = -\frac{1}{6}\kappa^2\lambda[h_\mu{}^\alpha h_\nu{}^\beta - \frac{1}{3} h^{\alpha\beta} h_{\mu\nu}]\mathcal{E}_{\alpha\beta}$$

on the brane, which carries Coulomb, gravito-magnetic and gravitational wave effects of the free gravitational field in the bulk. The effective nonlocal energy flux on the brane,

$$\mathcal{Q}_\mu = \frac{1}{6}\kappa^2\lambda \, h_\mu{}^\alpha \mathcal{E}_{\alpha\beta} u^\beta,$$

carries Coulomb and gravito-magnetic effects from the free gravitational field in the bulk.

The local and nonlocal bulk modifications may be consolidated into an effective total energy-momentum tensor,

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa^2 T_{\mu\nu}^{\text{tot}}, \tag{4}$$

where

$$T_{\mu\nu}^{\text{tot}} = T_{\mu\nu} + \frac{6}{\lambda} S_{\mu\nu} - \frac{1}{\kappa^2}\mathcal{E}_{\mu\nu}.$$

The effective total energy density, pressure, anisotropic stress and energy flux are [12]

$$\rho^{\text{tot}} = \rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{6\mathcal{U}}{\kappa^4\lambda}, \tag{5}$$

$$p^{\text{tot}} = p + \frac{\rho}{2\lambda}(\rho + 2p) + \frac{2\mathcal{U}}{\kappa^4\lambda}, \tag{6}$$

$$\pi_{\mu\nu}^{\text{tot}} = \frac{6}{\kappa^4\lambda}\mathcal{P}_{\mu\nu}, \tag{7}$$

$$q_\mu^{\text{tot}} = \frac{6}{\kappa^4\lambda}\mathcal{Q}_\mu. \tag{8}$$

The brane energy-momentum tensor separately satisfies the conservation equations, $\nabla^\nu T_{\mu\nu} = 0$. The Bianchi identities on the brane imply that the projected Weyl tensor obeys the constraint

$$\nabla^\mu \mathcal{E}_{\mu\nu} = \frac{6\kappa^2}{\lambda}\nabla^\mu S_{\mu\nu}. \tag{9}$$

This shows how nonlocal bulk effects are sourced by local bulk effects, which include spatial gradients and time derivatives; evolution and inhomogeneity in the matter fields can generate nonlocal gravitational effects in the bulk, which backreact on the brane. The brane energy-momentum tensor and the consolidated effective energy-momentum tensor are both conserved separately. These conservation equations, as well as the brane field equations and Bianchi identities, are given in covariant form in [12]. We are interested here in the particular case of a Bianchi type I brane geometry, the simplest anisotropic generalization of an FRW brane geometry.

III. ANISOTROPIC BRANE

A Bianchi type I brane has the induced metric

$$ds^2 = -dt^2 + R_i^2(t)(dx^i)^2, \tag{10}$$

and is covariantly characterized by

$$D_\mu f = 0, \quad A_\mu = 0 = \omega_\mu, \quad Q_\mu = 0, \quad R^*_{\mu\nu} = 0, \qquad (11)$$

where D_μ is the projected covariant spatial derivative, f is any physically defined scalar, A_μ is the 4-acceleration, ω_μ is the vorticity, and $R^*_{\mu\nu}$ is the Ricci tensor of the 3-surfaces orthogonal to u^μ. (Note that in the coordinates of Eq. (10), we have $u_\mu = -\delta_\mu{}^0$, $h_{\mu0} = 0$, and $D_\mu f = \delta_\mu{}^i \partial_i f$.)

The conservation equations [12] reduce to

$$\dot{\rho} + \Theta(\rho + p) = 0, \qquad (12)$$

$$\dot{\mathcal{U}} + \tfrac{4}{3}\Theta\mathcal{U} + \sigma^{\mu\nu}\mathcal{P}_{\mu\nu} = 0, \qquad (13)$$

$$D^\nu \mathcal{P}_{\mu\nu} = 0, \qquad (14)$$

where an overdot denotes $u^\nu \nabla_\nu$, Θ is the volume expansion rate, and $\sigma_{\mu\nu}$ is the shear. Introducing the directional Hubble parameters $H_i = \dot{R}_i / R_i$ and the mean expansion factor $a = (R_1 R_2 R_3)^{1/3}$, one gets $\Theta \equiv 3H = 3\dot{a}/a \equiv \Sigma_i H_i$.

There is no evolution equation for $\mathcal{P}_{\mu\nu}$, reflecting the fact that in general the equations do not close on the brane, and one needs bulk equations to determine brane dynamics. There are bulk degrees of freedom whose impact on the brane cannot be predicted by brane observers.

The generalized Raychaudhuri equation on the brane [12] becomes (with $\Lambda = 0$)

$$\dot{\Theta} + \tfrac{1}{3}\Theta^2 + \sigma^{\mu\nu}\sigma_{\mu\nu} + \tfrac{1}{2}\kappa^2(\rho + 3p)$$

$$= -\tfrac{1}{2}\kappa^2(2\rho + 3p)\frac{\rho}{\lambda} - \frac{6\mathcal{U}}{\kappa^2\lambda}, \qquad (15)$$

where the general relativistic case is recovered when the right-hand side is set to zero. The vanishing of $R^*_{\mu\nu}$ leads via the Gauss-Codazzi equations on the brane to

$$\dot{\sigma}_{\mu\nu} + \Theta\sigma_{\mu\nu} = \frac{6}{\kappa^2\lambda}\mathcal{P}_{\mu\nu}, \qquad (16)$$

$$-\tfrac{2}{3}\Theta^2 + \sigma^{\mu\nu}\sigma_{\mu\nu} + 2\kappa^2\rho = -\kappa^2\frac{\rho^2}{\lambda} - \frac{12\mathcal{U}}{\kappa^2\lambda}. \qquad (17)$$

The presence of the nonlocal bulk tensor $\mathcal{P}_{\mu\nu}$ on the right of Eq. (16) means that we cannot simply integrate to find the shear as in general relativity (see [13]). However, we can circumvent this problem in a special case: when the nonlocal energy density vanishes or is negligible, i.e.,

$$\mathcal{U} = 0. \qquad (18)$$

This assumption is often made in the case of FRW branes, and in that case, it leads to a conformally flat bulk geometry [14]. When Eq. (18) holds, then the conservation equation (13) implies $\sigma^{\mu\nu}\mathcal{P}_{\mu\nu} = 0$. This consistency condition implies a condition on the evolution of $\mathcal{P}_{\mu\nu}$, i.e., $\sigma^{\mu\nu}\dot{\mathcal{P}}_{\mu\nu} = 6\mathcal{P}^{\mu\nu}\mathcal{P}_{\mu\nu}/\kappa^2\lambda$, as follows from Eq. (16). Since there is no evolution equation for $\mathcal{P}_{\mu\nu}$ on the brane [12], this is consistent on the brane. However, we would need to check that the

brane metric with $\mathcal{U} = 0$ leads to a physical 5-dimensional bulk metric. This would have to be done numerically (the bulk metric for a Bianchi brane-world is not known), and is a topic for further investigation.

Equation (16) may be integrated after contracting it with the shear, to give

$$\sigma^{\mu\nu}\sigma_{\mu\nu} \equiv \sum_{i=1}^{3}(H_i - H)^2 = \frac{6\Sigma^2}{a^6}, \quad \dot{\Sigma} = 0. \qquad (19)$$

We now substitute into Eq. (17) to obtain the generalized Friedmann equation for the Bianchi type I brane (with $\Lambda = 0 = \mathcal{U}$):

$$H^2 = \frac{\kappa^2}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Sigma^2}{a^6}. \qquad (20)$$

When $\Sigma = 0$, this recovers the equation for an FRW brane [14]. When $\rho/\lambda \to 0$, we recover the equation for a Bianchi type I model in general relativity [13].

IV. INFLATION ON THE ANISOTROPIC BRANE

The evolution equation for a minimally coupled scalar field confined to the brane is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \qquad (21)$$

The energy density and pressure are, respectively,

$$\rho = \rho_{\text{kin}} + \rho_{\text{pot}}, \quad p = \rho_{\text{kin}} - \rho_{\text{pot}}, \qquad (22)$$

where $\rho_{\text{kin}} = \tfrac{1}{2}\dot{\phi}^2$ and $\rho_{\text{pot}} = V(\phi)$. Setting $\Sigma = 0$ in Eq. (20), we see that the extra-dimensional terms act to increase the Hubble rate, and hence the damping experienced by the scalar field as it rolls down its potential. Thus for a FRW brane, inflation at high energies ($\rho > \lambda$) proceeds at a higher rate than the corresponding rate in general relativity. This introduces important changes to the dynamics of the early Universe [9,10,14], and accounts for an increase in the amplitude of scalar [9] and tensor [15] fluctuations at Hubble-crossing, and for a change to the evolution of large-scale density perturbations during inflation [16].

The condition for inflation is $\ddot{a} > 0$, which is equivalent to $\dot{\Theta} + \tfrac{1}{3}\Theta^2 > 0$. By Eq. (15), with $\mathcal{U} = 0$, this gives

$$w < -\frac{1}{3}\left(\frac{2\rho + \lambda}{\rho + \lambda}\right) - \left(\frac{2}{1 + \rho/\lambda}\right)\frac{\sigma_{\mu\nu}\sigma^{\mu\nu}}{3\kappa^2\rho}, \qquad (23)$$

where $w = p/\rho$. When the shear vanishes, this reduces to the condition for FRW brane inflation given in [9]; if $\rho/\lambda \to \infty$, we have $w < -\tfrac{2}{3}$, while the general relativity condition $w < -\tfrac{1}{3}$ is recovered as $\rho/\lambda \to 0$. For the Bianchi type I brane, the condition becomes

$$w < -\frac{1}{3}\left(\frac{2\rho + \lambda}{\rho + \lambda}\right) - \frac{4\Sigma^2}{\kappa^2 a^6 \rho(1 + \rho/\lambda)}. \qquad (24)$$
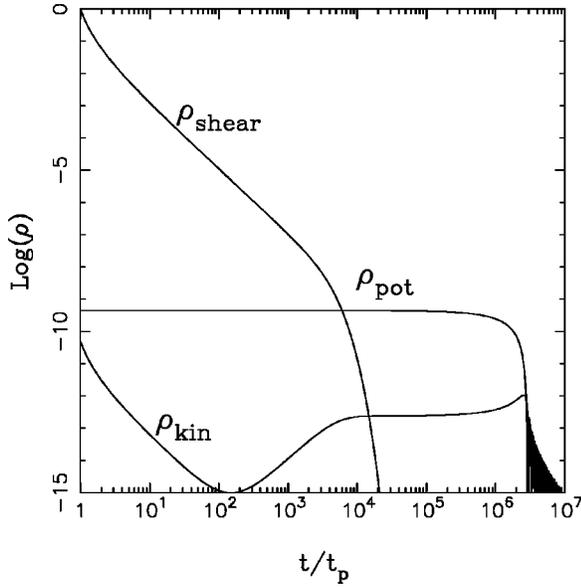
FIG. 1. This figure shows the evolution of the three components contributing to the effective energy density, when $V = \frac{1}{2}m\phi^2$. The Universe evolves from $t = t_p$ and $a = 1$, with initial conditions $\dot{\phi} = -10^{-5}$, $\phi = 3$, $\rho/\lambda = 5$ and $\rho_{shear} = 1$; we take $m = 10^{-5}$ ($M_p = 1$ is assumed). The kinetic term initially plunges under the influence of the shear and rises after the shear stops dominating the dynamics of the Universe. The oscillating phase at the end of inflation shows up as the dense patch at the right of the plot.

From Eqs. (20) and (24), one might naively expect the presence of shear to be detrimental to inflation since (i) Eq. (24) implies that a more negative equation of state is necessary to accelerate the Universe in the presence of shear, and (ii) Eq. (20) suggests that a large initial value of the anisotropy $\rho_{shear} = 3\Sigma^2/\kappa^2 a^6 \gg \rho_\phi$ might, by dominating the expansion dynamics of the early Universe, prevent inflation from occurring. A closer examination of the situation however reveals both these arguments to be flawed. From Eqs. (20) and (21), we see that the presence of expansion anisotropy (shear) acts in a manner which is actually *conducive* to inflation. For a Bianchi type I brane, the shear anisotropy term in Eq. (20) reinforces the increase of the Hubble rate. A larger value of initial shear dampens the kinetic energy of the scalar field, allowing the inflationary condition, Eq. (24), to be reached earlier. The important role played by anisotropy is illustrated by considering the worst case scenario for inflation when $\rho_{kin} = \dot{\phi}^2/2 \gg \rho_{pot} = V(\phi)$. If we assume that the Universe was initially very anisotropic, so that its expansion rate was dominated by the shear term, we get

$$a^3(t) \approx a_0^3 + 3\Sigma(t - t_0).$$

Consequently, at early times,

$$\rho_{shear}, \rho_{kin} \sim (t - t_0)^{-2}. \quad (25)$$

This estimate is confirmed by numerical integration (see Figs. 1–3). Remarkably, we find that anisotropy always disappears within a fixed interval of time, no matter what its initial value (compare [4]). The decrease in kinetic energy is
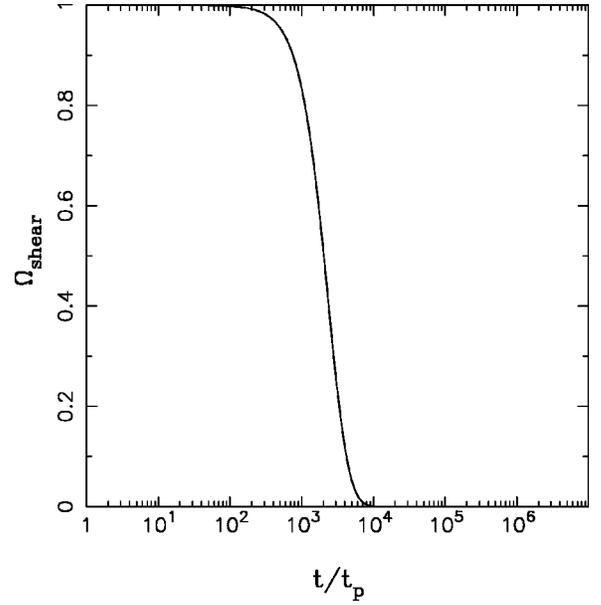

FIG. 2. The evolution of the dimensionless shear parameter $\Omega_{shear} = \sigma^2/6H^2$ is shown as a function of time for the model in Fig. 1. The shear decreases monotonically as the Universe expands and isotropises.

influenced by the value of the initial anisotropy; a larger value of Σ causes a more rapid decay of the kinetic term and therefore results in an earlier onset of inflation.

V. CONCLUSIONS

Our results illustrated in Figs. 1 and 3 show the kinetic, potential and anisotropy energy densities plotted as functions of time. The associated dimensionless shear parameter $\Omega_{shear} = \sigma^2/6H^2$ is shown in Figs. 2 and 4, and the expansion factor is shown in Fig. 5.
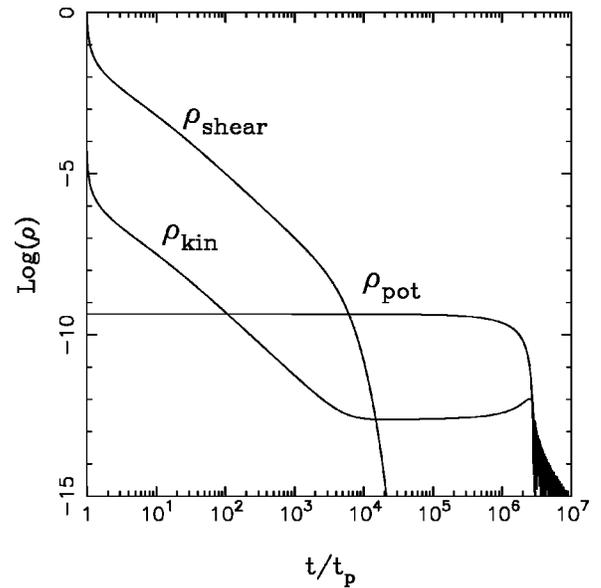

FIG. 3. Same as Fig. 1, but with a larger initial value of the field velocity, $\dot{\phi} = -10^{-2}$ and $\rho/\lambda = 5 \times 10^5$.
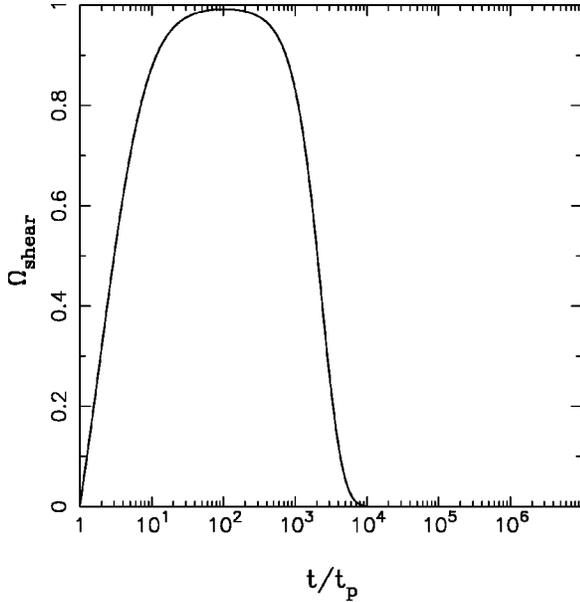
FIG. 4. The evolution of the dimensionless shear parameter $\Omega_{\text{shear}} = \sigma^2/6H^2$ is shown as a function of time for the model in Fig. 3. We find that the early and late-time expansion of the Universe is isotropic, but the shear term dominates during an intermediate anisotropic stage.

We find that the influence of anisotropy on the kinetic energy is particularly strong if $\Sigma^2/a^6\kappa^2 \gg \dot{\phi}^2$ initially. In this case the kinetic term drops to a very small value, then rises after the anisotropy has disappeared, gradually approaching its asymptotic slow-roll value

$$\dot{\phi} \simeq -V'/3H \ \Rightarrow \ \dot{\phi}^2 \simeq \frac{\lambda}{3\pi}\left(\frac{M_\text{p}}{\phi}\right)^2. \tag{26}$$

We assume here that the potential has the simple ''chaotic'' form $V = \frac{1}{2}m^2\phi^2$, for which the standard inflationary slow-roll condition is $\dot{\phi}^2 \simeq m^2 M_\text{p}^2/16\pi^2$. Comparing with Eq. (26), we find that dependence on m^2 has been replaced in the brane scenario by λ/ϕ^2. Thus the kinetic energy does not remain constant but gradually increases as the field amplitude decreases during slow-roll. We find that the decay of anisotropy is generically accompanied by a corresponding decrease in the kinetic energy of the scalar field. This effect leads to greater inflation (see Fig. 5).

Examining Eq. (20) closely, we find that the effective equation of state of matter when $\rho \gg \lambda$ is $w_{\text{eff}} = 2w + 1$. Consequently for matter with $w = p/\rho > 0$ the approach to the initial singularity is matter dominated and not shear dominated, due to the predominance of the matter term $\rho^2/2\lambda^2$
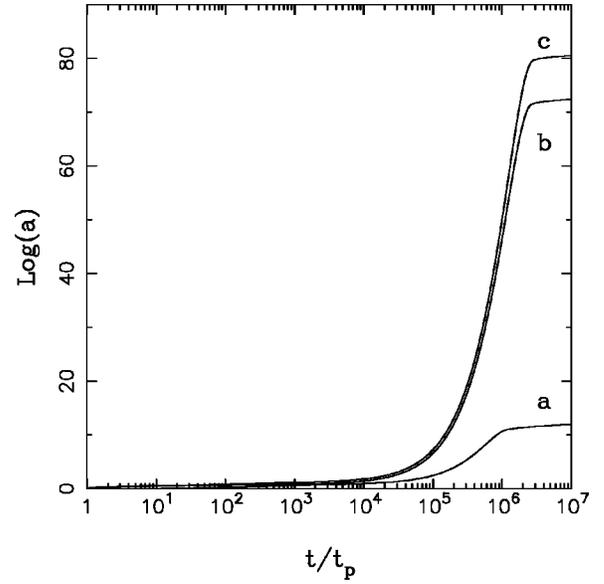

FIG. 5. The evolution of the scale factor with time is shown for the model in Fig. 3. The curves shown in increasing amplitude from bottom to top correspond to (a) $\Sigma = 0$, $1/2\lambda = 0$ (the general relativity case), (b) $\Sigma = 0$, $1/2\lambda = 10^{10}$ and (c) $\Sigma = 1$, $1/2\lambda = 10^{10}$. We see that the effect of both the extra-dimensional term and cosmic shear is to produce more inflation.

relative to the shear term Σ^2/a^6. [Within the framework of scalar field models this corresponds to $\dot{\phi}^2 > 2V(\phi)$.] The fact that the density effectively grows faster than $1/a^6$ for $w > 0$ is a uniquely brane effect. Within the framework of general relativity such behavior is clearly impossible since it would demand an ultra-stiff equation of state, $w > 1$. The expansion of the early Universe is therefore characterized by three successive expansion epochs during which the Bianchi type I model experiences (i) initial quasi-isotropic expansion: $\sigma^2/H^2 \to 0, t \to 0$, (ii) transient anisotropy dominated regime: $\sigma^2/H^2 \sim 1$, and (iii) anisotropy dissipation: $\sigma^2/H^2 \to 0, t \to \infty$. These three stages are illustrated in Fig. 4. For smaller values of $\dot{\phi}^2$ (corresponding to $w < 0$), stage (i) is absent and the shear decreases monotonically from a large initial value (see Fig. 2).

The decay of shear and the accompanying isotropization of the Universe significantly increases the class of initial conditions from which the present Universe could have originated.

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