Flavor symmetry breaking effects on the SU(3) Skyrmion

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We study the massive SU(3) Skyrmion model to investigate the flavor symmetry breaking (FSB) effects on the static properties of the strange baryons in the framework of the rigid rotator quantization scheme combined with the improved Dirac quantization one. Both the chiral symmetry breaking pion mass and FSB kinetic terms are shown to improve c, the ratio of the strange–light to light–light interaction strengths, and $\bar{c}$, that of the strange–strange to light–light interaction strengths.

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It is well known that baryons can be obtained from topological solutions, known as SU(2) Skyrmions, since the homotopy group $\Pi_3(\mathrm{SU}(2))=Z$ admits fermions [1–3]. Using the collective coordinates of isospin rotation of the Skyrmion, Adkins *et al.* [1] have performed a semiclassical quantization having static properties of baryons within 30% of the corresponding experimental data. The hyperfine splittings for the SU(3) Skyrmion [4] has been studied in two main schemes. First, the SU(3) cranking method exploits the rigid rotation of the Skyrmion in the collective space of SU(3) Euler angles with full diagonalization of the flavor symmetry breaking (FSB) terms [5]. Especially, Yabu and Ando [6] proposed the exact diagonalization of the symmetry breaking terms by introducing higher irreducible representation mixing in the baryon wave function, which was later interpreted in terms of the multiquark structure [7] in the baryon wave function. Second, Callan and Klebanov [8] suggested an interpretation of baryons containing a heavy quark as bound states of solitons of the pion chiral Lagrangian with mesons. In their formalism, the fluctuations in the strangeness direction are treated differently from those in the isospin directions [8,9].

On the other hand, the Dirac method [10] is a well-known formalism to quantize physical systems with constraints. In this method, the Poisson brackets in a second-class constraint system are converted into Dirac brackets to attain self-consistency. The Dirac brackets, however, are generically field dependent, nonlocal and contain problems related to ordering of field operators. These features are unfavorable for finding canonically conjugate pairs. To overcome the above problems, Batalin, Fradkin, and Tyutin (BFT) [11] developed a method which converts the second-class constraints into first-class ones by introducing auxiliary fields. Recently, this BFT scheme has been successively applied to several models of current interest [12,13]. Especially this BFT method [11] has given an additional energy term in the SU(2) Skyrmion model [14] and has been also applied [15] to an open string theory with D-branes.

The motivation of this paper is to generalize the standard flavor symmetric (FS) SU(3) Skyrmion rigid rotator approach [16] to the SU(3) Skyrmion case with the pion mass and FSB terms so that one can investigate the chiral breaking pion mass and FSB effects on c, the ratio of the strange–light to light–light interaction strengths, and $\bar{c}$, that of the strange–strange to light–light interaction strengths.

Now we start with the SU(3) Skyrmion Lagrangian of the form

$$\mathcal{L}=-\frac{1}{4}f_\pi^2\,\mathrm{tr}(l_\mu l^\mu)+\frac{1}{32e^2}\mathrm{tr}[l_\mu,l_\nu]^2+\mathcal{L}_{WZW}+\frac{1}{4}f_\pi^2\,\mathrm{tr}\,M(U+U^\dagger-2)+\mathcal{L}_{\mathrm{FSB}},$$

$$\mathcal{L}_{\mathrm{FSB}}=\frac{1}{6}(f_K^2m_K^2-f_\pi^2m_\pi^2)\mathrm{tr}((1-\sqrt{3}\lambda_8)(U+U^\dagger-2))-\frac{1}{12}(f_K^2-f_\pi^2)\mathrm{tr}((1-\sqrt{3}\lambda_8)(Ul_\mu l^\mu+l_\mu l^\mu U^\dagger)),\quad(1)$$

where f_π and f_K are the pion and kaon decay constants. Here e is the dimensionless Skyrme parameter and $l_\mu=U^\dagger\partial_\mu U$ with an SU(3) matrix U and M is proportional to the quark mass matrix given by

$$M=\mathrm{diag}(m_\pi^2,m_\pi^2,2m_K^2-m_\pi^2),$$

where $m_\pi=138$ MeV and $m_K=495$ MeV. Note that $\mathcal{L}_{\mathrm{FSB}}$ is the FSB correction term due to the relations $m_\pi\neq m_K$ and $f_\pi\neq f_K$ [17,18] and the Wess–Zumino–Witten (WZW) term [19] is described by the action

$$\Gamma_{\mathrm{WZW}}=-\frac{iN}{240\pi^2}\int_{\mathsf{M}}d^5r\,\epsilon^{\mu\nu\alpha\beta\gamma}\,\mathrm{tr}(l_\mu l_\nu l_\alpha l_\beta l_\gamma),$$

where N is the number of colors and the integral is done on the five-dimensional manifold $\mathsf{M}=V\times S^1\times I$ with the three-space volume V, the compactified time S^1 and the unit interval I needed for a local form of WZW term.

Now we consider only the rigid motions of the SU(3) Skyrmion:

$$U(\vec{x},t)=\mathcal{A}(t)U_0(\vec{x})\mathcal{A}(t)^\dagger.$$

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Assuming maximal symmetry in the Skyrmion, we describe the hedgehog solution U_0 embedded in the SU(2) isospin subgroup of SU(3):

$$U_0(\vec{x})=\begin{pmatrix} e^{i\vec{\tau}\cdot\hat{x}f(r)} & 0 \\ 0 & 1\end{pmatrix},$$

where the τ_i $(i=1,2,3)$ are Pauli matrices, $\hat{x}=\vec{x}/r$, and $f(r)$ is the chiral angle determined by minimizing the static mass E given below and for unit winding number $\lim_{r\to\infty} f(r)=0$ and $f(0)=\pi$.

Since $\mathcal{A}$ belongs to SU(3), $\mathcal{A}^\dagger\dot{\mathcal{A}}$ is anti-Hermitian and traceless to be expressed as a linear combination of $i\lambda_a$ as follows:

$$\mathcal{A}^\dagger\dot{\mathcal{A}}=ief_\pi v^a\lambda_a=ief_\pi\begin{pmatrix}\vec{v}\cdot\tau+\nu 1 & V \\ V^\dagger & -2\nu\end{pmatrix},$$

where

$$\vec{v}=(v^1,v^2,v^3),\quad V=\begin{pmatrix} v^4-iv^5 \\ v^6-iv^7\end{pmatrix},\quad \nu=\frac{v^8}{\sqrt{3}}. \tag{2}$$

After tedious algebraic manipulations, the FSB contribution to the Skyrmion Lagrangian is then expressed as

$$\begin{aligned}\mathcal{L}_{\rm FSB}=&-(f_K^2m_K^2-f_\pi^2m_\pi^2)(1-\cos f)\sin^2 d+\frac{1}{2}(f_K^2-f_\pi^2)\sin^2 d\\ &\times\left(\frac{8}{3}e^2f_\pi^2\vec{v}^2\sin^2 f-\frac{2\sin^2 f}{r^2}-\left(\frac{df}{dr}\right)^2\right)\cos f\\ &-(f_K^2-f_\pi^2)e^2f_\pi^2\frac{\sin^2 d}{d^2}((1-\cos f)^2\|D^\dagger V\|^2\\ &-\sin^2 f\|D^\dagger\tau\cdot\hat{r}V\|^2)\\ &+\frac{i\sqrt{2}}{3}(f_K^2-f_\pi^2)e^2f_\pi^2\frac{\sin 2d}{d}\sin^2 f(D^\dagger\vec{v}\cdot\tau V\\ &-(D^\dagger\vec{v}\cdot\tau V)^*)+(f_K^2-f_\pi^2)e^2f_\pi^2\cos^2 d\\ &\times(1-\cos f)V^\dagger V.\end{aligned} \tag{3}$$

In order to separate the SU(2) rotations from the deviations into strange directions, the time-dependent rotations can be written as [20]

$$\mathcal{A}(t)=\begin{pmatrix} A(t) & 0 \\ 0 & 1\end{pmatrix}S(t)$$

with $A(t)\in$ SU(2) and the small rigid oscillations $S(t)$ around the SU(2) rotations.[1] Furthermore, we exploit the time-dependent angular velocity of the SU(2) rotation through

$$A^\dagger\dot{A}=\frac{i}{2}\dot{\alpha}\cdot\vec{\tau}.$$

Note that one can use the Euler angles for the parametrization of the rotation [22]. On the other hand the small rigid oscillations S, which were also used in Ref. [16], can be described as

$$S(t)=\exp\left(i\sum_{a=4}^{7} d^a\lambda_a\right)=\exp(i\mathcal{D}),$$

where

$$\mathcal{D}=\begin{pmatrix} 0 & \sqrt{2}D \\ \sqrt{2}D^\dagger & 0\end{pmatrix},\quad D=\frac{1}{\sqrt{2}}\begin{pmatrix} d^4-id^5 \\ d^6-id^7\end{pmatrix}.$$

Including the FSB correction terms in Eq. (3), the Skyrmion Lagrangian to order $1/N$ is then given in terms of the angular velocity α_i and the strange deviations D:

$$\begin{aligned}L=&-E+\frac{1}{2}\mathcal{I}_1\dot{\alpha}\cdot\dot{\alpha}+(4\mathcal{I}_2+\Gamma_1)\dot{D}^\dagger\dot{D}+\frac{i}{2}N(D^\dagger\dot{D}-\dot{D}^\dagger D)+i\left(\mathcal{I}_1-2\mathcal{I}_2-\frac{1}{2}\Gamma_1+\Gamma_2\right)(D^\dagger\dot{\alpha}\cdot\vec{\tau}\dot{D}-\dot{D}^\dagger\dot{\alpha}\cdot\vec{\tau}D)-\frac{1}{2}ND^\dagger\dot{\alpha}\cdot\vec{\tau}D\\ &+2\left(\mathcal{I}_1-\frac{4}{3}\mathcal{I}_2-\frac{4}{3}\Gamma_1+3\Gamma_2\right)(D^\dagger D)(\dot{D}^\dagger\dot{D})-\frac{1}{2}\left(\mathcal{I}_1-\frac{4}{3}\mathcal{I}_2-\frac{1}{3}\Gamma_1+2\Gamma_2\right)(D^\dagger\dot{D}+\dot{D}^\dagger D)^2+\left(2\mathcal{I}_2+\frac{1}{2}\Gamma_1\right)(D^\dagger\dot{D}-\dot{D}^\dagger D)^2\\ &-\frac{i}{3}N(D^\dagger\dot{D}-\dot{D}^\dagger D)D^\dagger D-\frac{1}{2}\Gamma_0m_\pi^2-(\Gamma_0(\chi^2m_K^2-m_\pi^2)+\Gamma_3)\left(D^\dagger D-\frac{2}{3}(D^\dagger D)^2\right)-2(\Gamma_1-\Gamma_2)(D^\dagger\dot{D})(\dot{D}^\dagger D),\end{aligned} \tag{4}$$

where $\chi=f_K/f_\pi$. Here the soliton energy E, the moments of inertia $\mathcal{I}_1$ and $\mathcal{I}_2$, the strength Γ_0 of the chiral symmetry breaking and the inertia parameters Γ_i $(i=1,2,3)$ originated from the FSB term are, respectively, given by

[1] Here one notes that the fluctuations ϕ_a from collective rotations A can be also separated by the other suitable parametrization [21] $U=A\sqrt{U_0}A^\dagger\exp(i\Sigma_{a=1}^8\phi_a\lambda_a)A\sqrt{U_0}A^\dagger$.

$$E=4\pi\int_0^\infty dr\, r^2\left[\frac{f_\pi^2}{2}\left(\left(\frac{df}{dr}\right)^2+\frac{2\sin^2 f}{r^2}\right)+\frac{1}{2e^2}\frac{\sin^2 f}{r^2}\left(2\left(\frac{df}{dr}\right)^2+\frac{\sin^2 f}{r^2}\right)\right],$$

$$\mathcal{I}_1=\frac{8\pi}{3}\int_0^\infty dr\, r^2\sin^2 f\left[f_\pi^2+\frac{1}{e^2}\left(\left(\frac{df}{dr}\right)^2+\frac{\sin^2 f}{r^2}\right)\right],$$

$$\mathcal{I}_2=2\pi\int_0^\infty dr\, r^2(1-\cos f)\left[f_\pi^2+\frac{1}{4e^2}\left(\left(\frac{df}{dr}\right)^2+\frac{2\sin^2 f}{r^2}\right)\right],$$

$$\Gamma_0=8\pi f_\pi^2\int_0^\infty dr\, r^2(1-\cos f),$$

$$\Gamma_1=(\chi^2-1)\Gamma_0,$$

$$\Gamma_2=(\chi^2-1)\frac{8\pi}{3}f_\pi^2\int_0^\infty dr\, r^2\sin^2 f,$$

$$\Gamma_3=(\chi^2-1)4\pi f_\pi^2\int_0^\infty dr\, r^2\left(\left(\frac{df}{dr}\right)^2+\frac{2\sin^2 f}{r^2}\right)\cos f. \tag{5}$$

The momenta π_h^i and π_s^α, conjugate to the collective coordinates α_i and the strange deviation $D_\alpha^\dagger$ are given by

$$\vec{\pi}_h=\mathcal{I}_1\dot{\alpha}+i\left(\mathcal{I}_1-2\mathcal{I}_2-\frac{1}{2}\Gamma_1+\Gamma_2\right)(D^\dagger\vec{\tau}\dot{D}-\dot{D}^\dagger\vec{\tau})-\frac{1}{2}ND^\dagger\vec{\tau}D,$$

$$\begin{aligned}\pi_s=&(4\mathcal{I}_2+\Gamma_1)\dot{D}-\frac{i}{2}ND-i\left(\mathcal{I}_1-2\mathcal{I}_2-\frac{1}{2}\Gamma_1+\Gamma_2\right)\\&\times\dot{\alpha}\cdot\vec{\tau}D+2\left(\mathcal{I}_1-\frac{4}{3}\mathcal{I}_2-\frac{4}{3}\Gamma_1+3\Gamma_2\right)(D^\dagger D)\dot{D}\\&-\left(\mathcal{I}_1-\frac{4}{3}\mathcal{I}_2-\frac{1}{3}\Gamma_1+2\Gamma_2\right)(D^\dagger\dot{D}+\dot{D}^\dagger D)D\\&-(4\mathcal{I}_2+\Gamma_1)(D^\dagger\dot{D}-\dot{D}^\dagger D)D+\frac{i}{3}N(D^\dagger D)D\\&-2(\Gamma_1-\Gamma_2)(D^\dagger\dot{D})D,\end{aligned} \tag{6}$$

which satisfy the Poisson brackets

$$\{\alpha_i,\pi_h^j\}=\delta_i^j,\quad \{D_\alpha^\dagger,\pi_s^\beta\}=\{D^\beta,\pi_{s,\alpha}^\dagger\}=\delta_\alpha^\beta.$$

Performing Legendre transformation, we obtain the Hamiltonian to order $1/N$ as follows:

$$\begin{aligned}H=&E+\frac{1}{2}\Gamma_0 m_\pi^2+\frac{1}{2\mathcal{I}_1}\vec{\pi}_h^2+\frac{1}{4\mathcal{I}_2'}\pi_s^\dagger\pi_s-i\frac{N}{8\mathcal{I}_2'}(D^\dagger\pi_s-\pi_s^\dagger D)+\left[\frac{N^2}{16\mathcal{I}_2'}+\Gamma_0(\chi^2m_K^2-m_\pi^2)+\Gamma_3\right]D^\dagger D\\&+i\left[\frac{1}{2\mathcal{I}_1}-\frac{1}{4\mathcal{I}_2'}\left(1+\frac{\Gamma_2}{\mathcal{I}_1}\right)\right]\cdot(D^\dagger\vec{\pi}_h\cdot\vec{\tau}\pi_s-\pi_s{}^\dagger\vec{\pi}_h\cdot\vec{\tau}D)+\frac{N}{4\mathcal{I}_2'}\left(1+\frac{\Gamma_2}{\mathcal{I}_1}\right)D^\dagger\vec{\pi}_h\cdot\vec{\tau}D\\&+\left[\frac{1}{2\mathcal{I}_1}-\frac{1}{3\mathcal{I}_2'}\left(1+\frac{3}{2}\frac{\Gamma_2}{\mathcal{I}_1}\right)+\frac{\Gamma_2^2+\mathcal{I}_1(\Gamma_1-\Gamma_2)}{8\mathcal{I}_1\mathcal{I}_2'^2}\right](D^\dagger D)(\pi_s^\dagger\pi_s)+\left[\frac{1}{12\mathcal{I}_2'}\left(1+\frac{3}{2}\frac{\Gamma_2}{\mathcal{I}_1}\right)-\frac{1}{8\mathcal{I}_1}-\frac{\Gamma_2^2-\mathcal{I}_1(\Gamma_1-\Gamma_2)}{32\mathcal{I}_1\mathcal{I}_2'^2}\right]\\&\times(D^\dagger\pi_s+\pi_s^\dagger D)^2-\left(\frac{1}{8\mathcal{I}_2'}+\frac{\Gamma_1-\Gamma_2}{32\mathcal{I}_2'^2}\right)(D^\dagger\pi_s-\pi_s^\dagger D)^2-i\frac{N}{8}\left[\frac{1}{\mathcal{I}_2'}\left(1-\frac{\Gamma_2}{\mathcal{I}_1}\right)+\frac{\Gamma_2^2+2\mathcal{I}_1(\Gamma_1-\Gamma_2)}{2\mathcal{I}_1\mathcal{I}_2'^2}\right](D^\dagger\pi_s-\pi_s^\dagger D)(D^\dagger D)\\&+\left[\frac{N^2}{12\mathcal{I}_2'}-\frac{2}{3}\Gamma_0(\chi^2m_K^2-m_\pi^2)-\frac{2}{3}\Gamma_3+\frac{N^2}{32}\frac{\Gamma_2^2+2\mathcal{I}_1(\Gamma_1-\Gamma_2)}{\mathcal{I}_1\mathcal{I}_2'^2}\right](D^\dagger D)^2,\end{aligned} \tag{7}$$

where $\mathcal{I}_2'=\mathcal{I}_2+\frac{1}{4}\Gamma_1$.

Through the symmetrization procedure [14], we can obtain the Hamiltonian of the form

$$\begin{aligned}H=&E+\frac{1}{2}\Gamma_0 m_\pi^2+\frac{1}{2\mathcal{I}_1}\left(\vec{I}^2+\frac{1}{4}\right)+\frac{1}{4\mathcal{I}_2'}\pi_s{}^\dagger\pi_s-i\frac{N}{8\mathcal{I}_2'}(D^\dagger\pi_s-\pi_s^\dagger D)+\left[\frac{N^2}{16\mathcal{I}_2'}+\Gamma_0(\chi^2m_K^2-m_\pi^2)+\Gamma_3\right]D^\dagger D\\&+i\left[\frac{1}{2\mathcal{I}_1}-\frac{1}{4\mathcal{I}_2'}\left(1+\frac{\Gamma_2}{\mathcal{I}_1}\right)\right](D^\dagger\vec{I}\cdot\vec{\tau}\pi_s-\pi_s^\dagger\vec{I}\cdot\vec{\tau}D)+\frac{N}{4\mathcal{I}_2'}\left(1+\frac{\Gamma_2}{\mathcal{I}_1}\right)D^\dagger\vec{I}\cdot\vec{\tau}D+\cdots,\end{aligned} \tag{8}$$

TABLE I. The values of c and $\bar{c}$ in the massless pion and massive pion rigid rotator approaches to the SU(3) Skyrmions compared with experimental data. For the rigid rotator approaches, both the predictions in the flavor symmetric (FS) case and flavor symmetry breaking (FSB) one are listed.

Source	c	$\bar{c}$
Rigid rotator, massless and FS	0.92	0.86
Rigid rotator, massless and FSB	0.82	0.69
Rigid rotator, massive and FS	0.79	0.66
Rigid rotator, massive and FSB	0.67	0.56
Experiment	0.67	0.27

where the isospin operator $\vec{I}$ is given by $\vec{I}=\vec{\pi}_h$ and the ellipsis stands for the strange–strange interaction terms of order $1/N$ which can be readily read off from Eq. (7). Here one notes that the overall energy shift $1/8\mathcal{I}_1$ originates from the Weyl ordering correction in the BFT Hamiltonian scheme. (See Ref. [23] for details.)

Following the quantization scheme of Klebanov and Westerberg for the strangeness flavor direction [16], one can obtain the Hamiltonian of the form

$$H=E+\frac{1}{2}\Gamma_0 m_\pi^2+\frac{1}{2\mathcal{I}_1}\left(\vec{I}^2+\frac{1}{4}\right)+\frac{N}{8\mathcal{I}_2'}(\mu-1)a^\dagger a+\left[\frac{1}{2\mathcal{I}_1}-\frac{1}{4\mathcal{I}_2'\mu}\left(1+\frac{\Gamma_2}{\mathcal{I}_1}\right)(\mu-1)\right]a^\dagger\vec{I}\cdot\vec{\tau}a+\left[\frac{1}{8\mathcal{I}_1}-\frac{1}{8\mathcal{I}_2'\mu^2}\left(1+\frac{\Gamma_2}{\mathcal{I}_1}\mu-\frac{\Gamma_2^2+2\mathcal{I}_1(\Gamma_1-\Gamma_2)}{4\mathcal{I}_1\mathcal{I}_2'}\times(\mu-1)\right)(\mu-1)\right](a^\dagger a)^2, \tag{9}$$

where

$$\mu=\left(1+\frac{\chi^2 m_K^2-m_\pi^2+\Gamma_3/\Gamma_0}{m_0^2}\right)^{1/2},$$

$$m_0=\frac{N}{4(\Gamma_0\mathcal{I}_2')^{1/2}}$$

and $a^\dagger$ is creation operator for constituent strange quarks and we have ignored the irrelevant creation operator $b^\dagger$ for strange antiquarks [16]. Then, introducing the angular momentum of the strange quarks

$$\vec{J}_s=\tfrac{1}{2}a^\dagger\vec{\tau}a,$$

one can rewrite the Hamiltonian (9) as

$$H=E+\frac{1}{2}\Gamma_0 m_\pi^2+\omega a^\dagger a+\frac{1}{2\mathcal{I}_1}\left(\vec{I}^2+2c\vec{I}\cdot\vec{J}_s+\bar{c}\vec{J}_s^2+\frac{1}{4}\right), \tag{10}$$

where

$$\omega=\frac{N}{8\mathcal{I}_2'}(\mu-1),$$

$$c=1-\frac{\mathcal{I}_1}{2\mathcal{I}_2'\mu}\left(1+\frac{\Gamma_2}{\mathcal{I}_1}\right)(\mu-1),$$

$$\bar{c}=1-\frac{\mathcal{I}_1}{\mathcal{I}_2'\mu^2}\left(1+\frac{\Gamma_2}{\mathcal{I}_1}\mu-\frac{\Gamma_2^2+2\mathcal{I}_1(\Gamma_1-\Gamma_2)}{4\mathcal{I}_1\mathcal{I}_2'}(\mu-1)\right)\times(\mu-1).$$

Here note that the FSB effects are included in c and $\bar{c}$, through Γ_1, Γ_2, $\mathcal{I}_2'$ and χ and Γ_3 in μ.

The Hamiltonian (10) then yields the structure of the hyperfine splittings as follows:

$$\delta M=\frac{1}{2\mathcal{I}_1}\left[cJ(J+1)+(1-c)\left(I(I+1)-\frac{Y^2-1}{4}\right)+(1+\bar{c}-2c)\frac{Y^2-1}{4}+\frac{1}{4}(1+\bar{c}-c)\right], \tag{11}$$

where $\vec{J}=\vec{I}+\vec{J}_s$ is the total angular momentum of the quarks, and c and $\bar{c}$ are the modified quantities due to the existence of the FSB effect as shown above.

Now using the experimental values of the pion and kaon decay constants $f_\pi=93$ MeV and $f_K=114$ MeV, we fix the value of the Skyrmion parameter e to fit the experimental data of $c_{\rm exp}=0.67$ to yield the predictions for the values of c and $\bar{c}$

$$c=0.67,\quad \bar{c}=0.56 \tag{12}$$

which are contained in Table I, together with the experimental data and the SU(3) rigid rotator predictions without pion mass. For the massless and massive rigid rotator approaches we have used the above values for the decay constants f_π and f_K to obtain both the predictions in the FS and FSB cases. As a result, we have explicitly shown that the more realistic physics considerations via the pion mass and the FSB terms improve both the c and $\bar{c}$ values, as shown in Table I.

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