Circularly polarized gluons emitted in high energy QCD processes

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In the light of the recent report by the ALEPH Collaboration [D. Buskulic *et al.*, Phys. Lett. B **365**, 437 (1996)] where Λ_b polarization was measured, which involves polarization transfer from the b quark to Λ_b, we discuss the gluon circular polarization created in high energy e^+e^- annihilation, previously calculated, and give a calculation of gluon circular polarization creation in high energy electron-proton collisions. For high energies the gluon circular polarization is sizable and the polarization is directly transferred to $q\bar{q}$-vector mesons. We give energy spectra and angular distribution of the gluon circular polarization for various initial energies. Gluon elliptic polarization, inferred from linear and circular polarization, is displayed for two initial energies.

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I. INTRODUCTION

Quarks and antiquarks emitted in high energy electron-positron annihilation,

$$e^+ + e^- \to q\bar{q}, \tag{1}$$

and with gluon emission

$$e^+ + e^- \to q\bar{q} + g \tag{2}$$

are in general polarized. The quark longitudinal polarization without gluon emission, Eq. (1), is at the Z_0 resonance for zero-mass quarks given by

$$P_f^{q\bar{q}} = -\frac{2v_f a_f}{v_f^2 + a_f^2},$$

with v_f and a_f the vector and the axial vector couplings to the Z_0 boson for flavor *f*. The quark mass changes this formula to [1]

$$P_f^{q\bar{q}} = \frac{2v_f a_f \beta}{v_f^2(1+\bar{m}_f^2/2) + a_f^2\beta^2}, \tag{3}$$

where $\beta^2 = 1 - \bar{m}_f^2$, with $\bar{m}_f$ the scaled quark mass $\bar{m}_f = m_f/m_{Z_0}$. All quarks which may be produced at this energy, i.e., all except the top quark, have sizable longitudinal polarizations, ~93% for *d*, *s*, and *b* quarks and ~60% for *u* and *c* quarks. Emission of gluons as in Eq. (2) lowers the numerical value of the polarization $P_f^{q\bar{q}g}$, in particular at very high energies [1,2]. At the Z_0 resonance the effect of gluon emission on the quark longitudinal polarization $P_f^{q\bar{q}}$ is of the order of 3%.

An experiment to measure the Λ_b polarization at LEP and thereby test the theoretical prediction of $P_f^{q\bar{q}}$ has been performed by the ALEPH Collaboration [3]. The theoretical prediction of the Λ_b polarization, taking into account expected effects of reduction of polarization by transfer from the b quark to Λ_b, is estimated to -0.69 ± 0.06. The measured Λ_b polarization was, however, two standard deviations lower, $P_{\Lambda_b} \approx -0.23^{+0.23}_{-0.20}(\text{stat})\,^{+0.08}_{-0.07}(\text{syst})$. As expressed by the collaboration, this may point to depolarization mechanisms occurring during hadronization which are yet to be understood.

In the light of this result, it might be of interest to discuss also the circular polarization of gluons, P_c, emitted in e^+e^- annihilation, Eq. (2), and in deep inelastic scattering of electrons on protons:

$$e^- + P \to e + g + X. \tag{4}$$

Circular (and linear) polarization of gluons, Eq. (2), for zero-mass quarks was obtained in Ref. [4]. At the Z_0 resonance, the photon circular polarization is maximum for gluons emitted in the direction of the electron momentum [4]:

$$P_c^{\max} = -\frac{2va}{v^2 + a^2}, \tag{5}$$

where v and a are the electron vector and axial vector couplings to the Z_0 boson. In contrast to the quark polarization (3), the gluon polarization is small at the Z_0 resonance, $P_c^{\max} = -0.16$. At higher electron-positron energies, the polarization may be sizable, as shown in Fig. 1 up to −50%. The curve is taken from Ref. [4].

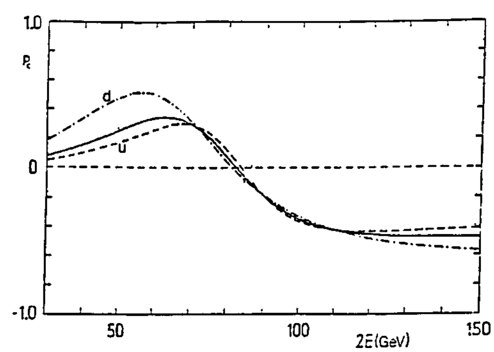

FIG. 1. Maximum gluon circular polarization in high energy e^+e^- annihilation, Eq. (2), as a function of the c.m. energy $2E$. The solid curve represents flavor-averaged polarization, and the curves marked *u* and *d* are polarizations for *u* and *c* quarks and *d*, *s*, and *b* quarks, respectively.

We do not make any attempts to explain the loss of polarization in the ALEPH experiment. What we do is to point out the possibility to measure the Υ polarization ($b\overline{b}$ spin-1 state), when Υ is produced by a polarized gluon. The mechanism here is simpler than is the case of Λ_b, also since the gluon polarization is directly transferred to Υ [Eq. (26)]. The implication is that this experiment may be simpler to understand (a two-quark instead of three-quark process). This could also be a first step to bring new insight into the hadronization process via polarization effects as pointed out at the end.

We obtain in the present paper the circular polarization of gluons produced in deep inelastic scattering of electrons on protons, Eq. (4). The cross section and linear gluon polarization were obtained in Refs. [4] and [5].

The paper is organized in the following way. In Sec. II we obtain the cross section including circular polarization effects at the quark level, where the specific calculations are given in the Appendix. The cross section for the scattering of protons is obtained in Sec. III, where the relevant gluon circular polarization effects are described. In Sec. IV we define the quark distribution functions $b_u^p(\eta)$, $b_d^p(\eta)$, and $b_s^p(\eta)$ for the proton including valence and sea quarks. Numerical results are given for various energies and angles. A discussion of the results is given in Sec. V.

II. CROSS SECTION AT THE QUARK LEVEL

The cross section for lepton-quark scattering with the emission of a circularly polarized gluon,

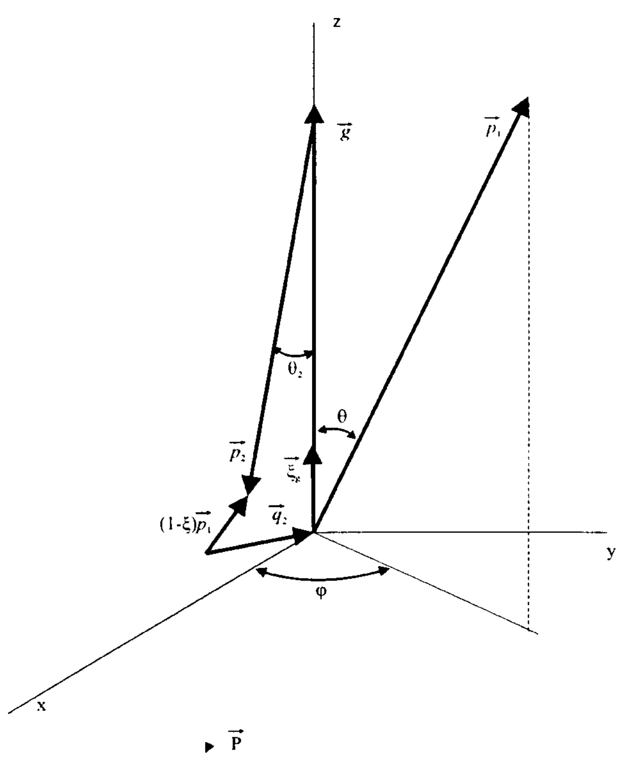

FIG. 2. Center-of-mass momenta and circular polarization vector $\vec{\xi}_g$.

$$l(p_1)+n(q_1)\rightarrow l(p_2)+q(q_2)+g(g,\vec{\xi}_g)+X, \qquad (6)$$

with $\vec{\xi}_g = i\mathbf{e}\times\mathbf{e}^*$ the gluon unit circular polarization vector (see Fig. 2), is for quark flavor f given by

$$d^6\sigma_f=\frac{2}{3}\frac{\alpha^2\alpha_s(Q^2)}{(2\pi)^2}\frac{d^3g}{\omega}\frac{d^3p_2}{E_2}\frac{1}{E_2'}\times\frac{1}{(p_1q_1)}\frac{1}{(q_1g)}\frac{1}{(q_2g)}\frac{1}{Q^2}[h_f^{(1)}(Q^2)A+h_f^{(2)}(Q^2)A'+\xi_g(h_f^{(5)}(Q^2)B+h_f^{(6)}(Q^2)B')]\times\delta(E_1+E_1'-E_2-E_2'-\omega), \qquad (7)$$

with

$$A/|Q^2|=(p_1q_1)^2+(p_2q_2)^2+(p_2q_1)^2+(p_1p_2)^2, \qquad (8)$$

$$A'/|Q^2|=(p_1q_1)^2+(p_2q_2)^2-(p_2q_1)^2-(p_1p_2)^2, \qquad (9)$$

and

$$B=\frac{1}{2\omega}\{q_2\cdot gq_2\cdot p_1F_1(q_1,p_2)+q_1\cdot p_2F_2(q_1,p_2)+(q_1+q_2)\cdot gq_1\cdot p_2F_1'(q_1,p_2)\}-\omega p_1g[p_2\cdot q_1q_1\cdot g-p_2\cdot q_2q_1\cdot g-p_2\cdot q_2q_2\cdot g-p_2\cdot q_1q_2\cdot g]-\frac{1}{2\omega}\{p_1\leftrightarrow p_2\}, \qquad (10)$$

$$B'=\frac{1}{2\omega}\{q_2\cdot gq_2\cdot p_1F_1(q_1,p_2)+q_1\cdot p_2F_2(q_1,p_2)+(q_1+q_2)\cdot gq_1\cdot p_2F_1'(q_1,p_2)\}-\omega p_1g[2p_2\cdot q_2q_1\cdot g-p_2\cdot gq_1\cdot q_2-p_2\cdot q_2q_2\cdot g-q_1\cdot p_2q_1\cdot g]-\frac{\omega}{2}[q_2\cdot g(2p_2\cdot gq_1\cdot p_1-p_1\cdot p_2q_1\cdot q_2)-(q_1\leftrightarrow q_2)]+\frac{1}{2\omega}\{p_1\leftrightarrow p_2\}, \qquad (11)$$

where

$$F_1(q_1,p_2)=\omega q_1\cdot p_2-E_2q_1\cdot g+E_1'p_2\cdot g, \qquad (12)$$

$$F_1'(q_1,p_2)=F_1(q_2,p_1)=\omega q_2\cdot p_1-E_1q_2\cdot g+E_2'p_1\cdot g, \qquad (13)$$

$$F_2(q_1,p_2)=\omega(q_2\cdot gq_1\cdot p_1-p_1\cdot gq_1\cdot q_2)-E_1q_1\cdot gq_2\cdot g+E_2'p_1\cdot gq_1\cdot g. \qquad (14)$$

Here A and A' are the coefficients for unpolarized gluons obtained in [5] and B and B' are the circular polarization coefficients derived in the Appendix.

In Eq. (7) the $h^{(n)}(Q^2)$ coupling parameters are

$$h^{(1)}(Q^2)=Q_f^2-2Q_f f(Q^2)vv_f+f^2(Q^2)(v^2+a^2)(v_f^2+a_f^2),$$

$$h^{(2)}(Q^2)=-2Q_f f(Q^2)aa_f+4f^2(Q^2)vav_f a_f,$$

$$h^{(5)}(Q^2)=2Q_f f(Q^2)av_f-2f^2(Q^2)va(v_f^2+a_f^2),$$

$$h^{(6)}(Q^2)=-2Q_f f(Q^2)va_f+2f^2(Q^2)(v^2+a^2)v_f a_f, \tag{15}$$

$$f(Q^2)=\frac{1}{4\sin^2 2\theta_W}\frac{Q^2}{Q^2-M_Z^2}, \tag{16}$$

with θ_W the weak mixing angle rleated to the Z_0 mass:

$$M_Z=\left(\frac{2\sqrt{2}\pi\alpha}{G_F}\right)^{1/2}\frac{1}{\sin^2 2\theta_W}.$$

Here $h^{(1)}$, $h^{(2)}$, and $h^{(5)}$ are identical to the definitions used earlier [4,7]; $h^{(3)}$ and $h^{(4)}$ are always related to initially transversally polarized leptons and are not used here. The coefficient $h^{(6)}$ was not given earlier.

III. SCATTERING ON PROTONS

The cross section for protons is as usual given by [5,6]

$$d^6\sigma=\int_0^1 d\eta\int d^4q_1\ \delta^4(q_1-\eta P)\sum_{f,\bar{f}} b_f(\eta)d^6\sigma_f, \tag{17}$$

where ηP is the momentum carried by quark momentum q_1 given by the spectral weights $b_f(\eta)$, the quark distribution functions. This gives the cross section

$$d^6\sigma_f=\frac{2}{3}\frac{\alpha^2\alpha_s(Q^2)}{(2\pi)^2}\frac{d^3g}{\omega}\frac{d^3p_2}{E_2}\int\frac{d\eta}{\eta^2}\frac{1}{P\cdot(Q-g)p_1\cdot P}\frac{1}{p_1\cdot gq_2\cdot g}\frac{1}{Q^4}\sum_{f,\bar{f}}([b_f(\eta)-b_{\bar{f}}(\eta)]h_f^{(1)}(Q^2)A$$
$$+[b_f(\eta)-b_f(\eta)]h_f^{(2)}(Q^2)A'+\xi_g\{|b_f(\eta)-b_{\bar{f}}(\eta)|h_f^{(5)}(Q^2)B+[b_f(\eta)-b_{\bar{f}}\eta]h_f^{(6)}(Q^2)B'\})\delta\left(\eta+\frac{(Q-g)^2}{2P\cdot(Q-g)}\right). \tag{18}$$

In Eq. (18), η has the fixed value

$$\eta=\frac{q_1\cdot q_2}{E[2(E-E_2)-\omega(1+\cos\theta)]+p_1\cdot p_2}. \tag{19}$$

Integrating over η and ϕ_2, the cross section can be written as

$$d^5\sigma=\frac{2}{3}\frac{\alpha^2\alpha_s(Q^2)}{(2\pi)^2}\frac{E_2\ dE_2\ d\cos\theta_2\ d\omega\ d\cos\theta\, d\phi}{E^3\eta^2\omega(1+\cos\theta)}\frac{1}{E[2-(1-\eta)(1+\cos\theta)-E_2(1+\cos\theta)]}$$
$$\times\frac{1}{E[4(E-E_2)-2\omega(1+\cos\theta)]-Q^2}\frac{1}{Q^4}\sum_{f,\bar{f}}([b_f(\eta)-b_{\bar{f}}(\eta)]h_f^{(1)}(Q^2)A+[b_f(\eta)-b_{\bar{f}}(\eta)]h_f^{(2)}(Q^2)A'$$
$$+\xi_g\{[b_f(\eta)-b_{\bar{f}}(\eta)]h_f^{(5)}(Q^2)B+[b_f(\eta)-b_{\bar{f}}\eta)]h_f^{(6)}(Q^2)B'\}). \tag{20}$$

The gluon circular polarization for the case that the gluon energy ω and angles θ and ϕ, together with the final electron energy E and polar angle θ_2, are recorded is given by

$$P_g=\frac{d^5\sigma(\xi_g=1)-d^5\sigma(\xi_g=-1)}{d^5\sigma(\xi_g=1)+d^5\sigma(\xi_g=-1)}$$
$$=\frac{\Sigma_{f,\bar{f}}\{h_f^{(5)}(Q^2)B+\Omega_f(\eta)h_f^{(6)}(Q^2)B'\}[b_f(\eta)+b_f(\overline{\eta})]}{\Sigma_{f,\bar{f}}\{h_f^{(1)}(Q^2)A+\Omega_f(\eta)h_f^{(2)}(Q^2)A'\}[b_f(\eta)+b_f(\overline{\eta})]}, \tag{21}$$

with

$$\Omega_f(\eta)=\frac{b_f(\eta)-b_{\bar{f}}(\eta)}{b_f(\eta)+b_{\bar{f}}(\eta)}, \tag{22}$$

which vanishes for nucleon s (sea) quarks and which is not much below the value 1 for most values of η for u and d quarks.

TABLE I. Barger and Phillips quark distribution functions for the proton [8].

f	u	d	s
a	0.594	0.072	
b	0.461	0.206	
c	0.621	0.621	
d			0.145

IV. NUMERICAL RESULTS

Convenient descriptions of the quark distribution functions for the proton are [8]

$$b_u^P(\eta) = \eta^{-1/2}[a_u(1-\eta^2)^3 + b_u(1-\eta^2)^5 + c_u(1-\eta^2)^7 + b_s^P],$$

$$b_d^P(\eta) = \eta^{-1/2}[a_d(1-\eta^2)^3 + b_d(1-\eta^2)^5 + c_d(1-\eta^2)^7 + b_s^P],$$

$$b_s^P = b_{\bar{u}}^P = b_{\bar{d}}^P = b_{\bar{s}}^P = \eta^{-1/2} d_s \frac{(1-\eta^2)^9}{(1+\eta)^9}. \tag{23}$$

We use here the values given by Barger and Phillips [8], here given in Table I. We are in particular interested in values of the gluon polarization for total energies $2E = 314$ GeV and the HERA I energy, together with the gluon production cross section. The latter is taken from Ref. [5] which is based on the same quark distribution functions as here. At the same time, we can calculate the elliptical polarization of the gluon [9]:

$$P_{\text{ell}} = (P_c^2 + P_l^2)^{1/2}. \tag{24}$$

The circular gluon polarization for several center-of-mass energies is given as functions of the gluon energy ω, the final electron energy E_2, and the gluon emission angle θ in Figs. 3, 4, and 5. In order to display the effect of the elliptic gluon polarization, we give in Fig. 6 the linear, circular, and elliptic gluon polarizations for center-of-mass energies 314 GeV and 1 TeV.

V. DISCUSSION

The results obtained here show that the gluon circular polarization may become sizable for large initial electron energies, in particular for gluon emission at large angles θ—comparable to the circular polarization results previously obtained in high energy e^+e^- annihilation at high energy, Fig. 1. Compared to the Λ_b polarization measurements [3], it could be possible to obtain vector meson polarization measurements, in particular for b quark mesons. Again, polarization transfer from the quark to the meson is in general not known. However, for gluon decay $g \to (q\bar{q})_{\text{bound}}$, the gluon polarization is completely transferred to the meson. This can be seen from the obvious analogy to the photoproduction of

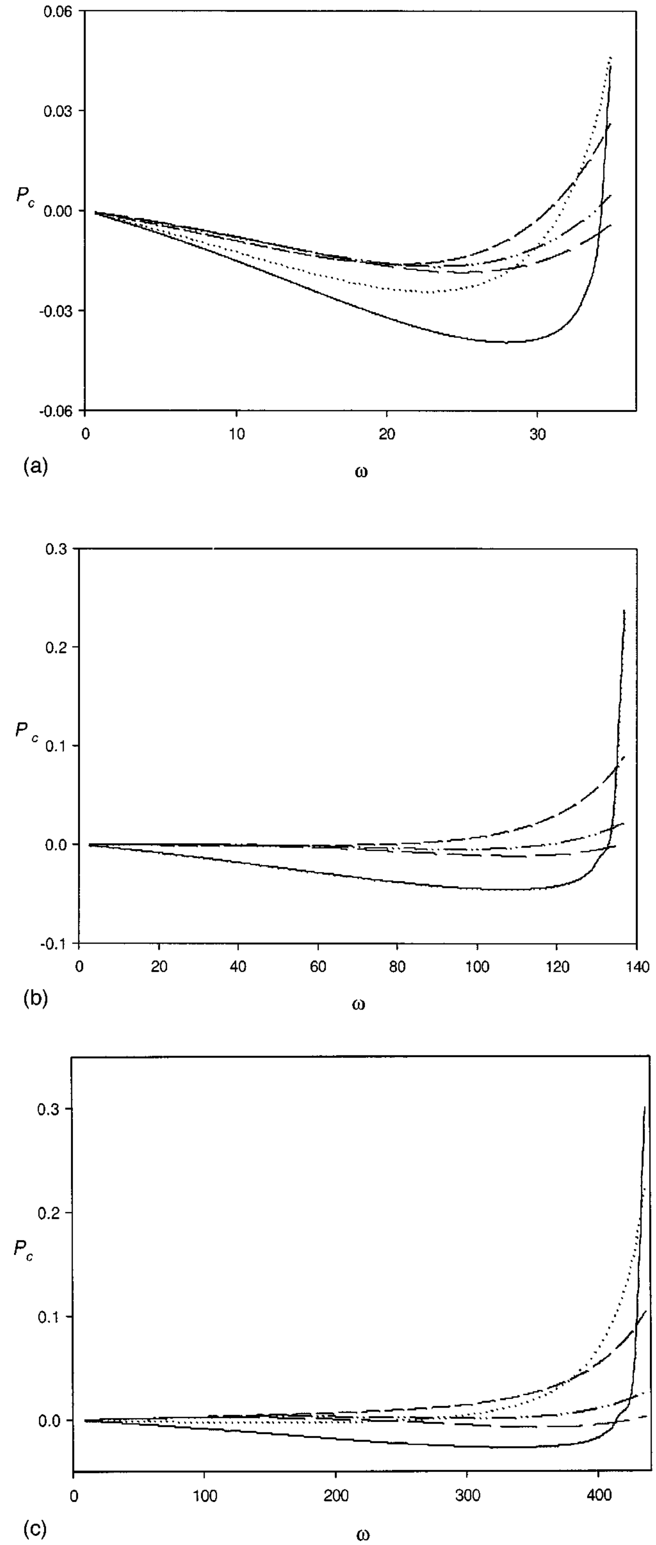

FIG. 3. (a) Gluon circular polarization as a function of the gluon energy ω for c.m. energy $2E = 80$ GeV and final electron energy $E_2 = 20$ GeV. Gluon emission angles $\theta = \pi/4$, and solid curve $\phi = 0$, dotted curve $\phi = \pi/4$, dashed curve $\phi = \pi/2$, dash-dotted curve $\phi = 3\pi/4$, long dashed curve $\phi = \pi$. Electron emission angle $\theta_2 = \pi/4$. (b) Same as (a) for $2E = 314$ GeV and $E_2 = 78.5$ GeV. (c) Same as (a) for $2E = 1$ TeV and $E_2 = 250$ GeV.

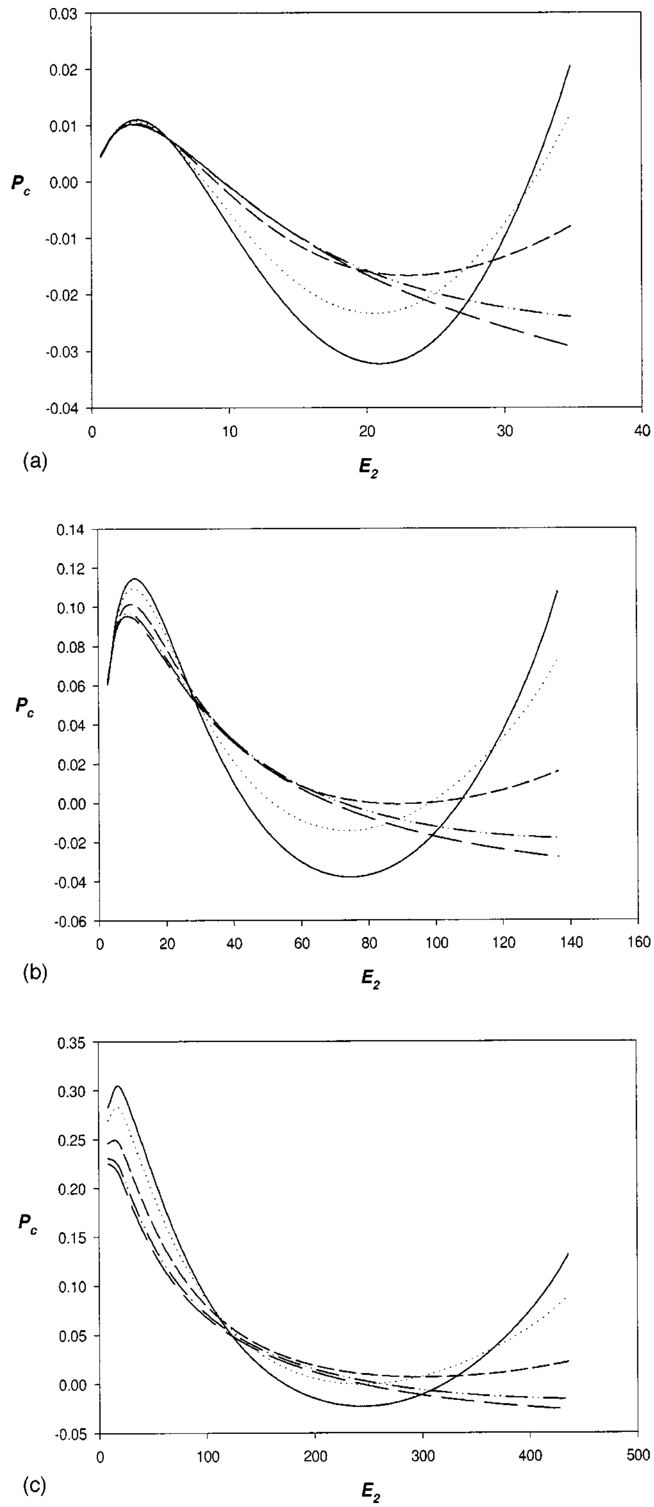

FIG. 4. (a) Same as Fig. 3, as a function of E_2, for $2E = 80$ GeV and $\omega = 20$ GeV. (b) Same as (a) for $2E = 314$ GeV and $\omega = 78.5$ GeV. (c) Same as (a) for $2E = 1$ TeV and $\omega = 250$ GeV.

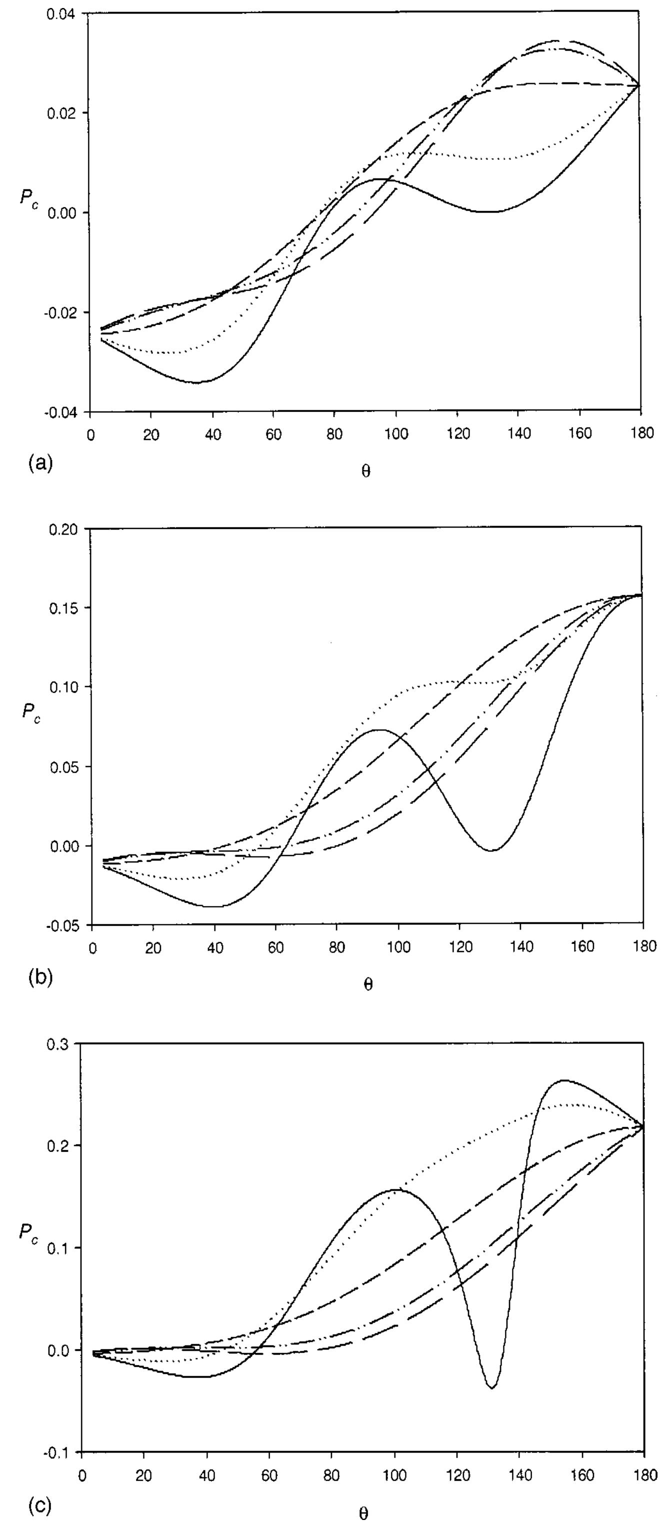

FIG. 5. (a) Same as Fig. 3, as a function of θ, for $2E = 80$ GeV and $E_2 = \omega = 20$ GeV. (b) Same as (a) for $2E = 314$ GeV and $E_2 = \omega = 78.5$ GeV. (c) Same as (a) for $2E = 1$ TeV and $E_2 = \omega = 250$ GeV.

ortho-positronium, where the polarization-dependent cross section is of the form [10]

$$d\sigma(s) \sim (1 + P_c \vec{s} \cdot \hat{p}_{\rm ps}), \tag{25}$$

where P_c here is the circular polarization of the photon, $\vec{s}$ is the unit longitudinal polarization vector of the spin-1 positronium, and $\hat{p}_{\rm ps}$ is the unit positronium momentum. The positronium polarization is obtained from

$$P_{\rm ps} = \frac{d\sigma(\vec{s} \cdot \hat{p}_{\rm ps} = 1) - d\sigma(\vec{s} \cdot \hat{p}_{\rm ps} = -1)}{d\sigma(\vec{s} \cdot \hat{p}_{\rm ps} = 1) + d\sigma(\vec{s} \cdot \hat{p}_{\rm ps} = 1)} = P_c. \tag{26}$$

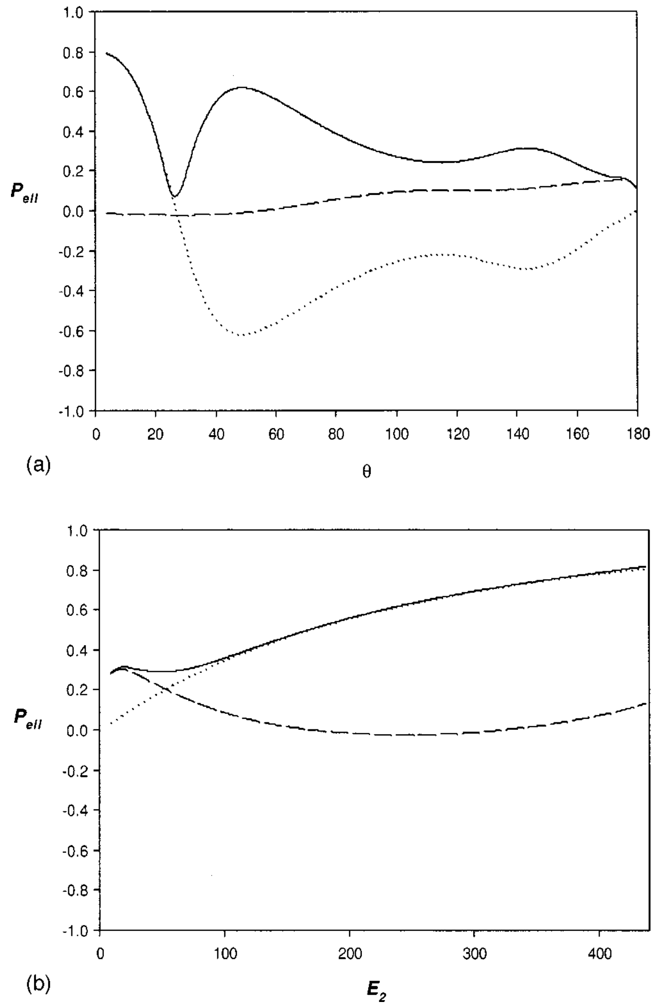

FIG. 6. (a) Elliptic polarization (solid curve) as a function of the gluon emission angle θ for $2E=314\,\mathrm{GeV}$, $E_2=\omega=78.5\,\mathrm{GeV}$, and $\theta_2=\phi=\pi/4$. (b) Same as (a) as a function of the final state electron energy, E_2 for $2E=1$ TeV, $\theta=\theta_2=\pi/4$, and $\phi=0$. Dashed curves: circular polarization. Dotted curves: linear polarization.

For the gluon-b quark system, the gluon polarization is then completely transferred to the Υ vector meson. This could mean that the loss of polarization would be reduced considerably compared to the b-quark polarization transfer to Λ_b.

Several models of quark jet fragmentation have been proposed. Very recently an analysis of the various models has been given by Dong [11]. These models do not take into account the transfer of the polarization dynamics. It is not unlikely that measurements of the polarization transfer could bring new insight into the mechanisms of hadronization of quarks in general, by including in the model polarization transfer.

APPENDIX: MATRIX ELEMENTS

We use here the procedure in Ref. [7], including here the gluon circular polarization which was not taken into account except in a special case in Ref. [4]. These calculations concern the electron-positron annihilation

$$p_+ + p_- \to q + \overline{q} + g,$$

which is related to our case by the substitution rules

$$p_+ \to -p_2, \quad p_- = p_1,$$

$$\overline{q}_+ \to -q_2, \quad q = q_1.$$

The matrix element squared including photon and Z_0 exchange summed over colors is given by

$$\sum_{\text{colors}} |M_\gamma + M_Z|^2 = \frac{(4\pi)^3\alpha^2\alpha_s}{Q^4}\{S_{\gamma\gamma}+S_{\gamma Z}+S_{ZZ}\}, \tag{A1}$$

where

$$S_{\gamma\gamma} = L^{\mu\nu}_{\gamma\gamma} H_{\gamma\gamma\mu\nu},$$

$$S_{\gamma Z} = 2\, f(Q^2) L^{\mu\nu}_{\gamma Z} H_{\gamma Z,\mu\nu},$$

$$S_{ZZ} = f^2(Q^2) L^{\mu\nu}_{ZZ} H_{ZZ,\mu\nu}, \tag{A2}$$

with $L^{\mu\nu}$ the lepton tensors:

$$L^{\mu\nu}_{\gamma\gamma} = \Xi L^{\mu\nu}_1 + \xi L^{\mu\nu}_2 - L^{\mu\nu}_3,$$

$$L^{\mu\nu}_{\gamma Z} = -v L^{\mu\nu}_{\gamma\gamma} + a[\xi L^{\mu\nu}_1 + \Xi L^{\mu\nu}_2 + L^{\mu\nu}_4],$$

$$L^{\mu\nu}_{ZZ} = (v^2+a^2)[\Xi L^{\mu\nu}_1 + \xi L^{\mu\nu}_2] - (v^2-a^2) L^{\mu\nu}_3$$
$$-2va[\xi L^{\mu\nu}_1 + \Xi L^{\mu\nu}_2], \tag{A3}$$

where v and a are the lepton vector and axial vector coupling constants, respectively, and

$$\Xi = 1 + P_1^{\parallel} P_2^{\parallel}, \quad \xi = P_1^{\parallel} + P_2^{\parallel}, \tag{A4}$$

with $P_1^{\parallel}$ and $P_2^{\parallel}$ the initial and final lepton longitudinal polarizations, respectively. We have not included transverse polarizations. These can be obtained from the formalism of Ref. [7].

The basic lepton tensors are found to be given by

$$L^{\mu\nu}_1 = -4t^{\mu\nu}{}_{\alpha\beta} p_1^\alpha p_2^\beta,$$

$$L^{\mu\nu}_1 = 4i\xi^{\mu\nu}{}_{\alpha\beta} p_1^\alpha p_2^\beta. \tag{A5}$$

$L^{\mu\nu}_3$ and $L^{\mu\nu}_3$ involve only transverse lepton polarizations and will not be given here.

The hadron tensors $H_{\mu\nu}$ are given by

$$H_{\gamma\gamma\mu\nu} = Q_f^2 H_{V\mu\nu},$$

$$H_{\gamma Z\mu\nu} = Q_f(v_f H_{V\mu\nu} - a_f H_{A\mu\nu}),$$

$$H_{ZZ\mu\nu} = (v_f^2 + a_f^2) HV_{\mu\nu} - 2v_f a_f H_{A_\mu \nu}, \tag{A6}$$

where Q_f, v_f, and a_f are the quark charge vector and axial vector coupling constant, respectively, and $H_{V\mu\nu}$ and $H_{A\mu\nu}$ the vector and axial vector hadronic matrix elements:

$$H_{V\mu\nu}=\frac{1}{4}\mathrm{Tr}\left\{ \not{q}_1\left[\not{e}^*\frac{\not{q}_1+\not{g}}{q_1\cdot g}\gamma_\mu-\gamma_\mu\frac{\not{q}_2-\not{q}}{q_2\cdot g}\not{e}^*\right] \times\not{q}_2\left[\gamma_\nu\frac{\not{q}_1+\not{g}}{q_1\cdot g}\not{e}-\not{e}\frac{\not{q}_2-\not{g}}{q_2\cdot g}\gamma_\nu\right]\right\},$$

$$H_{A\mu\nu}=\frac{1}{4}\mathrm{Tr}\left\{ \not{q}_1\left[\not{e}^*\frac{\not{q}_1+\not{g}}{q_1\cdot g}\gamma_\mu-\gamma_\mu\frac{\not{q}_2-\not{q}}{q_2\cdot g}\not{e}^*\right] \times\not{q}_2\left[\gamma_\nu\frac{\not{q}_1+\not{g}}{q_1\cdot g}\not{e}-\not{e}\frac{\not{q}_2-\not{g}\not{g}}{q_2\cdot g}\gamma_\nu\right]\gamma_5\right\}. \quad \text{(A7)}$$

Decomposition in symmetric and antisymmetric tensors under $e\leftrightarrow e^*$ gives

$$H_{V\mu\nu}=H_{1\mu\nu}+H_{2\mu\nu}H_{A\mu\nu}=H_{3\mu\nu}+H_{4\mu\nu},$$

with $H_{2\mu\nu}$ and $H_{4\mu\nu}$ antisymmetric under $e\leftrightarrow e^*$. One term in Eq. (A2) is

$$S_ZZ=f^2(Q^2)[(v^2+a^2)\Xi-2va\xi]L_1^{\mu\nu}(v_f^2+a_f^2)H_{2\mu\nu} +[(v^2+a^2)\Xi-2va\Xi y]L_2^{\mu\nu}(-2v_fa_f)H_{4\mu\nu},$$

where we have kept only terms which contribute to gluon circular polarization. Connecting the terms in Eq. (A1) leads to the result given in Eq. (8) with B and B' given by Eqs. (10) and (11), here with unpolarized leptons $\Xi=1$ and $\xi=0$.

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