# Solution to the $\mu$ problem in the presence of a heavy gluino LSP

Arash Mafi and Stuart Raby

Department of Physics, The Ohio State University, 174 West 18th Avenue, Columbus, Ohio 43210

(Received 20 September 2000; published 8 February 2001)

In this paper we present a solution to the  $\mu$  problem in an SO(10) supersymmetric grand unified model with gauge mediated and *D*-term supersymmetry breaking. Peccei-Quinn symmetry is broken at the messenger scale  $M \sim 10^{12}$  GeV and enables the generation of the  $\mu$  term. The boundary conditions defined at *M* lead to a phenomenologically acceptable version of the minimal supersymmetric standard model with novel particle phenomenology. Either the gluino or the gravitino is the lightest supersymmetric particle (LSP). If the gravitino is the LSP, then the gluino, with a mass in the range 25–35 GeV, can be treated as a stable particle with respect to experiments at high energy accelerators. Given the extensive phenomenological constraints we show that the model can only survive in a narrow region of parameter space resulting in a light neutral Higgs boson with a mass  $\sim (86-91)\pm 6$  GeV and  $\tan\beta \sim 9-14$ . In addition the lightest top squark and neutralino have masses  $\sim 100-122$  GeV and  $\sim 50-72$  GeV, respectively. Thus the model will soon be tested. Finally, the invisible axion resulting from Peccei-Quinn symmetry breaking is a cold dark matter candidate.

DOI: 10.1103/PhysRevD.63.055010

PACS number(s): 12.60.Jv

### I. INTRODUCTION

Supersymmetry (SUSY) is a strongly motivated candidate for new physics beyond the standard model (SM). It provides a natural framework for resolving the hierarchy problem. The minimal supersymmetric standard model (MSSM), with conserved *R* parity, has an economical particle content with well defined interactions most of which are already constrained by experiment. It has two Higgs doublets ( $H_u$  and  $H_d$ ) which are necessary for giving mass to both up and down quarks, respectively. In the MSSM, electroweak symmetry breaking (EWSB) occurs naturally since  $m_{H_u}^2$ , the soft SUSY breaking (SSB) mass of  $H_u$ , is automatically driven negative as a result of a large top quark Yukawa coupling.

The MSSM solves the hierarchy problem by allowing for dimensionful SSB parameters of the order of the electroweak scale and protecting scalar masses from large radiative corrections above the SSB scale. However, this in itself is not sufficient to solve the hierarchy problem. In addition, it is necessary to demand that the  $\mu$  parameter (where  $\mu$  is the bilinear Higgs coupling in the superpotential of the form  $\mu H_u H_d$ ) is also of order the electroweak scale. Consider the vacuum conditions obtained by minimizing the tree level Higgs potential, we have

$$\mu^{2} = -\frac{M_{Z}^{2}}{2} + \frac{m_{H_{d}}^{2} - m_{H_{u}}^{2} \tan^{2}\beta}{\tan^{2}\beta - 1},$$
(1)

where  $\tan\beta = \langle H_u \rangle / \langle H_d \rangle$ ,  $\sin 2\beta = 2B/m_A^2$ ,  $m_A$  is the mass of the *CP* odd Higgs boson, and *B* is the SSB Higgs bilinear coupling. On the left hand side of Eq. (1), the  $\mu$  parameter, multiplying a supersymmetric  $\mu$  term in the Lagrangian, breaks no SM symmetries; it could in principle be as large as the Planck or grand unified theory (GUT) scales. On the right side, the *Z* boson mass and the SSB Higgs boson masses are of order the electroweak scale. Clearly, all three scales, i.e.,  $\mu$ ,  $M_Z$ , and the SSB scales must be of the same order. This is the  $\mu$  problem [1].

In order to avoid large values for  $\mu$ , a symmetry is needed which prevents the  $\mu$  term at the tree level, but allows such a term once this symmetry is broken. Moreover, since in the MSSM the  $\mu$  term is contained in the superspace potential, there are two possibilities: (1) it can be generated via a term in the Kähler potential (at the tree level or radiatively) once supersymmetry is broken or (2) no supersymmetry breaking is required if it is generated through higher dimension operators in the superspace potential. Several simple mechanisms for generating a  $\mu$  term have been suggested [2,3].

In the context of gauge mediated supersymmetry breaking (GMSB) models [4], there is an additional problem.  $\mu$  can be generated at one loop order once supersymmetry is broken. However, the *B* parameter (the SSB scalar Higgs bilinear coupling) is usually generated at the same loop order and is too large. A solution generating *B* at higher loop order than  $\mu$  was proposed in Ref. [5].

In this paper we use an extension of the GMSB model discussed in Ref. [6] to solve the  $\mu$  problem. This model has GMSB with Higgs-messenger mixing in an SO(10) theory and naturally leads to a gluino lightest supersymmetric particle (LSP). The gluino LSP is stable due to *R*-parity conservation. The specific signature of a gluino LSP i.e. missing momentum has been analyzed in Ref. [7] for CERN  $e^+e^-$  collider LEP and Collider Detector at Fermilab (CDF) data and in Ref. [8] for CDF data. Reference [8] concludes that a stable gluino with mass in the range 25–35 GeV is still allowed by both the LEP and CDF data. Our model, with a modest adjustment of parameters, gives a gluino with mass in this range.

The  $\mu$  term is absent in this model (at the tree level) due to a U(1) Peccei-Quinn (PQ) symmetry. Both SUSY and PQ symmetry are broken when the chiral superfield *X* develops a vacuum expectation value (VEV)

$$\langle X \rangle = M + \theta^2 F_X. \tag{2}$$

Note that the following dimension five operator in the Kähler potential,

$$K \supset \frac{1}{M_P} X^{\dagger} \mathbf{10}_H^2 + \text{H.c.}, \qquad (3)$$

is, however, allowed by the symmetries. Thus we find  $\mu \sim F_X/M_P$ . *B*, on the other hand, is generated radiatively via renormalization group (RG) running below the messenger scale M.<sup>1</sup>

In addition to solving the  $\mu$  problem, the PQ symmetry provides a natural solution to the strong *CP* problem [9]. The strong *CP* violating  $\theta$  term dynamically tracks to zero. Moreover, as a bonus, the axion is a candidate for cold dark matter.

In the next section we discuss the model, saving some of the details for the appendices. We derive the low energy spectrum consistent with electroweak symmetry breaking, gauge coupling unification, and third generation quark and lepton masses. We then consider experimental constraints which constrain the available parameter space to a very narrow region. In this region we find a light neutral Higgs boson with mass  $\sim (86-91)\pm 6$  GeV and  $\tan\beta \sim 9-14$ . In addition the lightest top squark and neutralino have mass  $\sim 100-122$  GeV and  $\sim 50-72$  GeV, respectively. Finally, in an appendix we investigate the possibility of obtaining a reasonable mass for the tau neutrino in the model. Clearly this model is preeminently testable.

## **II. THE MODEL**

The theory at the GUT scale is defined by the SO(10) invariant superpotential  $W \supset W_1 + W_2 + W_3$  and a nonrenormalizable term in the Kähler potential *K* where

$$W_{1} = \mathbf{16}_{3}\mathbf{10}_{H}\mathbf{16}_{3},$$
  
$$W_{2} = \lambda_{a}\mathbf{10}_{H}A\mathbf{10}_{4} + \lambda_{X}X\mathbf{10}_{4}^{2},$$
 (4)

$$W_{3} = \lambda_{1} \,\overline{\eta}_{1} A \,\eta_{1} + \lambda_{2} \,\overline{\eta}_{2} A \,\eta_{2} + \lambda X \,\overline{\eta}_{1} \,\eta_{2} \,.$$
$$K \supset \lambda_{K} \frac{X^{\dagger}}{M_{P}} \mathbf{10}_{H}^{2} + \text{H.c.}$$
(5)

 $(\mathbf{16}_3, \eta_1, \eta_2)$  are  $\mathbf{16}$ 's,  $(\overline{\eta}_1, \overline{\eta}_2)$  are  $\overline{\mathbf{16}}$ 's,  $(\mathbf{10}_H, \mathbf{10}_A)$  are  $\mathbf{10}$ 's, (X) is a singlet, and (A) is an adjoint under SO(10).

At the GUT scale, the theory is invariant under a U(1) PQ and a *R* symmetry. The *R* symmetry is broken spontaneously at the GUT scale. The PQ symmetry, however, is not broken at the GUT scale and prevents a  $\mu$  term in the superpotential. The PQ and *R* charges of the fields are defined in Appendix A.  $W_1$  contains the coupling of the third family matter multiplet (16<sub>3</sub>) to the Higgs field (10<sub>*H*</sub>) which includes both the weak doublet and color triplet Higgs fields.

 $W_2$  serves two purposes. In the first case, it provides doublet-triplet splitting using the Dimopoulos-Wilczek mechanism [10]. The adjoint field A gets a VEV,

$$\langle A \rangle = (B - L) M_G, \qquad (6)$$

where B-L (baryon number minus lepton number) is nonvanishing on color triplets and zero on weak doublets and the singlet X gets a VEV

$$\langle X \rangle = M + \theta^2 F_X. \tag{7}$$

This gives mass of order  $M_G$  to the color triplet Higgs states and of order M to the weak doublets in  $\mathbf{10}_A$ . The Higgs doublets in  $\mathbf{10}_H$  remain massless. The SUSY breaking VEV, on the other hand, exhibits the second purpose for  $W_2$ .

In the second case,  $W_2$  and  $W_3$  also provide the messengers for SUSY breaking.<sup>2</sup> The auxiliary field  $\mathbf{10}_A$  and the fields  $\overline{\eta}_1, \eta_2$  feel SUSY breaking at tree level due to the VEV  $F_X$ . They are thus the messengers for GMSB [6,11]. We take the messenger scale  $M \sim 10^{12}$  GeV with the effective SUSY breaking scale in the observable sector given by

$$\Lambda = F_X / M \sim 10^5 \text{ GeV.} \tag{8}$$

Thus the Higgs field in this model plays a central role with regards to supersymmetry breaking. It is this central role which also provides a natural framework for solving the  $\mu$  problem using the PQ symmetry. When *X* gets a VEV, both SUSY and the PQ symmetry are broken. The  $\mu$  term is generated at the scale *M*:

$$\mu = \lambda_K \frac{F_X}{M_P},\tag{9}$$

while B remains zero at tree level.

The PQ symmetry solves the strong *CP* problem and produces an axion; the Goldstone boson of the broken PQ symmetry. The axion gets mass due to the QCD chiral anomaly of order  $m_a^2 = (f_\pi^2/f_a^2)m_\pi^2 N^2[Z/(1+Z)^2]$  [12] where  $Z = m_u/m_d \sim 0.56$ ,  $f_a \equiv M = 10^{12}$  GeV is the **PQ** symmetry breaking scale and N = 3 is the number of families. Putting in the numbers we find  $m_a \sim 2 \times 10^{-5}$  eV.<sup>3</sup>

We refer the interested reader to Appendix A for the complete model defined at the GUT scale. Note, in order to obtain realistic t, b, and  $\tau$  masses, we find it necessary to abandon Yukawa unification at  $M_G$ . How this is obtained in

<sup>&</sup>lt;sup>1</sup>Note that the mechanism used here to generate the  $\mu$  term is a combination of the ideas discussed in Refs. [2] and [3].

<sup>&</sup>lt;sup>2</sup>Because of an accidental cancellation, gluinos receive no mass at one loop from  $W_2$ . Thus  $W_3$  is introduced with additional messenger fields  $(\eta_1, \overline{\eta}_1, \eta_2, \overline{\eta}_2)$  contributing to the masses of gauginos and scalars at the scale  $M_G$ .

<sup>&</sup>lt;sup>3</sup>We take  $M = 10^{12}$  GeV, so that the energy density in the axion field does not over close the universe.

the complete model is discussed in Appendix B. Finally, a simple extension of the model to include a  $\tau$  neutrino mass is presented in Appendix C.

# III. BOUNDARY CONDITIONS AT THE MESSENGER SCALE

The boundary conditions at the messenger scale are determined by two sources of SUSY breaking, gauge mediation and D term [6]. The messengers give mass to the gauginos and Higgs at one loop and to squarks and sleptons at two loops. Since the color triplet messengers have mass of order the GUT scale, the gluino mass is suppressed compared to the other gauginos. The gaugino masses (at M) are given by

$$m_{\tilde{g}} = \frac{\alpha_3}{\pi} \Lambda b^2,$$

$$M_2 = \frac{\alpha_2}{4\pi} \Lambda \left( 1 + \frac{28}{9} b^2 \right),$$

$$M_1 = \frac{3}{5} \frac{\alpha_1}{4\pi} \Lambda (1 + 4b^2),$$
(10)

where

$$b^2 = -\frac{9\lambda^2}{\lambda_1 \lambda_2} \frac{M^2}{M_C^2} > 0.$$
(11)

The two loop GMSB contribution to the scalar masses is given by

$$\widetilde{m}^{2} = 2\Lambda^{2} \left\{ C_{3} \left( \frac{\alpha_{3}}{4\pi} \right)^{2} (a^{2} + 4b^{2}) + C_{2} \left( \frac{\alpha_{2}}{4\pi} \right)^{2} \left( 1 + \frac{28}{9}b^{2} \right) + C_{1} \left( \frac{\alpha_{1}}{4\pi} \right)^{2} \left( \frac{3}{5} + \frac{2}{5}a^{2} + \frac{12}{5}b^{2} \right) \right\},$$
(12)

where  $a = \lambda_X M / \lambda_a M_G$ ,  $C_3 = 4/3$  for color triplets and zero otherwise,  $C_2 = 3/4$  for weak doublets and zero otherwise and  $C_1 = (\frac{3}{5})(Y/2)^2$ . *a* and *b* are free independent parameters which we use to fit the data at the EWSB scale.

The gravitino mass is given by

$$m_{\tilde{G}} = \frac{\Lambda_{susy}^2}{\sqrt{3}M_P},\tag{13}$$

where  $\Lambda_{susy}$  is the scale of SUSY breaking and  $M_P = (8 \pi G_N)^{-1/2} = 2.4 \times 10^{18}$  GeV is the reduced Planck mass. For  $\Lambda_{susy}^2 = F_X \simeq 10^{17}$  GeV<sup>2</sup> the gravitino mass is

$$m_{\tilde{G}} = \frac{M}{\sqrt{3}M_P} \Lambda \simeq 0.024 \text{ GeV}, \qquad (14)$$

making the gravitino the LSP. However, this conclusion is model dependent.

For example, we show that in the particular model of SUSY breaking discussed in Ref. [13], the gravitino mass is

significantly larger. In this model the field which gets both a scalar and *F*-component VEV is the third component of an SU(2)<sub>*F*</sub> vector field S<sub>3</sub>. In this theory X is a composite field with  $\langle X \rangle = M = S_3^2/M_{st}$  and  $F_X = 2S_3F_{S_3}/M_{st}$ . The gravitino mass is therefore given by [11]

$$m_{\tilde{G}} = \frac{F_{S_3}}{\sqrt{3}M_P} = \frac{1}{2\sqrt{3}} \frac{\sqrt{MM_{st}}}{M_P} \Lambda,$$
 (15)

with  $\Lambda$  still given by  $\Lambda = F_X/M$ . The gravitino mass is thus enhanced by the factor  $1/2\sqrt{M_{st}/M}$ . For example, letting the string scale  $M_{st} = M_P$  and requiring the scale of PQ symmetry breaking  $M = 10^{12}$  GeV, we find  $m_{\tilde{G}} = 18.6$  GeV.<sup>4</sup> To conclude, in this model, with  $\Lambda = 10^5$  GeV, the gravitino, gluino and wino masses (at M) are given by

$$m_{\tilde{g}} = 18.6 \text{ GeV},$$
  
 $m_{\tilde{g}} = \left(\frac{b}{0.1}\right)^2 \times 14 \text{ GeV},$  (17)

$$M_2 = 340$$
 GeV.

Hence either the gluino or the gravitino is the LSP depending on the particular SUSY breaking model and the value of the parameter b.<sup>5</sup>

For phenomenological reasons we assume that SUSY is also broken by the *D* term of an anomalous  $U(1)_X$  gauge symmetry as already discussed in Ref. [6]. Moreover, the GMSB and *D* term contributions are necessarily comparable.<sup>6</sup> The *D*-term contribution to scalar masses is given by

$$\delta_D \tilde{m}_a^2 = dQ_a^X M_2^2, \qquad (18)$$

where  $Q_a^X$  is the U(1)<sub>X</sub> charge of the field *a* and *d* is an arbitrary parameter of order 1 which measures the strength of *D* term versus gauge-mediated SUSY breaking. The value of  $Q_a^X$  for a=16, 10, 1 of SO(10) is given by 1, -2, 4 [6].

We now summarize the messenger scale boundary conditions. The gaugino masses are given by Eq. (10) and scalar

<sup>4</sup>It is necessary to check that the supergravity contribution to squark and slepton masses is small compared to the GMSB contribution. This ratio scales as

$$\frac{m_{\tilde{G}}}{M_2} = \frac{2\pi}{\sqrt{3}\,\alpha_2} \frac{S_3}{M_P}.$$
(16)

Thus  $S_3$  cannot be much larger than  $10^{15}$  giving  $m_{\tilde{G}}/M_2 \approx 0.04$ . Taking the string scale  $M_{st} = M_P$  and requiring the scale of PQ symmetry breaking  $M = 10^{12}$  GeV, one gets  $S_3 \approx 1.5 \times 10^{15}$  and  $m_{\tilde{G}}/M_2 \approx 0.056$  which is reasonable.

<sup>5</sup>In order to have b = 0.1 with  $M/M_G \sim 10^{-4}$  we need to take  $\lambda^2/\lambda_1\lambda_2 \sim 10^5$ .

<sup>6</sup>Reference [13] presents a model which dynamically breaks SUSY and leads to comparable SUSY breaking effects from gauge-mediated and *D*-term sources.

mass by Eqs. (12) and (18).<sup>7</sup> D-term SUSY breaking only contributes to the scalar masses. The SSB trilinear scalar coupling A and scalar Higgs mass squared B vanish at tree level but are generated via RGE running below M. We have chosen  $a = 10^{-4}$ . We then determine the free parameters  $\Lambda$ , b, d, and  $\mu$  (at M) and  $\alpha_{GUT}$ ,  $\epsilon_3$ , the top  $(\lambda_t)$ , bottom  $(\lambda_b)$  and  $\tau$   $(\lambda_{\tau})$  Yukawa couplings (at  $M_G$ ) by fitting the low energy data which we take to include  $m_t$ ,  $m_b$ ,  $m_\tau$ ,  $\alpha_{em}$ ,  $\alpha_s$ , and  $\sin^2 \theta_W$ .<sup>8</sup> Imposing gauge coupling unification at the GUT scale, we renormalize the effective Lagrangian parameters to the EWSB scale using one (two) loop equations for dimensionful (dimensionless) parameters. We have also included the effect of a light gluino in the running of  $m_b$  and  $\alpha_s$  below the EWSB scale [14]. Finally the one loop SUSY threshold corrections for the b and  $\tau$  masses at the EWSB scale have also been included.

#### IV. PHENOMENOLOGY AT THE EWSB SCALE

The parameter *b* is varied to keep the gluino pole mass at 30 GeV.<sup>9</sup> *d* and  $\Lambda$  set the scale for squark, slepton, and gaugino masses. We have examined cases of fixed  $\Lambda$ , varying *d*, and vice versa in order to study the effects of these two SUSY breaking mechanisms on the low energy phenomenology separately. tan $\beta$  is solved from Eq. (1) and helps determine the quark and lepton masses. We have allowed for values of ( $\epsilon_3 < 4\%$ ).

In the first part of our analysis,  $\Lambda$  is fixed to the value  $\Lambda = 10^5$  GeV while *d* is varied. Figure 1 shows the values of  $|\mu|$  at the messenger scale giving the best low energy fit. Note that  $\mu < 0$  and  $|\mu|$  increases with *d*. The reason for negative  $\mu$  can best be seen by considering the following equation:

$$B = (m_{H_u}^2 - m_{H_d}^2) \frac{\tan 2\beta}{2} - M_Z^2 \frac{\sin 2\beta}{2}.$$
 (19)

At small tan  $\beta$ , the second term on the right-hand side of the equation is negligible. Although  $m_{H_u}^2$  and  $m_{H_d}^2$  have a common value at the messenger scale,  $m_{H_u}^2$  is always driven to smaller values than  $m_{H_d}^2$  by the RGE because of the larger top Yukawa coupling. We also know that tan2 $\beta$ <0 as long as tan $\beta$ >1. We therefore need B>0 at the EWSB scale. The one loop  $\beta$  function for B is given by



FIG. 1.  $-\mu$  at the messenger scale,  $-\mu$ ,  $-B/\mu$ , and  $\sqrt{B}$  at the EWSB scale are plotted in this figure.

$$\beta_{B}^{(1)} = B \left\{ 3|Y_{t}|^{2} + 3|Y_{b}|^{2} + |Y_{\tau}|^{2} - 3g_{2}^{2} - \frac{3}{5}g_{1}^{2} \right\} + \mu \left\{ 6A_{t}Y_{t}^{\dagger} + 6A_{b}Y_{b}^{\dagger} + 2A_{\tau}Y_{\tau}^{\dagger} + 6g_{2}^{2}M_{2} + \frac{6}{5}g_{1}^{2}M_{1} \right\}.$$
(20)

Since *B* and the trilinear couplings  $A_{t,b,\tau}$  vanish at the messenger scale, we must choose  $\mu < 0$  in order to get a positive *B* at the EWSB scale.

The values of  $|\mu|, \sqrt{B}$ , and  $|B/\mu|$  at the Z scale are also shown in the plot. It is notable that RGE running gives small values of B, eliminating fine tuning in the Higgs potential, and thus giving a good solution to the  $\mu$  problem. Note that  $|\mu|, \sqrt{B}$  are both increasing functions of d; with a sharper rise at small values of d. This can be understood using Eq. (1). For moderate values of d, the  $M_Z^2$  and  $m_{H_d}^2$  terms are negligible (the latter due to the fact  $|m_{H_{J}}^{2}| < |m_{H_{J}}^{2}|$  and the factor of  $\tan \beta^2 - 1$  in the denominator). Hence the approximate relation  $\mu^2 = -m_{H_u}^2 > 0$  holds. Increasing d leads to a linear increase in  $|m_{H_u}^2|$  [the  $U(1)_X$  charge of  $H_u$  is -2] and therefore in  $|\mu|$ . For very small values of d,  $|m_{H_{u}}^{2}|$  becomes comparable to  $M_Z^2/2$  and the significant cancellation in the relation  $\mu^2 \approx -M_Z^2/2 - m_{H_u}^2$  leads to a sharp decrease in  $\mu$ . Since *B* is generated from  $\mu$  via RGE's, the dependence of  $\sqrt{B}$  on d follows that of  $\mu$ .

We mentioned that  $m_{H_u}^2 - m_{H_d}^2$  is zero at the messenger scale but is negative at the Z scale. Decreasing d has a small effect on the value of  $m_{H_u}^2 - m_{H_d}^2$  at the Z scale. However, from Eq. (19) we see that a sharp decrease in B at small values of d has to be compensated by small values of  $|\tan 2\beta|$ . This is the reason for the sharp increase in tan  $\beta$ , plotted in Fig. 2, at low values of d.<sup>10</sup>

<sup>&</sup>lt;sup>7</sup>Note that the Higgs states also get a SSB mass correction at one loop due to their direct interaction with the messengers. This correction is negligible however because of the small values we have chosen for the free parameters a, b.

<sup>&</sup>lt;sup>8</sup>Note that we allow for a small one loop threshold correction to gauge coupling unification at  $M_G$  which we have parametrized by the free parameter  $\epsilon_3 = (\alpha_3 - \alpha_G)/\alpha_G$  evaluated at  $M_G$ .

<sup>&</sup>lt;sup>9</sup>Using the RGE's we evaluate  $m_{\tilde{g}}^{\overline{MS}}$  at  $m_{\tilde{g}}$  and then calculate the one loop corrected gluino pole mass. It is the pole mass which is constrained to lie between 25 and 35 GeV. Note, that an 18-GeV modified minimal subtraction scheme ( $\overline{MS}$ ) running mass defined at  $M_Z$  is roughly equivalent to a 30-GeV pole mass.

<sup>&</sup>lt;sup>10</sup>Note that a small  $|\tan 2\beta|$  gives a large  $\tan\beta$  assuming  $\pi/4 < \beta < \pi/2$ .



FIG. 2. Variation of  $\tan\beta$  versus *d* for fixed  $\Lambda = 10^5$  GeV.

In Fig. 3 we plot the masses of the Higgs states. The masses of  $A, H^0$ , and  $H^+$  are determined at tree level while we have included the one loop SUSY threshold corrections to the mass of h. Since B increases with d, the masses of  $A, H^0$  and  $H^+$  also increase while A stays the lightest. One might think that since the mass of the lightest Higgs state h is an increasing function of  $\tan\beta$ , it should decrease when increases with d, the effect of B dominates over that of  $\tan\beta$  and the mass of h slowly increases with d. The reason for the slow increase is that the mass of h is determined by EWSB and hence it cannot be a strong function of d.

In Fig. 4, we have magnified the small d and  $\Lambda$  regions in order to show the rapid variation in the value of  $m_h$  here. The solid line shows the change in the mass of h versus d for a fixed  $\Lambda = 10^5$  GeV while the dashed line shows the variation versus  $\Lambda$  with a fixed d. As discussed above, B decreases with decreasing d. This decrease in B is directly reflected in the decrease in the tree level value of  $m_h$ . Similarly the



FIG. 3. MSSM Higgs bosons masses versus d for fixed  $\Lambda = 10^5$  GeV.



FIG. 4. A magnified variation of the mass of the lightest neutral Higgs boson *h* for small *d* and  $\Lambda$ . The solid line shows the variation versus *d* for a fixed  $\Lambda = 10^5$  GeV while the dashed line shows the variation versus  $\Lambda$  with a fixed d = 1.

value of *B* decreases when  $\Lambda$  decreases and thus so does  $m_h$ .<sup>11</sup>

For completeness, we show the behavior of  $\tan\beta$  with changing  $\Lambda$  and fixed d=1 in Fig. 5;  $\tan\beta$  decreases rapidly for  $\Lambda < 10^5$  GeV. This is because  $|\tan 2\beta|$  must increase to compensate for  $|m_{H_u}^2 - m_{H_d}^2|$  which is decreasing proportional to  $\Lambda^2$  [see Eq. (19)].

Figure 6 contains plots of the masses of charginos and neutralinos versus *d*. The mass of the heavy chargino  $(\chi_2^+)$  and the two heaviest neutralinos  $(\chi_3^0, \chi_4^0)$  increase, as  $|\mu|$  increases with *d*. However, for most values of *d*, the mass of  $\chi_1^0$  scales as  $M_1$  while the masses of  $\chi_2^0$  and  $\chi_1^+$  scale as  $M_2$  and do not run with *d*. At very small *d*, tan $\beta$  gets very large and the off diagonal elements, proportional to sin  $\beta$  in the chargino and neutralino mass matrices, become larger than the diagonal elements including  $\mu$  resulting in a sharp drop in the masses.

Squark and slepton masses are plotted in Fig. 7. The *D* term contributes positively to the masses of squarks and sleptons since the  $U(1)_X$  charges of these fields are +1. Consequently, we see an increase in their masses with increasing *d*. The mixing due to *A* and  $\mu$  terms for the third generation squarks and sleptons is very small. Nevertheless, due to the different boundary conditions at *M*, the right handed squarks and sleptons are lighter than the left handed ones. The right handed stop is always the lightest and below  $d \approx 0.7$  it becomes lighter than the top. This is because the right handed stop mass squared is driven negative by RGE as a conse-

<sup>&</sup>lt;sup>11</sup>The Higgs mass apparently increases without bound as  $\Lambda$  increases. We believe this result is due to the fact that we only use the tree level Higgs potential for EWSB. Note that we have checked that we cannot obtain reasonable fits to the data for  $\Lambda > 1.1 \times 10^5$  GeV; hence we believe that our range for the allowed Higgs mass will not be significantly affected when including higher order corrections to the Higgs potential.



FIG. 5. Variation of  $\tan\beta$  versus  $\Lambda$  for fixed d=1.

quence of the large top Yukawa coupling.

Finally in Fig. 8 we plot the masses of squarks, sleptons, charginos, neutralinos, and Higgs fields versus  $\Lambda$ . As one expects, all these masses increase with  $\Lambda$  except the lightest Higgs whose mass is determined by EWSB. Recall that all scalar and gaugino masses at the messenger scale are directly proportional to  $\Lambda$ .

### V. LABORATORY AND COSMOLOGICAL CONSTRAINTS

Let us first consider the heavy gluino. If it is the LSP it was shown that it can survive in a narrow window with mass between 25 and 35 GeV [7,8]. We note that to get this limit, Refs. [7] and [8] assume very large squark masses. Reference [8] however also argues that lowering the squark masses increases the allowed range for the gluino mass. Now consider the possibility that the gravitino is the LSP and the gluino is the NLSP. In this case we must check whether the analysis of Refs. [7] and [8] still applies, i.e., whether the gluino lifetime is greater than  $\sim 10^{-8}$  s. The decay rate of the gluino to a gluon and a gravitino is given by



FIG. 6. The chargino and neutralino masses versus d for fixed  $\Lambda = 10^5$  GeV.



FIG. 7. Squark and slepton masses versus d for fixed  $\Lambda = 10^5$  GeV.

$$\Gamma_{\tilde{g}\to g\tilde{G}} = \frac{\alpha_s^2}{48\pi} \frac{m_{\tilde{g}}^5}{M_P^2 m_{\tilde{G}}^2} \left(1 - \frac{m_{\tilde{G}}^2}{m_{\tilde{g}}^2}\right)^3.$$
 (21)

Even in the case that  $\Lambda_{susy}^2 = F_X \approx 10^{17} \text{ GeV}^2$ , which gives a very light gravitino as in Eq. (13), a gluino mass of 30 GeV has a lifetime of  $\tau_{\tilde{g}} \approx 2 \times 10^6 \text{ s} \approx 1$  month. We therefore conclude that the gluino is in all cases a stable particle with regards to detector experiments. Hence the missing momentum analysis of Refs. [7] and [8] is relevant and a heavy gluino NLSP with mass in the range 25–35 GeV is still viable.

Note that there are several significant advantages for having a heavy gluino LSP or NLSP.

It reduces the fine tuning necessary for EWSB, since the dominant contribution to scalar masses due to RG running from  $M_G$  to  $M_Z$  comes from color corrections proportional to the gluino mass squared [15].



FIG. 8. Squark, slepton, chargino, neutralino, and Higgs masses versus  $\Lambda$  for fixed d=1. Many of the masses are almost degenerate, therefore one representative is shown from each set.

Even if the gluino is the NLSP, its lifetime is long enough for it to be a candidate for the UHECRon, i.e., the source of the ultra high energy cosmic rays [16].

The model with a Higgs mass of order 90 GeV and a stop mass less than the top satisfies some of the dynamical constraints necessary for electroweak baryogenesis in supersymmetric theories [17].

We now consider the LEP constraints on other SUSY parameters in our model. The most important constraints come from the latest Higgs search results at LEP [20]. The light neutral Higgs h and the CP odd Higgs A in the MSSM are produced at LEP via the Higgs-strahlung process  $e^+e^ \rightarrow hZ$  or the pair production process  $e^+e^- \rightarrow hA$ . h and A decay predominantly into  $b\bar{b}$  and  $\tau^+\tau^-$ . Thus LEP experiments search for either a  $b\bar{b}$  or  $\tau^+\tau^-$  plus the decay products of the Z; or for  $bb\overline{b}\overline{b}$  and  $\tau^+\tau^-b\overline{b}$ . In our model the offdiagonal elements in the stop mass-squared matrix are very small, thus the LEP limits for the neutral Higgs in the no stop-quark mixing scenario are most applicable [20,21]. These limits are very severe. Looking at our data in Figs. 2 and 4, it can be seen that only  $d \sim 0.40-0.45$  survives the LEP constraint for  $\Lambda = 10^5$  GeV. With these values of the parameters, the mass of the lightest neutral Higgs resides in the narrow range  $\sim (86-91)\pm 6$  GeV with  $\tan\beta \sim 9-14$ .<sup>12</sup> At this point, our model also survives the limit on the mass of A as indicated in Ref. [20].  $H^+$  and  $H^0$  are also too massive to be constrained. For  $d \sim 0.40 - 0.45$  and  $\Lambda = 10^5$  GeV we find the lightest stop and neutralino with mass in the range 100–122 and 50–72 GeV, respectively. Note, if  $\Lambda$ increases, then the Higgs mass and  $\tan\beta$  remain unchanged while all other masses increase.<sup>13</sup>

This narrow region of parameter space is obtained with the fit values of the parameters  $\alpha_G = 5.28-5.49 \times 10^{16}$  GeV,  $\epsilon_3 = 2.44-2.42\%$ ,  $\lambda_{b,t,\tau} = (0.065, 0.42, 0.096) - (0.042, 0.42, 0.061)$  at  $M_G$  and  $\mu = (-85.4) - (-139)$  GeV and b = 0.06 at *M*. Note that we are not able to get a good fit to the data assuming Yukawa coupling unification. This was not the case in Ref. [6] and is due to the fact that, unlike the model presented here, the parameter *B* was a free parameter. In Appendix B, we show how to complete the model described earlier in order to accommodate



FIG. 9. The ratio of the SUSY contribution to the amplitude of  $b \rightarrow s \gamma$  to the SM contribution. The dashed line shows the variation versus *d* for a fixed  $\Lambda = 10^5$  GeV while the solid line shows the variation versus  $\Lambda$  with a fixed d=1.

nonunification of Yukawa couplings in this SO(10) theory. Clearly this feature of the model is not very satisfying.

The process  $e^+e^- \rightarrow$  hadrons can constrain the chargino mass based on the OPAL 2  $\sigma$  bound on new physics at  $\sqrt{s}$ = 172 GeV [23]. We analyzed the contributions to  $e^+e^ \rightarrow$  hadrons coming from chargino and neutralino pair production using (*SPYTHIA*, *A SUPERSYMMETRIC EXTENSION OF PYTHIA* 5.7 [24]), which has been modified by S. Mrenna and K. Tobe to accommodate a gluino LSP. The result is that except for very small values of *d* or  $\Lambda$  which are strongly ruled out by the Higgs constraint, the cross section is below the limit of 3 *pb* reported in Ref. [23].<sup>14</sup>

We have evaluated the rate for  $b \rightarrow s \gamma$ .<sup>15</sup> The ratio of the SUSY amplitude to the SM amplitude for (varying *d*-fixed  $\Lambda$ ) and (varying  $\Lambda$ -fixed *d*) are given in Fig. 9. The main SUSY contribution in our model comes from the charged Higgs-top loop. Figure 9 shows that we always get an amplitude larger than the SM amplitude approaching it at large  $\Lambda$  since  $\mu < 0$ . Note that our result is at the limiting edge of values allowed by CLEO data [25]. Our result is, however, incomplete. We approximate flavor mixing using the known CKM elements as a crude approximation to squark-quark flavor mixing and we only use a one-loop analysis. With regards to the latter, it is clear that a one loop analysis is not sufficient since the higher order corrections have recently been shown to be very important (see, for example, Ref.

<sup>&</sup>lt;sup>12</sup>We use the tree level Higgs potential in our analysis and the masses of  $A, H^0$  and  $H^+$  are determined only at tree level. We have however included one loop SUSY threshold corrections to the mass of the lightest Higgs scalar h. In order to ascertain the theoretical errors associated with the Higgs boson mass, we have also used FEYNHIGGSFAST [18] to perform a two loop calculation of the Higgs boson mass at d=0.45. We find  $m_h=95$  GeV; instead of 91 GeV found at one loop. Moreover FEYNHIGGSFAST has a quoted error margin of  $\pm 2$  GeV. Hence our prediction for the range of the light Higgs boson mass includes a theoretical uncertainty of  $\pm 6$  GeV. Note our model thus survives a very recent limit on the Higgs mass by the ALEPH Collaboration [19].

<sup>&</sup>lt;sup>13</sup>Note that a recent LEP bound on a heavy gluino LSP using stop production and decay [22] does not constrain the model since the stop mass in our case is larger than the values probed in this search.

<sup>&</sup>lt;sup>14</sup>For example, when  $\Lambda = 10^5$  GeV, the cross section of the process  $e^+e^- \rightarrow$  hadrons through charginos and neutralinos exceeds the 3 pb limit only for d < 0.37.

<sup>&</sup>lt;sup>15</sup>In order to do this calculation we need to assume some values for flavor mixing elements. As a rough estimate we use the observed CKM matrix elements in the appropriate places. This is clearly just a rough estimate which gives us at best an order of magnitude approximation.

[26]). Hence within the approximations considered here, the model is roughly consistent with the observed rate for  $b \rightarrow s \gamma$ .

Now consider the low energy consequences of the PQ symmetry which is spontaneously broken at the scale  $f_A \equiv M = 10^{12}$  GeV. This generates an invisible axion which is a significant component of the energy density of the universe and thus a candidate for cold dark matter [27]. The axion is predominantly the angular part of the complex scalar field component of the chiral superfield X. The radial part of this scalar field and the fermion component of the supermultiplet are named saxino ( $\tilde{S}$ ) and axino ( $\tilde{A}$ ) in the literature, respectively. In our model, these fields obtain large radiative masses via a messenger  $\mathbf{10}_A$  loop,

$$M_{\widetilde{A}} = \frac{\lambda_X^2}{8\pi^2}\Lambda, \quad M_{\widetilde{S}}^2 = \frac{\lambda_X^2}{6\pi^2}\Lambda^2.$$
(22)

The axino decays mainly to a gluino and a gluon. The decay rate is calculated to be [28,29]

$$\Gamma_{\tilde{A} \to g\tilde{g}} = \frac{\alpha_s^2}{16\pi^3} f^2 \left(\frac{m_t^2}{\tilde{m}_{t_1}\tilde{m}_{t_2}}\right) \frac{m_{\tilde{A}}^3}{f_A^2}, \quad f(x) = \frac{x}{1-x} + \frac{x\ln x}{(1-x)^2}$$
(23)

and we find  $10^{-22}$  GeV $<\Gamma_{\tilde{A}\to g\tilde{g}}<10^{-17}$  GeV for 0.2  $< m_t^2/\tilde{m}_{t_1}\tilde{m}_{t_2}<1$  and  $0.5<\lambda_X<3$ . This translates into an axino lifetime of  $10^{-7}$  s $<\tau_{\tilde{A}\to g\tilde{g}}<10^{-2}$  s. The saxino decays either to 2 gluons or 2 gluinos. The

The saxino decays either to 2 gluons or 2 gluinos. The decay rate of the saxino to 2 gluons is given in Ref. [30],

$$\Gamma_{\tilde{S}\to 2g} = \frac{\alpha_s^2}{32\pi^3} \frac{M_{\tilde{S}}^3}{f_A^2}.$$
(24)

For  $0.5 < \lambda_X < 3$ , the decay rate is  $10^{-18}$  GeV  $< \Gamma_{\tilde{S} \rightarrow 2g} < 10^{-15}$  GeV with a comparable decay rate into two gluinos. This gives the saxino an approximate lifetime of  $10^{-9} < \tau_{\tilde{S} \rightarrow 2g} < 10^{-6}$  s. The heavy axino and saxino have very short lifetimes and decay before nucleosynthesis starts. Their decay therefore does not affect the standard nucleosynthesis calculations.

Finally, the relic gluino density, assuming the gluino is the LSP, should be small enough that it does not constitute a significant fraction of the dark matter halo density (see, for example, Ref. [31]). References [7] and [11] show that taking into account the nonperturbative effects of gluino-gluino annihilation into quark-antiquark and gluon-gluon the relic gluino density is extremely small;  $\Omega_{\tilde{g}}h^2 \sim 10^{-8}-10^{-11}$ . There are, however, stringent limits on a gluino LSP coming from searches for anomalous heavy isotopes [32] and from energetic neutrinos due to gluino annihilations in the Sun [33]. Both of these latter constraints apparently rule out an absolutely stable gluino LSP. They do not, however, constrain the case of a gluino NLSP and gravitino LSP also considered in this paper.

## **VI. CONCLUSIONS**

In this paper we have presented a solution to the  $\mu$  and strong *CP* problems in the presence of a heavy gluino LSP. The model has a natural Peccei-Quinn symmetry which prevents the  $\mu$  term at tree level. However, when the PQ symmetry is broken at the messenger scale  $M \sim 10^{12}$  GeV the  $\mu$ term is generated.

The particle phenomenology of the model is quite novel. Either the gluino or the gravitino is the LSP. If the gravitino is the LSP, then the gluino is the NLSP with a lifetime on the order of one month or longer. In either case this heavy gluino, with mass in the range 25–35 GeV, can be treated as a stable particle with respect to experiments at high energy accelerators.

We have studied some of the phenomenological constraints on the model. The most significant comes from LEP searches for the neutral Higgs boson. Our model is most like the no stop-quark mixing benchmark which is severely constrained by the data. In fact the model only survives in a narrow region of parameter space resulting in a light neutral Higgs with mass  $\sim (86-91)\pm 6$  GeV and  $\tan\beta \sim 9-14$ . In addition the lightest stop and neutralino have mass  $\sim 100-122$  GeV and  $\sim 50-72$  GeV, respectively. Thus the model will soon be tested. Finally, the invisible axion resulting from PQ symmetry breaking is a cold dark matter candidate.

#### ACKNOWLEDGMENTS

This work was supported by the DOE/ER/01545-796. A. Mafi thanks K. Tobe for helpful discussions. S. Raby thanks N. Polonsky and L. Roszkowski for useful conversations at an early stage in this work.

#### **APPENDIX A: COMPLETE MODEL**

In this section we present the complete model with all the symmetries and charges. The model is defined at the GUT scale by the SO(10) invariant superpotential  $W \supset W_1 + W_2 + W_3 + W_4 + W_5$  and a nonrenormalizable term in the Kahler potential *K* where

$$W_{1} = \mathbf{16}_{3}\mathbf{10}_{H}\mathbf{16}_{3},$$

$$W_{2} = \lambda_{a}\mathbf{10}_{H}A\mathbf{10}_{A} + \lambda_{X}X\mathbf{10}_{A}^{2},$$

$$W_{3} = \lambda_{1}\overline{\eta}_{1}A\eta_{1} + \lambda_{2}\overline{\eta}_{2}A\eta_{2} + \lambda_{X}\overline{\eta}_{1}\eta_{2},$$
(A1)
$$W_{4} = \lambda_{Y_{3}}\overline{\eta}_{1}Y\mathbf{16}_{3},$$

$$W_{5} = \lambda_{H}\mathbf{10}_{H}\overline{\psi}\overline{\psi}' + \lambda_{Y_{1}}Y\overline{\psi}'\psi',$$

$$K \supseteq \frac{X^{\dagger}}{M_{P}}\mathbf{10}_{H}^{2} + \text{H.c.} \qquad (A2)$$

TABLE I. The *R* and PQ charges of different fields in the complete model.

Fields	<b>10</b> <sub>H</sub>	<b>10</b> <sub>A</sub>	А	Х	$oldsymbol{\eta}_1$	$\overline{\eta}_1$	$\eta_2$	$\bar{\eta}_2$	<b>16</b> <sub>3</sub>	ψ	$\overline{\psi}$	$\psi'$	$\overline{\psi}'$	Y
R	+1	+1	+2	+2	+1	+1	+1	+1	+3/2	0	0	-1/2	+3	+3/2
PQ	+1	-1	0	+2	-1/2	+1/2	-5/2	+5/2	-1/2	0	0	+1	-1	0

 $(\mathbf{16}_3, \eta_1, \eta_2, \psi, \psi')$  are all  $\mathbf{16}$ 's,  $(\bar{\eta}_1, \bar{\eta}_2, \bar{\psi}, \bar{\psi}')$  are  $\overline{\mathbf{16}}$ 's,  $(\mathbf{10}_H, \mathbf{10}_A)$  are  $\mathbf{10}$ 's, (X, Y) are singlets, and (A) is an adjoint under SO(10).  $W_1, W_2$ , and  $W_3$  were discussed earlier in Sec. II.

The theory is invariant under U(1) PQ and *R* symmetries. The charges of the fields under these symmetries are given in Table I.

The *R* symmetry is broken by the VEVs of several different fields at the GUT scale. However, the PQ symmetry is not broken at the GUT scale and prevents a  $\mu$  term in the superpotential.

 $W_4$  contains the field Y which gets a VEV

$$\langle Y \rangle = M_G \,. \tag{A3}$$

 $W_4$  also results in a bottom- $\tau$  Yukawa coupling nonunification at the GUT scale as discussed in Sec. 1 of Appendix B.

We use  $W_5$  in a standard way to split the top and bottom Yukawa couplings by giving VEV's to  $\psi$  and  $\overline{\psi}$  of order  $M_G$ . This mechanism is discussed in Sec. 2 of Appendix B.

## APPENDIX B: YUKAWA COUPLING NONUNIFICATION IN AN SO(10) SUSY GUT

In minimal SO(10), all standard model fermions in a given generation are contained in a single spinor (16) representation of SO(10). The coupling of the form  $W_1$  results in a unified Yukawa couplings for the top and bottom quarks and the  $\tau$  lepton, i.e.,  $\lambda_t = \lambda_b = \lambda_\tau$  at  $M_G$ . It is interesting, and necessary for our model, to see if it is possible to relax this condition in a simple way. The low tan $\beta$  fit of our SO(10) SUSY GUT to the infrared-scale physical observables requires  $\lambda_t - \lambda_b$  splitting at the GUT scale. We are also interested in the possibility of splitting  $\lambda_b - \lambda_\tau$ . The reason is that our best fits to the data come from bottom Yukawa couplings which are ~30% smaller than the  $\tau$  Yukawa coupling at the GUT scale.

### 1. $\lambda_b - \lambda_{\tau}$ nonunification

 $W_1, W_3$  and  $W_4$  contain interaction terms

$$\overline{\eta}_1(\lambda_1 A \eta_1 + \lambda_{\gamma_3} Y \mathbf{16}_3 + \lambda X \eta_2), \tag{B1}$$

resulting in a heavy (16'') and a massless (at the GUT scale) (16') multiplet given by

$$\mathbf{16}'' \propto (\lambda_1 \langle A \rangle \eta_1 + \lambda_{Y_2} \langle Y \rangle \mathbf{16}_3) \tag{B2}$$

and

$$\mathbf{16}' \propto (\lambda_{Y3} \langle Y \rangle \eta_1 - \lambda_1 \langle A \rangle \mathbf{16}_3). \tag{B3}$$

Note that we can safely ignore the last term in Eq. (B1) since the VEV of X is much smaller than  $M_G$ .

The massless  $\mathbf{16}'$  is identified with the matter multiplet containing the third generation quarks and leptons. Since *A* gets a VEV in the *B*-*L* direction and *B*-*L* quantum numbers are different for quarks and leptons,  $\lambda_b - \lambda_\tau$  unification will be lost, but we still have  $\lambda_t - \lambda_b$  unification. In fact this mechanism is an SO(10) version of the one introduced in Ref. [34]. Considering Eqs. (6) and (A3) we define

$$s_a = (\sin \theta_{B-L})_a = \frac{\rho T_a^{B-L}}{\sqrt{1 + \rho^2 (T_a^{B-L})^2}},$$
 (B4)

where  $T_a^{B-L}$  is the B-L quantum number of the state  $a = Q, \overline{U}, \overline{D}, L, \overline{E}, \overline{\nu}$  in the matter multiplet and  $\rho = \lambda_1 / \lambda_{\gamma_2}$ .<sup>16</sup>

From  $W_1$  we see that the fermions in the matter multiplet **16**' obtain mass at the electroweak scale due to the term

$$(-s_a \mathbf{16}') \mathbf{10}_H (-s_a \mathbf{16}').$$
 (B5)

We therefore have

$$\frac{\lambda_b}{\lambda_\tau} = \frac{s_Q s_{\bar{D}}}{s_L s_{\bar{E}}} = \frac{1}{9} \frac{1+\rho^2}{1+\rho^{2/9}},$$
(B6)

where  $\lambda_b$  and  $\lambda_{\tau}$  are the effective bottom and  $\tau$  Yukawa couplings. We thus find

$$\frac{1}{9} < \frac{\lambda_b}{\lambda_\tau} < 1, \tag{B7}$$

depending on the choice of  $\rho = \lambda_1 / \lambda_{Y_3}$ . For  $\rho \simeq 4.2$  we get  $(\lambda_\tau - \lambda_b) / \lambda_\tau \simeq 30\%$ .

# 2. $\lambda_t - \lambda_b$ nonunification

The splitting between  $\lambda_t$  and  $\lambda_b$ ,  $\lambda_\tau$  Yukawa couplings is best achieved in the Higgs sector using  $W_5$ . Since  $\psi$  and  $\overline{\psi}$ get VEV's of order  $M_G$ , the Higgs doublet mass term can be written as

<sup>&</sup>lt;sup>16</sup>In calculating the one loop gaugino (two loop scalar) mass contributions from  $W_3$  in Eq. (10) [(12)], we have ignored an order  $1/\rho$  correction coming from  $W_4$ . Since *b* is a small free parameter, this does not affect the wino, bino, and scalar masses while the gluino mass is varied by the free parameter *b*.

$$\begin{pmatrix} d_H & d_A & d_{\bar{\psi}'} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda_X \langle X \rangle & 0 \\ \lambda_H \langle \bar{\psi} \rangle & 0 & \lambda_{Y_1} \langle Y \rangle \end{pmatrix} \begin{pmatrix} \bar{d}_H \\ \bar{d}_A \\ \bar{d}_{\psi'} \end{pmatrix}.$$
(B8)

From the above we see that the two light Higgs doublets are  $d_H$  and  $\cos \gamma \bar{d}_H - \sin \gamma \bar{d}_{\psi'}$  which are identified with the light Higgs doublets of the MSSM,  $(H_u, H_d)$ , respectively.  $\cos \gamma$  is given by

$$\cos \gamma = \frac{\rho'}{\sqrt{1 + {\rho'}^2}},\tag{B9}$$

where

$$\rho' = \frac{\lambda_{Y_1} \langle Y \rangle}{\lambda_H \langle \bar{\psi} \rangle}.$$
 (B10)

Note that the rest of the Higgs doublets remain very heavy. From  $W_1$  we see that

$$\frac{\lambda_b}{\lambda_t} = \cos \gamma, \tag{B11}$$

where  $\lambda_t$  and  $\lambda_b$  are the effective Yukawa couplings of the top and bottom quarks. A hierarchy of 50 is easily achieved by choosing  $\rho' \simeq 1/50$ .

### APPENDIX C: $\tau$ NEUTRINO MASS

In this section we show that it is possible to get a reasonable  $\tau$  neutrino mass, in agreement with atmospheric neutrino oscillations. Let us add two SO(10) singlets *N* and *P* to the model with *R* and PQ charges given in Table II.

The only possible couplings for these singlets are

TABLE II. The R and PQ charges of the new fields in the neutrino sector.

Fields	Ν	Р
R	+5/2	-1
PQ	+1/2	-1

$$\lambda_{n_1} \overline{\psi} N \mathbf{16}_3 + \lambda_{n_2} N^2 P. \tag{C1}$$

We assume that *P* gets a VEV of order the messenger scale  $(10^{12} \text{ GeV})$ . The neutrino mass matrix is then given by

$$\left( \begin{array}{ccc} \nu_{L} & \overline{\nu}_{R} & N \end{array} \right) \left( \begin{array}{ccc} 0 & m_{t} \frac{s_{L} s_{\overline{E}}}{s_{Q} s_{\overline{U}}} & 0 \\ m_{t} \frac{s_{L} s_{\overline{E}}}{s_{Q} s_{\overline{U}}} & 0 & -s_{\overline{E}} \lambda_{n_{1}} \langle \overline{\psi} \rangle \\ 0 & -s_{\overline{E}} \lambda_{n_{1}} \langle \overline{\psi} \rangle & \lambda_{n_{2}} \langle P \rangle \end{array} \right)$$

$$\times \left( \begin{array}{c} \nu_{L} \\ \overline{\nu}_{R} \\ N \end{array} \right).$$

$$(C2)$$

The mass of the lightest neutrino, identified as  $\nu_{\tau}$ , is given by

$$m_{\nu_{\tau}} = \left(\frac{s_L}{s_Q s_{\bar{U}}}\right)^2 \frac{\lambda_{n_2}}{\lambda_{n_1}^2} \frac{m_t^2 \langle P \rangle}{\langle \bar{\psi} \rangle^2} \simeq 6.3 \times 10^{-7} \left(\frac{\lambda_{n_2}}{\lambda_{n_1}^2}\right) \\ \times \left(\frac{10^{16} \text{ GeV}}{\langle \bar{\psi} \rangle}\right)^2 \text{ eV} \sim 6 \times 10^{-2} \text{ eV}.$$
(C3)

Thus one gets a reasonable value for  $m_{\nu_{\tau}}$  with  $\lambda_{n_1} \sim 3 \times 10^{-3}$  and  $\lambda_{n_2} \sim 1$ .

- For a recent review, see N. Polonsky, in "Proceedings of the International Symposium on Supersymmetry, Supergravity and Superstring," Seoul, Korea, 1999, hep-ph/9911329.
- [2] J. E. Kim and H. P. Nilles, Phys. Lett. 138B, 150 (1984).
- [3] G. F. Giudice and A. Masiero, Phys. Lett. B 206, 480 (1988).
- [4] S. Dimopoulos and S. Raby, Nucl. Phys. B192, 353 (1981); M. Dine, W. Fischler, and M. Srednicki, *ibid.* B189, 575 (1981);
  M. Dine and W. Fischler, Phys. Lett. 110B, 227 (1982); M. Dine and M. Srednicki, Nucl. Phys. B202, 238 (1982); L. Alvarez-Gaumé, M. Claudson, and M. Wise, *ibid.* B207, 96 (1982); C. Nappi and B. Ovrut, Phys. Lett. 113B, 175 (1982).
- [5] G. Dvali, G. F. Giudice, and A. Pomarol, Nucl. Phys. B478, 31 (1996).
- [6] S. Raby and K. Tobe, Nucl. Phys. B539, 3 (1999).
- [7] H. Baer, K. Cheung, and J. F. Gunion, Phys. Rev. D 59, 075002 (1999).
- [8] A. Mafi and S. Raby, Phys. Rev. D 62, 035003 (2000).

- [9] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977).
- [10] S. Dimopoulos and F. Wilczek, Proceedings Erice Summer School, edited by A. Zichichi (1981).
- [11] S. Raby, Phys. Rev. D 56, 2852 (1997); Phys. Lett. B 422, 158 (1998).
- [12] See, for example, M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981).
- [13] S. Raby and K. Tobe, Phys. Lett. B 437, 337 (1998).
- [14] L. Clavelli and L. R. Surguladze, Phys. Lett. B 446, 153 (1999).
- [15] M. Bastero-Gil, G. L. Kane, and S. F. King, Phys. Lett. B 474, 103 (2000).
- [16] G. Farrar, Invited talk at the Workshop on "Observing the Highest Energy Particles (>10<sup>20</sup> eV) from Space," College Park, Maryland, 1997; I. F. M. Albuquerque, G. R. Farrar, and E. W. Kolb, Phys. Rev. D **59**, 015021 (1999).

- [17] M. Carena, M. Quiros, and C. E. M. Wagner, Phys. Lett. B 380, 81 (1996); Nucl. Phys. B524, 3 (1998).
- [18] S. Heinemeyer, W. Hollik, and G. Weiglein, hep-ph/0002213.
- [19] ALEPH Collaboration, R. Barate *et al.*, CERN-EP/2000-131, 2000.
- [20] ALEPH, DELPHI, L3, and OPAL Collaborations, Note for ICHEP'2000, Osaka, Japan, 2000, http://lephiggs.web.cern.ch/ LEPHIGGS/papers/index.html
- [21] T. Junk, talk presented at SUSY2K.
- [22] DELPHI Collaboration, S. Katsanevas and P. Verdier, DELPHI 2000-019 CONF 340, 2000.
- [23] OPAL Collaboration, K. Ackerstaff *et al.*, Eur. Phys. J. C 2, 441 (1998).
- [24] S. Mrenna, Comput. Phys. Commun. 101, 232 (1997).
- [25] For a recent report, see CLEO Collaboration, S. Ahmed et al.,

CLEO CONF 99-10, hep-ex/9908022.

- [26] W. de Boer, M. Huber, A. Gladyshev, and D. I. Kazakov, hep-ph/0007078.
- [27] L. J. Rosenberg, Nucl. Phys. B (Proc. Suppl.) 72, 209 (1999).
- [28] Jihn E. Kim, A. Masiero, and D. V. Nanopoulos, Phys. Lett. 139B, 346 (1984).
- [29] J. F. Nieves, Phys. Lett. B 174, 411 (1986).
- [30] T. Asaka and T. Yanagida, Phys. Lett. B 494, 297 (2000).
- [31] G. D. Starkman, A. Gould, R. Esmailzadeh, and S. Dimopoulos, Phys. Rev. D 41, 3594 (1990).
- [32] R. N. Mohapatra and V. L. Teplitz, Phys. Rev. Lett. 81, 3079 (1998).
- [33] A. E. Faraggi, K. A. Olive, and M. Pospelov, Astropart. Phys. 13, 31 (2000).
- [34] S. Dimopoulos and A. Pomarol, Phys. Lett. B 353, 222 (1995).