

## Anomaly-free flavor symmetry and neutrino anarchy

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We show that one can describe the quark and lepton masses with a single anomaly-free  $U(1)$  flavor symmetry provided a single order one parameter is enhanced by roughly 4–5. The flavor symmetry can be seen to arise from inside the  $E_6$  symmetry group in such a way that it commutes with the  $SU(5)$  grand unified gauge group. The scenario does not distinguish between the left-handed lepton doublets and hence is a model of neutrino anarchy. It can therefore account for the large mixing observed in atmospheric neutrino experiments and predicts that the solar neutrino oscillation data are consistent with the large mixing angle solution of matter-enhanced oscillations.

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The traditional approach to expressing the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the quark and lepton matrices is to expand in the small parameter,  $\lambda \sim |V_{us}| \approx 0.22$ . This is well justified because the experimental data for the mass ratios and CKM elements appear to be well-described by integer exponents of this expansion parameter:

$$|V_{us}| \sim \lambda, \quad |V_{cb}| \sim \lambda^2, \quad |V_{ub}| \sim \lambda^4. \quad (1)$$

One also has a constraint on the CKM elements from  $B_d^0 - \bar{B}_d^0$  mixing [1],  $|V_{tb}^* V_{td}| = 0.0084 \pm 0.0018$ , which implies that

$$|V_{td}| \sim \lambda^3. \quad (2)$$

In the quark sector, one has the mass ratios

$$\frac{m_u}{m_c} \sim \lambda^4, \quad \frac{m_c}{m_t} \sim \lambda^4, \quad \frac{m_d}{m_s} \sim \lambda^2, \quad \frac{m_s}{m_b} \sim \lambda^2, \quad (3)$$

while in the lepton sector, the mass ratios  $m_\mu/m_\tau \sim \lambda^2$ ,  $m_e/m_\tau \sim \lambda^4$ . These mass ratios are valid at a high (grand unified) scale and are consistent with the experimental constraints near the electroweak scale after including renormalization group scaling [2]. The remaining constraints on leptons involve the neutrino masses and mixings. The most interesting aspect of the neutrino data is that the atmospheric neutrino mixing appears to be large, perhaps even maximal. If the neutrino masses also obey some hierarchical relations, it seems at first sight to be difficult to understand such a pattern for the neutrino masses, since large mixing should result when the neutrino masses are of roughly the same order of magnitude. The Super-Kamiokande data [3] suggest that  $\Delta m_{23}^2 \sim 2.2 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{23}^v \sim 1$ , where the subscripts indicate the generations of neutrinos involved in the mixing [We assume the mixing is between  $\nu_\mu$  and  $\nu_\tau$ , and not some sterile neutrino. We are also not using the Liquid Scintillation Neutrino Detector (LSND) data.]

The solar neutrino flux can be accounted for by distinct solutions. Two of these involve matter-enhanced oscillation [Mikheyev-Smirnov-Wolfenstein (MSW) type], while the third involves vacuum oscillations (VO). The two MSW solutions are differentiated by the size of the mixing angle, so one is usually called the small mixing angle (SMA) solution, and the other is called the large mixing angle (LMA) solu-

tion. The most recent data disfavors the VO and SMA solution at the 95% confidence level [4,5]. The rough values required for the mixing parameters in the two MSW cases are shown in the table below.

	$\Delta m_{1x}^2 [\text{eV}^2]$	$\sin^2 2\theta_{1x}$
MSW(SMA)	$5 \times 10^{-6}$	$6 \times 10^{-3}$
MSW(LMA)	$2 \times 10^{-5}$	$\sim 1$

Since the most recent data favors the LMA solution for solar neutrinos, there has been a great deal of effort recently to explain the neutrino data with various approaches like bimaximal mixing models or neutrino anarchy [6–9]. The LMA solution is the most interesting from the standpoint of neutrino factories [10], but requires us to understand how the lepton sector differs in terms of hierarchies (of masses and mixing angles) from the quark sector. In this note we present a model that exhibits neutrino anarchy.

Flavor symmetries have been useful tools for modeling the patterns of fermion masses and mixings [11]. The Froggatt-Nielsen mechanism [12] is a popular method for systematically generating a hierarchy in the Yukawa couplings. Consider a horizontal  $U(1)_\theta$  symmetry under which the standard model fields carry charges. Yukawa interactions are now required to respect this horizontal (or flavor) symmetry, and they can arise in two ways: (1) as renormalizable interactions  $\Psi_i \Psi_j H$ , or (2) as nonrenormalizable interactions involving a gauge singlet superfield  $\theta$  which we can assume without loss of generality has a flavor charge  $-1$ ,  $\Psi_i \Psi_j H (\theta/M)^{n_{ij}}$ . For this effective term to respect the flavor symmetry, the charges of the fields must sum to zero. The assumed smallness of the parameter  $\epsilon = \langle \theta \rangle / M$  can give rise then to a hierarchy of masses from factors of the form  $\epsilon^{n_{ij}}$ . Following the reasoning given in the previous section, we take  $\epsilon \sim \lambda^2$  where again  $\lambda$  is identified with the (sine of) Cabibbo angle, 0.22. The Froggatt-Nielsen mechanism does not by itself determine the order one coefficients. A fundamental theory would presumably fix their values.

A flavor symmetry can suppress the entries in a systematic way compared to order one entries. By assigning charges to the various fields one can obtain Yukawa matrices in reasonable agreement with the experimentally measured values.

We suggest in this Brief Report that the true expansion parameter is actually  $\epsilon \sim \lambda^2$ , and the largeness of  $|V_{us}|$  in comparison to  $\epsilon$  comes about because of a presumably order one coefficient,  $C$ , that turns out to be of order 4–5. The same

large coefficient can contribute to  $|V_{td}|$  yielding a value  $(4-5)\epsilon^2$  which is consistent with previous expansions in terms of  $\lambda$ , Eq. (2). This also resolves the problem of the discrepancy between the relative sizes of  $|V_{ub}|$  and  $|V_{td}|$  where the former is best described by  $\lambda^4$  and the latter is best described by  $\lambda^3$ ; this mismatch has proven to be a challenge to model builders employing a  $U(1)$  flavor symmetry. In most unified models the quark charges are related to the lepton charges and odd exponents appear in the mixing angles in the neutrino sector, in disagreement with the data. It can be overcome but usually requires a more complicated flavor symmetry than might otherwise be the case.

If one takes the hierarchy in Eq. (1) seriously and insists on obtaining  $|V_{us}| \sim \lambda$  and  $|V_{td}| \sim \lambda^3$ , then there is a unique solution implementing a  $U(1)$  flavor symmetry. This solution is the one found by Elwood, Irges, and Ramond [13]. The lepton sector can be given charge assignments, and neutrino masses and mixings can be derived. We emphasize here that the neutrino data prefers even exponents of  $\lambda$  to best describe the experimental data. The data suggests that  $\sin \theta_{1x} \sim \lambda^0$  (which henceforth we will denote simply by 1) for the LMA solution, or  $\sin \theta_{1x} \sim \lambda^2$  for the SMA solution. The model gives, however, that  $\sin \theta_{1x} \sim \lambda^3$  [13], a value that is somewhat small compared to the MSW(SMA) solution. It can be shown that this odd exponent results from grand unified relations between quarks and leptons and the insistence that there are odd exponents in the CKM elements. We show below that the odd exponents in the quark sector can be seen as arising from just one order one coefficient that has fluctuated upward enough to disturb the naive hierarchy by one inverse power of  $\lambda$ . Furthermore it seems likely that if any of the underdetermined order one coefficients does fluctuate to a large value, it is most likely to be ones arising in the lighter two generations.

Large mixing for neutrinos is problematic for two reasons: (1) it seems that without fine tuning, large mixing should be associated with mass eigenvalues of the same order of magnitude, and (2) the CHOOZ data [14] indicates that one of the three mixing angles is small ( $|U_{e3}| < 0.15$ , where  $U$  is the neutrino mixing matrix), of order  $\lambda$  or smaller. Both of these issues have been discussed recently in Refs. [6,8] where it was shown that it is not so unnatural for a neutrino mass matrix with all entries of order one to give acceptable mixing angles in agreement with the neutrino data. This idea has been called neutrino anarchy. Our model extends the results of Ref. [8] by embedding the neutrino anarchy model in a larger symmetry in an anomaly-free way, and suggests a theoretical framework for a unique assignment of fermion flavor charges arising from an  $E_6$  symmetry. The charge assignment of Ref. [8] is not unique, and the neutrino anarchy can be achieved for a number of different charge assignments to the lepton doublets of the standard model, and the flavor charges of the Majorana neutrinos is not determined. The model suggested in this note establishes the charges of the Majorana neutrinos as well as all the standard model particles. The flavor charges for the Higgs bosons is also justified on the basis of their position in the representations of the  $E_6$  symmetry.

We show that the quark sector can be explained in terms of a single small expansion parameter,  $\epsilon = \lambda^2$ , provided one order one coefficient  $C = 4-5$  is somewhat large. We em-

phasize in this paper that only one such factor is needed to explain all the quark mass and mixing data, and this point suggests that the correct expansion parameter is  $\epsilon \sim \lambda^2$  rather than  $\lambda$ . This result is independent of the particular flavor charge suggested in this paper, and is true generally in models incorporating Abelian flavor symmetries in the Froggatt-Nielsen mechanism.

Our approach is to find an anomaly-free flavor symmetry. We assume that this  $U(1)$  symmetry breaks, despite the absence of a Green-Schwarz mechanism, so as to give a hierarchical mass matrix pattern according to the Froggatt-Nielsen mechanism. The large mixing angles for neutrinos can be achieved by having the lepton electroweak doublets be indistinguishable under the flavor symmetry. Embedding these doublets into the  $\mathbf{5}^*$  multiplet of  $SU(5)$  and assigning flavor charges to the  $\mathbf{10}$  multiplet, one can then assign charges that yield the correct mass ratios for the quarks, Eq. (3). The charges for the quark fields in  $SU(5)$  multiplets are as follows ( $i=1,2,3$ ) where we use  $\lambda$  as the expansion parameter and require the flavor charges to be even integers:

$$\mathbf{10}_i (4,2,0), \quad \mathbf{5}^*_i (0,0,0). \quad (4)$$

This gives rise to the Yukawa matrices

$$\mathbf{U} \sim \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \quad \mathbf{D} \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix}, \quad (5)$$

for the quark sector. This set of matrices has been suggested previously [8,15] as giving a reasonable fit to the data apart from the measured values of  $|V_{us}|$  and  $|V_{td}|$  seemed to be enhanced by roughly one inverse power of  $\lambda$ .

The enhancement of  $|V_{td}|$  and  $|V_{us}|$  can easily arise from the same large order one coefficient. Given the down-type Yukawa matrix in Eq. (5), one finds the following expression for the leading contribution to the sine of the mixing angle,  $s_{12}^d$ , for generations 1 and 2,

$$s_{12}^d \sim \left( \frac{d_{12}}{\tilde{d}_{22}} - \frac{d_{13}d_{32}}{\tilde{d}_{22}} \right) + \frac{1}{\tilde{d}_{22}^2} [d_{11}d_{21} - d_{11}d_{31}(d_{23} + d_{22}d_{32})] - \frac{1}{\tilde{d}_{22}^2} (d_{32}d_{12} + d_{13}) [d_{21}d_{31} + d_{31}^2(d_{23} + d_{22}d_{32})], \quad (6)$$

where  $d_{ij} = \mathbf{D}_{ij}/\mathbf{D}_{33}$  and  $\tilde{d}_{22} = d_{22} - d_{23}d_{32}$ . In Eq. (6),  $s_{12}^d$  is expressed for the case in which all of the mixing angles in the right-handed transformation matrix needed to diagonalize  $\mathbf{D}$  are large as in Eq. (5). This generalized expressions given in the literature [16,17] for which only the mixing angle in the second and third generation is of order one. The CKM matrix can be characterized by four mixing angles which are given to leading order by

$$|V_{us}| = s_{12}^d - s_{12}^u, \quad |V_{cb}| = s_{23}^d - s_{23}^u,$$

$$|V_{ub}| = s_{13}^d - s_{13}^u - s_{12}^u |V_{cb}|, \quad |V_{td}| = -s_{13}^d + s_{13}^u + s_{12}^d |V_{cb}|.$$

There are phases in the Yukawa entries which do not affect the expansion in terms of  $\lambda$ , and so we omit them here. A large order one coefficient that enhances  $s_{12}^d$  from its expected magnitude of  $\lambda^2$  to order  $\lambda$  will enhance just those

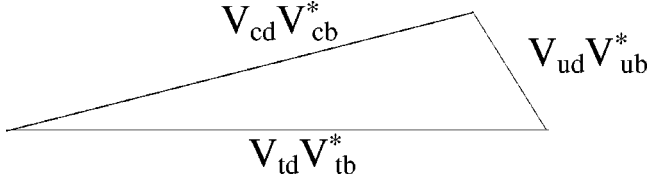


FIG. 1. Unitarity triangle corresponding to Eq. (7).

CKM elements that the data tells us are large. One can easily see that the large order one coefficient  $C$  contributes to  $|V_{us}|$  and  $|V_{td}|$  by looking at the constraints from the unitarity of the CKM matrix. Consider the following unitarity relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \quad (7)$$

From this relation one immediately sees that an enhancement of  $|V_{cb}|$  by a factor 4–5 that doesn't at the same time enhance  $|V_{ub}|$  must be compensated by an enhancement of  $|V_{td}|$  (see Fig. 1).

The Yukawa matrix for the charged lepton sector can be deduced from the charge assignments in Eq. (4),

$$\mathbf{E} \sim \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \quad (8)$$

which yields good agreement with the known mass ratios. The indistinguishability of the  $\mathbf{5}^*$  multiplets with respect to the  $U(1)$  flavor symmetry gives rise to large mixing in the lepton sector.

In the usual chain of embedding  $SO(10)$  inside  $E_6$  and  $SU(5)$  inside  $SO(10)$ , there arise two  $U(1)$  symmetries [18]:  $E_6 \rightarrow SO(10) \times U(1)_\psi$ ,  $SO(10) \rightarrow SU(5) \times U(1)_\chi$ . Both of these  $U(1)$  symmetries as well as any linear combination is guaranteed to be anomaly free by virtue of that fact that  $E_6$  is an anomaly free group. We can assign charges to the fields in a way that is  $SU(5)$ -symmetric, and that also respects the  $U(1)$  flavor symmetry.

The charges of the  $SU(5)$  multiplets under the  $U(1)_\psi$  and  $U(1)_\chi$  are

	$\mathbf{10}_a$	$\mathbf{5}_a^*$	$\mathbf{1}_a$	$\mathbf{5}_b$	$\mathbf{5}_b^*$	$\mathbf{1}_c$
$2\sqrt{10}Q_\chi$	-1	3	-5	2	-2	0
$2\sqrt{6}Q_\psi$	1	1	1	-2	-2	4

The charges are grouped into the  $\mathbf{16}$ ,  $\mathbf{10}$  and  $\mathbf{1}$  representations of  $SO(10)$  inside a  $\mathbf{27}$  representation of  $E_6$ . Now it is easy to see that the charge assignments given in the previous section are obtained by taking the linear combination  $Q_\theta = 3 \times (2\sqrt{6}Q_\psi)/16 - (2\sqrt{10}Q_\chi)/16$ . This yields the  $U(1)_\theta$  charges

	$\mathbf{10}_a$	$\mathbf{5}_a^*$	$\mathbf{1}_a$	$\mathbf{5}_b$	$\mathbf{5}_b^*$	$\mathbf{1}_c$
$4Q_\theta$	1	0	2	-2	-1	3.

 (9)

The charge assignments in Eq. (4) can now be identified as  $(16Q_\theta, 8Q_\theta, 0)$ . This does not then represent a strict embedding of the representations inside  $E_6$ , but since each generation has  $U(1)_\theta$  charges that are multiples (modings) of the

$SU(5) \times U(1)_\theta$  inside  $E_6$ , the gauge anomalies are guaranteed to be absent as long as the extra matter content of the  $\mathbf{27}$  is present (either near the electroweak scale or at some higher scale). The construction of the flavor charge from the two  $U(1)$  subgroups of  $E_6$  guarantees that the couplings  $\mathbf{10}_a - \mathbf{5}_a^* - \mathbf{5}_b^*$  and  $\mathbf{5}_b - \mathbf{5}_b^* - \mathbf{1}_c$  are allowed by the  $U(1)_\theta$  flavor symmetry.

As is well known, supersymmetric gauge coupling unification requires that in addition to complete multiplets of  $SU(5)$  with masses at the electroweak scale, there must be a pair of states with the quantum numbers of the two Higgs doublets of the minimal supersymmetric model (MSSM). These should arise outside of three generations of  $\mathbf{27}$  giving rise to the standard model matter multiplets and the exotic  $\mathbf{5}_b$ ,  $\mathbf{5}_b^*$  and  $\mathbf{1}_c$ . These Higgs fields could arise [19] as components from either a  $\mathbf{27} + \mathbf{27}^*$  or a  $\mathbf{78}$ . A particularly interesting possibility for the  $U(1)_\theta$  charges occurs in the former case where one can choose these Higgs fields from the  $\mathbf{5}_a^*(\mathbf{5}_a)$  of  $SU(5)$  inside the  $\mathbf{16}(\mathbf{16}^*)$  of  $SO(10)$ . Then the (unnormalized)  $Q_\chi$  and  $Q_\psi$  charges are  $\mathbf{5}_a^*(3,1)$  and  $\mathbf{5}_a(-3, -1)$ , so that the Higgs fields have vanishing  $Q_\theta$  charge [like the lepton doublets in Eq. (9)].

From the charge assignment for the  $SU(5)$  singlet state inside the  $\mathbf{16}$  of  $SO(10)$ , one obtains the Majorana mass matrix of the right handed neutrinos

$$\mathbf{M}_R \sim \begin{pmatrix} \lambda^{16} & \lambda^{12} & \lambda^8 \\ \lambda^{12} & \lambda^8 & \lambda^4 \\ \lambda^8 & \lambda^4 & 1 \end{pmatrix} M_P,$$

and the Yukawa matrix for the Dirac masses of the neutrinos

$$\mathbf{Y}_\nu \sim \begin{pmatrix} \lambda^8 & \lambda^4 & 1 \\ \lambda^8 & \lambda^4 & 1 \\ \lambda^8 & \lambda^4 & 1 \end{pmatrix}.$$

(This strong hierarchy in the Majorana mass matrix is useful in explaining the baryon asymmetry of the universe via the process of leptogenesis [20].) Finally via the seesaw mechanism  $\mathbf{m}_\nu = \mathbf{Y}_\nu^T \mathbf{M}_R^{-1} \mathbf{Y}_\nu v_2^2$ , where  $v_2$  is the vacuum expectation value of the Higgs field that couples to the up-type quarks, one obtains the mass matrix of the light neutrinos

$$\mathbf{m}_\nu \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{v_2^2}{M_P}.$$

Of course, it is not necessary that the right handed neutrinos have the charge assignment in Eq. (9) to obtain this light neutrino mass matrix. It is determined solely by the universal flavor charge assigned to the  $\mathbf{5}_a^*$  multiplets in Eq. (4).

Large mixing in the atmospheric neutrino data and LMA solution for solar neutrinos together with the constraint from CHOOZ indicates that the neutrino mixing matrix form is

$$U \sim \begin{pmatrix} 1 & 1 & \lambda^k \\ 1 & 1 & 1 \\ \lambda^k & 1 & 1 \end{pmatrix} \quad (10)$$

when expanded in powers of  $\lambda$ . However the naive expectation from power counting for the  $U(1)_\theta$  symmetry indicates that the exponent is  $k=0$ . The model seems to suggest that all three mixing angles are large, and the largeness of  $U_{e3}$  element is problematic in light of the CHOOZ reactor data [14], which places an upper limit on the mixing  $U_{e3}$  from the constraints on  $\bar{\nu}_e$  disappearance. It appears that, in terms of the expansion in terms of  $\lambda$ , this element of the mixing matrix in Eq. (10) should be of order  $\lambda$  or smaller to account for the lack of  $\bar{\nu}_e$  disappearance in the reactor neutrino experiment.

Additionally it seems that such a neutrino mixing matrix implies large mixing between all three generations and neutrinos with masses which do not exhibit a hierarchy. One approach to remedy this situation is to introduce an additional discrete component to the flavor symmetry which can enhance or suppress mass eigenvalues or mixing angles in comparison to their values in a model in which the flavor symmetry is simply  $U(1)$  [17,21]. However it has been argued recently [6,8] that it is not so improbable that experimentally acceptable values for neutrino masses and mixings can result, even when naively on the basis of power counting the neutrino mixing matrix appears to give  $U_{e3}$  typically of order one. This scenario has been dubbed neutrino anarchy. It is argued in Ref. [8], where a proper weighting of the random order one coefficients has been justified on the assumption of basis-independence of the neutrino states, one can, without much fine tuning, find a result where  $U_{e3}$  is just below the current experimental limit. Indeed, this can be considered the most characteristic feature of neutrino anarchy that can be experimentally tested in the near future. While an acceptable value for  $U_{e3}$  occurs only in a

10% tail of its probability distribution [8], the other observables do not need any fine-tuning, and the argument that has developed is that this situation is not an improbable one.

We have suggested in this note that if one is willing to give up the assumption that  $|V_{us}| \sim \lambda$  is the correct expansion parameter for the hierarchies evidenced in the fermion masses and mixings, then one can get a complete description of the masses and mixings in the quark, charged lepton, and neutrino sectors. A particular model in which the  $U(1)$  horizontal symmetry is nonanomalous is easily constructed by taking the appropriate linear combination of the  $U(1)_\psi$  and  $U(1)_\chi$  subgroups of  $E_6$ . The CKM elements satisfy  $|V_{us}| \approx C\lambda^2$  and  $|V_{ts}| \approx C\lambda^4$  where  $C$  is a relatively large order one parameter of 4–5, and the neutrino sector that results falls into the class exhibiting the neutrino anarchy of Refs. [6,8].

More generally we feel an interpretation of the hierarchy in terms of a expansion parameter  $\epsilon \sim \lambda^2$  is quite reasonable and makes the resulting model-building much easier since only even powers of  $\lambda$  will appear.

The set of  $U(1)$  flavor charges for the standard model particles has appeared in the literature. Here we have shown how to embed this  $U(1)$  flavor symmetry as a subgroup of the  $E_6$  symmetry that commutes with the standard  $SU(5)$  subgroup. Together with recent insights on the viability of models where the three lepton doublets are indistinguishable, the model is in reasonable agreement with all the current experimental data.

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- [1] Particle Data Group, C. Caso *et al.*, *Eur. Phys. J. C* **3**, 1 (1998).
- [2] V. Barger, M. S. Berger, and P. Ohmann, *Phys. Rev. D* **47**, 1093 (1993); **47**, 2038 (1993); Y. Koide and H. Fusaoka, *ibid.* **59**, 053004 (1999).
- [3] The Super-Kamionkande Collaboration, Y. Fukuda *et al.*, *Phys. Rev. Lett.* **81**, 1562 (1998).
- [4] Y. Suzuki, for the Super-Kamionkande Collaboration, talk at Neutrino 2000.
- [5] J. Ellis, ‘‘Summary of Neutrino 2000,’’ hep-ph/0008334.
- [6] L. Hall, H. Murayama, and N. Weiner, *Phys. Rev. Lett.* **84**, 2572 (2000).
- [7] A. Nelson and M. Strassler, *J. High Energy Phys.* **9**, 030 (2000).
- [8] N. Haba and H. Murayama, *Phys. Rev. D* (to be published) hep-ph/0009174.
- [9] J. Sato and T. Yanagida, *Phys. Lett. B* **493**, 356 (2000).
- [10] Neutrino Factory and Muon Collider Collaboration, D. Ayres *et al.*, physics/9911009.
- [11] Some recent attempts in this direction can be found in Z. Berezhiani and A. Rossi, hep-ph/0003084; Q. Shafi and Z. Tavartkiladze, *Phys. Lett. B* **487**, 145 (2000).
- [12] C. D. Froggatt and H. B. Nielsen, *Nucl. Phys.* **B147**, 277 (1979).
- [13] J. K. Elwood, N. Irges, and P. Ramond, *Phys. Lett. B* **413**, 322 (1997); N. Irges, S. Lavignac, and P. Ramond, *Phys. Rev. D* **58**, 035003 (1998); J. K. Elwood, N. Irges, and P. Ramond, *Phys. Rev. Lett.* **81**, 5064 (1998).
- [14] M. Apollonio *et al.*, *Phys. Lett. B* **466**, 415 (1999).
- [15] K. S. Babu and S. M. Barr, *Phys. Lett. B* **381**, 202 (1996); S. M. Barr, *Phys. Rev. D* **55**, 1659 (1997); M. Strassler, talk presented at International Workshop on Perspectives of Strong Coupling Gauge Theories (SCGT 96), 1996, Nagoya, Japan; N. Haba, *Phys. Rev. D* **59**, 035011 (1999); K. Yoshioka, *Mod. Phys. Lett. A* **15**, 29 (2000); J. Hisano, K. Kurosawa, and Y. Nomura, *Nucl. Phys.* **B584**, 3 (2000).
- [16] M. Leurer, Y. Nir, and N. Seiberg, *Nucl. Phys.* **B420**, 468 (1994); M. Tanimoto, *Phys. Lett. B* **456**, 220 (1999).
- [17] M. S. Berger and K. Siyeon, *Phys. Rev. D* **62**, 033004 (2000); hep-ph/0005249.
- [18] J. L. Hewett and T. G. Rizzo, *Phys. Rep.* **183**, 193 (1989).
- [19] T. G. Rizzo, *Phys. Rev. D* **59**, 015020 (1999).
- [20] M. Plumacher, hep-ph/9807557; W. Buchmuller and M. Plumacher, hep-ph/0007176.
- [21] Y. Grossman, Y. Nir, and Y. Shadmi, *J. High Energy Phys.* **10**, 007 (1998); Y. Nir and Y. Shadmi, *ibid.* **05**, 023 (1999).