Existence of the σ meson below 1 GeV and chiral symmetry

Yurii S. Surovtsev*
Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna 141 980, Russia

Dalibor Krupa† and Miroslav Nagy‡
Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 842 28 Bratislava, Slovakia
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On the basis of a simultaneous description of the isoscalar s-wave channel of $\pi\pi$ scattering (from the threshold up to 1.9 GeV) and the $\pi\pi\rightarrow K\overline{K}$ process (from the threshold to $\sim$1.4 GeV where the 2-channel unitarity is valid), a confirmation of the σ meson at $\sim$665 MeV and an indication for the glueball nature of the $f_0(1500)$ state are obtained. These results are achieved in the model-independent approach consisting of the immediate application of such general principles as analyticity and unitarity to the analysis of experimental data. A parameterless description of the $\pi\pi$ background is first given by allowance for the left-hand branch point in the proper uniformizing variable. It is shown that the large $\pi\pi$ background, usually obtained, combines, in reality, the influence of the left-hand branch point and the contribution of a wide resonance at $\sim$665 MeV. The coupling constants of the observed states with the $\pi\pi$ and $K\overline{K}$ systems and lengths of the $\pi\pi$ and $K\overline{K}$ scattering are obtained. The existence of the $f_0(665)$ state and the obtained $\pi\pi$-scattering length ($a_0^0 = 0.27\pm0.06,[m_{\pi^+}^{-1}]$) seem to suggest the linear realization of chiral symmetry.

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I. INTRODUCTION

The problem of scalar mesons is the most troublesome and long lived in light meson spectroscopy. The main difficulty in understanding the scalar-isoscalar sector seems to be related with the possible considerable influence of the vacuum and such effects as the instanton contributions that are difficult to take into account. But there is another difficulty related to a strong model dependence of information on multichannel states obtained in analyses based on specific dynamic models or on an insufficiently flexible representation of states (e.g., the standard Breit-Wigner form). Especially, this concerns scalar mesons due to the weakest kinematic diminution of their widths. It was observed that the scalar mesons are either very large or, if narrow, lie near the channel thresholds. Earlier, we have shown [1] that an inadequate description of multichannel states gives not only their distorted parameters when analyzing data but also can cause the fictitious states when one neglects important (even energetic closed) channels. In this paper conversely, we are going, to demonstrate that the large background (e.g., this happens in analyzing $\pi\pi$ scattering) can hide low-lying states (even such important for theory as a σ meson [2]). With this object, a very interesting and instructive history is related. Majority of analyses rejected this meson by resolving the known ''up-down ambiguity'' in the 700–900 MeV region in solutions of the $\pi\pi$ phase-shift analyses for δ_0^0 in favor of the ''down'' one, because to the ''up'' solution, one related the $\epsilon(800)$ resonance of width $\sim$150–200 MeV. However, some of theorists continued to insist that this state exists, because it is required by most models (such as the linear σ models or the Nambu–Jona-Lasinio models [3]) for spontaneous breaking of chiral symmetry. Since all the analyses of the s-wave $\pi\pi$ scattering gave a large $\pi\pi$ background, it was said that this state (if exists) is ''unobservably wide.'' Recently, new analyses of the old and new experimental data have been performed which give a very wide scalar-isoscalar state in the energy region 500–850 MeV [4,8]. However, these analyses use either the Breit-Wigner form (even if modified) or specific forms of interactions in a quark model, unitarized by taking the relevant process thresholds into account, or in a multichannel approach to the considered processes. Therefore, there one cannot talk about results being model independent. In addition, in these analyses, a large $\pi\pi$ background is obtained. We are going to show that a proper detailing of the background (as allowance for the left-hand branch point) permits us to extract, from the latter, a very wide (but observable) state below 1 GeV even in the ''down'' solution for the $\pi\pi$ phase shift, and, therefore, it is highly important in studying lightest states.

An adequate consideration of multichannel states and a model-independent information on them can be obtained on the basis of the first principles (analyticity, unitarity, and Lorentz invariance) immediately applied to analyzing experimental data. The way of realization is a consistent allowance for the nearest singularities on all sheets of the Riemann surface of the S matrix. The Riemann-surface structure is taken into account by a proper choice of the uniformizing variable. Earlier, we have proposed this method for 2- and 3-channel resonances and developed the concept of standard clusters (poles on the Riemann surface) as a qualitative characteristic of a state and a sufficient condition of its existence as well as a criterion of a quantitative description of the coupled-process amplitudes when all the complications of the analytic structure due to a finite width of resonances and

*Email address: surovcev@thsun1.jinr.ru
†Email address: krupa@savba.sk
‡Email address: fyzinami@nic.savba.sk

crossing channels and high-energy ‘‘tails’’ are accumulated in quite a smooth background [1,9,10]. Let us stress that for a wide state, the pole position (the pole cluster one for multichannel states) is a more stable characteristic than the mass and width which are strongly dependent on a model. The cluster kind is determined from the analysis of experimental data and is related to the state nature. At all events, we can, in a model-independent manner, discriminate between bound states of particles and the ones of quarks and gluons, qualitatively predetermine the relative strength of coupling of a state with the considered channels, and obtain an indication on its gluonium nature.

Since, in this work, a main stress is laid on studying lowest states, it is sufficient to make use of a two-channel approach when considering simultaneously the coupled processes $\pi\pi\to\pi\pi,K\bar{K}$. Furthermore, in the uniformizing variable, one must take into account, in addition to the branch points corresponding to the thresholds of the processes $\pi\pi\to\pi\pi,K\bar{K}$, also the left-hand branch-point at $s=0$ related to the background which contains the crossing-channel contributions.

The layout of the paper is as follows. In Sec. II, we outline the two-coupled-channel formalism, determine the pole clusters on the Riemann surface as characteristics of multichannel states, and introduce a new uniformizing variable, allowing for the branch points of the right-hand (unitary) and left-hand cuts of the $\pi\pi$-scattering amplitude. In Sec. II, we analyze simultaneously experimental data on the processes $\pi\pi\to\pi\pi,K\bar{K}$ in the isoscalar s wave on the basis of the presented approach. In the Conclusion, the obtained results are discussed.

II. TWO-COUPLED-CHANNEL FORMALISM

Considering the multichannel problem (here the 2-channel one), we pursue two aims: to obtain a model-independent information about the multichannel resonances and an indication about their QCD nature, and to describe the experimental data on the coupled processes. The first purpose is achieved through the account of the nearest (to the physical region of interest) singularities of the S matrix. Herewith it is important to analyze simultaneously experimental data on the coupled processes.

Here we consider the coupled processes of $\pi\pi$ and $K\bar{K}$ scattering and $\pi\pi\to K\bar{K}$. Therefore, we have the two-channel S matrix determined on the 4-sheeted Riemann surface. The S-matrix elements $S_{\alpha\beta}$, where $\alpha,\beta=1(\pi\pi),2(K\bar{K})$, have the right-hand (unitary) cuts along the real axis of the s-variable complex plane, starting at the points $4m_\pi^2$ and $4m_K^2$ and extending to ∞, and the left-hand cuts, which are related to the crossing-channel contributions and extend along the real axis towards $-\infty$ and begin at $s=0$ for S_{11} and at $4(m_K^2-m_\pi^2)$ for S_{22} and S_{12}. We number the Riemann-surface sheets according to the signs of analytic continuations of the channel momenta

$$k_1=(s/4-m_\pi^2)^{1/2},\quad k_2=(s/4-m_K^2)^{1/2} \tag{1}$$

as follows: signs $(\mathrm{Im}\,k_1,\mathrm{Im}\,k_2)=++,-+,--,+-$ correspond to the sheets I, II, III, IV. Then, for instance, from the physical region on sheet I we pass across the cut below the $K\bar{K}$ threshold to sheet II; above the $K\bar{K}$ threshold, to sheet III.

To elucidate the resonance representation on the Riemann surface, we express analytic continuations of the matrix elements to the unphysical sheets $S_{\alpha\beta}^L$ ($L=\mathrm{II,III,IV}$) in terms of those on the physical sheet $S_{\alpha\beta}^I$. Those expressions are convenient for our purpose because, on sheet I (the physical sheet), the matrix elements $S_{\alpha\beta}^I$ can have only zeroes beyond the real axis. Using the reality property of the analytic functions and the 2-channel unitarity, one can obtain

$$\begin{gathered}
S_{11}^{\mathrm{II}}=\frac{1}{S_{11}^I},\quad S_{11}^{\mathrm{III}}=\frac{S_{22}^I}{\det S^I},\quad S_{11}^{\mathrm{IV}}=\frac{\det S^I}{S_{22}^I},\\
S_{22}^{\mathrm{II}}=\frac{\det S^I}{S_{11}^I},\quad S_{22}^{\mathrm{III}}=\frac{S_{11}^I}{\det S^I},\quad S_{22}^{\mathrm{IV}}=\frac{1}{S_{22}^I},\\
S_{12}^{\mathrm{II}}=\frac{iS_{12}^I}{S_{11}^I},\quad S_{12}^{\mathrm{III}}=\frac{-S_{12}^I}{\det S^I},\quad S_{12}^{\mathrm{IV}}=\frac{iS_{12}^I}{S_{22}^I}.
\end{gathered} \tag{2}$$

Here $\det S^{\mathrm{I}}=S_{11}^{\mathrm{I}}S_{22}^{\mathrm{I}}-(S_{12}^{\mathrm{I}})^2$. Provided a resonance has the only decay mode (1-channel case), in the matrix element, the resonance (in the limit of its narrow width) is represented by a pair of complex conjugate poles on the second sheet and by a pair of conjugate zeroes on the physical sheet at the same points of complex energy. This model-independent statement about the poles as the nearest singularities holds also when taking account of the finite width of a resonance. In the case of two coupled channels, formulas (2) immediately give the resonance representation by poles and zeroes on the 4-sheeted Riemann surface. Here one must discriminate between three types of resonances which have the following representstions: (a) a pair of complex conjugate poles on sheet II and, therefore, a pair of complex conjugate zeroes on the first sheet in S_{11}; (b) a pair of conjugate poles on sheet IV and, therefore, a pair of complex conjugate zeroes on sheet I in S_{22}; (c) a pair of conjugate poles on each of sheets II and IV, that is, a pair of conjugate zeroes on the physical sheet in each of matrix elements S_{11} and S_{22}.

As seen from Eq. (2), to the resonances of types (a) and (b) one has to make correspond a pair of complex conjugate poles on sheet III which are shifted relative to a pair of poles on sheet II and IV , respectively [if the coupling among channels were absent, i.e., $S_{12}=0$, the poles on sheet III would lay exactly (a) under the poles on the second sheet, (b) above the poles on the fourth sheet]. For the states of type (c) one must consider corresponding two pairs of conjugate poles on sheet III which are reasonably expected to be a pair of the complex conjugate compact formations of poles. Thus, we arrive at the notion of three standard pole clusters which represent two-channel bound states of quarks and gluons. It is convenient to distinguish between those clusters according to the presence of zeroes, corresponding to the state, on the physical sheet in matrix element S_{11} (a), S_{22} (b), or in both (c).

Note that this resonance division into types is not formal. In paricular, the resonance, coupled relatively more strongly to the first ($\pi\pi$) channel than to the second ($K\overline{K}$) one, is described by the pole cluster of type (a); in the opposite case it is represented by the cluster of type (b) (say, if it has a dominant $s\overline{s}$ component). Finally, since the most noticeable property of a glueball is the flavor-singlet structure of its wave function and, therefore, (except the factor $\sqrt{2}$ for a channel with neutral particles) it has practically equal coupling with all the members of the nonet, then a glueball must be represented by the pole cluster of type (c) as a necessary condition.

Just as in the 1-channel case, the existence of a particle bound-state means the presence of a pole on the real axis under the threshold on the physical sheet, so in the 2-channel case, the existence of a bound state in channel 2 ($K\overline{K}$ molecule), which, however, can decay into channel 1 ($\pi\pi$ decay), would imply the presence of a pair of complex conjugate poles on sheet II under the threshold of the second channel without an accompaniment of the corresponding shifted pair of poles on sheet III. Namely, according to this test, earlier an interpretation of the $f_0(980)$ state as $K\overline{K}$ molecule has been rejected [1,9,11].

Generally, formulas (2) are a solution of the 2-channel problem in the sense of giving a chance to predict (on the basis of the data on one process) the coupled-process amplitudes under a certain conjecture about the background. We made this earlier in the 2-channel approach [9]. It was a success to describe ($\chi^2/ndf \approx 1.06$) the experimental isoscalar s wave of $\pi\pi$ scattering from the threshold to 1.9 GeV, to predict satisfactorily (on the basis of data on $\pi\pi$ scattering) the behavior of the s wave of $\pi\pi \rightarrow K\overline{K}$ process approximately up to 1.25 GeV. To take account of the proper right-hand branch points, the corresponding uniformizing variable has been used. However, for the simultaneous analysis of experimental data on the coupled processes it is more convenient to use the Le Couteur–Newton relations [12] representing compactly all features given by formulas (2) and expressing the S-matrix elements of all coupled processes in terms of the Jost matrix determinant $d(k_1,k_2) \equiv d(s)$, the real analytic function with the only square-root branch points at the process thresholds $k_i = 0$ [13]. These branch points should be taken into account in the corresponding uniformizing variable. Earlier, this was done by us in the 2-channel consideration [9] with the uniformizing variable

$$z = \frac{k_1 + k_2}{\sqrt{m_K^2 - m_\pi^2}}, \tag{3}$$

which was proposed in Ref. [13] and maps the 4-sheeted Riemann surface with two unitary cuts, starting at the points $4m_\pi^2$ and $4m_K^2$, onto the plane. (Note that other authors have used the parametrizations with the Jost functions in analyzing the s-wave $\pi\pi$ scattering in the one-channel approach [14] and in the two-channel one [11]. In Ref. [11], the uniformizing variable k_2 has been used, therefore, their approach cannot be employed near the $\pi\pi$ threshold.)

When analyzing the processes $\pi\pi \rightarrow \pi\pi, K\overline{K}$ by the above methods in the 2-channel approach, two resonances [$f_0(975)$ and $f_0(1500)$] are found to be sufficient for a satisfactory description ($\chi^2/N_{DF} \approx 1.00$). However, in this case, a large $\pi\pi$-background has been obtained. The character of the background representation (the pole of second order on the imaginary axis on sheet II and the corresponding zero on sheet I) suggests that a wide light state is possibly hidden in the background. To check this, one must work out the background in some detail.

Now we will take into account also the left-hand branch-point at $s=0$ in the uniformizing variable

$$v = \frac{m_K \sqrt{s - 4m_\pi^2} + m_\pi \sqrt{s - 4m_K^2}}{\sqrt{s(m_K^2 - m_\pi^2)}}. \tag{4}$$

The variable v maps the 4-sheeted Riemann surface, having (in addition to the two above-indicated unitary cuts) also the left-hand cut starting at the point $s=0$, onto the v plane.[1] It is convenient to write also the function $s(v)$

$$s = -\frac{16 m_K^2 m_\pi^2 v^2}{(m_K^2 - m_\pi^2)(v-b)(v+b)(v-b^{-1})(v+b^{-1})}, \tag{5}$$

where $b = \sqrt{(m_K + m_\pi)/(m_K - m_\pi)}$ is the point into which $s = \infty$ is mapped on the v plane. The symmetry properties of this function

$$s(v) = s(-v) = s(v^{-1}) = s(-v^{-1}) = s^*(v^*) \tag{6}$$

demonstrate which points on the v plane correspond to the same point on the s plane.

In Fig. 1, the plane of the uniformizing variable v for the $\pi\pi$-scattering amplitude is depicted. The Roman numerals (I, . . . ,IV) denote the images of the corresponding sheets of the Riemann surface; the thick line represents the physical region; the points i, 1, and b correspond to the $\pi\pi, K\overline{K}$ thresholds, and $s=\infty$, respectively; the shaded intervals $(-\infty, -b]$, $[-b^{-1}, b^{-1}]$, $[b, \infty)$ are the images of the corresponding edges of the left-hand cut. The depicted positions of poles (*) and of zeroes (○) give the representation of the type (a) resonance in S_{11}. In Fig. 1, a very symmetric picture is shown which ensures the known fact that the $\pi\pi$ interaction is practically elastic up to the $K\overline{K}$ threshold [the contribution of the multiparticle states (4π, 6π) is negligible within the up-to-date experiment accuracy]. This property of the $\pi\pi$ interaction is satisfied since the poles and zeroes are symmetric to each other with respect to the unit circle. If the $\pi\pi$ scattering were elastic also above the $K\overline{K}$ threshold, there would be the symmetry of the poles and zeroes with

[1] The analogous uniformizing variable has been used, e.g., in Ref. [15] in studying the forward elastic $p\overline{p}$ scattering amplitude.

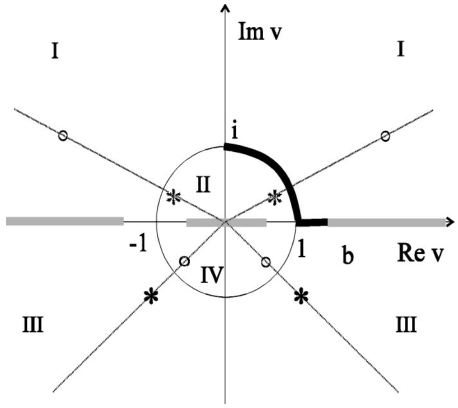

FIG. 1. Uniformization plane for the $\pi\pi$-scattering amplitude. The Roman numerals (I, . . . ,IV) denote the images of the corresponding sheets of the Riemann surface; the thick line represents the physical region (the points i, 1, and b correspond to the $\pi\pi, K\bar{K}$ thresholds, and $s=\infty$, respectively); the shaded lines are the images of the corresponding edges of the left-hand cut. The depicted positions of poles (*) and of zeroes (○) give the representation of the type (a) resonance in S_{11}.

respect to the real axis. The symmetry of the whole picture relative to the imaginary axis ensures the property of the real analyticity.

On the v plane the Le Couteur–Newton relations are [9,13]

$$S_{11}=\frac{d(-v^{-1})}{d(v)}, \quad S_{22}=\frac{d(v^{-1})}{d(v)}, \quad S_{11}S_{22}-S_{12}^2=\frac{d(-v)}{d(v)}. \tag{7}$$

Then, the condition of the real analyticity implies

$$d(-v^*)=d^*(v) \tag{8}$$

for all v, and the unitarity needs the following relations to hold true for the physical v values:

$$|d(-v^{-1})|\leqslant|d(v)|, \quad |d(v^{-1})|\leqslant|d(v)|,$$
$$|d(-v)|=|d(v)|. \tag{9}$$

The d function that on the v plane does not already possess branch points is taken as

$$d=d_B d_{\mathrm{res}}, \tag{10}$$

where $d_B=B_\pi B_K$; B_π contains the possible remaining $\pi\pi$-background contribution, related to exchanges in crossing channels; B_K is that part of the $K\bar{K}$ background which does not contribute to the $\pi\pi$-scattering amplitude. The most considerable part of the background of the considered coupled processes related to the influence of the left-hand branch point at $s=0$ is taken into account already in the uniformizing variable v (4). The function $d_{res}(v)$ represents the contribution of resonances, described by one of three types of the pole-zero clusters, i.e., except for the point $v=0$, it consists of zeroes of clusters

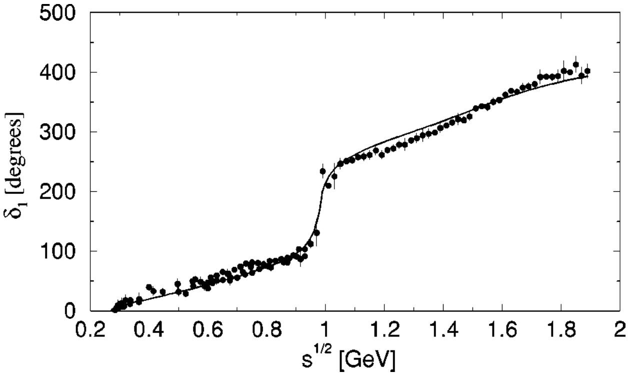

FIG. 2. The energy dependence of the phase shift (δ_1) of the $\pi\pi$-scattering amplitude obtained on the basis of a simultaneous analysis of the experimental data on the coupled processes $\pi\pi\to\pi\pi, K\bar{K}$ in the channel with $I^G J^{PC}=0^+0^{++}$. The data on the $\pi\pi$ scattering are taken from Refs. [16].

$$d_{\mathrm{res}}=v^{-M}\prod_{n=1}^{M}(1-v_n^* v)(1+v_n v), \tag{11}$$

where n runs over the independent zeroes; therefore, for resonances of the types (a) and (b), n has two values, for the type (c), four values; M is the number of pairs of the conjugate zeroes.

III. ANALYSIS OF EXPERIMENTAL DATA.

Using the described 2-channel approach, we analyze simultaneously the available experimental data on the $\pi\pi$ scattering [16] and the process $\pi\pi\to K\bar{K}$ [17] in the channel with $I^G J^{PC}=0^+0^{++}$. As data, we use the results of phase analyses which are given for phase shifts of the amplitudes (δ_1 and δ_{12}) and for moduli of the S-matrix elements η_1 (the elasticity parameter) and ξ:

$$S_a=\eta_a e^{2i\delta_a} \quad (a=1,2), \quad S_{12}=i\xi e^{i\delta_{12}}. \tag{12}$$

(Remember that ''1'' denotes here the $\pi\pi$ channel and ''2'' denotes $K\bar{K}$.) The 2-channel unitarity condition gives

$$\eta_1=\eta_2=\eta, \quad \xi=(1-\eta^2)^{1/2}, \quad \delta_{12}=\delta_1+\delta_2. \tag{13}$$

We have taken the data on the $\pi\pi$ scattering from the threshold up to 1.89 GeV. Then, comparing experimental data for ξ with values of ξ, calculated by Eq. (13) with the use of experimental points for the elasticity parameter η, one can see that the 2-channel unitarity takes place to $\sim$1.4 GeV.

To obtain the satisfactory description of the s-wave $\pi\pi$ scattering from the threshold to 1.89 GeV (Figs. 2 and 3), we have taken $B_\pi=1$ in Eq. (10), and three multichannel resonances turned out to be sufficient: the two ones of the type (a) [$f_0(665)$ and $f_0(980)$] and $f_0(1500)$ of the type (c). Therefore, in Eq. (11) $M=8$ and the following zero positions on the v plane, corresponding to these resonances, have been established in this situation with the parameterless description of the background:

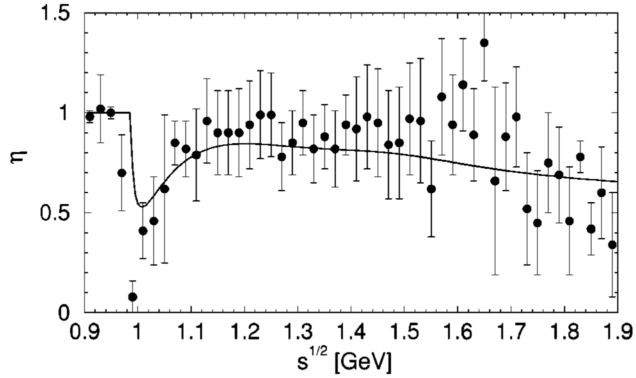

FIG. 3. The same as in Fig. 2 but for the elasticity parameter η.

$$\text{for } f_0(665): \quad v_1 = 1.36964 + 0.208632i,$$

$$v_2 = 0.921962 - 0.25348i,$$

$$\text{for } f_0(980): \quad v_3 = 1.04834 + 0.0478652i,$$

$$v_4 = 0.858452 - 0.0925771i,$$

$$\text{for } f_0(1500): \quad v_5 = 1.2587 + 0.0398893i,$$

$$v_6 = 1.2323 - 0.0323298i,$$

$$v_7 = 0.809818 - 0.019354i,$$

$$v_8 = 0.793914 - 0.0266319i.$$

Here for the phase shift δ_1 and the elasticity parameter η, 113 and 50 experimental points [16], respectively, are used; when rejecting the points at energies 0.61, 0.65, and 0.73 GeV for δ_1 and at 0.99, 1.65, and 1.85 GeV for η which give an anomalously large contribution to χ^2, we obtain for χ^2/N_{DF} the values 2.7 and 0.72, respectively; the total χ^2/N_{DF} in the case of $\pi\pi$ scattering is 1.96.

With the presented picture, the satisfactory description for the modulus (ξ) of the $\pi\pi \to K\bar{K}$ matrix element is given from the threshold to $\sim$ 1.4 GeV (Fig. 4). Here 35 experimental points [17] are used; $\chi^2/N_{DF} \approx 1.11$ when eliminating the points at energies 1.002, 1.265, and 1.287 GeV (with an especially large contribution to χ^2). However, for the phase shift $\delta_{12}(s)$, a slightly excessive curve is obtained. Therefore, keeping the *parameterless* description of the $\pi\pi$ background, one must take into account the part of the $K\bar{K}$ background that does not contribute to the $\pi\pi$-scattering amplitude. Furthermore, this contribution is to be elastic. Note that the variable v is uniformizing for the $\pi\pi$-scattering amplitude, i.e., on the v plane, S_{11} has no cuts, however, the amplitudes of the $K\bar{K}$ scattering and $\pi\pi \to K\bar{K}$ process do have the cuts on the v plane, which arise from the left-hand cut on the s plane, starting at the point $s = 4(m_K^2 - m_\pi^2)$. Under the $s \to v$ conformal mapping (4), this left-hand cut is mapped into cuts which begin at the points

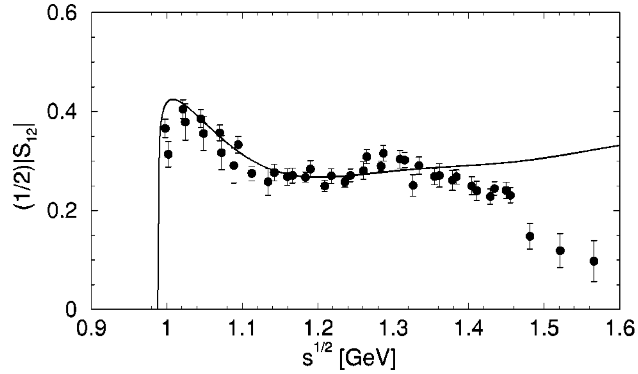

FIG. 4. The energy dependence of the ($|S_{12}|$) obtained on the basis of a simultaneous analysis of the experimental data on the coupled processes $\pi\pi \to \pi\pi, K\bar{K}$ in the channel with $I^G J^{PC} = 0^+ 0^{++}$. The data on the process $\pi\pi \to K\bar{K}$ are taken from Ref. [17].

$$v = \frac{m_K \sqrt{m_K^2 - 2m_\pi^2} \pm i m_\pi}{m_K^2 - m_\pi^2}$$

on the unit circle on the v plane, go along it up to the imaginary axis, and occupy the latter. This left-hand cut will be neglected in the Riemann-surface structure, and the contribution on the cut will be taken into account in the $K\bar{K}$ background as a pole on the real s axis on the physical sheet in the sub-$K\bar{K}$-threshold region. On the v plane, this pole gives two poles on the unit circle in the upper half-plane, symmetric to each other with respect to the imaginary axis, and two zeroes, symmetric to the poles with respect to the real axis, i.e., at describing the process $\pi\pi \to K\bar{K}$, one additional parameter is introduced, say, a position p of the zero on the unit circle. Therefore, for B_K in Eq. (10) we take the form

$$B_K = v^{-4}(1 - pv)^4 (1 + p^* v)^4. \tag{14}$$

The fourth power in Eq. (14) is stipulated by the following. First, a pole on the real s axis on the physical sheet in S_{22} is accompanied by a pole in sheet II at the same s value [as is seen from Eqs. (2)]. On the v plane, this implies the pole of second order (and also zero of the same order, symmetric to the pole with respect to the real axis). Second, for the s-channel process $\pi\pi \to K\bar{K}$, the crossing u and t channels are the π-K and $\bar{\pi}$-K scattering (exchanges in these channels give contributions on the left-hand cut). This results in an additional doubling of the multiplicity of the indicated pole on the v plane. Zeroes of the fourth order in B_K (and, respectively, poles of the fourth order in the $K\bar{K}$ amplitude) provide a better description of the $K\bar{K}$ background than the ones of the first order in our recent work [18]. One can verify that the expression (14) does not contribute to S_{11}, i.e., the parameterless description of the $\pi\pi$ background is kept. A satisfactory description of the phase shift $\delta_{12}(\sqrt{s})$ (Fig. 5) is obtained to $\sim$1.52 GeV with the value of the parameter $p = 0.948201 + 0.31767i$ (this corresponds to the pole position on the s plane at $s = 0.434$ GeV^2). Here 59 experimental

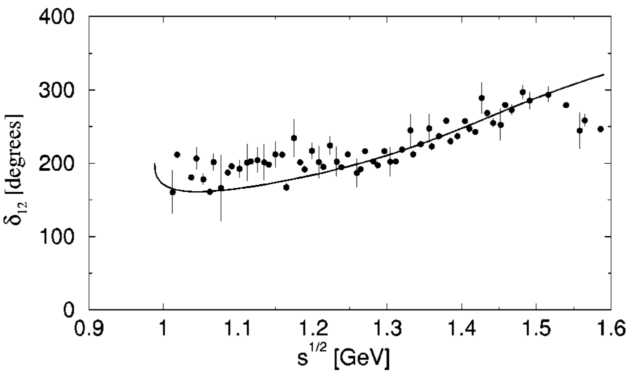

FIG. 5. The same as in Fig. 4 but for the phase shift (δ_{12}).

points [17] are considered; $\chi^2/N_{\rm DF}\approx 3.05$ when eliminating the points at energies 1.117, 1.247, and 1.27 GeV (with an especially large contribution to χ^2). The total $\chi^2/N_{\rm DF}$ for four analyzed quantities to describe the coupled processes $\pi\pi\to\pi\pi, K\overline{K}$ is 2.12. The number of adjusted parameters is 17, where they all (except the single one related to the $K\overline{K}$ background) are positions of poles describing resonances.

In Table I, the obtained poles on the corresponding sheets of the Riemann surface are shown on the complex energy plane ($\sqrt{s_r}=E_r-i\Gamma_r$). We stress that these are not masses and widths of resonances. Since, for wide resonances, values of masses and widths are very model dependent, it is reasonable to report characteristics of pole clusters which must be rather stable for various models.

TABLE I. Pole clusters for obtained resonances.

	$f_0(665)$		$f_0(980)$		$f_0(1500)$	
Sheet	E, MeV	Γ, MeV	E, MeV	Γ, MeV	E, MeV	Γ, MeV
II	610 ± 14	620 ± 26	988 ± 5	27 ± 8	1530 ± 25	390 ± 30
III	720 ± 15	55 ± 9	984 ± 16	210 ± 22	1430 ± 35	200 ± 30
					1510 ± 22	400 ± 34
IV					1410 ± 24	210 ± 38

Now we can calculate the coupling constants of the obtained states with the $\pi\pi$-''1'' and $K\overline{K}$-''2'' systems through the residues of amplitudes at the pole on sheet II. Expressing the T matrix via the S matrix as

$$S_{ii}=1+2i\rho_i T_{ii}, \quad S_{12}=2i\sqrt{\rho_1\rho_2}T_{12}, \tag{15}$$

where $\rho_i=\sqrt{(s-4m_i^2)/s}$, and taking the resonance part of the amplitude in the form

$$T_{ij}^{\rm res}=\sum_r g_{ir}g_{rj}D_r^{-1}(s), \tag{16}$$

where $D_r(s)$ is an inverse propagator $[D_r(s)\propto s-s_r]$, we define the coupling constants as

$$g_i g_j=\frac{16m_K^2m_\pi^2}{3(m_K^2-m_\pi^2)}\left|\frac{v_r^{*2}-v_r^{*-2}}{(v_r^{*2}-b^2)(v_r^{*2}-b^{-2})(v_r^{*-2}-b^2)(v_r^{*-2}-b^{-2})}\lim_{v\to v_r^{*-1}}(1-v_r^* v)\frac{S_{ij}(v)}{\sqrt{\rho_i\rho_j}}\right|. \tag{17}$$

Here we denote the coupling constants with the $\pi\pi$ and $K\overline{K}$ systems through g_1 and g_2, respectively. The obtained values of the coupling constants of the observed states are given in Table II.

In this 2-channel approach, there is no point in calculating the coupling constant of the $f_0(1500)$ state with the $K\overline{K}$ system, because the 2-channel unitarity is valid only to 1.4 GeV and above this energy there is a considerable disagreement between the calculation of the amplitude modulus S_{12} and the experimental data.

Let us indicate also scattering lengths calculated in our approach. For the $K\overline{K}$ scattering, we obtain

$$a_0^0(K\overline{K})=-1.188\pm 0.13+(0.648\pm 0.09)i, \quad [m_{\pi^+}^{-1}].$$

A presence of the imaginary part in $a_0^0(K\overline{K})$ reflects the fact, that already at the threshold of the $K\overline{K}$ scattering, other channels (2π, 4π, etc.) are opened.

In Table III, we compare our result for the $\pi\pi$ scattering length a_0^0 with results of some other works both theoretical and experimental. We here presented model-independent results: the pole positions, coupling constants, and scattering lengths. The former can be used further for calculating masses and widths of these states in various models.

If we suppose that the obtained state $f_0(665)$ is the σ meson, then from the known relation of the σ model between the coupling constant of the σ with the $\pi\pi$ system and masses

$$g_{\sigma\pi\pi}=\frac{m_\sigma^2-m_\pi^2}{\sqrt{2}f_{\pi^0}}$$

(here f_{π^0} is the constant of the weak decay of the π^0: f_{π^0}

TABLE II. Coupling constants of the observed states with the $\pi\pi$ (g_1) and $K\overline{K}$ (g_2) systems.

	$f_0(665)$	$f_0(980)$	$f_0(1500)$
g_1, GeV	0.7477 ± 0.095	0.1615 ± 0.03	0.899 ± 0.093
g_2, GeV	0.834 ± 0.1	0.438 ± 0.028	

TABLE III. Comparison of results of various works for the $\pi\pi$ scattering length a_0^0.

a_0^0, $m_{\pi^+}^{-1}$	References	Remarks
0.27 ± 0.06	our paper	model-independent approach
0.26 ± 0.05	L. Rosselet *et al.* [16]	analysis of the decay $K\to\pi\pi e\nu$ using Roy's model
0.24 ± 0.09	A.A. Bel'kov *et al.* [16]	analysis of the process $\pi^- p\to\pi^+\pi^- n$ using the effective range formula
0.23	S. Ishida *et al.* [6]	modified analysis of $\pi\pi$ scattering using Breit-Wigner forms
0.16	S. Weinberg [19]	current algebra (nonlinear σ model)
0.20	J. Gasser, H. Leutwyler [20]	theory with one-loop corrections, nonlinear realization of chiral symmetry
0.217	J. Bijnens *et al.* [21]	theory with two-loop corrections, nonlinear realization of chiral symmetry
0.26	M.K. Volkov [22]	theory with linear realization of chiral symmetry

$=93.1$ MeV), we obtain $m_\sigma\approx 342$ MeV. That small value of the σ mass can be in part a result of the mixing with the $f_0(980)$ state [23].

IV. CONCLUSIONS

In the present work, in the model-independent approach consisting in the immediate application of first principles (analyticity causality and unitarity) to the analysis of experimental data, a satisfactory simultaneous description of the isoscalar s-wave channel of the processes $\pi\pi\to\pi\pi,K\overline{K}$ from the thresholds to the energy values, where the 2-channel unitarity is valid, is obtained with three states [$f_0(665)-\sigma$ meson, $f_0(980)$ and $f_0(1500)$] that are sufficient to describe the analyzed data. A parameterless description of the $\pi\pi$ background is first given by allowance for the left-hand branch-point in the proper uniformizing variable. It is shown that the large $\pi\pi$ background, usually obtained in various analyses, combines, in reality, the influence of the left-hand branch-point and the contribution of a wide resonance at $\sim$665 MeV. Thus, a model-independent confirmation of the state, already discovered in other works [5–8] (or claiming this discovery) and denoted in the PDG issues by $f_0(400-1200)$ [2], is obtained. This is the σ meson required by majority of models for spontaneous breaking of chiral symmetry. Note also that a light σ meson is needed, for example, for an explanation of $K\to\pi\pi$ transitions using the Dyson-Schwinger model [24].

The discovery of the $f_0(665)$ state solves one important mystery of the scalar-meson family that is related to the Higgs boson of the hadronic sector. This is a result of principle, because the schemes of nonlinear realization of the chiral symmetry have been considered which do without the Higgs mesons. One can think that a linear realization of the chiral symmetry (at least, for the lightest states and related phenomena) is valid. First, this is a simple and beautiful mechanism that works also in other fields of physics, for example, in superconductivity. Second, the effective Lagrangians obtained on the basis of this mechanism (the Nambu–Jona-Lasinio and other models) describe perfectly the ground states and related phenomena. The only weak link of this approach was the absence of the σ-meson below 1 GeV. Note also that the $f_0(665)$ changes but does not solve the problem of unusual properties of the scalar mesons that prevent the scalar nonet to be made up.

Let us also notice that the character of the $f_0(665)$ pole cluster (namely, a considerable shift of the pole on sheet III towards the imaginary axis) can point to the unconsidered channel with which this state is, possibly, strongly coupled and the threshold of which is situated below 600 MeV. In this energy region, only one channel is opened: this is the 4π channel. It is interesting to verify this assumption, because it concerns the above important state.

This analysis does not reveal the $f_0(1370)$ resonance. Therefore, if this meson exists, it must be weakly coupled with the $\pi\pi$ channel, i.e., be described by the pole cluster of type (b) [this would testify to the dominant $s\overline{s}$ component in this state; as to that assignment of the $f_0(1370)$ resonance, we agree, e.g., with the work [25]].

The $f_0(1500)$ state is represented by the pole cluster on the Riemann surface of the S matrix of type (c) which corresponds to a glueball. This type of cluster (i.e., the presence of zeroes, corresponding to the state, on the physical sheet of both $\pi\pi$ and $K\overline{K}$ scattering) reflects the flavor-singlet structure of the glueball wave function and is only a necessary condition of the glueball nature of the $f_0(1500)$ state. Let us also pay attention to the strong coupling of the $f_0(1500)$ state with the $\pi\pi$ system, and to that in the model-independent approach, one can obtain a qualitative indication—how large is the admixture of other states ($q\overline{q},q\overline{q}g$, etc.)? To this end, one must consider the $\pi\pi\to K\overline{K}$ process in our 3-channel approach [1] and determine the coupling constants of the $f_0(1500)$ with the other members of the pseudoscalar nonet.

We emphasize that the obtained results are model independent, since they are based on the first principles and on the mathematical fact that a local behavior of analytic functions, determined on the Riemann surface, is governed by the nearest singularities on all sheets.

We think that multichannel states are most adequately represented by clusters, i.e., by the pole positions on all corresponding sheets. The pole positions are rather stable characteristics for various models, whereas masses and widths are very model dependent for wide resonances.

Finally, note that in the model-independent approach, there are many adjusted parameters (although, e.g., for the $\pi\pi$ scattering, they all are positions of poles describing resonances). The number of these parameters can be diminished by some dynamic assumptions, but this is another approach and of other value.

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