# Finite sum of gluon ladders and high energy cross sections

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A model for the Pomeron at $t=0$ is suggested. It is based on the idea of a finite sum of ladder diagrams in QCD. Accordingly, the number of $s$-channel gluon rungs and correspondingly the powers of logarithms in the forward scattering amplitude depends on the phase space (energy) available, i.e., as energy increases, progressively new prongs with additional gluon rungs in the $s$-channel open. Explicit expressions for the total cross section involving two and three rungs or, alternatively, three and four prongs [with $\ln^2(s)$ and $\ln^3(s)$ as highest terms, respectively] are fitted to the proton-proton and proton-antiproton total cross section data in the accelerator region. Both QCD calculation and fits to the data indicate fast convergence of the series. In the fit, two terms (a constant and a logarithmically rising one) almost saturate the whole series, the $\ln^2(s)$ term being small and the next one, $\ln^3(s)$, negligible. Theoretical predictions for the photon-photon total cross section are also given.

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## I. INTRODUCTION

It is widely accepted that the Pomeron in QCD corresponds to an infinite sum of gluon ladders with Reggeized gluons on the vertical lines (see Fig. 1), resulting [1–3] in the so-called supercritical behavior $\sigma_t \sim s^{\alpha_P(0)}$, $\alpha_P(0)>1$, where $\alpha_P(0)$ is the intercept of the Pomeron trajectory. In that approach, the main contribution to the inelastic amplitude and to the absorptive part of the elastic amplitude in the forward direction arises from the multi-Regge kinematics in the limit $s\to\infty$ and leading logarithmic approximation. In the next-to-leading logarithmic approximation (NLLA), corrections require also the contribution from the quasi-multi-Regge kinematics [4]. Hence, the subenergies between neighboring $s$-channel gluons must be large enough to be in the Regge domain. At finite total energies, this implies that the amplitude is represented by a finite sum of $N$ terms [5], where $N$ increases like $\ln s$, rather than by the solution of the Balitskiĭ-Fadin-Kuraev-Lipatov (BFKL) integral equation [1–3]. The interest in the first few terms of the series is related to the fact that the energies reached by the present accelerators are not high enough to accommodate a large number of $s$-channel gluons that eventually hadronize and give rise to clusters of secondary particles.

The lowest order diagram is that of two-gluon exchange, first considered by Low and Nussinov [6]. The next order, involving an $s$-channel gluon rung was studied, e.g., in the papers [2,7]. The problem of calculating these diagrams is twofold. The first one is connected with the nonperturbative contributions to the scattering amplitude in the "soft" region. It may be ignored by "freezing" the running coupling constant at some fixed value of the momenta transferred and assuming that the forward amplitude can be cast by a smooth interpolation to $t=0$. More consistently, one introduces a nonperturbative model [8] of the gluon propagator valid also in the forward direction. The second problem is more technical: as $s\to\infty$ the number of Feynman diagrams that contribute to the leading order rapidly increases and, in each of them, only the leading contribution is usually evaluated. At any order in the coupling, subleading terms coming both from the neglected diagrams and from the calculated ones are present. Although functionally the result is always the sum of increasing powers of logarithms, the numerical values of the coefficients entering the sum is lost unless all diagrams are calculated.

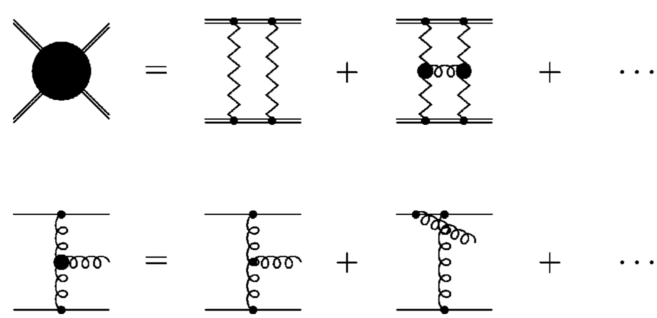

FIG. 1. Schematic representation of the total cross section in the leading $\ln(s)$ approximation (first row). Double lines represent protons or antiprotons, vertical zig-zag lines are Reggeized gluons, and horizontal wavy lines are gluons. The effective vertex for two Reggeized gluons and one gluon is defined in the second row. Here external lines can represent quark or gluons.

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Conversely, one can expand the ''supercritical'' Pomeron $\sim s^{\alpha(0)}$ in powers of ln($s$). Such and expansion is legitimate within the range of active accelerators, i.e., near and below the TeV energy region, where fits to total cross sections by a power or logarithms are known [9] to be equivalent numerically. Moreover, forward scattering data (total cross sections and the ratio of the real to the imaginary part of the forward scattering amplitude) do not discriminate even between a single and quadratic fit in ln($s$) to the data.

Phenomenologically, more information on the nature of the series can be gained if the $t$ dependence is also involved. The well-known (diffractive) dip-bump structure of the differential cross section can be roughly imitated by the Glauber series, although more refined studies within the dipole Pomeron (DP) model [linear behavior in ln($s$)] [10] show that the relevant series is not just the Glauber one. A generalization of the DP model including higher powers of ln($s$) was considered in [11]. In a recent paper [12] the Pomeron was considered as a finite series of ladder diagrams, including one gluon rung besides the Low-Nussinov ''Born term'' and resulting in a constant plus logarithmic term in the total cross section. With a subleading Regge term added, good fits to $pp$ and $p\bar{p}$ total as well as differential cross section were obtained in [12]. There is however a substantial difference between our approach and that of Ref. [12] or simple decomposition in powers of ln($s$), namely that we consider the opening channels (in $s$) as threshold effects, the relevant prongs being separated in rapidity by $\ln s_0$, $s_0$ being a parameter related to the average subenergy in the ladder. Although such an approach inevitably introduces new parameters, we consider it more adequate in the framework of the finite-ladder approach. We mention these attempts only for the sake of completeness, although we stick to the simplest case of $t=0$, where there are hopes to have some connection with the QCD calculations.

In Sec. II we consider a new parametrization for total cross sections based on the contribution from a finite series of QCD diagrams with relative weights (coefficients) and rapidity gaps to be determined from the data. Each set of the diagrams is ''active'' in ''its zone,'' i.e., the parameters should be fitted in each energy interval separately and the relevant solutions should match. The matching procedure will be similar to that known for the wave functions in quantum mechanics, i.e., we require continuity of the total cross section and of its first derivative. In Sec. III we present the result of fits to the $p\bar{p}$ and $pp$ experimental data. Section IV is devoted to a discussion of the truncated series in QCD and to the calculation of the coefficients of the powers of $\ln s$. Finally, in Sec. V we will draw our conclusions.

## II. DESCRIPTION OF THE MODEL

The Pomeron contribution to the total cross section is represented in the form

$$\sigma_t^{(P)}(s)=\sum_{i=0}^{N} f_i\,\theta(s-s_0^i)\,\theta(s_0^{i+1}-s), \tag{1}$$

where

$$f_i=\sum_{j=0}^{i} a_{ij}L^j, \tag{2}$$

$s_0$ is the prong threshold, $\theta(x)$ is the step function and $L \equiv \ln(s)$. Here and in the following, by $s$ and $s_0$ respectively, $s/(1\ \mathrm{GeV}^2)$ and $s_0/(1\ \mathrm{GeV}^2)$ is implied. The main assumption in Eq. (1) is that the widths of the rapidity gaps $\ln(s_0)$ are the same along the ladder. The functions $f_i$ are polynomials in $L$ of degree $i$, corresponding to finite gluon ladder diagrams in QCD, where each power of the logarithm collects all the relevant diagrams. When $s$ increases and reaches a new threshold, a new prong opens adding a new power in $L$. In the energy region between two neighboring thresholds, the corresponding $f_i$, given in Eq. (1), is supposed to represent adequately the total cross section. With reference to Fig. 1, in our model a new threshold is reached every time the imaginary part of the amplitude takes the contribution from the cut of an additional $s$-channel gluon [2].

In Eq. (1) the sum over $N$ is a finite one, since $N$ is proportional to ln($s$), where $s$ is the present squared c.m. energy. Hence this model is quite different from the usual approach where, in the limit $s\to\infty$, the infinite sum of the leading logarithmic contributions gives rise to an integral equation for the amplitude.

To make the idea clearer, we describe the mechanism in the case of three gaps (two rungs). To remedy the effect of the first threshold and get a smooth behavior at low energies, we have included also a Pomeron daughter, going like $\sim 1/s$ in the first two gaps with parameters $b_0$ and $b_1$ respectively. Then

$$f_0(s)=a_{00}+b_0/s \quad \text{for} \quad s\leqslant s_0, \tag{3}$$

$$f_1(s)=a_{10}+b_1/s+a_{11}L \quad \text{for} \quad s_0\leqslant s\leqslant s_0^2, \tag{4}$$

$$f_2(s)=a_{20}+a_{21}L+a_{22}L^2 \quad \text{for} \quad s_0^2\leqslant s\leqslant s_0^3. \tag{5}$$

By imposing the requirement of continuity (of the cross section and of its first derivative) one constrains the parameters. E.g., from the conditions $f_1(s_0)=f_0(s_0)$ and $f_1'(s_0)=f_0'(s_0)$ the relations

$$b_1=a_{11}s_0+b_0,$$

$$a_{10}=a_{00}-a_{11}\ln(s_0)-a_{11}$$

follow. Furthermore, from $f_2(s_0^2)=f_1(s_0^2)$ and $f_2'(s_0^2)=f_1'(s_0^2)$ one gets

$$a_{20}=a_{22}\ln^2(s_0^2)+a_{10}+b_1[1+\ln(s_0^2)]/s_0^2,$$

$$a_{21}=a_{11}-2a_{22}\ln(s_0^2)-b_1/s_0^2.$$

The same procedure can be repeated for any number of gaps.

In fitting the model to the data, we rely mainly on $p\bar{p}$ data that extend to the highest (accelerator) energies, to which the Pomeron is particularly sensitive. To increase the confidence level, $pp$ data were included in the fit as well. To keep the

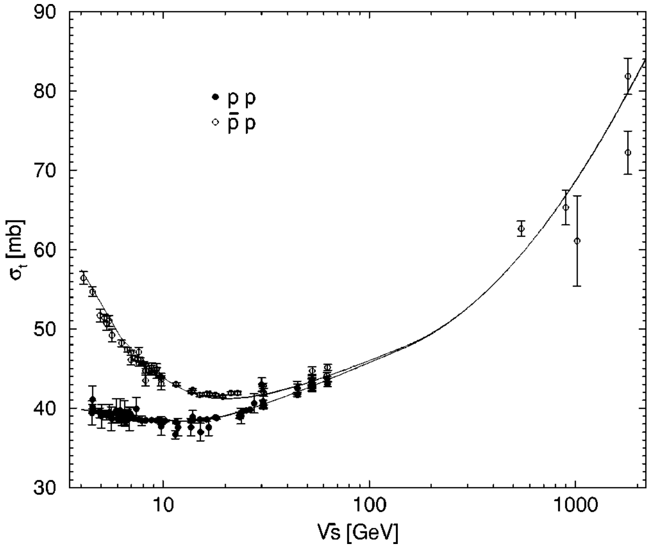


FIG. 2. Total cross section calculated up to 2 gluon rungs and fitted to the $p\bar{p}$ and $pp$ data.

number of the free parameters as small as possible and following the successful phenomenological approach of Donnachie and Landshoff [13], a single ''effective'' Reggeon trajectory with intercept $\alpha(0)$ will account for nonleading contributions, thus leading to the following form for the total cross section:

$$\sigma_t(s)=\sigma_t^{(P)}(s)+R(s), \tag{6}$$

where $\sigma_t^{(P)}(s)$ is given by Eq. (1) and $R(s)=as^{\alpha(0)-1}$ (the parameter $a$ is different for $p\bar{p}$ and $pp$ and is considered as an additional free parameter).

Ideally, one would let free the width of the gap $s_0$ and consequently the number of gluon rungs (highest power of $L$). Although possible, technically this is very difficult. Therefore we considered only the cases of two and three rungs and, for each of them, we treated $s_0$ as a free parameter.

Notice that the values of the parameters depend on the energy range of the fitting procedure. For example, the values of the parameters in $f_0$ if fitted in ''its'' range, i.e., for $s\leqslant s_0$, will get modified in $f_1$ with the higher energy data and correspondingly higher order diagrams included.

### III. FITS TO THE $p\bar{p}$ AND $pp$ DATA

As a first attempt, only three rapidity gaps, that correspond to two gluon rungs in the ladder were considered. Fits to the $p\bar{p}$ and $pp$ data were performed from $\sqrt{s}=4$ GeV up to the highest energy Tevatron data (for $p\bar{p}$), including all the results from there [14]. The resulting fit is shown in Fig. 2 with a $\chi^2$/d.o.f.$\approx$1.71. The values of the fitted parameters are quoted in Table I. Interestingly, the value of $s_0$ turns out to be very close to 144 GeV$^2$, i.e., the value for which the energy range considered is covered with equal rapidity gaps uniformly.

Next, we covered the energy span available in the accelerator region by four gaps, resulting in 3 gluon rungs and consequently $L^3$ as the maximal power. After the matching procedure, we are left with ten free parameters: first of all $s_0$, then $a_{00},b_0,a_{11},a_{22},a_{32},a_{33}$, each determined in its range, while the two $a$'s and $\alpha(0)$ are fitted in the whole range of the data. The final value for $s_0$ turned out to be $s_0\simeq42.5$ GeV$^2$ resulting in a sequence of energy intervals ending at $\sqrt{s}=1800$ GeV. It is amusing to notice that a search for the phase space region where the production amplitude in the multicluster configuration has a maximum leads, with the help of cosmic ray data, to an average ''subenergy'' $\langle s_i\rangle\sim44$ GeV$^2$ [15], that is very near to the value of $s_0$ found in the fit.

TABLE I. Value of the parameters in the case of 2 rungs. The parameters $b_i$ and $a_{\ldots}$ are given in units of 1 mb. The quantities in round parentheses represent the errors. Parameters without error are derived from the matching condition.

| $s_0$ | $\alpha(0)$ | $a_{p\bar{p}}$ | $a_{pp}$ |
|---|---|---|---|
| 147.97(93) | 0.441(11) | 71.7(3.4) | 0.00(37) |
| $b_0$ | $a_{00}$ | $a_{11}$ | $a_{22}$ |
| 35.6(1.5) | 38.097(23) | 2.300(38) | 0.857(61) |
| $a_{10}$ | $a_{20}$ | $a_{21}$ | $\chi^2$/DOF |
| 24.3 | 110.1 | −14.85 | 1.71 |

Figure 3 shows our fit to the $p\bar{p}$ and $pp$ total cross section data. The values of the fitted parameters are quoted in Table II. The value of the $\chi^2$/DOF is $\sim$1.38, much better than in the case of two gluon rungs. It is interesting to observe that the coefficients in front of the leading logarithms are related roughly by a factor of 1/10. Moreover, the coefficient $a_{33}$ turns out to be compatible with zero, in contrast with the expectation from QCD calculation in the leading logarithmic

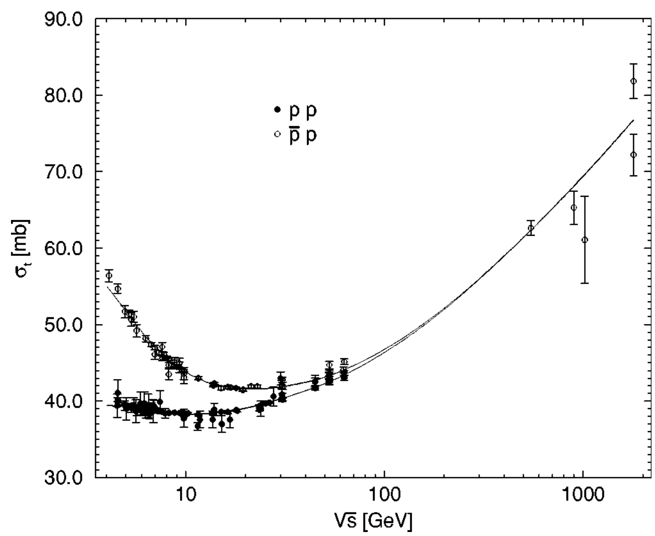


FIG. 3. Total cross section calculated up to 3 gluon rungs and fitted to the $p\bar{p}$ and $pp$ data.

TABLE II. Value of the parameters in the case of 3 rungs. The parameters $b_i$ and $a_{\dots}$ are given in units of 1 mb. The quantities in round parenthesis represent the errors. Parameters without error are derived from the matching condition.

| $s_0$ | $\alpha(0)$ | $a_{p\bar{p}}$ | $a_{pp}$ | $b_0$ | $a_{00}$ |
|---|---|---|---|---|---|
| 42.43(53) | 0.4295(95) | 160.9(8.6) | 85.2(6.6) | −180(18) | 33.20(29) |
| $a_{11}$ | $a_{22}$ | $a_{32}$ | $a_{33}$ | $a_{10}$ | $a_{20}$ |
| 2.631(86) | 0.324(55) | 0.19(22) | 0.000(38) | 20.7 | 38.6 |
| $a_{21}$ | $a_{30}$ | $a_{31}$ | $b_1$ | $\chi^2$/DOF | |
| −2.19 | 22.3 | 0.72 | −68.42 | 1.38 | |

appoximation (see next section). The value of the effective Reggeon intercept remains rather low, close to 0.45, comparable with the value found in Ref. [16].

## IV. EXPLICIT ITERATIONS OF BFKL

From the theoretical point of view, the phenomenological model of Sec. II corresponds to the explicit evaluation in QCD of gluonic ladders with an increasing number of $s$-channel gluons. This correspondence is far from literal since each term of the BFKL series takes into account only the dominant logarithm in the limit $s\rightarrow\infty$. In the following we give concrete expressions for the forward high energy scattering amplitudes for photons and hadrons in the form of an expansion in powers of large logarithms, taking for simplicity only the leading logarithmic terms.

We start from known results obtained in paper [2] where an explicit expression for the total cross section for hadron-hadron scattering has been obtained. In the high energy limit, it is convenient to introduce the Mellin transform of the amplitude

$$A(\omega,t)=\int_0^\infty d\left(\frac{s}{m^2}\right)\left(\frac{s}{m^2}\right)^{-\omega-1}\frac{\mathrm{Im}_s A(s,t)}{s}$$

and its inverse

$$\frac{\mathrm{Im}_s A(s,t)}{s}=\frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty}d\omega\left(\frac{s}{m^2}\right)^{\omega}A(\omega,t).$$

The general expression of $A(\omega,t)$ in the leading logarithmic approximation (LLA) has the form

$$A(\omega,t)=\int d^2k\frac{\Phi^a(k,q)F_\omega^b(k,q)}{k^2(q-k)^2},$$

where $\Phi^a(k,q)$ and $\Phi^b(k,q)$ (see next equation) are the impact factors of the colliding hadrons $a$ and $b$, obeying the gauge conditions $\Phi^j(0,q)=\Phi^j(q,q)=0$ $(j=a,b)$. The quantity $F_\omega^b(k,q)$ obeys the BFKL equation:

$$\omega F_\omega^b(k,q)=\Phi^b(k,q)+\gamma\int\frac{d^2k'}{2\pi}\times\frac{A(k,k',q)F_\omega^b(k',q)-B(k,k',q)F_\omega^b(k,q)}{(k-k')^2},$$

with

$$A(k,k',q)=\frac{-q^2(k-k')^2+k^2(q-k')^2+k'^2(q-k)^2}{k'^2(q-k')^2},$$

$$B(k,k',q)=\frac{k^2}{k'^2+(k'-k)^2}+\frac{(q-k)^2}{(q-k')^2+(k-k')^2},$$

and

$$\gamma=3\frac{\alpha_s}{\pi}.$$

The strong coupling $\alpha_s$ is assumed to be frozen at a suitable scale set, for example, by the external particles. The iteration procedure and the reciprocal Mellin transform give (besides we put $q=0$)

$$\sigma_t(s)=\frac{\mathrm{Im}_s\ A(s,0)}{s}=\int d^2k\frac{\Phi^a(k,0)}{(k^2)^2}\left[\Phi_0^b(k)+\rho\Phi_1^b(k)+\frac{1}{2!}\rho^2\Phi_2^b+\cdots\right],$$

where

$$\rho=\frac{3\alpha_s}{\pi}\ln\left(\frac{s}{m^2}\right) \tag{7}$$

and the subsequent iterations begin from $\Phi_0^b(k)=\Phi^b(k,0)$. In the previous integral and everywhere in the following, all the momenta are 2-dimensional Euclidean vectors, living in the plane transverse to the one formed by the momenta of the colliding particles.

For the case of photon-photon scattering one has [2] (see also the references quoted there)

$$\Phi^{\gamma}=\sum_{i=1}^{2}\tau_{ii}^{\gamma\gamma}(k,0)=\frac{2}{3}\alpha\alpha_s T\left(\frac{k^2}{m^2}\right),$$

where

$$T\left(\frac{k^2}{m^2}\right)=\frac{1}{4}+\frac{5-\beta^2}{8\beta}\ln\left(\frac{\beta+1}{\beta-1}\right),$$

$$\beta^2=1+\frac{4m^2}{k^2}>1.$$

From the integrals[1]

$$\frac{1}{\pi}\int d^2k\frac{T^2\left(\frac{k^2}{m^2}\right)}{(k^2)^2}=\frac{0.673}{m^2},$$

$$\int_0^\infty\frac{dt}{t}\int_0^\infty\frac{dt'}{t'}T(t)\left[\frac{T(t')-T(t)}{|t-t'|}+\frac{T(t)}{\sqrt{t^2+4t'^2}}\right]=1.443,$$

we conclude

$$\sigma^{\gamma\gamma\to 2q2\bar{q}}(s)=\sigma_0\left[1+6.4\frac{\alpha_s}{\pi}\ln\left(\frac{s}{m^2}\right)\right], \tag{8}$$

where $\sigma_0$ is a constant and we used the approximated equality $(3\times1.44)/0.67=6.4$.

To obtain the cross section of proton-proton scattering, we use the ansatz of Ref. [17] for the impact factor of a hadron in terms of its form factor $F(q^2)$:

$$\Phi^p(k,q)=F^p\left(\frac{q^2}{4}\right)-F^p\left(\left(k-\frac{q}{2}\right)^2\right),\quad \Phi^p(0,q)=\Phi(q,q)=0.$$

Here the 2-dimensional Euclidean vector $q$ is related to the 4-dimensional transferred momentum $Q$ by the relation $Q^2=-q^2<0$. Using the formulas given above, we obtain, for a simplified but experimentally acceptable form of proton's impact factor in the forward direction, $q=0$,

$$\Phi_0(k)=ak^2e^{-bk^2}, \tag{9}$$

where $a$ and $b$ are in $\mathrm{GeV}^{-2}$. It is convenient to define

$$\psi_n(k^2)=\frac{\Phi_n(k)}{k^2},$$

then (for $n\geqslant1$)

$$\psi_n(k^2)=\int_0^1\frac{dx}{1-x}(\psi_{n-1}(k^2x)-\psi_{n-1}(k^2))+\int_1^\infty\frac{dx}{x-1}\left(\psi_{n-1}(k^2x)-\frac{1}{x}\psi_{n-1}(k^2)\right)$$

and

$$\sigma_t(s)=\pi\int_0^\infty dk^2\psi_0(k^2)\sum_n\psi_n(k^2)\frac{\rho^n}{n!}. \tag{10}$$

The integrations can be performed analytically, due to the simple choice of the impact factor in Eq. (9), and the final result is

$$\sigma_t(s)=\frac{\pi a^2}{2b}\left\{1+2(\ln 2)\rho+\left[\frac{\pi^2}{12}+2(\ln 2)^2\right]\rho^2+\frac{1}{3}\left[\frac{\pi^2}{2}(\ln 2)+4(\ln 2)^3-\frac{3}{4}\zeta(3)\right]\rho^3+\cdots\right\}, \tag{11}$$

where $\rho$ is defined in Eq. (7).

As stressed above, the coefficients of different powers of $\ln(s/m^2)$ in Eq. (11) refer to the dominant contribution, at asymptotic energies, for each perturbative order. In the fit, instead, the Pomeron contribution is determined only from the experimental data at high but finite energies. However, we can obtain a rough estimate of the importance of the subleading contributions by comparing Eq. (11) with the phenomenological fit of the previous section, in particular with the amplitude $f_3$ relative to the last gap of the three rungs case. If we assume a commonly used value for the strong coupling, $\alpha_s\sim0.5-0.7$ [17], we must conclude that subleading contributions to the QCD Pomeron are important. Moreover, the coefficient $a_{33}$ turns out to be compatible with zero, in contrast with the result of Eq. (11). These findings may be related to the fact that energies reached by the present accelerators are not yet asymptotic.

An approach similar the one followed in this section was considered in Ref. [18]. However, although in both cases the LLA BFKL equation is used, our results are not comparable with those of Ref. [18], first of all because we use a definite ansatz for the impact factors, different from a constant [see, for instance, Eq. (9)]. Moreover, differently from [18], where all orders of perturbation theory were taken into account, in our case we consider a truncated iteration of the BFKL equation.

## V. CONCLUSIONS

Although high quality fits were not the primary goal of the present study, we still may conclude that they are compatible with those existing [13], and there is still room for further improvement. Our main goal instead was to seek for an adequate picture of the Pomeron exchange at $t=0$. In our opinion, it is neither an infinite sum of gluon ladders as in the BFKL approach [1–3], nor its power expansion. In fact, the finite series—call it ''threshold approach''—considered in this and our previous paper [5] realizes a nontrivial dynami-

[1] S. Gevorkyan (private communication).

cal balance between the total reaction energy and the subenergies equally partitioned between the multiperipheral ladders.

The results of our fits, whose quality will be definitely improved when future data from RHIC and LHC will be available, show also that few terms of powers of $\ln(s)$ are sufficient to describe the data at all realistic energies. As a guess, we do not exclude that higher terms cancel completely, but anyway they are negligibly small. The case of two terms (logarithmic rise in $s$) is particularly interesting as it corresponds to a dipole Pomeron with a number of attractive features [10] such as self-reproducibility with respect of unitarity corrections. In case of a $\ln^2(s)$ rise (three terms) we still should not worry about the Froissart bound, so ultimately the Pomeron as viewed in this paper does not need to be unitarized. This conclusion is an important by-product of our paper. For the dipole Pomeron, relevant calculations for $t \neq 0$ are interesting and important but difficult. In the case of a single gluon rung they were performed in Ref. [12] and, with a nonperturbative gluon propagator, in the last reference of [8].

The role and the value of the width of the gap, $s_0$, is an important physical parameter *per se*, independent of the model presented above. We have fitted it and compared successfully with the prediction from cosmic-ray data. However its value may be estimated, e.g., as the lowest energy where the Pomeron exchange is manifest, although the latter is also a matter of debate. Further fits of the model to new experimental data may settle some details left open by this paper.

The results from the LLA BFKL equation do not resemble closely those from the numerical fits, thus leading to the conclusion that next-to-leading corrections cannot be omitted in the analytical calculations.

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