

## “Neutrinoless double beta decay” at a neutrino factory

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We examine the prospects of detecting an analogous process of neutrinoless double beta decay at a neutrino factory from a high energy muon storage ring. Limits from CERN LEP experiments, neutrinoless double beta decay as well as from global fits have to be incorporated and they severely restrict the results. We investigate what limits on light and heavy effective Majorana neutrino masses can be obtained and compare them with existing ones. Contributions from right-handed neutrinos and purely right-handed interactions are also discussed. We also comment on conspiracy in the mixing matrix, which might reduce the results within orders of magnitude. However, other “new physics” contributions to the same final state might produce large event numbers.

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### I. INTRODUCTION

The physics potential of a muon storage ring is rich and exciting. The option of using the neutrinos from the  $\mu$  decay has especially gathered much attention [1]. Typically the main focus lies in long baseline oscillation experiments [2] with source-detector distances from 730 up to 10000 km. This development is driven by the urge to find out about oscillation phenomena in more detail and to gain additional information, be it about  $CP$  violation, the sign of  $\Delta m^2$ , the size of  $|U_{e3}|$ , or the existence of sterile neutrinos.

An additional option is the use of a detector directly at the storage ring site. Neutrino interactions of up to  $10^{13}/\text{yr}$  provide the possibility of high precision experiments regarding Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, structure functions, electroweak parameters, charm physics, or other phenomena; see [3] for some possibilities. As in any other new experiment, new physics may lurk in the results. In the light of recent developments in oscillation experiments, effects of massive neutrinos are hot candidates. Evidence for massive neutrinos and therefore physics beyond the standard model (SM) comes from the up-down asymmetry of atmospheric muon neutrinos, the deficit of solar neutrinos, and the Liquid Scintillation Neutrino Detector (LSND) experiment. See [4] for more complete surveys. For example, the seesaw mechanism [5] might connect the very light known neutrinos to heavy neutrinos, which are usually assumed to be of a Majorana nature. Majorana particles can show their presence not only by being directly produced, but also via indirect effects stemming from their  $B-L$  violating mass term. The best known example for such a process is neutrinoless double beta decay ( $0\nu\beta\beta$ ) [6], which results in limits on the effective electron neutrino Majorana mass  $\langle m_{ee} \rangle$ . The complete  $3 \times 3$  matrix of (light) effective Majorana masses is defined as

$$\begin{aligned} \langle m_{\alpha\beta} \rangle &= |(U \text{diag}(m_1, m_2, m_3) U^T)_{\alpha\beta}| \\ &= \left| \sum m_i U_{\alpha i} U_{\beta i} \right| \\ &\leq \sum m_i |U_{\alpha i} U_{\beta i}| \quad \text{with } \alpha, \beta = e, \mu, \tau, \end{aligned} \quad (1)$$

where the sum goes over the mass eigenstates  $m_i$ . Conversely, the “inverse effective mass” is defined as

$$\begin{aligned} \left\langle \frac{1}{m_{\alpha\beta}} \right\rangle &= \left| \left( U \text{diag} \left( \frac{1}{m_1}, \frac{1}{m_2}, \frac{1}{m_3}, \dots \right) U^T \right)_{\alpha\beta} \right| \\ &= \left| \sum \frac{1}{m_i} U_{\alpha i} U_{\beta i} \right| \\ &\leq \sum \frac{1}{m_i} |U_{\alpha i} U_{\beta i}| \quad \text{with } \alpha, \beta = e, \mu, \tau. \end{aligned} \quad (2)$$

The sum over  $i$  is not the same in Eqs. (1) and (2): For  $\langle m_{\alpha\beta} \rangle$  it goes over all “light” mass eigenstates and in  $\langle 1/m_{\alpha\beta} \rangle$  over all “heavy” states. The attribute “light” or “heavy” depends on the energy scale of the process one considers to obtain information about the respective element. Note that we indicated that the sum goes up to a number greater than 3; it is, however, also possible that only one additional very heavy neutrino exists. Apart from theoretical prejudices, *a priori* we do not know how many there are.

The latter matrix might seem somewhat artificial; its form comes from the fact that heavy Majorana neutrinos force cross sections or branching ratios (typically processes in analogy to  $0\nu\beta\beta$ ) into a mass<sup>-2</sup> behavior. The knowledge of the elements of the matrices is rather poor, of course with the exception of  $\langle m_{ee} \rangle$  and  $\langle 1/m_{ee} \rangle$ . At a neutrino factory, the following processes (see Fig. 1) can be used to gain information about the other elements:

$$\nu_l N \rightarrow l^\mp \alpha^\pm \beta^\pm X,$$

$$\text{where } l = e, \mu \quad \text{and } \alpha, \beta = e, \mu, \tau. \quad (3)$$

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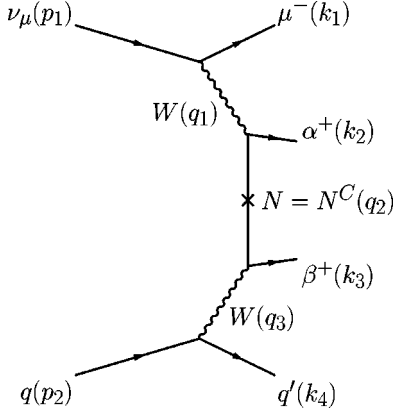


FIG. 1. Diagram for  $\nu_\mu q \rightarrow \mu^- \alpha^+ \beta^+ q'$ . Note that there is a crossed term and for  $\alpha \neq \beta$  there are two possibilities for the leptons to be emitted from. The leptonic part also can be replaced by the corresponding other neutrino species.

Here  $X$  denotes the hadronic final state. Because of very general arguments [7,8], the pure observation of this process guarantees a  $B-L$  violating Majorana mass and in connection with supersymmetry (SUSY) a  $B-L$  violating sneutrino mass term. This connection is depicted in Fig. 2 for the non-SUSY case. The precise determination of the mass of the intermediate Majorana neutrino will be very difficult; however, even the demonstration of Majorana mass terms will be an exciting and important result, since different models predict different mass matrices. In some models  $\langle m_{ee} \rangle$  is zero and therefore the only direct information about the mass matrix might come from neutrino oscillations. This complicates the situation, since only mass squared differences are measured and the additional phases induced by the Majorana nature are unobservable. Other experiments or cosmological arguments give total mass scales but the precise matrix is highly non-trivial to find [9]. Thus, information about entries in  $\langle m_{\alpha\beta} \rangle$  is very important. Similar arguments hold for the existence of heavy Majorana neutrinos. Note that the seesaw formula predicts their mass  $m_N$  to lie in the range

$$m_N \simeq \frac{m_D^2}{m_\nu} \simeq 10^2 - 10^{18} \text{ GeV}, \quad (4)$$

where  $m_D$  is a charged lepton or quark mass (i.e., electron to top quark) and  $m_\nu$  the mass of a light neutrino ( $10^{-5} - 1$  eV as indicated by oscillation experiments). It turns out that the highest cross section of process (3) is obtained for the lower region of this mass range.

In addition, if a  $B-L$  violating process is detected, it is helpful to know if the ‘‘mildly extended’’ (i.e., just additional Majorana neutrinos) SM can provide the signal or if another theory, such as SUSY, has to be considered.

The paper is organized as follows: In Sec. II we discuss some properties of process (3) and its application to neutrino factory kinematics. We review in Sec. III the status of direct experimental limits on  $\langle m_{\alpha\beta} \rangle$ . For the first time we give—using DESY  $ep$  collider HERA data—bounds on elements of  $\langle 1/m_{\alpha\beta} \rangle$  other than  $\langle 1/m_{ee} \rangle$  and examine what new limits might be accomplished for different neutrino factories. It

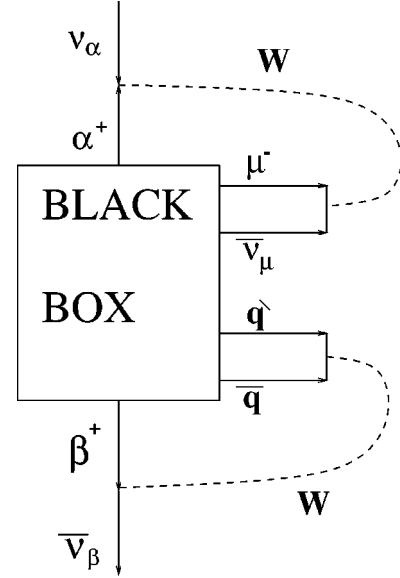


FIG. 2. Connection between Majorana mass term of  $\nu_\alpha$  and  $\nu_\beta$  and the existence of process  $\nu_\mu N \rightarrow \mu^- \alpha^+ \beta^+ X$ .

turns out that for muon energies higher than 500 GeV physically meaningful limits on  $\langle m_{\alpha\beta} \rangle$  can be obtained. Then we summarize limits on heavy Majorana neutrinos and their mixing with SM particles. Regarding the prospects of detecting events from process (3) we apply in Sec. IV all these limits, which severely restricts the results. We then discuss what significance lies in the bounds in the sense that even for not too baroque models the results might be reduced within one order of magnitude for final states containing muons and taus. For electron channels, a reduction of several orders of magnitude is easily possible. This latter fact results from  $0\nu\beta\beta$  bounds, which prohibit finding these signals anyway.

## II. THE PROCESS AND A NEUTRINO FACTORY

### A. Kinematics

The Feynman diagram is shown in Fig. 1; the calculation is straightforward and described in more detail in [10]. For one eigenstate  $m_i$  we find

$$\begin{aligned} & |\bar{\mathcal{M}}|^2(\nu_l q \rightarrow l^- \alpha^+ \alpha^+ q') \\ & \equiv |\overline{\mathcal{M}}_{--}|^2 \\ & = m_i^2 U_{\alpha i}^4 G_F^4 M_W^8 2^{12} \frac{1}{(q_1^2 - M_W^2)^2 (q_3^2 - M_W^2)^2} (p_1 \cdot p_2) \\ & \times \left[ \frac{1}{(q_2^2 - m_i^2)^2} (k_1 \cdot k_2)(k_3 \cdot k_4) + \frac{1}{(\tilde{q}_2^2 - m_i^2)^2} (k_1 \cdot k_3) \right. \\ & \times (k_2 \cdot k_4) - \frac{1}{(q_2^2 - m_i^2)(\tilde{q}_2^2 - m_i^2)} ((k_2 \cdot k_3)(k_1 \cdot k_4) \\ & \left. - (k_1 \cdot k_2)(k_3 \cdot k_4) - (k_1 \cdot k_3)(k_2 \cdot k_4)) \right]. \quad (5) \end{aligned}$$

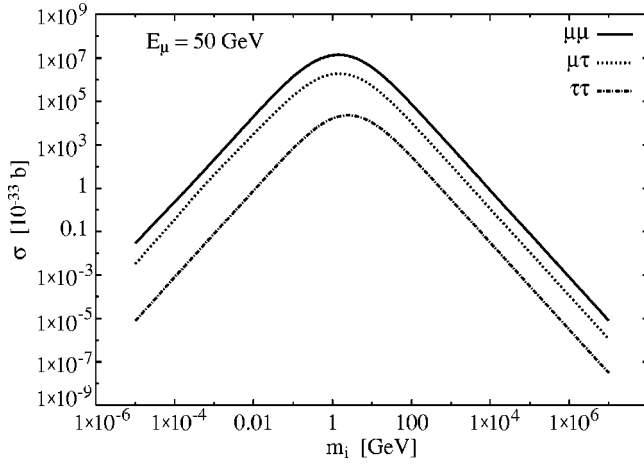


FIG. 3. Total cross section for  $\nu_\mu N \rightarrow \mu^- \alpha^+ \beta^+ X$  as a function of the Majorana mass for a  $\mu^-$  energy of 50 GeV. No limits on  $U_{ai}$  are applied.

Here  $\tilde{q}_2$  denotes the momentum of the Majorana neutrino in the crossed diagram, which has a relative sign due to the interchange of two identical fermion lines. In addition one has to include a factor  $\frac{1}{2}$  to avoid double counting in the phase space integration. Scattering with an antiquark and with an antineutrino is equivalent to the following simple replacements:

$$\begin{aligned} |\overline{\mathcal{M}}|^2(\nu_l \bar{q} \rightarrow l^- \alpha^+ \alpha^+ \bar{q}') &\equiv |\overline{\mathcal{M}}_{-+}|^2 = |\overline{\mathcal{M}}_{--}|^2(p_2 \leftrightarrow k_4), \\ |\overline{\mathcal{M}}|^2(\bar{\nu}_l q \rightarrow l^+ \alpha^- \alpha^- q') &\equiv |\overline{\mathcal{M}}_{+-}|^2 = |\overline{\mathcal{M}}_{-+}|^2, \\ |\overline{\mathcal{M}}|^2(\bar{\nu}_l \bar{q} \rightarrow l^+ \alpha^- \alpha^- \bar{q}') &\equiv |\overline{\mathcal{M}}_{++}|^2 = |\overline{\mathcal{M}}_{--}|^2. \end{aligned} \quad (6)$$

Of course, the two leptons from the intermediate “ $WW \rightarrow \alpha\beta$ ” diagram do not have to be of the same flavor: An interesting statistical effect [8] occurs when one considers the relative difference between, say, the  $\mu\mu$  and the  $\mu e$  final state (mass effects play no role for  $e$  and  $\mu$ ): First, there is no phase space factor  $\frac{1}{2}$  for the latter case. Then, there is the possibility that an electron is produced at the (“upper”)  $\nu l W$  vertex or at the (“lower”)  $q q' W$  vertex. Both diagrams are topologically distinct and thus have to be treated separately. This means, four diagrams lead to the  $\mu e$  final state, whereas only two lead to the  $\mu\mu$  final state. We see that there is a relative factor 4 between the two cases. Note though that now the interference terms are *added* to the two squared amplitudes since there is no relative sign between the two. This reduces the relative factor to about 3.

The details of our Monte Carlo program are given in [10]; now an integration over the incoming neutrino energy spectrum has to be included. A simple phase space calculation for the  $\mu \rightarrow e \nu_e \nu_\mu$  decay gives for the normalized distribution in the laboratory frame:

$$\frac{dN}{dE_\nu}(\nu_\mu) = \frac{2}{E_\mu^3} E_\nu^2 \left( 3 - 2 \frac{E_\nu}{E_\mu} \right),$$

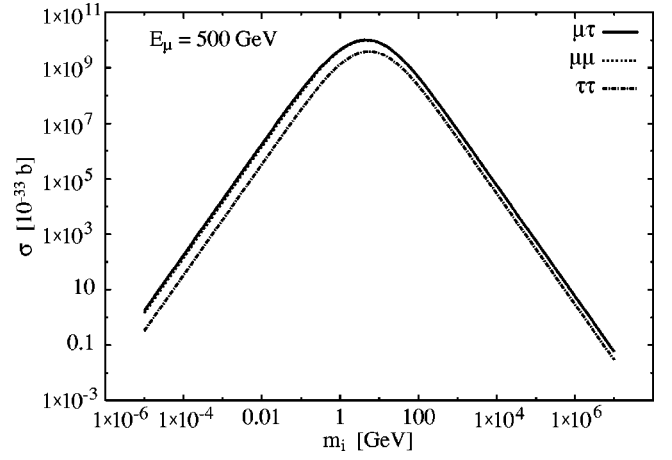


FIG. 4. Total cross section for  $\nu_\mu N \rightarrow \mu^- \alpha^+ \beta^+ X$  as a function of the Majorana mass for a  $\mu^-$  energy of 500 GeV. No limits on  $U_{ai}$  are applied.

$$\frac{dN}{dE_\nu}(\nu_e) = \frac{12}{E_\mu^3} E_\nu^2 \left( 1 - \frac{E_\nu}{E_\mu} \right). \quad (7)$$

The maximal neutrino energy is  $E_\mu$  and the mean value is  $\langle E_\nu \rangle = 7/10 (3/5) E_\mu$  for  $\nu_\mu$  ( $\nu_e$ ).

In Figs. 3, 4, and 5 we show the total cross section for the reaction  $\nu_\mu N \rightarrow \mu^- \alpha^+ \beta^+ X$  with the  $\mu\mu$ ,  $\mu\tau$ , and  $\tau\tau$  final states for three different muon energies. For the above given statistical arguments to be valid, the mass of the final state leptons has to be negligible. This is the case for muon energies higher than about 3 TeV, as can be seen from the figures, where the  $\mu\tau$  final state is the leading signal for  $E_\mu = 4$  TeV. We set  $U_{ai} = 1$  in order to show the mass dependence of the signal; the slope of the curves is easily understood from the two extreme limits of

$$\sigma \propto \frac{m_i^2}{(q^2 - m_i^2)^2} \rightarrow \begin{cases} m_i^2 & \text{for } m_i^2 \ll q^2, \\ m_i^{-2} & \text{for } m_i^2 \gg q^2, \end{cases} \quad (8)$$

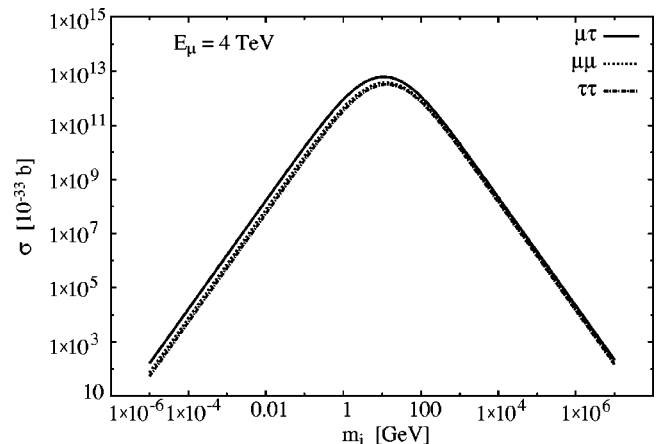


FIG. 5. Total cross section for  $\nu_\mu N \rightarrow \mu^- \alpha^+ \beta^+ X$  as a function of the Majorana mass for a  $\mu^-$  energy of 4 TeV. No limits on  $U_{ai}$  are applied.

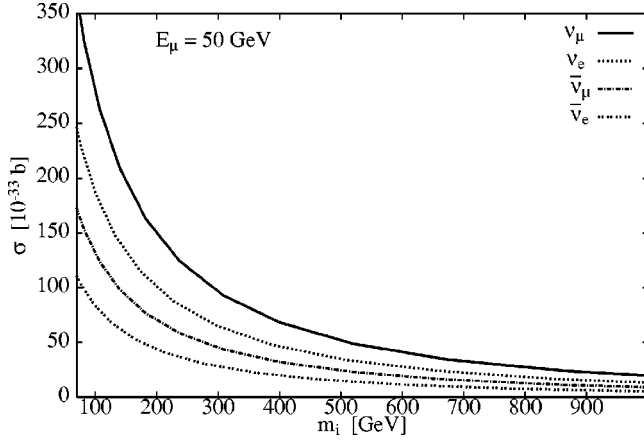


FIG. 6. Total cross section for  $\nu_l N \rightarrow l \mu \mu X$  for  $\nu_\mu, \nu_e, \bar{\nu}_\mu,$  and  $\bar{\nu}_e$  as a function of the Majorana mass for a  $\mu$  energy of 50 GeV. The limits on  $U_{ai}$  are applied.

where  $q$  is the momentum of the Majorana neutrino.

For a  $\mu^- \mu^+$  collider or a muon storage ring four different signals are possible (corresponding to incoming  $\nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu,$  or  $\nu_e$ ); Fig. 6 shows that the muon neutrino from the  $\mu^-$  decay gives the highest cross section. In this figure we plotted the interesting area of the mass range given by the see-saw formula (4) and applied also the limits on heavy neutrino mixing as explained in Sec. III. Finally, we give in Fig. 7 the cross section for  $\nu_\mu N \rightarrow \mu^- \mu^+ \mu^+ X$  with two possible other realizations, namely, via an intermediate right-handed Majorana neutrino and via right-handed interactions with a  $W_R$  mass of 720 GeV, the current lower bound [11]. For the latter case, the matrix elements are identical whereas for the former one has to make the replacement ( $p_2 \leftrightarrow k_4, p_1 \leftrightarrow k_1$ ) in Eqs. (5) and (6). It can be seen that a left-handed heavy neutrino gives the highest contribution. Of course it is possible that all these realizations contribute and thus interfere.

We checked the dependence of the results on oscillation parameters by integrating over two-flavor formulas. Even for  $E_\mu = 50$  GeV, a detector-source distance of 1 km and LSND-like values of  $\Delta m^2$  ( $\approx 0.1$  eV<sup>2</sup>), and  $\sin^2 2\theta$  ( $\approx 10^{-3}$ ) the relative suppression of the signal was not more than  $\mathcal{O}(10^{-6})$ . Inserting typical parameters of atmospheric or solar experiments has even less effect.

### B. Neutrino factories

Several proposals for a muon storage ring have been discussed; the number of expected neutrino interactions differs. The formula used for the luminosity in units of  $\text{cm}^{-2} \text{s}^{-1}$  is [12]

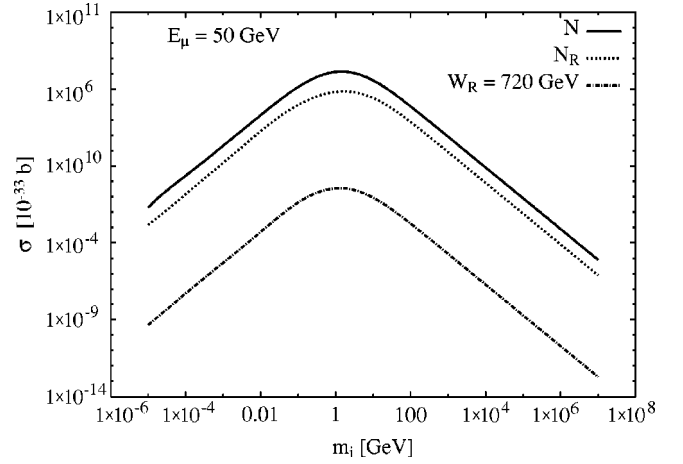


FIG. 7. Total cross section for  $\nu_\mu N \rightarrow \mu^- \alpha^+ \beta^+ X$  as a function of the Majorana mass for a  $\mu^-$  energy of 50 GeV and different possible realizations of the process.  $N$  is a left-handed,  $N_R$  a right-handed Majorana, and  $W_R$  denotes the process with a right-handed  $W$  boson and a left-handed Majorana. No limits on  $U_{ai}$  are applied.

$$\mathcal{L} = N_A f N_\mu l, \quad (9)$$

where  $N_A$  is the Avogadro number,  $N_\mu$  the number of muons injected in the ring per second,  $f$  the fraction of the ring occupied by the production straight section, and  $l$  the mass depth of the target in  $\text{g cm}^{-2}$ . Typical numbers are  $f = 0.02$ ,  $l = 1000$   $\text{g cm}^{-2}$ , and  $N_\mu = 10^{12} - 10^{14} \text{ s}^{-1}$ . Let us be optimistic and assume  $N_\mu = 10^{14} \text{ s}^{-1}$  with a ‘‘year’’ of  $10^7$  s running time. With this parameter set one gets  $\mathcal{L} \approx 10^{39} \text{ cm}^{-2} \text{ s}^{-1}$ . The neutrinos from the  $\mu$  decay will all end inside the detector since their opening angle is just  $\theta \approx 1/\gamma_\mu = m_\mu/E_\mu$ . Typical distances between detector and muon ring are  $10^2$  to  $10^3$  m, discussed energies go up to  $10^6$  GeV. A complete scope of all possible options is not our aim. If definite plans for machines are made our results can easily be rescaled with the help of relation (9).

### III. LIMITS ON NEUTRINO PARAMETERS

As expected,  $0\nu\beta\beta$  provides us with the best limit of all entries in  $\langle m_{\alpha\beta} \rangle$  and  $\langle 1/m_{\alpha\beta} \rangle$ . Recently, other elements of the mass matrix were investigated and for the first time limits on the  $\tau$  sector of  $\langle m_{\alpha\beta} \rangle$  were given [13]. The process discussed was  $e^+ p \rightarrow \bar{\nu}_e \alpha^+ \beta^+ X$  at HERA and gave bounds on  $\langle m_{e\tau} \rangle$ ,  $\langle m_{\mu\tau} \rangle$ , and  $\langle m_{\tau\tau} \rangle$ . In [14] improved limits on  $\langle m_{\mu\mu} \rangle$  (via  $K^+ \rightarrow \pi^- \mu^+ \mu^+$ ) and  $\langle m_{e\mu} \rangle$  ( $\mu^- - e^+$  conversion on titanium) are given. Together with the  $0\nu\beta\beta$  limit [15] the current situation is as follows:

$$\langle m_{\alpha\beta} \rangle \lesssim \begin{pmatrix} 2 \times 10^{-10} [15] & 1.7(8.2) \times 10^{-2} [14] & 4.2 \times 10^3 [13] \\ & 5.0 \times 10^2 [14] & 4.4 \times 10^3 [13] \\ & & 2.0 \times 10^4 [13] \end{pmatrix} \text{ GeV}. \quad (10)$$

TABLE I. Obtainable limits for  $\langle m_{\alpha\beta} \rangle$  and  $\langle 1/m_{\alpha\beta} \rangle$  (in GeV and  $\text{GeV}^{-1}$ ) for different muon energies in GeV. It holds  $\langle m_{ee} \rangle \approx \langle m_{\mu\mu} \rangle \approx 1/3 \langle m_{e\mu} \rangle$  and  $\langle 1/m_{ee} \rangle \approx \langle 1/m_{\mu\mu} \rangle \approx 1/3 \langle 1/m_{e\mu} \rangle$ . For the number of events the parameter set given in Sec. II B is used. For  $\langle 1/m_{\alpha\beta} \rangle$  the limits on  $|U_{ai}|^2$  from Eq. (11) are used.

$E_\mu$	$\langle m_{\mu\mu} \rangle$	$\langle m_{\mu\tau} \rangle$	$\langle m_{\tau\tau} \rangle$	$\langle 1/m_{\mu\mu} \rangle$	$\langle 1/m_{\mu\tau} \rangle$	$\langle 1/m_{\tau\tau} \rangle$
50	25.2	57.6	$1.2 \times 10^3$	12.4	32.6	199.5
100	12.9	21.9	128.2	3.1	1.7	13.4
200	6.6	8.1	26.9	0.8	1.1	1.7
300	4.6	4.7	14.2	0.3	0.4	0.7
400	3.4	3.3	8.1	0.2	0.2	0.3
500	2.8	2.6	5.8	0.2	0.2	0.2
$10^3$	2.0	1.5	3.2	$3.7 \times 10^{-2}$	$3.7 \times 10^{-2}$	0.1
$2 \times 10^3$	0.7	0.5	3.1	$7.4 \times 10^{-3}$	$7.4 \times 10^{-3}$	$7.4 \times 10^{-3}$
$4 \times 10^3$	0.4	0.3	0.2	$2.6 \times 10^{-3}$	$2.3 \times 10^{-3}$	$2.8 \times 10^{-3}$
$10^4$	0.2	0.1	0.2	$5.3 \times 10^{-4}$	$4.9 \times 10^{-4}$	$5.8 \times 10^{-4}$
$10^5$	$2.9 \times 10^{-2}$	$1.4 \times 10^{-2}$	$1.4 \times 10^{-2}$	$2.2 \times 10^{-5}$	$1.6 \times 10^{-5}$	$1.6 \times 10^{-5}$
$10^6$	$4.6 \times 10^{-4}$	$2.9 \times 10^{-4}$	$3.2 \times 10^{-4}$	$1.4 \times 10^{-6}$	$7.0 \times 10^{-7}$	$1.4 \times 10^{-6}$

There is a spread over 14 orders of magnitude. For  $\langle m_{e\mu} \rangle$  two values are given, depending on the spin configuration of the final state protons. Note that for all entries except for the  $ee$ , element the limits lie in the unphysical region; e.g., for  $\langle m_{e\tau} \rangle = 4.2 \times 10^3$  GeV the cross section is proportional to  $m^{-2}$  and not to  $m^2$  as assumed to get the limit. Improvement on most values might be expected from  $B$  decays [14,16,17].

For  $\langle 1/m_{ee} \rangle$  a limit from  $0\nu\beta\beta$  exists [18], for which a heavy neutrino has  $m_i \geq 1$  GeV. Besides neutrinoless double beta decay there are other ways to get information about heavy neutrinos: The LEP machine can produce heavy neutral leptons via  $e^+e^- \rightarrow N\bar{N}$ ; the most stringent limits come from the L3 collaboration [19]; they exclude masses below 70 to 80 GeV, depending on the charged lepton they couple to ( $e$ ,  $\mu$ , or  $\tau$ ). On the other hand, if heavy neutrinos mix with their light SM counterparts, they should alter the results for  $\mu$  decay,  $\nu$  scattering, and so on. Global fits then limit the mixing parameters [20], in total the limits read

$$\begin{aligned} \sum |U_{ei}|^2 &< 6.6 \times 10^{-3}, \quad m_i > 81.8 \text{ GeV}, \\ \sum |U_{\mu i}|^2 &< 6.0 \times 10^{-3}, \quad m_i > 84.1 \text{ GeV}, \quad (11) \\ \sum |U_{\tau i}|^2 &< 1.8 \times 10^{-2}, \quad m_i > 73.5 \text{ GeV}. \end{aligned}$$

In Sec. IV B we will discuss how these bounds restrict the possibilities of observing the  $0\nu\beta\beta$  analogue at a neutrino factory. Before that we apply the procedure from [13] again to the HERA data and gain limits on the other elements of  $\langle 1/m_{\alpha\beta} \rangle$ . Here, heavy neutrinos must have  $m_i \geq 100$  GeV. The matrix reads

$$\left\langle \frac{1}{m_{\alpha\beta}} \right\rangle \leq \begin{pmatrix} 1.1 \times 10^{-8} & 5.4 \times 10^{-3} & 8.6 \times 10^{-3} \\ & 8.4 \times 10^{-3} & 9.0 \times 10^{-3} \\ & & 0.1 \end{pmatrix} \text{ GeV}^{-1}. \quad (12)$$

Now there is only a spread of 8 orders of magnitude. All non- $ee$  entries are unphysical, e.g., for the  $\mu\mu$  element one gets, with the bound from Eq. (11),

$$m_i \geq \frac{|U_{\mu i}|^2}{\left\langle \frac{1}{m_{\mu\mu}} \right\rangle} \approx 0.7 \text{ GeV}. \quad (13)$$

All limits on the same quantities for a *right-handed* Majorana neutrino with the usual couplings to the SM particles lie in the same order of magnitude. Table I shows what limits could be achieved for a luminosity per year given by the parameter set after Eq. (9). The improvement would be tremendous and already for muon energies higher than 500 GeV the bounds on  $\langle m_{\alpha\beta} \rangle$  lie in the physical region: The limit on  $\langle m_{\mu\mu} \rangle$  is about 3 GeV, where the slope of the cross section is still rising, i.e., proportional to  $m_i^2$ . For  $\langle 1/m_{\alpha\beta} \rangle$  the situation is different: For  $E_\mu = 500$  GeV the limit on  $\langle 1/m_{\tau\tau} \rangle$  is about 0.2 GeV, which translates into  $m_i \geq 0.1$  GeV, which is a light neutrino, i.e., in that region is  $\sigma \propto m_i^2$ . Here, energies around 10 TeV are required to get physical meaningful values.

#### IV. DETECTION OF THE PROCESS

##### A. Experimental considerations

Because of the smallness of the cross section of the process discussed here, one might ask if SM processes exist which fake the signal. A discussion of this kind has already been done for trimuon production in  $\nu N$  scattering at previous fixed target experiments, both experimentally [21] and theoretically [22]. This trimuon production has a  $(-++)$  signature. Due to the principle creation of conventional neutrino beams by using pion and kaon decays, there is always a “ $\bar{\nu}_\mu$  pollution” in the beam which can give a  $(-++)$  signal through muon pair production, be it radiatively or in the hadronic final state via, e.g., vector meson production. These effects exist on the level of about  $10^{-4}$  of the total observed charged current events. Kinematical cuts to suppress this

background, e.g., using the invariant mass or angular isolation, have been developed. For previous experiments, however, it was found [10] that for trimuon production via Majorana neutrinos the signal-to-background ratio is far too small. However, for a muon storage ring we know exactly what neutrino flavor is coming in and thus in the case of  $\mu^-$  decay there is no SM process to give a  $(-++)$  event. The only exception is  $\nu_\mu N \rightarrow \mu^- e^+ \mu^+ X$  which might be faked by a  $\bar{\nu}_e$  CC event with  $\mu^+ \mu^-$  production in the jet or via bremsstrahlung. However, as we will show below, final states with electrons cannot be expected due to the severe limits from  $0\nu\beta\beta$ . Because of the  $\nu_\mu \bar{\nu}_e$  or  $\bar{\nu}_\mu \nu_e$  structure of the beam, ratios between observed types of events (coming from each neutrino species) could be used to establish a signal. Also polarization of the muon beam could be useful because it allows to change the neutrino spectra and therefore the event ratios in a predictable way. Possible channels involving  $\tau$  leptons in the final state might be investigated by topological and kinematical methods as used by CHORUS and NOMAD. Finally, it could even be possible to obtain information about  $CP$  violation by comparing event numbers from different channels.

Up to now we ignored in this section effects of neutrino oscillations. An incoming  $\bar{\nu}_e$  could oscillate into a  $\bar{\nu}_\mu$  and create via the aforementioned processes a  $(-++)$  signal. The relevant oscillation parameters are now given by atmospheric ( $\Delta m^2 \approx 10^{-3} \text{ eV}^2$ ) and CHOOZ ( $\sin^2 2\theta \lesssim 0.2$ ) data. Integrating the CC cross section of the  $\bar{\nu}_e$  over a two-flavor formula and also taking into account the factor  $10^{-4}$  yields numbers smaller than the ratio of process (3) with the (CC + NC) cross section by at least one order of magnitude, even for a  $L=1 \text{ km}$  and  $E_\mu=50 \text{ GeV}$  option of the experiment.

For the other final states there is no SM background: Typical events with additional leptons are production of gauge bosons, which, however, are always accompanied with neutrinos or extra jets, and thus in principle distinguishable.

### B. Is it observable?

Unfortunately, the bounds on neutrinos and their mixing severely restrict the prospects of detecting a signal from  $B$ - $L$  violating mass terms at the discussed experiments: For example, let us consider a 4 TeV muon source (be it a collider or just a storage ring) and the  $\mu\mu$  channel. For the moment, we stick to one  $m_i$ . The maximum cross section is achieved for a mass eigenstate of about 10 GeV,  $\sigma_{\mu\mu}(m_i \approx 10 \text{ GeV}) \approx 10^{-20} \text{ b}$ . A few years of running with  $\mathcal{L} \approx 10^{39} \text{ cm}^2 \text{ s}^{-1}$  per year could establish an observation. However, for the minimal allowed mass of 84.1 GeV, the cross section reduces to  $\sigma_{\mu\mu}(m_i = 84.1 \text{ GeV}) \approx 2.0 \times 10^{-21} \text{ b}$ , which is then further suppressed by the  $U_{\mu i}$  limit to  $7.3 \times 10^{-26} \text{ b}$ . Roughly the same number holds for the  $ee$  channel, and for the  $e\mu$  channel about three times this number. However, now the value from Eq. (12) comes into play: Assuming one mass eigenstate of 81.8 GeV one gets  $|U_{ei}|^2 < 9 \times 10^{-7}$ , resulting in  $\sigma_{ee}(m_i = 81.8 \text{ GeV}) \approx 10^{-33} \text{ b}$ . The cross section stays constant till  $m_i = 6 \times 10^5 \text{ GeV}$  and scales with  $m_i^{-2}$  for larger

TABLE II. Maximal ratio of  $\nu_\mu N \rightarrow \mu^- \alpha^+ \beta^+ X$  and sum of CC and NC for  $\nu_\mu$  for different final states and muon energies in GeV. Indirect bounds on mixing matrix elements are applied. The last column displays the expected number of (CC+NC) events from  $\nu_\mu$  with the optimistic parameter set given in Sec. II B.

$E_\mu$	$\sigma_{\mu\mu}/\sigma^{(CC+NC)}$	$\sigma_{\mu\tau}/\sigma^{(CC+NC)}$	$\sigma_{\tau\tau}/\sigma^{(CC+NC)}$	$N^{CC+NC}$
50	$1 \times 10^{-20}$	$7 \times 10^{-21}$	$7 \times 10^{-22}$	$4 \times 10^9$
100	$1 \times 10^{-19}$	$1 \times 10^{-19}$	$7 \times 10^{-20}$	$8 \times 10^9$
200	$7 \times 10^{-19}$	$2 \times 10^{-18}$	$2 \times 10^{-18}$	$2 \times 10^{10}$
300	$3 \times 10^{-18}$	$7 \times 10^{-18}$	$1 \times 10^{-17}$	$2 \times 10^{10}$
400	$7 \times 10^{-18}$	$2 \times 10^{-17}$	$4 \times 10^{-17}$	$4 \times 10^{10}$
500	$1 \times 10^{-17}$	$4 \times 10^{-17}$	$7 \times 10^{-17}$	$4 \times 10^{10}$
$10^3$	$7 \times 10^{-17}$	$4 \times 10^{-16}$	$7 \times 10^{-16}$	$8 \times 10^{10}$
$2 \times 10^3$	$7 \times 10^{-16}$	$3 \times 10^{-15}$	$7 \times 10^{-15}$	$1 \times 10^{11}$
$4 \times 10^3$	$5 \times 10^{-15}$	$2 \times 10^{-14}$	$5 \times 10^{-14}$	$2 \times 10^{11}$
$10^4$	$4 \times 10^{-14}$	$2 \times 10^{-13}$	$5 \times 10^{-13}$	$5 \times 10^{11}$
$10^5$	$2 \times 10^{-12}$	$1 \times 10^{-11}$	$2 \times 10^{-11}$	$3 \times 10^{12}$
$10^6$	$4 \times 10^{-11}$	$8 \times 10^{-10}$	$4 \times 10^{-10}$	$8 \times 10^{12}$

masses. For the  $e\mu$  and  $e\tau$  channel the cross sections are  $\sigma_{e\mu} \approx 2 \times 10^{-31} \text{ b}/m_i [\text{GeV}]$  and  $\sigma_{e\tau} \approx 8 \times 10^{-31} \text{ b}/m_i [\text{GeV}]$ , respectively. Thus, the electron final states of process (3) provide no real chance for observation.

Now we investigate possible event numbers: to be independent on the concrete values of the experimental parameters we calculate the charged and neutral current cross section by integrating over the energy spectrum (7) with the GRV 92 and 98 [23] parton distributions including  $c$  and  $b$  quark contributions. With this we give the maximal ratio [i.e., applying all limits of Eq. (11) for the cross section] of the process (3) as shown in Table II. We considered only the muonic and tauonic final states and took for the  $\mu\tau$  channel the value  $m_i = 84.1 \text{ GeV}$ . The last column gives the number of  $\nu_\mu(\text{CC+NC})$  events with the parameter set given after Eq. (9). Only the highest discussed energy provides a chance for observation. However, the realization of this kind of machine remains doubtful, but might be realized in a form of a new high-energy physics laboratory [24].

Though the numbers are no reason to be overoptimistic, the same final state we discussed might have contributions from other channels as the ones plotted in Fig. 7. For  $0\nu\beta\beta$  many limits on beyond-SM parameters were derived; see [25] for a review. A simple estimation shows the power of such a neutrino factory: For a 4 TeV energy and a 100 keV neutrino, the cross section is about  $10^{-29} \text{ b}$ . Other contributions might not need a helicity flip and are thus larger by roughly a factor of  $(m_\nu/E_m)^2 \approx 10^{13}$ , where  $E_m$  is the energy of the Majorana neutrino. With the mentioned  $10^{39} \text{ cm}^2 \text{ s}^{-1}$  luminosity we would have  $10^6$  events per year, a “new physics factory.”

### C. Mixing matrix for more than one Majorana neutrino

It might be interesting to see what happens if there is more than one heavy Majorana neutrino contributing to the signal. Note that every heavy mass eigenstate contributes

with  $m_i^{-2}U_{ai}^4$  to the cross section. For example, in a degenerate scheme with three mass eigenvalues  $m_i (i=1,2,3)$ , each of 90 GeV mass, the mixing matrix elements could conspire to be  $U_{\mu 1}^2=10^{-3}$ ,  $U_{\mu 2}^2=U_{\mu 3}^2=2.5 \times 10^{-3}$ , thus making the cross section lower by a factor of approximately 1/3. This reduction of the cross section gets worse the more additional mass eigenstates one introduces. For only two heavy neutrinos and  $U_{\mu 1}^2=10^{-3}$  we lose only a factor of 2/3.

For the electron sector the limit from  $0\nu\beta\beta$  in Eq. (12) modifies this procedure a bit: e.g., for two Majorana neutrinos with  $U_{e1}^2=-10^{-4}$  and  $m_1=90$  GeV we get  $U_{e2}^2=6.5 \times 10^{-3}$ , but from that follows  $m_2 \geq 5.8$  TeV. The decrease is by a factor of 2600, which has to be compared with the reduction one would have achieved without the constraint from  $0\nu\beta\beta$ , namely, a factor of 4/5. On the other hand, choosing  $U_{e1}^2=-6.5 \cdot 10^{-3}$  for  $m_1=90$  GeV leads to  $U_{e2}^2=10^{-4}$  but then follows  $m_2 \geq 1.4$  GeV, which is excluded. There are of course a lot of other allowed possibilities, which, however, all result in a reduction of the cross section. The importance of using the heavy neutrino bound from  $0\nu\beta\beta$  was first stressed in [25,26]. A detailed analysis of the topic of cancellation in the effective masses is (for the case of  $\langle m_{ee} \rangle$ ) given in [9].

Will the situation change with future improved mass and mixing limits? First, the high number of neutrino interactions might have impact on the global fits for the mixing matrix elements. However, the limits cannot be expected to be improved by factors larger than  $\mathcal{O}(1)$ . The LEP bound on the neutrino mass from [19] corresponds to about 40% of the used center of mass energy of 189 GeV; for simplicity we can assume that this will also hold for the upgrade energies. As other machines are concerned, at the CERN Large Hadron Collider (LHC) [28,29] or HERA [30], investigation of masses of a few 100 GeV might be possible, but mostly only the electron channels were considered. Applying the  $0\nu\beta\beta$  limit on heavy neutrinos (which has not been done in the analyses) to those results reduces the mass limits in those works. For the muon channels the results are significantly lower [29] or, as for HERA, not yet discussed. Regarding the Next Linear Collider, the pair production  $e^+e^- \rightarrow NN$  produces too small event numbers [31]. Recently it was shown in [32] that for  $\sqrt{s} \geq 500$  GeV the “indirect” process  $e^+e^- \rightarrow \nu e^\pm W^\mp$  might probe Majorana masses up to the center of mass energy. The same holds for the  $e\mu$  option of future colliders via  $e^\pm \mu^\mp \rightarrow \nu t^\pm W^\mp$  [33] and also Majorana neutrino pair production will be observable [34]. Processes such as  $e^- \gamma \rightarrow \nu_e \alpha^- \beta^- W^+$  [27] with  $\alpha, \beta = \mu, \tau$ , or  $e^- e^- \rightarrow \mu^- \mu^-$  [35] might also evade the  $0\nu\beta\beta$  constraint but

require center of mass energies in the same region as the ones discussed here.

## V. CONCLUSIONS

To conclude, we did a full analysis of the analogue of neutrinoless double beta decay at a neutrino factory while applying several experimental limits. The use of a detector right at the muon storage ring provides a very large number of neutrino interactions and for current and future mass limits the signals are perhaps observable at very high muon energies. Furthermore, if observed at lower energies, it is important to know how heavy Majorana neutrinos with SM couplings contribute to the events. The cross sections are very small but at least unaffected by oscillation phenomena. The limits on the effective Majorana mass matrix can be pushed down to physical values even from energies of  $E_\mu = 500$  GeV on. For its “inverse,” however, energies higher by a factor of more than 20 are required. Signatures of the discussed events might be the only chance to find out about Majorana mass terms since most other related  $B-L$  violating processes suffer from tiny ratios to the respective standard model events. Information on future limits on Majorana masses is mostly found in works concentrating on electron final states and thus unimportant when one incorporates  $0\nu\beta\beta$  bounds on heavy Majorana neutrinos. Anyway, our results should not change dramatically even for new CERN  $e^+e^-$  collider LEP limits on direct production or modified global fits. Finally, if there are several neutrinos contributing, the mixing matrix might reduce the results significantly; i.e., if there is only one Majorana neutrino, the cross section is maximal. The reduction of the signal can be worst for the electron channels, which provide, however, no chance for observation anyway, due to the results from neutrinoless double beta decay. Contributions to the same final state without intermediate Majorana neutrinos (e.g., SUSY particles), however, are a very realistic candidate for observation and the prospects for this will be addressed in future works. The process considered in this paper is then the relevant background signal.

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