

# Penguin enhancement and $B \rightarrow K\pi$ decays in perturbative QCD

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We compute the branching ratios of  $B \rightarrow K\pi$  decays in the framework of the perturbative QCD factorization theorem. Decay amplitudes are classified into the topologies of tree, penguin, and annihilation amplitudes, all of which contain both factorizable and nonfactorizable contributions. These contributions are expressed as the convolutions of hard  $b$  quark decay amplitudes with universal meson wave functions. It is shown that (1) matrix elements of penguin operators are dynamically enhanced compared to those employed in the factorization assumption, (2) annihilation diagrams are not negligible, contrary to common belief, (3) annihilation diagrams contribute large strong phases, and (4) the uncertainty of the current data of the ratio  $R = \text{Br}(B_d^0 \rightarrow K^\pm \pi^\mp) / \text{Br}(B^\pm \rightarrow K^0 \pi^\pm)$  and of  $CP$  asymmetries is too large to give a constraint of the unitarity angle  $\phi_3$ . Assuming  $\phi_3 = 90^\circ$  which is extracted from the best fit to the data of  $R$ , predictions for the branching ratios of the four  $B \rightarrow K\pi$  modes are consistent with data.

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## I. INTRODUCTION

$B$  factories at KEK and SLAC are taking data to probe the origin of  $CP$  violation. Within the Kobayashi-Maskawa (KM) ansatz [1],  $CP$  violation is organized in the form of a unitarity triangle shown in Fig. 1. The angle  $\phi_1$  can be extracted from the  $CP$  asymmetry in the  $B \rightarrow J/\psi K_S$  decays, which arises from the  $B-\bar{B}$  mixing. Because of the similar mechanism of  $CP$  asymmetry, the decays  $B^0 \rightarrow \pi^+ \pi^-$  are appropriate for the extraction of the angle  $\phi_2$ . However, these modes contain penguin contributions such that the extraction suffers large uncertainty. Additional measurements of the decays  $B^\pm \rightarrow \pi^\pm \pi^0$  and  $B^0 \rightarrow \pi^0 \pi^0$  and the use of isospin symmetry may resolve the uncertainties [2]. It has been proposed that the angle  $\phi_3$  can be determined from the decays  $B \rightarrow K\pi$ ,  $\pi\pi$  [3–6]. Contributions to these modes involve interference between penguin and tree amplitudes, and relevant strong phases have been formulated in terms of several independent parameters. Progress can be made along this direction, if one learns to compute nonleptonic two-body decay amplitudes including strong phases.

The conventional approach to exclusive nonleptonic  $B$  meson decays relies on the factorization assumption (FA) [7], in which nonfactorizable and final-state-interaction (FSI) effects are assumed to be absent. That is, this approach requires simplify-

ing assumptions. Though analyses are easier under this assumption, estimations of many important ingredients, such as tree and penguin (including electroweak penguin) contributions, and strong phases are not reliable. Moreover, it suffers the problem of scale dependence [8]. It is also difficult to resolve some controversies in the FA approach, such as the branching ratios of the  $B \rightarrow J/\psi K^{(*)}$  decays [9].

Perturbative QCD (PQCD) factorization theorem for exclusive heavy-meson decays [10] has been proved some time ago, and applied to the semileptonic  $B \rightarrow D^{(*)}(\pi)l\bar{\nu}$  decays [11,12], the nonleptonic  $B \rightarrow D^{(*)}\pi(\rho)$  decays [9,13], the penguin-induced radiative  $B \rightarrow K^* \gamma$  decay [14,15] and the charmless  $B \rightarrow \pi\phi$  decay [16]. PQCD is a method to separate hard components from a QCD process, which can be treated by perturbation theory. Nonperturbative components are organized in the form of hadron wave functions, which can be extracted from experimental data. Here we shall extend the PQCD formalism to more challenging charmless decays such as  $B \rightarrow K\pi$ ,  $\pi\pi$ . It will be shown that the difficulties encountered in the FA approach can be resolved in the PQCD formalism.

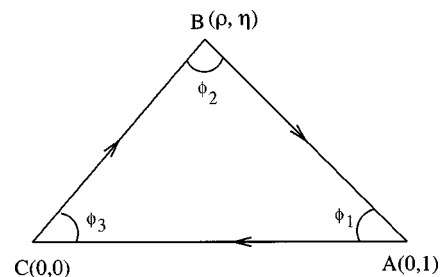


FIG. 1. Unitarity triangle and the definition of the angles  $\phi_i$ .

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In this paper we shall evaluate the branching ratios of the following modes:

$$\begin{aligned} B^\pm \rightarrow K^0 \pi^\pm, \quad B_d^0 \rightarrow K^\pm \pi^\mp, \\ B^\pm \rightarrow K^\pm \pi^0, \quad B_d^0 \rightarrow K^0 \pi^0. \end{aligned} \quad (1)$$

Contributions from various topologies, such as tree, penguin, and annihilation, including both factorizable and nonfactorizable contributions, can all be calculated. That is, FA is in fact not necessary. It has been argued that annihilation diagrams should be included in order to retain the covariance of decay amplitudes in the light-cone Fock representation [17]. In our approach strong phases arise from nonpinched singularities of quark and gluon propagators in nonfactorizable and annihilation diagrams. As explicitly shown in Sec. VII, strong phases from the Bander-Silverman-Soni (BSS) mechanism [18], which is a source of strong phases in the FA approach, are of next-to-leading order and negligible.

As an application, we derive the ratio  $R$  and the  $CP$  asymmetries defined by

$$R = \frac{\text{Br}(B_d^0 \rightarrow K^\pm \pi^\mp)}{\text{Br}(B^\pm \rightarrow K^0 \pi^\pm)}, \quad (2)$$

$$A_{CP}^0 = \frac{\text{Br}(\bar{B}_d^0 \rightarrow K^- \pi^+) - \text{Br}(B_d^0 \rightarrow K^+ \pi^-)}{\text{Br}(\bar{B}_d^0 \rightarrow K^- \pi^+) + \text{Br}(B_d^0 \rightarrow K^+ \pi^-)}, \quad (3)$$

$$A_{CP}^c = \frac{\text{Br}(B^- \rightarrow K^0 \pi^-) - \text{Br}(B^+ \rightarrow K^0 \pi^+)}{\text{Br}(B^- \rightarrow K^0 \pi^-) + \text{Br}(B^+ \rightarrow K^0 \pi^+)}, \quad (4)$$

$$A_{CP}'^0 = \frac{\text{Br}(\bar{B}_d^0 \rightarrow K^0 \pi^0) - \text{Br}(B_d^0 \rightarrow \bar{K}^0 \pi^0)}{\text{Br}(\bar{B}_d^0 \rightarrow K^0 \pi^0) + \text{Br}(B_d^0 \rightarrow \bar{K}^0 \pi^0)}, \quad (5)$$

$$A_{CP}'^c = \frac{\text{Br}(B^- \rightarrow K^- \pi^0) - \text{Br}(B^+ \rightarrow K^+ \pi^0)}{\text{Br}(B^- \rightarrow K^- \pi^0) + \text{Br}(B^+ \rightarrow K^+ \pi^0)}, \quad (6)$$

as functions of the unitarity angle  $\phi_3$  using PQCD factorization theorem. In the above expressions  $\text{Br}(B_d^0 \rightarrow K^\pm \pi^\mp)$  represents the  $CP$  average of the branching ratios  $\text{Br}(B_d^0 \rightarrow K^+ \pi^-)$  and  $\text{Br}(\bar{B}_d^0 \rightarrow K^- \pi^+)$ , and the definition of  $\text{Br}(B^\pm \rightarrow K^0 \pi^\pm)$  is similar. It will be shown that the uncertainty in the data for  $R$ ,  $A_{CP}^0$ , and  $A_{CP}^c$  [19,20],

$$R = 0.95 \pm 0.30, \quad A_{CP}^0 = -0.04 \pm 0.16, \quad A_{CP}^c = 0.17 \pm 0.24, \quad (7)$$

is still too large to provide useful information of  $\phi_3$ . Using the central values of the CLEO data for  $R$ , we obtain  $\phi_3 = 90^\circ$ .

An essential difference between the FA and PQCD approaches is that the hard scale at which Wilson coefficients are evaluated is chosen arbitrarily as  $m_b$  or  $m_b/2$  in the former,  $m_b$  being the  $b$  quark mass, but dynamically determined in the latter. It has been shown that choosing this dynamically determined scale minimizes higher-order cor-

rections to exclusive QCD processes [24]. We observe that this choice leads to an enhancement of penguin contributions by nearly 50% compared to those in the FA approach. As elaborated in Sec. V, this penguin enhancement is crucial for the explanation of the data of all  $B \rightarrow K\pi$ ,  $\pi\pi$  modes using a smaller angle  $\phi_3 \sim 90^\circ$ . Note that an angle  $\phi_3$  larger than  $110^\circ$  must be adopted in order to explain the above data in the FA approach [21].

Recently, Beneke *et al.* proposed an alternative approach to exclusive nonleptonic  $B$  meson decays [22]. In this formalism factorizable contributions (transition form factors), assumed to be dominated by soft dynamics, are not calculable and treated as nonperturbative inputs. Nonfactorizable contributions, being infrared safe, are evaluated in the PQCD framework. Annihilation contributions are still neglected. Therefore, this approach can be regarded as a mixture of the FA and PQCD ones. The comparison among the above approaches will be made briefly in Sec. VII. For a detailed comparison, including predictions which can be distinguished experimentally in the future, refer to [23].

PQCD factorization theorem for exclusive nonleptonic  $B$  meson decays are reviewed in Sec. II. The factorization formulas for various  $B \rightarrow K\pi$  decay modes are derived in Sec. III. The numerical analysis, including the determination of meson wave functions, is performed in Sec. IV. We emphasize the importance of the penguin enhancement in the PQCD approach in Sec. V. FSI effects are discussed in Sec. VI. The PQCD approach is compared with other approaches in Sec. VII. Section VIII is the conclusion.

## II. FACTORIZATION THEOREM IN BRIEF

We first sketch the rough idea of PQCD factorization theorem and of its application to two-body  $B$  meson decays. Take the  $B \rightarrow \pi$  transition form factor in the fast recoil region of the pion as an example [11]. Obviously, this process involves two scales: the  $b$  quark mass  $m_b$ , which provides the large energy release to the fast pion, and the QCD scale  $\Lambda_{\text{QCD}}$ , which is associated with bound-state mesons. Therefore, the  $B \rightarrow \pi$  transition form factor contains both perturbative and nonperturbative dynamics.

In perturbation theory nonperturbative dynamics is reflected by infrared divergences in radiative corrections. It has been shown order by order that these infrared divergences can be separated and absorbed into a  $B$  meson wave function or a pion wave function [11]. A formal definition of the meson wave functions as matrix elements of nonlocal operators can be constructed, which, if evaluated perturbatively, reproduces the infrared divergences. Certainly, one cannot derive a wave function using a perturbative method, but has to parametrize it as a parton model, which describes how a parton (valence quark, if a leading-twist wave function is referred) shares meson momentum. The meson wave functions, characterized by  $\Lambda_{\text{QCD}}$ , must be determined by nonperturbative means, such as lattice gauge theory and QCD sum rules, or extracted from experimental data. In Sec. IV we shall make explicit the determination of the  $B$  meson, kaon, and pion wave functions from currently available data and phenomenological arguments. In the practical calcula-

tion below, small parton transverse momenta  $k_T$  are included, and the characteristic scale is replaced by  $1/b$  with  $b$  being a variable conjugate to  $k_T$ .

After absorbing infrared divergences into the meson wave functions, the remaining part of radiative corrections is infrared finite. This part, called a hard amplitude, can be evaluated perturbatively in terms of Feynman diagrams with four on-shell external quarks, one of which is the  $b$  quark. Note that the  $b$  quark carries various momenta, whose distribution is described by the parton model introduced above. The analysis of next-to-leading-order corrections to the pion form factor [24] has suggested that the characteristic scale should be chosen as the virtuality  $t$  of internal particles, which is of order  $m_b$ , in order to minimize higher-order corrections to the hard amplitudes. This scale reflects the specific dynamics of a decay mode.

The  $B \rightarrow \pi$  transition form factor is then expressed as the convolution of three factors: the  $B$  meson and pion wave functions, and the hard  $b$  quark decay amplitude. This is so called factorization theorem. Note that the separation of non-perturbative and perturbative dynamics is quite arbitrary. This arbitrariness implies that a renormalization-group (RG) improvement of the factorization formula for the  $B \rightarrow \pi$  transition form factor can be implemented. The RG evolution from the all-order summation of large logarithmic corrections to the above convolution factors, along with Sudakov resummation [25], will be made explicit below.

A salient feature of PQCD factorization theorem is the universality of nonperturbative wave functions. Briefly speaking, the infrared divergences associated with the  $B$  meson are process-independent, and the formal definition of the  $B$  meson wave function in terms of matrix elements of non-local operators is universal for all  $B$  meson decay modes. It is not difficult to understand this universality: infrared divergences correspond to long-distance effects, while the hard  $b$  quark decay occurs in a very short space-time. It is natural that these two dramatically different subprocesses decouple from each other. That is, the long-distance dynamics is insensitive to specific decays of the  $b$  quark with large energy release. Because of universality, a  $B$  meson wave function extracted from some decay modes can be employed to make predictions for other modes. This is the reason PQCD factorization theorem possesses a predictive power. We emphasize that PQCD is a theory, instead of a model, since higher-order and higher-twist contributions can be included systematically. The model independence of PQCD predictions can be achieved, once wave functions are determined precisely.

PQCD factorization theorem for nonleptonic  $B$  meson decays, such as  $B \rightarrow K(\pi)\pi$  and  $B \rightarrow D^{(*)}\pi(\rho)$ , is similar, though more complicated. These decays involve three scales: the  $W$  boson mass  $M_W$ , at which the matching conditions of the effective weak Hamiltonian to the full Hamiltonian are defined, the typical scale  $t$  (of order  $m_b$ ), and the factorization scale  $1/b$  (of order  $\Lambda_{\text{QCD}}$ ) introduced above. The dynamics below  $1/b$  is regarded as being completely nonperturbative, and parametrized into meson wave functions  $\phi(x,b)$ ,  $x$  being the momentum fraction. Above the factorization scale, decay dynamics involves two characteristic scales,  $M_W$  and  $t$ , differing from the case of the  $B \rightarrow \pi$  tran-

sition form factor, and further factorization is necessary.

Radiative corrections produce two types of large logarithms:  $\ln(M_W/t)$  and  $\ln(tb)$ . The former are summed by RG equations to give the evolution from  $M_W$  down to  $t$  described by the Wilson coefficients  $C(t)$ , while the latter are summed to give the evolution from  $t$  to  $1/b$ . The matching between the full Hamiltonian and the effective Hamiltonian in the above three-scale factorization theorem is similar to that in the standard effective field theory. The difference is that diagrams in the full theory contain not only  $W$  boson emissions but hard gluon emissions from spectator quarks [8]. One can show that the effective operators, in the presence of the hard gluons from spectators, still form a complete basis, and that the Wilson coefficients derived in the three-scale factorization theorem are the same as those derived in the standard effective theory.

Because of the inclusion of parton transverse momenta, double logarithms  $\ln^2(Pb)$  from the overlap of two types of infrared divergences, collinear and soft, are generated in radiative corrections to meson wave functions [25], where  $P$  denotes the dominant light-cone component of a meson momentum. The resummation [25,26] of these double logarithms leads to a Sudakov form factor  $\exp[-s(P,b)]$ , which suppresses the long-distance contributions from the large  $b$  region, and vanishes as  $b = 1/\Lambda_{\text{QCD}}$ . This factor guarantees the applicability of PQCD to exclusive decays around the energy scale of the  $b$  quark mass [11]. For a detailed derivation of the relevant Sudakov form factors, we refer the readers to [11,12]. With all the large logarithms organized, the remaining finite contributions are absorbed into the hard  $b$  quark decay amplitude  $H(t)$ . In the case of nonleptonic decays  $H(t)$  contains all possible diagrams with six on-shell quarks.

A three-scale factorization formula for exclusive nonleptonic  $B$  meson decays possesses the typical expression,

$$C(t) \otimes H(t) \otimes \phi(x,b) \otimes \exp \left[ -s(P,b) - 2 \int_{1/b}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})) \right], \quad (8)$$

where the exponential involving the quark anomalous dimension  $\gamma = -\alpha_s/\pi$  describes the evolution from  $t$  to  $1/b$  mentioned above. Note that Eq. (8) is a convolution relation, with internal parton kinematics  $x$  and  $b$  integrated out. The hard scale  $t$ , related to the virtuality of internal particles in hard amplitudes, depends on  $x$  and  $b$ . All the convolution factors, except for the wave functions  $\phi(x,b)$ , are calculable in perturbation theory. The wave functions, though not calculable, are universal. If choosing  $t$  as the  $b$  quark mass  $m_b$ , the Wilson coefficient  $C(m_b)$  is a constant, and Eq. (8) reduces to the simple product of the Wilson coefficient and a hadronic matrix element.

### III. $B \rightarrow K\pi$ AMPLITUDES

The effective Hamiltonian for the flavor-changing  $b \rightarrow s$  transition is given by [27]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_q \left[ C_1(\mu) O_1^{(q)}(\mu) + C_2(\mu) O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right], \quad (9)$$

with the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements  $V_q = V_{qs}^* V_{qb}$  and the operators

$$\begin{aligned} O_1^{(q)} &= (\bar{s}_i q_j)_{V-A} (\bar{q}_j b_i)_{V-A}, \\ O_2^{(q)} &= (\bar{s}_i q_i)_{V-A} (\bar{q}_j b_j)_{V-A}, \\ O_3 &= (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A}, \\ O_4 &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}, \\ O_5 &= (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}, \\ O_6 &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \\ O_7 &= \frac{3}{2} (\bar{s}_i b_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V+A}, \\ O_8 &= \frac{3}{2} (\bar{s}_i b_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V+A}, \\ O_9 &= \frac{3}{2} (\bar{s}_i b_i)_{V-A} \sum_q e_q (\bar{q}_j q_j)_{V-A}, \\ O_{10} &= \frac{3}{2} (\bar{s}_i b_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V-A}, \end{aligned} \quad (10)$$

$i, j$  being the color indices. Using the unitarity condition, the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements for the penguin operators  $O_3$ - $O_{10}$  can also be expressed as  $V_u + V_c = -V_t$ . We define the angle  $\phi_3$  via

$$V_{ub} = |V_{ub}| \exp(-i\phi_3). \quad (11)$$

Here we adopt the Wolfenstein parametrization for the CKM matrix up to  $\mathcal{O}(\lambda^3)$ :

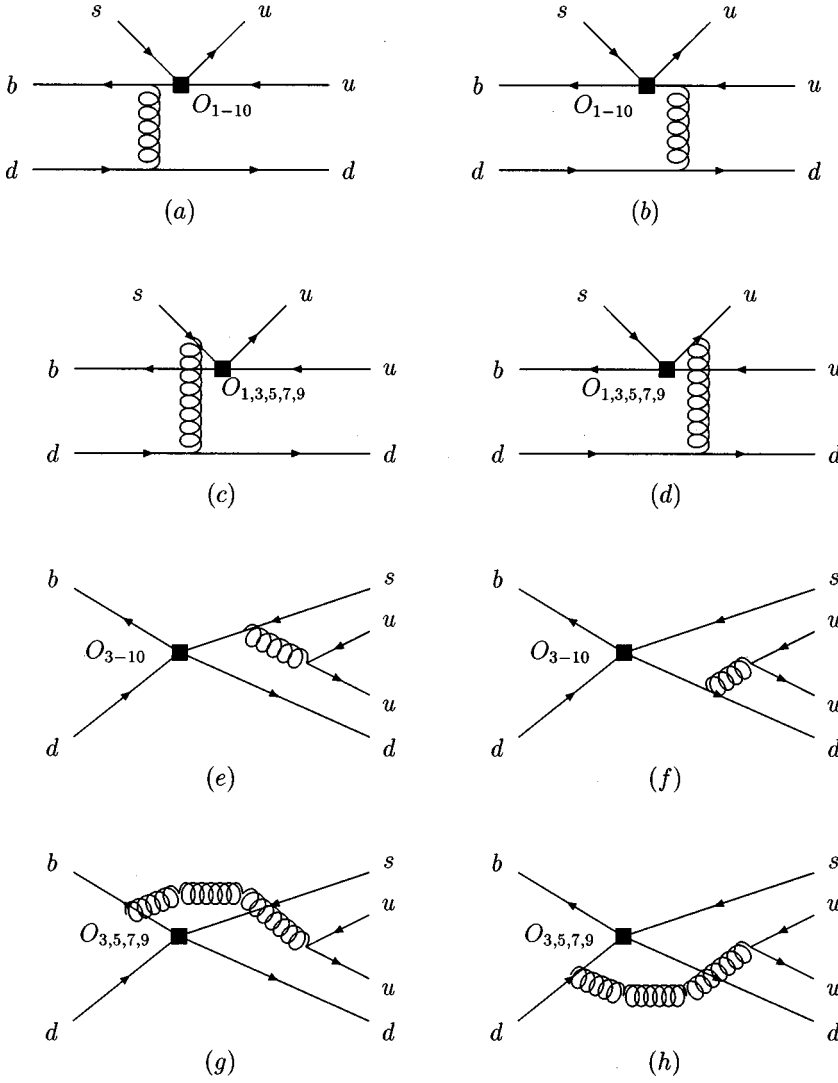
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \quad (12)$$

A recent analysis of quark-mixing matrix yields [28]

$$\begin{aligned} \lambda &= 0.2196 \pm 0.0023, \\ A &= 0.819 \pm 0.035, \\ R_b &\equiv \sqrt{\rho^2 + \eta^2} = 0.41 \pm 0.07. \end{aligned} \quad (13)$$

For the  $B^\pm \rightarrow K^0 \pi^\pm$  decays, the operators  $O_{1,2}^{(u)}$  contribute via an annihilation topology, and  $O_{1,2}^{(c)}$  do not contribute at leading order of  $\alpha_s$ . The absorptive part of the charm quark loop integral computed by BSS is thus of higher order.  $O_{3,4,5,6}$  contribute via tree and annihilation topologies, and the tree topology involves the  $B \rightarrow \pi$  form factor.  $O_{3,5}$  gives both factorizable and nonfactorizable (color-suppressed) contributions, while  $O_{4,6}$  gives only factorizable ones because of the color flow. The contributions from  $O_{7,8,9,10}$  are the same as  $O_{3,4,5,6}$  except for an additional factor  $(3/2)e_q$  with the light quark  $q=d$  in the tree topology and with  $q=u$  in the annihilation topology. For the  $B_d^0 \rightarrow K^\pm \pi^\mp$  decays, the operators  $O_{1,2}^{(u)}$  contribute via a tree topology, and  $O_{1,2}^{(c)}$  do not contribute at leading order of  $\alpha_s$ . The penguin operators contribute in the same way as in the  $B^\pm \rightarrow K^0 \pi^\pm$  decays but with the light quark  $q=u$  in the tree topology and with  $q=d$  in the annihilation topology. The lowest-order hard  $b$  quark decay amplitudes are summarized in Fig. 2 for  $B_d^0 \rightarrow K^\mp \pi^\pm$  decays and in Fig. 3 for  $B^\pm \rightarrow \bar{K}^0 \pi^\pm$  decays.

For the  $B^\pm \rightarrow K^\pm \pi^0$  decays, the operators  $O_{1,2}^{(u)}$  contribute via tree and annihilation topologies, where the tree topology involves both the  $B \rightarrow \pi$  and  $B \rightarrow K$  form factors. The penguin operators also contribute via tree and annihilation topologies with the light quark  $q=u$  in the annihilation topology. The tree topology involves both the  $B \rightarrow \pi$  form factor with  $q=u$  and the  $B \rightarrow K$  form factor, to which only the electroweak penguins with  $q=u$  and  $d$  contribute. For the  $B_d^0 \rightarrow K^0 \pi^0$  decays, the operators  $O_{1,2}^{(u)}$  contribute via the tree topology, which involves only the  $B \rightarrow K$  form factor. The penguin operators contribute via tree and annihilation topologies with the light quark  $q=d$  in the annihilation topology. The tree topology involves both the  $B \rightarrow \pi$  form factor with  $q=d$  and the  $B \rightarrow K$  form factor, to which only the electroweak penguins with  $q=u$  and  $d$  contribute. Their lowest-order diagrams for the hard  $b$  quark decay amplitudes are basically similar to those in Figs. 2 and 3.


 FIG. 2. Feynman diagrams for the  $B^\pm \rightarrow \bar{K}^0 \pi^\pm$  decays.

The momenta of the  $B$  and  $K$  mesons in light-cone coordinates are written as  $P_1 = (M_B/\sqrt{2})(1, 1, \mathbf{0}_T)$  and  $P_2 = (M_B/\sqrt{2})(1, 0, \mathbf{0}_T)$ , respectively. The  $B$  meson is at rest with the above parametrization of momenta. We define the momenta of light valence quark in the  $B$  meson as  $k_1$ , where  $k_1$  has a plus component  $k_1^+$ , giving the momentum fraction  $x_1 = k_1^+/P_1^+$ , and small transverse components  $\mathbf{k}_{1T}$ . The light valence quark and the  $s$  quark in the kaon carry the longitudinal momenta  $x_2 P_2$  and  $(1-x_2)P_2$ , and small transverse momenta  $\mathbf{k}_{2T}$  and  $-\mathbf{k}_{2T}$ , respectively. The pion momentum is then  $P_3 = P_1 - P_2$ , whose nonvanishing component is only  $P_3^-$ . The two light valence quarks in the pion carry the longitudinal momenta  $x_3 P_3$  and  $(1-x_3)P_3$ , and small transverse momenta  $\mathbf{k}_{3T}$  and  $-\mathbf{k}_{3T}$ , respectively. The kinematic variables associated with each meson are indicated in Fig. 4.

The Sudakov resummations of the large logarithmic corrections to the  $B$ ,  $K$ , and  $\pi$  meson wave functions  $\phi_B$ ,  $\phi_K$ , and  $\phi_\pi$  lead to the exponentials  $\exp(-S_B)$ ,  $\exp(-S_K)$ , and  $\exp(-S_\pi)$ , respectively, with the exponents [11,29]

$$S_B(t) = s(x_1 P_1^+, b_1) + 2 \int_{1/b_1}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})),$$

$$S_K(t) = s(x_2 P_2^+, b_2) + s((1-x_2)P_2^+, b_2)$$

$$+ 2 \int_{1/b_2}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})),$$

$$S_\pi(t) = s(x_3 P_3^-, b_3) + s((1-x_3)P_3^-, b_3)$$

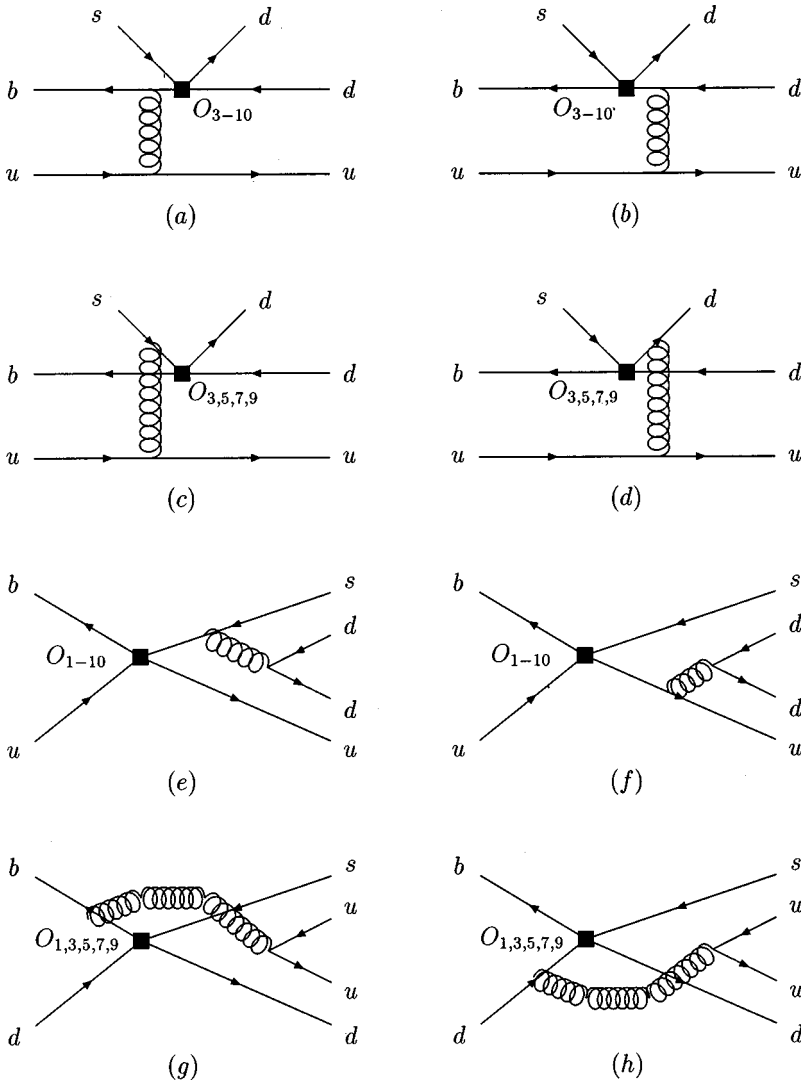
$$+ 2 \int_{1/b_3}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})). \quad (14)$$

The variables  $b_1$ ,  $b_2$ , and  $b_3$  conjugate to the parton transverse momentum  $k_{1T}$ ,  $k_{2T}$ , and  $k_{3T}$  represent the transverse extents of the  $B$ ,  $K$ , and  $\pi$  meson, respectively.

The exponent  $s$  is written as [25,26]

$$s(Q, b) = \int_{1/b}^Q \frac{d\mu}{\mu} \left[ \ln \left( \frac{Q}{\mu} \right) A(\alpha_s(\mu)) + B(\alpha_s(\mu)) \right], \quad (15)$$

where the anomalous dimensions  $A$  to two loops and  $B$  to one loop are

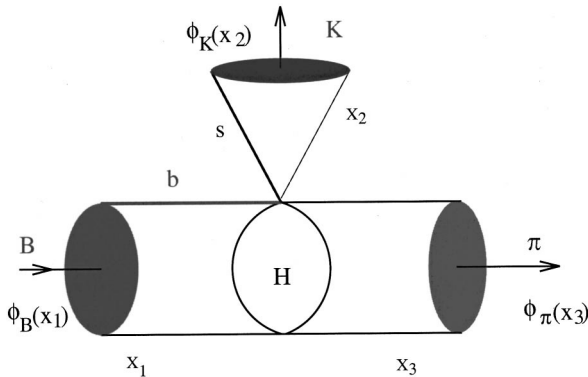

 FIG. 3. Feynman diagrams for the  $B_d^0 \rightarrow K^\pm \pi^\mp$  decays.

$$A = C_F \frac{\alpha_s}{\pi} + \left[ \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}f + \frac{2}{3}\beta_0 \ln\left(\frac{e^{\gamma_E}}{2}\right) \right] \left(\frac{\alpha_s}{\pi}\right)^2,$$

$$B = \frac{2}{3} \frac{\alpha_s}{\pi} \ln\left(\frac{e^{2\gamma_E-1}}{2}\right), \quad (16)$$

with  $C_F=4/3$  a color factor,  $f=4$  the active flavor number, and  $\gamma_E$  the Euler constant. The one-loop expression of the running coupling constant,

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)}, \quad (17)$$


 FIG. 4. Factorization of the  $B \rightarrow K\pi$  decays in the PQCD approach.

is substituted into Eq. (15) with the coefficient  $\beta_0=(33-2f)/3$ .

The decay rates of  $B^\pm \rightarrow K^0 \pi^\pm$  have the expressions

$$\Gamma = \frac{G_F^2 M_B^3}{128\pi} |\mathcal{A}|^2. \quad (18)$$

The decay amplitudes  $\mathcal{A}^+$  and  $\mathcal{A}^-$  corresponding to  $B^+ \rightarrow K^0 \pi^+$  and  $B^- \rightarrow K^0 \pi^-$ , respectively, are written as

$$\begin{aligned} \mathcal{A}^+ = & f_K V_t^* F_e^P + V_t^* \mathcal{M}_e^P + f_B V_t^* F_a^P + V_t^* \mathcal{M}_a^P - f_B V_u^* F_a \\ & - V_u^* \mathcal{M}_a, \end{aligned} \quad (19)$$

$$\begin{aligned} \mathcal{A}^- = & f_K V_t F_e^P + V_t \mathcal{M}_e^P + f_B V_t F_a^P + V_t \mathcal{M}_a^P - f_B V_u F_a \\ & - V_u \mathcal{M}_a, \end{aligned} \quad (20)$$

with the  $B$  meson (kaon) decay constant  $f_{B(K)}$ . The notations  $F$  represent factorizable contributions (form factors), and  $\mathcal{M}$  represent nonfactorizable (color-suppressed) contributions. The subscripts  $a$  and  $e$  denote the annihilation and tree topologies, respectively. The superscript  $P$  denotes contributions from the penguin operators.  $F_a$ , associated with the timelike  $K \rightarrow \pi$  form factor, and  $\mathcal{M}_a$  are from the operators  $O_{1,2}^{(u)}$ .

The decay rates of  $B_d^0 \rightarrow K^\pm \pi^\mp$  have the similar expressions with amplitudes

$$\begin{aligned} \mathcal{A} = & f_K V_t^* F_e^P + V_t^* \mathcal{M}_e^P + f_B V_t^* F_a^P + V_t^* \mathcal{M}_a^P - f_K V_u^* F_e \\ & - V_u^* \mathcal{M}_e, \end{aligned} \quad (21)$$

$$\begin{aligned} \bar{\mathcal{A}} = & f_K V_t F_e^P + V_t \mathcal{M}_e^P + f_B V_t F_a^P + V_t \mathcal{M}_a^P - f_K V_u F_e \\ & - V_u \mathcal{M}_e, \end{aligned} \quad (22)$$

for  $B_d^0 \rightarrow K^+ \pi^-$  and  $\bar{B}_d^0 \rightarrow K^- \pi^+$ , respectively. The notations are similar to those in Eqs. (19) and (20).  $F_e$ , associated with the  $B \rightarrow \pi$  form factor, and  $\mathcal{M}_e$  are from the operators  $O_{1,2}^{(u)}$ .

The decay amplitudes for  $B^\pm \rightarrow K^\pm \pi^0$  are given by

$$\begin{aligned} \sqrt{2} \mathcal{A}^{'+} = & f_K V_t^* F_e^P + V_t^* \mathcal{M}_e^P + f_B V_t^* F_a^P + V_t^* \mathcal{M}_a^P \\ & + f_\pi V_t^* F_{eK}^P + V_t^* \mathcal{M}_{eK}^P - f_K V_u^* F_e - V_u^* \mathcal{M}_e \\ & - f_B V_u^* F_a - V_u^* \mathcal{M}_a - f_\pi V_u^* F_{eK} - V_u^* \mathcal{M}_{eK}, \end{aligned} \quad (23)$$

$$\begin{aligned} \sqrt{2} \mathcal{A}'^- = & f_K V_t F_e^P + V_t \mathcal{M}_e^P + f_B V_t F_a^P + V_t \mathcal{M}_a^P + f_\pi V_t F_{eK}^P \\ & + V_t \mathcal{M}_{eK}^P - f_K V_u F_e - V_u \mathcal{M}_e - f_B V_u F_a \\ & - V_u \mathcal{M}_a - f_\pi V_u F_{eK} - V_u \mathcal{M}_{eK}, \end{aligned} \quad (24)$$

which correspond to  $B^+ \rightarrow K^+ \pi^0$  and  $B^- \rightarrow K^- \pi^0$ , respectively. The factorizable contribution  $F_{eK}^P$  ( $F_{eK}$ ) is associated with the  $B \rightarrow K$  form factor from the penguin (tree) operators, and  $\mathcal{M}_{eK}^P$  ( $\mathcal{M}_{eK}$ ) is the corresponding nonfactorizable contribution.

Similarly, the decay rates of  $B_d^0 \rightarrow K^0 \pi^0$  are obtained from the amplitudes

$$\begin{aligned} \sqrt{2} \mathcal{A}' = & f_K V_t^* F_e^P + V_t^* \mathcal{M}_e^P + f_B V_t^* F_a^P + V_t^* \mathcal{M}_a^P + f_\pi V_t^* F_{eK}^P \\ & + V_t^* \mathcal{M}_{eK}^P - f_\pi V_u^* F_{eK} - V_u^* \mathcal{M}_{eK}, \end{aligned} \quad (25)$$

$$\begin{aligned} \sqrt{2} \bar{\mathcal{A}}' = & f_K V_t F_e^P + V_t \mathcal{M}_e^P + f_B V_t F_a^P + V_t \mathcal{M}_a^P + f_\pi V_t F_{eK}^P \\ & + V_t \mathcal{M}_{eK}^P - f_\pi V_u F_{eK} - V_u \mathcal{M}_{eK}, \end{aligned} \quad (26)$$

for  $B_d^0 \rightarrow K^0 \pi^0$  and  $\bar{B}_d^0 \rightarrow \bar{K}^0 \pi^0$ , respectively.

Basically, one needs to derive the factorization formulas only for the tree and annihilation topologies. Wilson coefficients corresponding to different operators are then inserted into the factorization formulas. The form factors are written as

$$F_e^P = F_{e4}^P + F_{e6}^P,$$

$$\begin{aligned} F_{e4}^P = & 16\pi C_F M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \{ [(1+x_3)\phi_\pi(x_3) + r_\pi(1-2x_3)\phi'_\pi(x_3)] E_{e4}(t_e^{(1)}) h_e(x_1, x_3, b_1, b_3) \\ & + 2r_\pi \phi'_\pi(x_3) E_{e4}(t_e^{(2)}) h_e(x_3, x_1, b_3, b_1) \}, \end{aligned} \quad (27)$$

$$\begin{aligned} F_{e6}^P = & 32\pi C_F M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) r_K \{ [\phi_\pi(x_3) + r_\pi(2+x_3)\phi'_\pi(x_3)] E_{e6}(t_e^{(1)}) h_e(x_1, x_3, b_1, b_3) \\ & + [x_1 \phi_\pi(x_3) + 2r_\pi(1-x_1)\phi'_\pi(x_3)] E_{e6}(t_e^{(2)}) h_e(x_3, x_1, b_3, b_1) \}, \end{aligned} \quad (28)$$

$$F_a^P = F_{a4}^P + F_{a6}^P,$$

$$\begin{aligned} F_{a4}^P = & 16\pi C_F M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{ [-x_3 \phi_K(x_2) \phi_\pi(x_3) - 2r_\pi r_K(1+x_3) \\ & \times \phi'_K(x_2) \phi'_\pi(x_3)] E_{a4}(t_a^{(1)}) h_a(x_2, x_3, b_2, b_3) + [x_2 \phi_K(x_2) \phi_\pi(x_3) + 2r_\pi r_K(1+x_2) \phi'_K(x_2) \phi'_\pi(x_3)] \\ & \times E_{a4}(t_a^{(2)}) h_a(x_3, x_2, b_3, b_2) \}, \end{aligned} \quad (29)$$

$$\begin{aligned}
F_{a6}^P = & 32\pi C_F M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{ [r_\pi x_3 \phi_K(x_2) \phi'_\pi(x_3) + 2r_K \phi'_K(x_2) \phi_\pi(x_3)] E_{a6}(t_a^{(1)}) h_a(x_2, x_3, b_2, b_3) \\
& + [2r_\pi \phi_K(x_2) \phi'_\pi(x_3) + r_K x_2 \phi'_K(x_2) \phi_\pi(x_3)] E_{a6}(t_a^{(2)}) h_a(x_3, x_2, b_3, b_2) \}, \tag{30}
\end{aligned}$$

with the evolution factors

$$E_{ei}(t) = \alpha_s(t) a_i(t) \exp[-S_B(t) - S_\pi(t)], \tag{31}$$

$$E_{ai}(t) = \alpha_s(t) a_i(t) \exp[-S_K(t) - S_\pi(t)]. \tag{32}$$

The expression of  $F_e$  ( $F_a$ ) for the  $O_{1,2}$  contributions is the same as  $F_{e4}^P$  ( $F_{a4}^P$ ) but with the Wilson coefficient  $a_1(t_e)$  ( $a_1(t_a)$ ). The factorization formula of  $F_{eK}^P$  is written as

$$\begin{aligned}
F_{eK}^P = & 16\pi C_F M_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \{ [(1+x_2) \phi_K(x_2) + r_K(1-2x_2) \phi'_K(x_2)] \\
& \times [E_9(t_{eK}^{(1)}) - E_7(t_{eK}^{(1)})] h_e(x_1, x_2, b_1, b_2) + 2r_K \phi'_K(x_2) [E_9(t_{eK}^{(2)}) - E_7(t_{eK}^{(2)})] h_e(x_2, x_1, b_2, b_1) \}, \tag{33}
\end{aligned}$$

with the evolution factor

$$E_i(t) = \alpha_s(t) a_i(t) \exp[-S_B(t) - S_K(t)]. \tag{34}$$

The factorization formula of  $F_{eK}$  is the same as Eq. (33) but with the evolution factor  $E_9 - E_7$  replaced by  $E_2$ , which contains the Wilson coefficient  $a_2$ .

The hard functions  $h$ 's in Eqs (27)–(30) and in Eq. (33), are given by

$$h_e(x_1, x_3, b_1, b_3) = K_0(\sqrt{x_1 x_3} M_B b_1) [\theta(b_1 - b_3) K_0(\sqrt{x_3} M_B b_1) I_0(\sqrt{x_3} M_B b_3) + \theta(b_3 - b_1) K_0(\sqrt{x_3} M_B b_3) I_0(\sqrt{x_3} M_B b_1)], \tag{35}$$

$$\begin{aligned}
h_a(x_2, x_3, b_2, b_3) = & \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(\sqrt{x_2 x_3} M_B b_2) [\theta(b_2 - b_3) H_0^{(1)}(\sqrt{x_3} M_B b_2) J_0(\sqrt{x_3} M_B b_3) \\
& + \theta(b_3 - b_2) H_0^{(1)}(\sqrt{x_3} M_B b_3) J_0(\sqrt{x_3} M_B b_2)]. \tag{36}
\end{aligned}$$

The derivation of  $h$ , from the Fourier transformation of the lowest-order  $H$ , is the same as that for the  $B \rightarrow D\pi$  decays [30], but with a vanishing  $D$  meson mass. The hard scales  $t$  are chosen as the maxima of the virtualities of internal particles involved in  $b$  quark decay amplitudes, including  $1/b_i$ :

$$\begin{aligned}
t_e^{(1)} &= \max(\sqrt{x_3} M_B, 1/b_1, 1/b_3), \\
t_e^{(2)} &= \max(\sqrt{x_1} M_B, 1/b_1, 1/b_3), \\
t_a^{(1)} &= \max(\sqrt{x_3} M_B, 1/b_2, 1/b_3), \\
t_a^{(2)} &= \max(\sqrt{x_2} M_B, 1/b_2, 1/b_3), \tag{37}
\end{aligned}$$

which decrease higher-order corrections. The hard scales  $t_{eK}$  are the same as  $t_e$  but with  $x_3$  and  $b_3$  replaced by  $x_2$  and  $b_2$ , respectively. The Sudakov factor in Eq. (14) suppresses long-distance contributions from the large  $b$  region, and improves the applicability of PQCD to  $B$  meson decays.

For the nonfactorizable amplitudes, the factorization formulas involve the kinematic variables of all the three mesons, and the Sudakov exponent is given by  $S = S_B + S_K + S_\pi$ . One of the integrations over  $b_i$  can be performed trivially, leading to  $b_3 = b_1$ ,  $b_3 = b_2$ , or  $b_2 = b_1$ . Their expressions are



$$\mathcal{M}_e^P = \mathcal{M}_{e4}^P + \mathcal{M}_{e6}^P,$$

$$\begin{aligned} \mathcal{M}_{e4}^P &= 32\pi C_F \sqrt{2N_c} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_K(x_2) \\ &\quad \times \{ [(x_1 - x_2 - x_3) \phi_\pi(x_3) + r_\pi x_3 \phi'_\pi(x_3)] E'_{e4}(t_d^{(1)}) h_d^{(1)}(x_1, x_2, x_3, b_1, b_2) \\ &\quad + [(1 - x_1 - x_2) \phi_\pi(x_3) - r_\pi x_3 \phi'_\pi(x_3)] E'_{e4}(t_d^{(2)}) h_d^{(2)}(x_1, x_2, x_3, b_1, b_2) \}, \end{aligned} \quad (38)$$

$$\begin{aligned} \mathcal{M}_{e6}^P &= 32\pi C_F \sqrt{2N_c} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi'_K(x_2) r_K \{ [(x_1 - x_2) \phi_\pi(x_3) + r_\pi (x_1 - x_2 - x_3) \phi'_\pi(x_3)] \\ &\quad \times E'_{e6}(t_d^{(1)}) h_d^{(1)}(x_1, x_2, x_3, b_1, b_2) + [(1 - x_1 - x_2) \phi_\pi(x_3) + r_\pi (1 - x_1 - x_2 + x_3) \phi'_\pi(x_3)] \\ &\quad \times E'_{e6}(t_d^{(2)}) h_d^{(2)}(x_1, x_2, x_3, b_1, b_2) \}, \end{aligned} \quad (39)$$

$$\mathcal{M}_a^P = \mathcal{M}_{a4}^P + \mathcal{M}_{a6}^P,$$

$$\begin{aligned} \mathcal{M}_{a4}^P &= 32\pi C_F \sqrt{2N_c} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \{ [x_3 \phi_K(x_2) \phi_\pi(x_3) - r_\pi r_K (x_1 - x_2 - x_3) \phi'_K(x_2) \phi'_\pi(x_3)] \\ &\quad \times E'_{a4}(t_f^{(1)}) h_f^{(1)}(x_1, x_2, x_3, b_1, b_2) - [(x_1 + x_2) \phi_K(x_2) \phi_\pi(x_3) + r_\pi r_K (2 + x_1 + x_2 + x_3) \phi'_K(x_2) \phi'_\pi(x_3)] \\ &\quad \times E'_{a4}(t_f^{(2)}) h_f^{(2)}(x_1, x_2, x_3, b_1, b_2) \}, \end{aligned} \quad (40)$$

$$\begin{aligned} \mathcal{M}_{a6}^P &= 32\pi C_F \sqrt{2N_c} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \{ [-r_\pi x_3 \phi_K(x_2) \phi'_\pi(x_3) - r_K (x_1 - x_2) \phi'_K(x_2) \phi_\pi(x_3)] \\ &\quad \times E'_{a6}(t_f^{(1)}) h_f^{(1)}(x_1, x_2, x_3, b_1, b_2) - [r_\pi (2 - x_3) \phi_K(x_2) \phi'_\pi(x_3) - r_K (2 - x_1 - x_2) \phi'_K(x_2) \phi_\pi(x_3)] \\ &\quad \times E'_{a6}(t_f^{(2)}) h_f^{(2)}(x_1, x_2, x_3, b_1, b_2) \}, \end{aligned} \quad (41)$$

with the number of colors  $N_c = 3$ , the definition  $[dx] \equiv dx_1 dx_2 dx_3$ , and the evolution factors

$$E'_{ei}(t) = \alpha_s(t) a'_i(t) \exp[-S(t)|_{b_3=b_1}], \quad (42)$$

$$E'_{ai}(t) = \alpha_s(t) a'_i(t) \exp[-S(t)|_{b_3=b_2}]. \quad (43)$$

The expression of  $\mathcal{M}_e$  ( $\mathcal{M}_a$ ) is the same as  $\mathcal{M}_{e4}^P$  ( $\mathcal{M}_{a4}^P$ ) but with the Wilson coefficient  $a'_1(t_d)$  ( $a'_1(t_f)$ ).

The nonfactorizable amplitude  $\mathcal{M}_{eK}^P$  is written as

$$\mathcal{M}_{eK}^P = -32\pi C_F \sqrt{2N_c} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \phi_\pi(x_3) x_2 \phi_K(x_2) [E'_9(t_{dK}^{(1)}) + E'_7(t_{dK}^{(1)})] h_d^{(1)}(x_1, x_3, x_2, b_1, b_3) \quad (44)$$

with the evolution factor

$$E'_i(t) = \alpha_s(t) a'_i(t) \exp[-S(t)|_{b_2=b_1}]. \quad (45)$$

The expression of  $\mathcal{M}_{eK}$  is the same as Eq. (44) but with the

evolution factor  $E'_9 + E'_7$  replaced by  $E'_2$ , which contains the Wilson coefficient  $a'_2$ .

The functions  $h^{(j)}$ ,  $j = 1$  and  $2$ , appearing in Eqs. (38)–(41) and in Eq. (44), are written as

$$\begin{aligned}
h_d^{(j)} = & [\theta(b_1 - b_2)K_0(DM_B b_1)I_0(DM_B b_2) \\
& + \theta(b_2 - b_1)K_0(DM_B b_2)I_0(DM_B b_1)] \\
& \times K_0(D_j M_B b_2), \quad \text{for } D_j^2 \geq 0, \\
& \times \frac{i\pi}{2} H_0^{(1)}(\sqrt{|D_j^2|} M_B b_2), \quad \text{for } D_j^2 \leq 0, \quad (46)
\end{aligned}$$

$$\begin{aligned}
h_f^{(j)} = & \frac{i\pi}{2} [\theta(b_1 - b_2)H_0^{(1)}(FM_B b_1)J_0(FM_B b_2) \\
& + \theta(b_2 - b_1)H_0^{(1)}(FM_B b_2)J_0(FM_B b_1)] \\
& \times K_0(F_j M_B b_1), \quad \text{for } F_j^2 \geq 0, \\
& \times \frac{i\pi}{2} H_0^{(1)}(\sqrt{|F_j^2|} M_B b_1), \quad \text{for } F_j^2 \leq 0, \quad (47)
\end{aligned}$$

with the variables

$$\begin{aligned}
D^2 &= x_1 x_3, \\
D_1^2 &= F_1^2 = (x_1 - x_2) x_3, \\
D_2^2 &= -(1 - x_1 - x_2) x_3, \\
F^2 &= x_2 x_3, \\
F_2^2 &= x_1 + x_2 + (1 - x_1 - x_2) x_3. \quad (48)
\end{aligned}$$

For details of the derivation of  $h^{(j)}$ , refer to [30]. The hard scales  $t^{(j)}$  are chosen as

$$\begin{aligned}
t_d^{(1)} &= \max(DM_B, \sqrt{|D_1^2|} M_B, 1/b_1, 1/b_2), \\
t_d^{(2)} &= \max(DM_B, \sqrt{|D_2^2|} M_B, 1/b_1, 1/b_2), \\
t_f^{(1)} &= \max(FM_B, \sqrt{|F_1^2|} M_B, 1/b_1, 1/b_2), \\
t_f^{(2)} &= \max(FM_B, \sqrt{|F_2^2|} M_B, 1/b_1, 1/b_2), \quad (49)
\end{aligned}$$

The hard scale  $t_{dK}^{(1)}$  is similar to  $t_d^{(1)}$  with  $x_2$  and  $x_3$  interchanged and with  $b_2$  replaced by  $b_3$ .

In the above expressions the Wilson coefficients are defined by

$$\begin{aligned}
a_1 &= C_2 + \frac{C_1}{N_c}, \\
a'_1 &= \frac{C_1}{N_c}, \\
a_2 &= C_1 + \frac{C_2}{N_c}, \\
a'_2 &= \frac{C_2}{N_c},
\end{aligned}$$

$$\begin{aligned}
a_4 &= C_4 + \frac{C_3}{N_c} + \frac{3}{2} e_q \left( C_{10} + \frac{C_9}{N_c} \right), \\
a'_4 &= \frac{1}{N_c} \left( C_3 + \frac{3}{2} e_q C_9 \right), \\
a_6 &= C_6 + \frac{C_5}{N_c} + \frac{3}{2} e_q \left( C_8 + \frac{C_7}{N_c} \right), \\
a'_6 &= \frac{1}{N_c} \left( C_5 + \frac{3}{2} e_q C_7 \right), \\
a_7 &= \frac{3}{2} \left( C_7 + \frac{C_8}{N_c} \right), \\
a'_7 &= \frac{3}{2} \frac{C_8}{N_c}, \\
a_9 &= \frac{3}{2} \left( C_9 + \frac{C_{10}}{N_c} \right), \\
a'_9 &= \frac{3}{2} \frac{C_{10}}{N_c}. \quad (50)
\end{aligned}$$

Both QCD and electroweak penguin contributions have been included as shown in Eq. (50). It is then expected that electroweak penguin contributions are small, as concluded in [31].

The factors  $r_\pi$  and  $r_K$ ,

$$\begin{aligned}
r_\pi &= \frac{m_{0\pi}}{M_B}, \quad m_{0\pi} = \frac{M_\pi^2}{m_u + m_d}, \\
r_K &= \frac{m_{0K}}{M_B}, \quad m_{0K} = \frac{M_K^2}{m_s + m_d}, \quad (51)
\end{aligned}$$

with  $m_u$ ,  $m_d$ ,  $m_s$ ,  $M_\pi$ , and  $M_K$  being the masses of the  $u$  quark, the  $d$  quark, the  $s$  quark, the pion and the kaon, respectively, are associated with the normalizations of the pseudoscalar wave functions  $\phi'$ . The pseudovector and pseudoscalar pion wave functions  $\phi_\pi$  and  $\phi'_\pi$  are defined by

$$\phi_\pi(x) = \int \frac{dy^+}{2\pi} e^{-ixP_3^- y^+} \frac{1}{2} \langle 0 | \bar{d}(y^+) \gamma^- \gamma_5 u(0) | \pi \rangle, \quad (52)$$

$$\frac{m_{0\pi}}{P_3^-} \phi'_\pi(x) = \int \frac{dy^+}{2\pi} e^{-ixP_3^- y^+} \frac{1}{2} \langle 0 | \bar{d}(y^+) \gamma_5 u(0) | \pi \rangle, \quad (53)$$

satisfying the normalization

$$\int_0^1 dx \phi_\pi(x) = \int_0^1 dx \phi'_\pi(x) = \frac{f_\pi}{2\sqrt{2}N_c}, \quad (54)$$

with the pion decay constant  $f_\pi$ . The kaon wave functions  $\phi_K$  and  $\phi'_K$  possess similar definitions and normalizations

with the  $d$  quark field,  $m_{0\pi}$ , and  $f_\pi$  replaced by the  $s$  quark field,  $m_{0K}$  and  $f_K$ , respectively.

Note that we have included the intrinsic  $b$  dependence for the heavy meson wave function  $\phi_B$  but not for the light meson wave functions  $\phi_\pi$  and  $\phi_K$ . It has been shown that the intrinsic  $b$  dependence of the light meson wave functions, resulting in only 4% reduction of the predictions for the  $B \rightarrow \pi$  form factor, is not important [11]. It is reasonable to assume that the intrinsic  $b$  dependence of the kaon wave function, which is still unknown, is not essential either. As the transverse extent  $b$  approaches zero, the  $B$  meson wave function  $\phi_B(x, b)$  reduces to the standard parton model  $\phi_B(x)$ , i.e.,  $\phi_B(x) = \phi_B(x, b=0)$ , which satisfies the normalization

$$\int_0^1 \phi_B(x) dx = \frac{f_B}{2\sqrt{2N_c}}. \quad (55)$$

We do not distinguish the pseudovector and pseudoscalar components of the  $B$  meson wave functions under the heavy quark approximation.

#### IV. NUMERICAL ANALYSIS

In the factorization formulas derived in Sec. IV, the Wilson coefficients evolve with the hard scale  $t$  that depends on the internal kinematic variables  $x_i$  and  $b_i$ . Wilson coefficients at a scale  $\mu < M_W$  are related to the corresponding ones at  $\mu = M_W$  through usual RG equations. In our analysis we adopt the leading-order expressions for the Wilson coefficients with QCD and electroweak penguin contributions included,

$$\vec{C}(\mu) = T_g \left[ \exp \left( \int_{g(M_W)}^{g(\mu)} dg' \frac{\hat{\gamma}^{(0)T}(g')}{\beta(g')} \right) \right] \cdot \vec{C}(M_W), \quad (56)$$

where the leading-order anomalous dimension matrices  $\hat{\gamma}^{(0)}$  are referred to [27]. The matching conditions at  $\mu = M_W$  [32] and the choices of the relevant parameters are given in Appendix A.

Since the typical scale  $t$  of a hard amplitude is smaller than the  $b$  quark mass  $m_b$ , we further evolve the Wilson coefficients from  $\mu = m_b$  down to  $\mu = t$  using the RG equation,

$$\mu \frac{d}{d\mu} \vec{C}(\mu) = \left[ \frac{\alpha_s(\mu)}{4\pi} \hat{\gamma}_s^{(0)T} + \frac{\alpha_{em}(\mu)}{4\pi} \hat{\gamma}_e^{(0)T} \right] \cdot \vec{C}(\mu), \quad (57)$$

where the anomalous dimensions  $\hat{\gamma}_{s,e}^{(0)}$  for  $f=4$  are referred to [27]. The solution to Eq. (57) and the values of the Wilson coefficients  $C_i(m_b)$  are also listed in the Appendix A. For the scale  $t$  below the  $c$  quark mass  $m_c = 1.5$  GeV, we still employ the evolution function with  $f=4$ , instead of with  $f=3$ , for simplicity, since the matching at  $m_c$  is less essential. Therefore, we set  $f=4$  in the RG evolution between  $t$  and  $1/b$  governed by the quark anomalous dimension  $\gamma$ .

For the  $B$  meson wave function, we adopt the model

$$\phi_B(x, b) = N_B x^2 (1-x)^2 \times \exp \left[ -\frac{1}{2} \left( \frac{xM_B}{\omega_B} \right)^2 - \frac{(0.4 \text{ GeV})^2 b^2}{2} \right], \quad (58)$$

with the shape parameter  $\omega_B = 0.3$  GeV. The normalization constant  $N_B$ , which is related to the decay constant  $f_B$ , will be determined below. As to the pion wave functions, we employ the models

$$\phi_\pi(x) = \frac{3}{\sqrt{2N_c}} f_\pi x (1-x) [1 + c_\pi (5(1-2x)^2 - 1)], \quad (59)$$

$$\phi'_\pi(x) = \frac{3}{\sqrt{2N_c}} f_\pi x (1-x) [1 + c'_\pi (5(1-2x)^2 - 1)], \quad (60)$$

with the shape parameters  $c_\pi$  and  $c'_\pi$ .

The kaon wave functions are chosen as

$$\phi_K(x) = \frac{3}{\sqrt{2N_c}} f_K x (1-x) [1 + 0.51(1-2x) + 0.3(5(1-2x)^2 - 1)], \quad (61)$$

$$\phi'_K(x) = \frac{3}{\sqrt{2N_c}} f_K x (1-x) [1 + c'_K (5(1-2x)^2 - 1)]. \quad (62)$$

$\phi_K$  is derived from QCD sum rules [33], where the second term  $1-2x$ , rendering  $\phi_K$  a bit asymmetric, corresponds to  $SU(3)$  symmetry breaking effect. The decay constant  $f_K$  is set to 160 MeV (in the convention  $f_\pi = 130$  MeV). Since predictions for the  $B \rightarrow K\pi$  decays are insensitive to the kaon wave functions, we simply adopt the result of QCD sum rules. For the same reason, we assume that  $\phi'_K$  and  $\phi'_\pi$  possess the same functional form and that the shape parameter  $c'_K$  of the term  $5(1-2x)^2 - 1$  in  $\phi'_K$  is equal to  $c'_\pi$ .

We propose to determine  $c_\pi$  from the branching ratios of the  $B \rightarrow D\pi$  decays:

$$R_D = \frac{\text{Br}(B^- \rightarrow D^0 \pi^-)}{\text{Br}(\bar{B}_d^0 \rightarrow D^+ \pi^-)}, \quad (63)$$

because this quantity is insensitive to  $m_{0\pi}$  and  $\phi'_\pi$ . In order to render PQCD predictions reach the central value of the data of  $R_D = 1.61$  [34], a large  $c_\pi = 0.8$ , which enhances nonfactorizable contributions to the  $B^- \rightarrow D^0 \pi^-$  decay, is preferred. On the other hand, the data of the  $B \rightarrow \rho\pi$  decays also imply a large  $c_\pi$ . To further enhance nonfactorizable contributions relative to factorizable ones, the  $B$  meson wave function with  $\phi_B \rightarrow x^2$  as  $x \rightarrow 0$  has been assumed as shown in Eq. (58). This behavior, different from that of the model  $\phi_B \rightarrow \sqrt{x}$  as  $x \rightarrow 0$  proposed in [35], decreases factorizable contributions. Note that nonfactorizable contributions are insen-

sitive to the variation of the  $B$  meson wave function. The details for the above numerical study will be published elsewhere.

The extracted pion wave function  $\phi_\pi$  with  $c_\pi=0.8$  is close to the Chernyak-Zhitnitsky model with  $c_\pi=1.0$  [36]. It differs from the asymptotic model with  $c_\pi=0$ , which has been extracted from the data of the pion transition form factor involved in the process  $\pi\gamma^*\rightarrow\gamma$  [37]. We shall argue that the infrared structures of the above processes are different [38]. Hence, there is no contradiction between  $\phi_\pi$  determined from the  $B\rightarrow D\pi$  decays and from the pion transition form factor.

We then extract  $c'_\pi$  from the data of the pion form factor, whose factorization formula is written as [39]

$$F_\pi(Q^2) = 16\pi C_F Q^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \alpha_s(t) \\ \times \exp[-S_{\pi\pi}(t)] [x_2 \phi_\pi(x_1) \phi_\pi(x_2) \\ + 2r_\pi^2 (1-x_2) \phi'(x_1) \phi'_\pi(x_2)] h(x_1, x_2, b_1, b_2), \quad (64)$$

with

$$S_{\pi\pi}(t) = s(x_1 P_{\pi 1}^+, b_1) + s((1-x_1) P_{\pi 1}^+, b_1) \\ + s(x_2 P_{\pi 2}^-, b_2) + s((1-x_2) P_{\pi 2}^-, b_2) \\ + 2 \int_{1/b_1}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})) + 2 \int_{1/b_2}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})), \quad (65)$$

$$h(x_1, x_2, b_1, b_2) \\ = K_0(\sqrt{x_1 x_2} Q b_1) \\ \times [\theta(b_1 - b_2) K_0(\sqrt{x_2} Q b_1) I_0(\sqrt{x_2} Q b_2) \\ + \theta(b_2 - b_1) K_0(\sqrt{x_2} Q b_2) I_0(\sqrt{x_2} Q b_1)], \quad (66)$$

$$t = \max(\sqrt{x_1 x_2} Q, 1/b_1, 1/b_2). \quad (67)$$

The momentum transfer is defined by  $Q^2 = 2P_{\pi 1} \cdot P_{\pi 2}$ ,  $P_{\pi 1}$  and  $P_{\pi 2}$  being the momenta of the initial and final pions, respectively. Useful references for the derivation of the above expression are [29,40,41]. The data are  $Q^2 F_\pi(Q^2) \sim 0.4 \pm 0.2$  GeV<sup>2</sup> for  $Q^2 > 4$  GeV<sup>2</sup> [42,43]. Adopting  $m_{0\pi} = 1.4$  GeV, we find that the choice  $c'_\pi = 0$  gives the pion form factor  $Q^2 F_\pi(Q^2) \sim 0.4$  GeV<sup>2</sup> for  $Q^2 = 6.3$  GeV<sup>2</sup>. Hence, we choose  $c'_K = c'_\pi = 0$  as stated before.

With the pion wave functions fixed in the above procedures, we determine the  $B$  meson decay constant  $f_B$  [or  $N_B$  in Eq. (58)] from the Belle data of the  $B_d^0 \rightarrow \pi^\pm \pi^\mp$  decay [44],

$$\text{Br}(B_d^0 \rightarrow \pi^\pm \pi^\mp) = (6.3^{+3.9}_{-3.5} \pm 1.6) \times 10^{-6}, \quad (68)$$

where  $\text{Br}(B_d^0 \rightarrow \pi^\pm \pi^\mp)$  represents the  $CP$  average of  $\text{Br}(B_d^0 \rightarrow \pi^+ \pi^-)$  and  $\text{Br}(\bar{B}_d^0 \rightarrow \pi^- \pi^+)$ . The CLEO data

$\text{Br}(B_d^0 \rightarrow \pi^\pm \pi^\mp) = (4.3^{+1.6}_{-1.4} \pm 0.5) \times 10^{-6}$ , with a lower central value, overlap with the Belle data. We employ  $G_F = 1.16639 \times 10^{-5}$  GeV<sup>-2</sup>, the Wolfenstein parameters  $\lambda = 0.2196$ ,  $A = 0.819$ , and  $R_b = 0.38$ , the masses  $M_B = 5.28$  GeV, and the  $\bar{B}_d^0$  ( $B^-$ ) meson lifetime  $\tau_{B^0} = 1.55$  ps ( $\tau_{B^-} = 1.65$  ps) [28]. For the factorization formulas of the  $B \rightarrow \pi\pi$  decays, refer to [45]. Using the angle  $\phi_3 = 90^\circ$ , we obtain  $f_B = 190$  MeV, which corresponds to  $\text{Br}(B_d^0 \rightarrow \pi^\pm \pi^\mp) = 6.3 \times 10^{-6}$  and the  $B \rightarrow \pi$  transition form factor

$$F^{B\pi}(q^2=0) = 0.3. \quad (69)$$

Here  $q$  stands for the momentum carried away by the external  $W$ -emission. The value of  $f_B$  is close to that adopted in the PQCD studies of the  $B \rightarrow D\pi$  and  $B \rightarrow K^* \gamma$  decays [14,15], and consistent with those from lattice calculations [46] and from QCD sum rules [47] in the literature. The motivation to choose  $\phi_3 = 90^\circ$  will be explained later.

We emphasize that the decay constant  $f_B$  can not be determined unambiguously in the current analysis. The above value  $f_B = 190$  MeV corresponds to the shape parameter  $\omega_B = 0.3$  GeV. Changing  $\omega_B$ , different  $f_B$  will be obtained when fitting PQCD predictions to the data in Eq. (68). However, if more data, such as the  $CP$  asymmetry in the  $B_d^0 \rightarrow \pi^\pm \pi^\mp$  decays, are available, both  $\omega_B$  and  $f_B$  can be uniquely determined. The reason is that tree and penguin contributions depend on  $\omega_B$  and  $f_B$  simultaneously, while annihilation contributions, the most important source of strong phases as shown below, depend only on  $f_B$ . Because the branching ratio, mainly determined by tree and penguin contributions, and the  $CP$  asymmetry, related to strong phases of annihilation contributions, vary with the  $B$  meson wave function in a different way, their data can fix  $\omega_B$  and  $f_B$  uniquely.

Note that the above parameters are obtained by fitting predictions to the central values of the available data. If taking into account the uncertainty of the data, the allowed range of the parameters is in fact huge. For example, any value of the shape parameter  $c_\pi$  in the pion wave function  $\phi_\pi$  between 0.4 and 1.0 is acceptable for the data of  $R_D$ . The shape parameter  $c'_K$  in the pseudoscalar kaon wave function  $\phi'_K$  can differ from  $c'_\pi$  in  $\phi'_\pi$ . In this work we do not intend to determine the range of parameters, but adopt representative parameters to make predictions for the  $B \rightarrow K\pi$  decays, and examine whether the predictions are consistent with the data. For a summary of the parameters we have adopted in the numerical analysis, refer to Appendix B.

With all the meson wave functions fixed, we predict the branching ratios and the  $CP$  asymmetries of the  $B \rightarrow K\pi$  decays. We choose  $m_{0K} = 1.7$  GeV, and derive the branching ratios of the four  $B \rightarrow K\pi$  modes in Eq. (1) for different  $\phi_3$ , which are shown in Fig. 5. The branching ratios of the  $K^\pm \pi^0$  and  $K^\mp \pi^\pm$  modes increase with  $\phi_3$ , while those of the  $K^0 \pi^\pm$  and  $K^0 \pi^0$  modes are insensitive to the variation of  $\phi_3$ . The increase with  $\phi_3$  is mainly a consequence of the interference between the penguin contribution  $F_e^P$  and the tree contribution  $F_e$ . Predictions for the ratio  $R$  in Eq. (2)

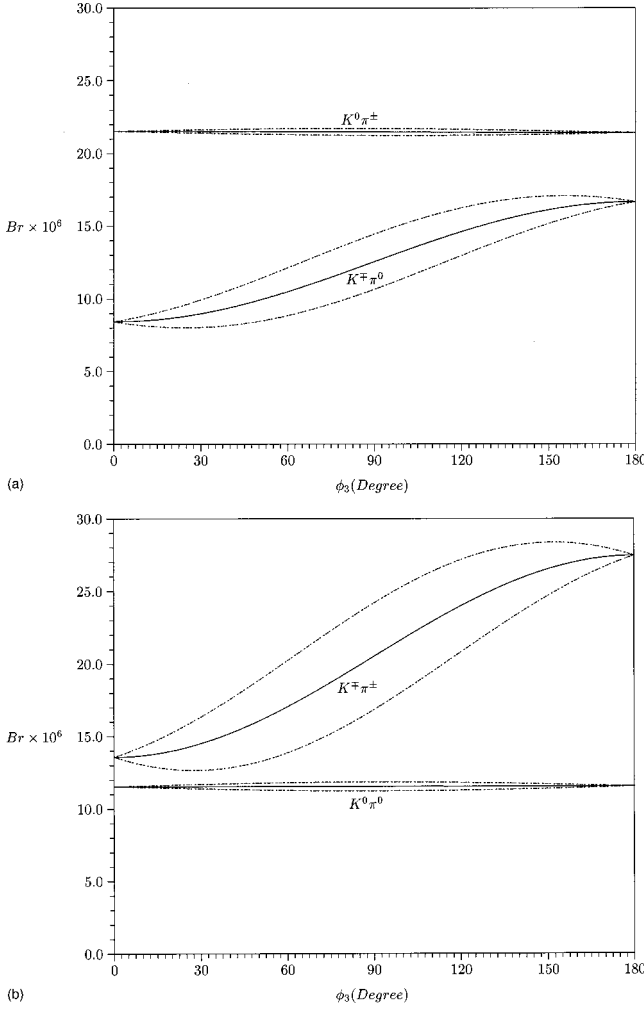


FIG. 5. Dependence of the branching ratios of the  $B \rightarrow K\pi$  decays on  $\phi_3$  with the upper (lower) dashed line corresponding to the  $\bar{B}$  ( $B$ ) meson decays.

and the  $CP$  asymmetries  $A_{CP}$  in Eqs. (3)–(6) for different  $\phi_3$  are displayed in Fig. 6 and Fig. 7, respectively. The prediction of  $R$  increases from 0.7 to 1.2 when  $\phi_3$  moves from 0 to  $180^\circ$ . Unfortunately, the large uncertainty of the current data does not give a constraint of  $\phi_3$ . Comparing with the central value of the CLEO data of  $R$  in Eq. (7), we extract  $\phi_3 = 90^\circ$ . The data of  $A_{CP}$  have also large uncertainties, and do not constraint  $\phi_3$  either. Our analysis shows that the magnitude of  $A_{CP}$  and  $A_{CP}'$  is negligible, smaller than 3%, while the magnitude of  $A_{CP}^c$  and  $A_{CP}'^c$  can reach 20%.

Our predictions for the branching ratio of each mode corresponding to  $\phi_3 = 90^\circ$ ,

$$\text{Br}(B^+ \rightarrow K^0 \pi^+) = 21.72 \times 10^{-6},$$

$$\text{Br}(B^- \rightarrow \bar{K}^0 \pi^-) = 21.25 \times 10^{-6},$$

$$\text{Br}(B_d^0 \rightarrow K^+ \pi^-) = 24.19 \times 10^{-6},$$

$$\text{Br}(\bar{B}_d^0 \rightarrow K^- \pi^+) = 16.84 \times 10^{-6},$$

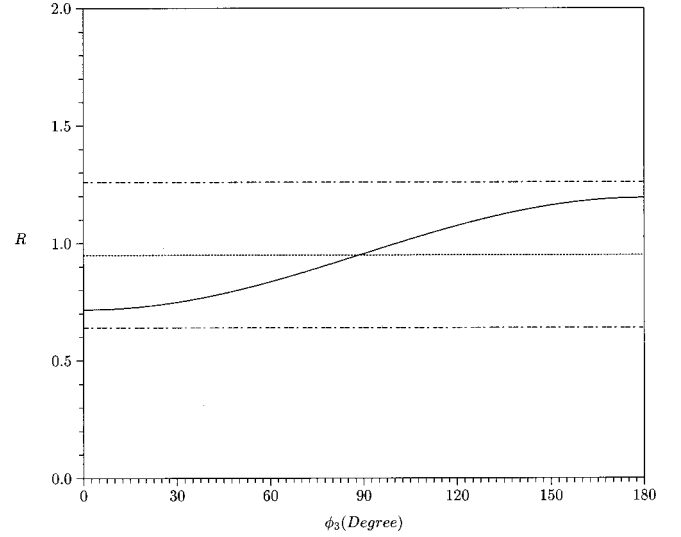


FIG. 6. Dependence of the ratio  $R$  on  $\phi_3$ . The dashed (dotted) lines correspond to the bounds (central value) of the data.

$$\begin{aligned} \text{Br}(B^+ \rightarrow K^+ \pi^0) &= 14.44 \times 10^{-6}, \\ \text{Br}(B^- \rightarrow K^- \pi^0) &= 10.65 \times 10^{-6}, \\ \text{Br}(B_d^0 \rightarrow K^0 \pi^0) &= 11.23 \times 10^{-6}, \\ \text{Br}(\bar{B}_d^0 \rightarrow \bar{K}^0 \pi^0) &= 11.84 \times 10^{-6}, \end{aligned} \quad (70)$$

are consistent with the CLEO data [19],

$$\begin{aligned} \text{Br}(B^\pm \rightarrow K^0 \pi^\pm) &= (18.2_{-4.0}^{+4.6} \pm 1.6) \times 10^{-6}, \\ \text{Br}(B_d^0 \rightarrow K^\pm \pi^\mp) &= (17.2_{-2.4}^{+2.5} \pm 1.2) \times 10^{-6}, \\ \text{Br}(B^\pm \rightarrow K^\pm \pi^0) &= (11.6_{-2.7}^{+3.0} \pm 1.4) \times 10^{-6}, \\ \text{Br}(B_d^0 \rightarrow K^0 \pi^0) &= (14.6_{-5.1}^{+5.9} \pm 2.4) \times 10^{-6}. \end{aligned} \quad (71)$$

The PQCD results of each form factor and nonfactorizable amplitude involved in the  $B^0 \rightarrow K^+ \pi^-$  decay are listed in Table I. It indicates that nonfactorizable contributions are only few percents of factorizable ones, consistent with the conclusion in [48]. This is the reason FA works well for most two-body  $B$  meson decay modes. However, there are exceptions. For modes whose factorizable contributions are proportional to the small Wilson coefficient  $a_2$ , such as  $B \rightarrow J/\psi K^{(*)}$ , nonfactorizable contributions become important. Similarly, the term  $F_{eK}$ , proportional to  $a_2$ , is small. Hence, the branching ratios of the  $K\pi^0$  modes are about half of those of the  $K\pi^\pm$  modes. Table I also indicates that the factorizable annihilation diagrams contribute dominant strong phases. The reason has been discussed in [49]. If expressing the amplitude of the  $B_d^0 \rightarrow K^+ \pi^-$  decay as

$$A = V_t^* P e^{i\delta_P} - V_u^* T, \quad (72)$$

with the penguin contribution  $P = |f_K F_e^P + f_B F_d^P|$  and the tree contribution  $T = |f_K F_e|$ , the strong phase  $\delta_P$  is as large as

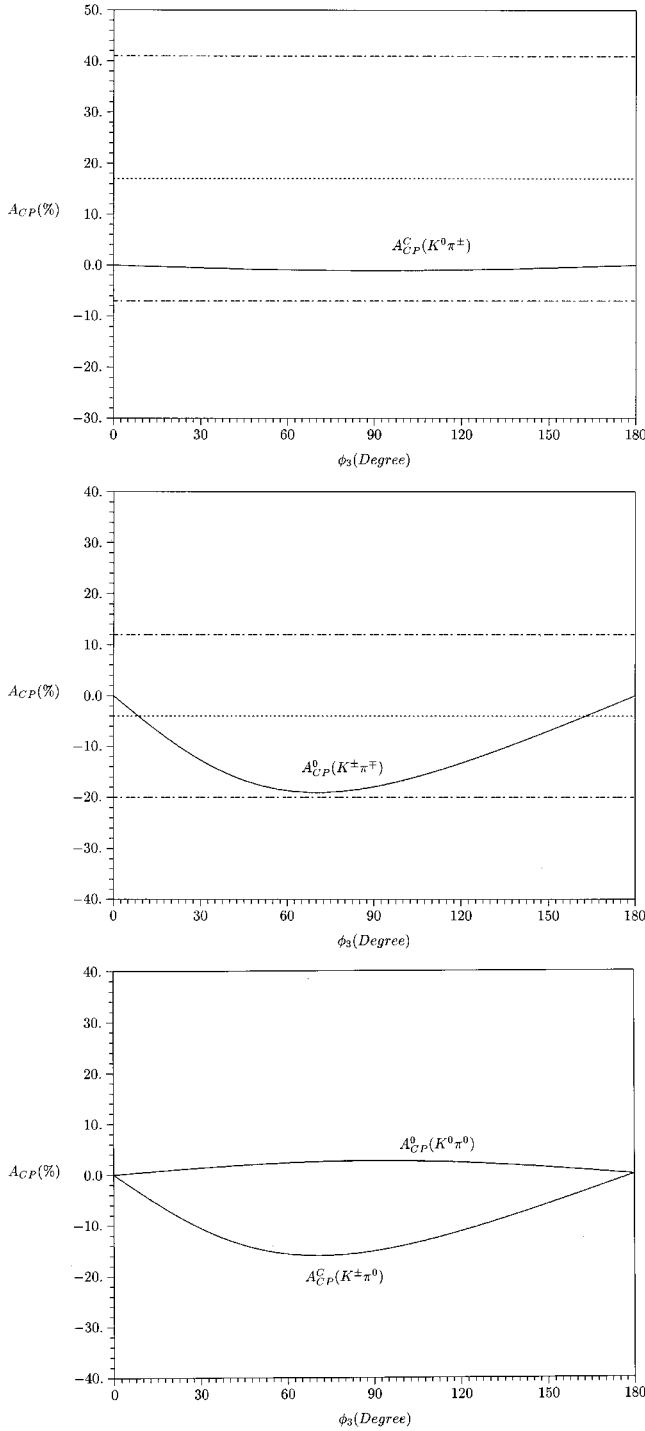


FIG. 7. Dependence of  $CP$  asymmetries on  $\phi_3$ . The dashed (dotted) lines correspond to the bounds (central value) of the data of the  $B^\pm \rightarrow K^0 \pi^\pm$  decays in (a) and the  $B_d^0 \rightarrow K^\pm \pi^\mp$  in (b).

$$\delta_p = 152^\circ. \quad (73)$$

This result is consistent with the conclusion drawn from a global fit to data of two-body charmless  $B$  meson decays [50], where the strong phase was introduced as a free parameter.

To test the sensitivity of our predictions to different choices of model wave functions and parameters, we have

TABLE I. Contribution to the  $B^0 \rightarrow K^+ \pi^-$  decay from each form factor and nonfactorizable amplitude.

$F_e$	$7.16 \times 10^{-1}$
$F_e^P$	$-6.18 \times 10^{-2}$
$F_a^P$	$3.01 \times 10^{-3} + i2.58 \times 10^{-2}$
$M_e$	$-1.89 \times 10^{-3} + i4.13 \times 10^{-3}$
$M_e^P$	$5.84 \times 10^{-5} - i1.54 \times 10^{-4}$
$M_a^P$	$-8.63 \times 10^{-5} - i2.23 \times 10^{-4}$

varied the shape parameter  $\omega_B$  for the  $B$  meson wave function from 0.3 to 0.5, the shape parameter  $c'_K$  for the kaon wave function from 0 to 0.8, the masses  $m_{0K(\pi)}$  from 1.3 GeV to 2.7 GeV, the forms of the meson wave functions, such as

$$\phi_B^{\text{test}}(x, b) = N_B^{\text{test}} x(1-x) \exp \left[ -\frac{1}{2} \left( \frac{xM_B}{\omega_B} \right)^2 - \frac{\omega_B^2 b^2}{2} \right], \quad (74)$$

$$\phi_K^{\text{test}}(x) = \frac{\sqrt{6}}{2} f_K x(1-x) [1 + 0.3(5(1-2x)^2 - 1)], \quad (75)$$

for the  $B$  meson and the kaon, and the asymptotic model

$$\phi_\pi^{\text{AS}}(x) = \frac{3}{\sqrt{2}N_c} f_\pi x(1-x), \quad (76)$$

for the pion, and the Wolfenstein parameters  $\lambda$  from 0.21 to 0.22. It is found that our predictions for  $R$  change by less than 5%, and are very stable. That is,  $R$  is an appropriate quantity for the determination of  $\phi_3$ .

There are other theoretical uncertainties from higher-order  $O(\alpha_s^2)$  and higher-twist  $O(1/M_B)$  corrections. As a simple estimation, we examine the fractional contribution to the form factor  $F^{B\pi}$  as a function of  $\alpha_s(t)/\pi$ . It is observed that 90% and 97% of the contributions arise from the region with  $\alpha_s(t)/\pi < 0.2$  and with  $\alpha_s(t)/\pi < 0.3$ , respectively. Therefore, our PQCD results are well within the perturbative region. It is reasonable to assume that  $O(\alpha_s^2)$  corrections to the decay amplitudes are about 15%. In the derivation of the hard functions, we have neglected the mass difference  $\bar{\Lambda} = M_B - m_b$  to obtain the leading-twist factorization formulas. Next-to-leading-twist corrections, proportional to  $\bar{\Lambda}/M_B$ , are then about 10%.

At last, we investigate the effects of  $SU(3)$  symmetry breaking in the  $B \rightarrow K\pi$  decays, taking the  $B^\pm \rightarrow K\pi^\pm$  and  $B_d^0 \rightarrow K^\pm \pi^\mp$  modes as an example. We assume  $f_K = f_\pi = 130$  MeV, which causes 20% reduction in the  $B \rightarrow K\pi$  decay amplitudes, and  $m_{0K} = m_{0\pi} = 1.4$  GeV, which causes 12% reduction. We also assume that the kaon wave functions  $\phi_K$  and  $\phi'_K$  have the same forms as the pion wave functions  $\phi_\pi$  and  $\phi'_\pi$ , respectively. However, this effect is mild. The  $SU(3)$  breaking effects are then found to be 32% in the amplitude level, and the branching ratios become

$$\begin{aligned}
 \text{Br}(B^+ \rightarrow K^0 \pi^+) &= 11.50 \times 10^{-6}, \\
 \text{Br}(B^- \rightarrow \bar{K}^0 \pi^-) &= 11.21 \times 10^{-6}, \\
 \text{Br}(B_d^0 \rightarrow K^+ \pi^-) &= 13.87 \times 10^{-6}, \\
 \text{Br}(\bar{B}_d^0 \rightarrow K^- \pi^+) &= 8.77 \times 10^{-6}.
 \end{aligned} \tag{77}$$

It is observed that the magnitude of  $CP$  asymmetry in the  $B_d^0 \rightarrow K^\pm \pi^\mp$  modes increases by 26% from  $-17.9\%$  to  $-22.5\%$ , which is due to the smaller branching ratios, i.e., the smaller denominator in Eq. (3).

## V. PENGUIN ENHANCEMENT

In this section we shall highlight the enhancement of penguin contributions observed in the PQCD approach, and its role in the explanation of the  $B \rightarrow \pi\pi$  and  $B \rightarrow K\pi$  data. For simplicity, we demonstrate our observation by means of the FA approach. Consider the ratios  $R$  in Eq. (2) and  $R_\pi$  defined by

$$R_\pi = \frac{\text{Br}(B_d^0 \rightarrow K^\pm \pi^\mp)}{\text{Br}(B_d^0 \rightarrow \pi^\pm \pi^\mp)}, \tag{78}$$

which can be written as

$$R = \frac{a_K^2 + 2a_K \lambda^2 R_b \cos \phi_3}{a_K^2}, \tag{79}$$

$$R_\pi = \frac{a_K^2 + 2a_K \lambda^2 R_b \cos \phi_3}{\lambda^2 R_b [R_b + 2a_\pi (R_b - \cos \phi_3)]}. \tag{80}$$

The factors

$$a_K = \frac{a_4 + 2r_K a_6}{a_1}, \quad a_\pi = \frac{a_4 + 2r_\pi a_6}{a_1}, \tag{81}$$

being negative values, represent the ratios of the penguin contribution to the tree contribution in the  $K\pi$  and  $\pi\pi$  modes, respectively. It is obvious that the data  $R \sim 1$  imply  $\phi_3 \sim 90^\circ$ , no matter what  $a_K$ ,  $\lambda$ , and  $R_b$  are. It is the reason when we vary all the parameters in the analysis in Sec. IV, the extraction of  $\phi_3$  remains invariant.

While to determine  $\phi_3$  from the data of the ratio  $R_\pi$ , one must have precise information of  $a_K$  and  $a_\pi$ , and of the parameters  $\lambda$  and  $R_b$ . It can be shown that the extraction of  $\phi_3$  from  $R_\pi$  depends on these parameters sensitively. Hence,  $R_\pi$  is not an appropriate quantity for the determination of  $\phi_3$ . To explain the data of  $R_\pi \sim 3-4$  in the FA approach, an unreasonably large  $m_0 \sim 4$  GeV corresponding to  $m_d = 2m_u = 3$  MeV, i.e., large  $|a_{K(\pi)}| \sim 0.09$  and a large  $\phi_3 \sim 130^\circ$  must be postulated [21]. This is obvious from Eq. (80), since a large  $|a_{K(\pi)}|$  enhances  $R_\pi$ , and a large  $\phi_3$  leads to constructive interference between the two terms in the numerator of  $R_\pi$ . The determination  $\phi_3 \sim 114^\circ$  from global fits to charmless  $B$  meson decays [21], located between the two

extreme cases  $90^\circ$  and  $130^\circ$ , is then understood. The result of  $\phi_3$  will become even larger, if reasonable  $m_0 \sim 1.4$  GeV are employed. The huge difference between  $90^\circ$  and  $130^\circ$  extracted from different data renders the determination of  $\phi_3$  in the FA approach less convincing. In the modified FA approach with effective number of colors  $N_c^{\text{eff}}$ , a large unitarity angle  $\phi_3 \sim 105^\circ$  is also concluded [51].

An interesting question is as follows. If we give higher weight to the extraction of  $\phi_3$  from  $R$ , which is more model-independent than that from  $R_\pi$ , can we explain the data  $R_\pi \sim 3-4$  using a smaller  $\phi_3$ ? The answer is positive in the PQCD approach. Table I shows that the ratio of the penguin contribution to the tree contribution reaches

$$|a_K| = \left| \frac{F_e^P}{F_e} \right| \sim 0.1, \tag{82}$$

even with a reasonable value of  $m_0 = 1.4$  GeV. The reason is that we do not assume the same form factors for the operators  $O_{1,2,3,4}$  and for  $O_{5,6}$ . These form factors, evaluated explicitly in the PQCD formalism, possess different factorization formulas as shown in Eqs. (27) and (28). It is easy to observe that the integrands in the two factorization formulas become identical, if the terms associated with the pseudo-scalar wave function  $\phi'_\pi$  and the factors  $x_3$  are dropped. The  $x_3 \rightarrow 0$  limit corresponds to the kinematic configuration, in which the light quark emitted from the  $b$  quark decay vertex carries the full meson momentum. This is the configuration, on which the equality of the two form factors in the FA approach is constructed. Therefore, the larger ratio of the penguin contribution to the tree contribution is achieved dynamically, instead of by increasing  $m_0$ . With this penguin enhancement, the observed branching ratios of the  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$  decays and  $R_\pi \sim 3-4$  can be explained simultaneously in the PQCD approach using  $m_0 = 1.4$  GeV and a smaller  $\phi_3 = 90^\circ$ . That is, the data of  $R_\pi$  do not demand large  $m_0$  and  $\phi_3$ . Such a dynamical enhancement of penguin contributions can not be obtained in the FA approach.

One of the sources responsible for the penguin enhancement is the RG evolution effect caused by the running hard scale  $t$ . In Fig. 8 we display the RG evolution of the Wilson coefficients  $a_i(\mu)$ ,  $i=1,4,6$ . It is found that  $a_1$  is almost constant for  $\mu = 500$  MeV to  $M_B$ . In contrast,  $|a_4|$  and  $|a_6|$  dramatically increase as  $\mu$  evolves to below  $M_B/2$ . If choosing  $t = M_B/2$  with  $m_0 = 1.4$  GeV, the ratio  $|a_{K(\pi)}| \sim 0.06$ , close to that in the FA approach with the same value of  $m_0$ , is too small to explain  $R_\pi \sim 3-4$ . As stated before, PQCD provides a prescription for choices of the hard scales  $t$ :  $t$  should be chosen as the virtualities of internal particles in Eq. (37) in order to decrease higher-order corrections. It reflects the fact that energy releases and evolution effects involved in different  $B$  meson decay modes are different. These hard scales can then reach lower values, at which  $|a_6(t)|$  is enhanced over  $|a_6(M_B/2)|$ . This evolution effect increases  $|a_{K(\pi)}|$  by about 50% as indicated by Eq. (82).

The enhancement due to the increase of  $C_6(t)$  with decreasing  $t$  makes us worry that the contribution from the small  $t$  region may be important. This will invalidate the

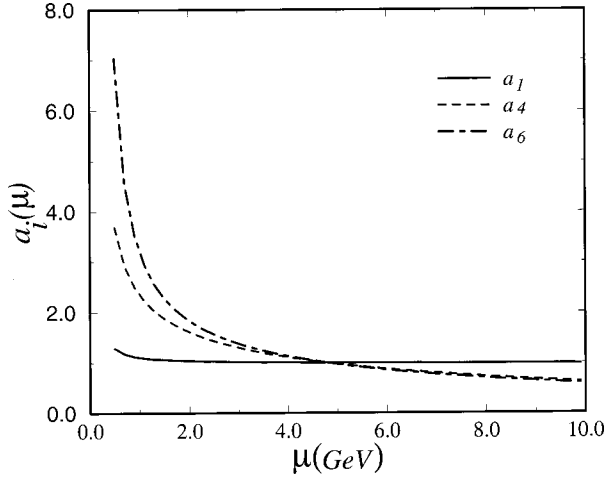


FIG. 8. RG evolution of the Wilson coefficients  $a_i(\mu)$ ,  $i=1, 4, 6$ , normalized by their values at  $\mu=m_b$ .

perturbative expansion of the hard amplitudes. As a check, we examine the fractional contribution to  $F_{e6}^P$  as a function of  $\alpha_s(t)/\pi$ . The results are displayed in Fig. 9, which indicate that about 80% (90%) of the contributions come from the region with  $\alpha_s(t)/\pi < 0.2$  (0.3). Therefore, exchanged gluons are still hard enough to guarantee the applicability of PQCD.

Another source of penguin enhancement is the behavior of the  $B$  meson wave function at  $x \rightarrow 0$ . As shown in Eqs. (27) and (28), the factorization formulas consist of two terms. It can be easily verified that when the two terms are roughly equal, the ratio of the penguin contribution to the tree contribution reaches its maximum. A simple investigation reveals the approximate expressions of the hard functions at small momentum fractions,

$$h_e(x_1, x_3, b_1, b_3) \sim \ln(x_1 x_3) \ln x_3,$$

$$h_e(x_3, x_1, b_3, b_1) \sim \ln(x_1 x_3) \ln x_1. \quad (83)$$

A  $B$  meson wave function with other behaviors, say,  $\phi_B \sim x$  or  $\sqrt{x}$  [35] as  $x \rightarrow 0$ , leads to the dominance of the second term, and the penguin contribution becomes relatively smaller. While the  $B$  meson wave function in Eq. (58), which vanishes like  $x^2$  as  $x \rightarrow 0$ , renders the contributions from the above two terms approximately the same. The penguin contribution corresponding to Eq. (58) is about 10% larger than that corresponding to the model in [35].

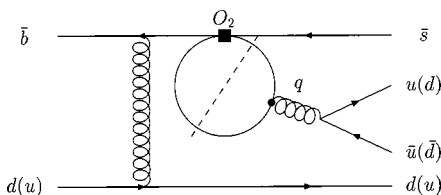


FIG. 9. Fraction contribution to  $F_{e6}^P$  as a function of  $\alpha_s(t)/\pi$ .

## VI. FINAL STATE INTERACTION

Two-body final state interaction (FSI) effects have been studied in various ways [52]. It was found that these effects enhance the  $CP$  asymmetry in the  $B^\pm \rightarrow K^0 \pi^\pm$  modes from order 0.5% under the FA [53] up to order (10–20)%. However, Kamal has pointed out that the large  $CP$  asymmetry is due to an overestimation of FSI effects by a factor of 20 [54]. For a critical assessment on the analyses of FSI effects in the literature, refer to [54].

We briefly sketch the methods used in most of the estimates of FSI effects. For simplicity, we consider only the  $B^+ \rightarrow K^0 \pi^+$  decay. The unitarity relation for the amplitude  $\mathcal{A}(B^+ \rightarrow K^0 \pi^+)$  is written as

$$\Im \mathcal{A}(B^+ \rightarrow K^0 \pi^+) = \frac{1}{2} \sum_N 2\pi \delta(M_B - E_N) \times \mathcal{A}(N \rightarrow K^0 \pi^+) \mathcal{A}^*(B^+ \rightarrow N). \quad (84)$$

If only the elastic channel  $K^+ \pi^0$  contributes, Watson's theorem tells that the phase of  $\mathcal{A}(B^+ \rightarrow K^0 \pi^+)$  is given by the  $S$  wave  $I=3/2$  phase shift. This argument works for the  $K \rightarrow \pi\pi$  decays but not for  $B$  meson decays. For  $M_B \sim 5$  GeV, many channels contribute and Watson's theorem says nothing about the strong phase of  $\mathcal{A}(B^+ \rightarrow K^0 \pi^+)$ . In fact, even if the phases of  $\mathcal{A}(N \rightarrow K^0 \pi^+)$  for all  $N$  are known, the unitarity relation does still not fix the phase of  $\mathcal{A}(B^+ \rightarrow K^0 \pi^+)$  uniquely.

In spite of this difficulty, some authors computed  $\mathcal{A}(N \rightarrow K^0 \pi^+)$  for few  $N$ . Certainly, more than the unitarity relation is needed to obtain the strong phase of  $\mathcal{A}(B^+ \rightarrow K^0 \pi^+)$ . The phases of  $\mathcal{A}(N \rightarrow K^0 \pi^+)$  are often estimated by a Regge analysis. However, this method is reliable only near the forward direction. In our problem we need  $S$  wave amplitudes, i.e., scattering amplitudes for all angles. A big assumption of a straight line trajectory has been adopted. This is highly questionable, especially for Pomerons. For these reasons, we believe that the above analyses are qualitative at most.

It is our viewpoint that if a strong phase cannot be determined in QCD, there is no other way to compute it. A simple physical picture of FSI, the color-transparency argument [55], has been put forward by Bjorken [56].

Since products of a  $B$  meson decay into two light mesons are quite energetic, the quark-antiquark pair inside a meson remains a state of small size with a correspondingly small chromomagnetic moment until it is far from the other meson. It is then more realistic that the two quark pairs group individually into final-state mesons without further exchanging soft gluons.

This picture is consistent with our observation: Sudakov suppression is strong for large meson momenta as shown in Eq. (14), which then demands final-state mesons of small transverse extent  $b$ , i.e., of small chromomagnetic moment.

The effects from soft gluon exchanges among mesons in two-body heavy meson decays have been analyzed quantitatively by means of RG methods, which sum up large loga-



rithms produced by infinite gluon emissions. It was found that these effects generate only small FSI phases for  $B$  meson decays, in agreement with the color-transparency argument, but large FSI phases for  $D$  meson decays [57]. That soft gluon effects are large in  $D$  meson decays is expected, since Sudakov suppression is weaker, two quarks in a final-state meson is separated by a larger distance, and soft gluons can resolve the color structure of that meson. Based on the above reasonings, we have neglected FSI effects in the PQCD approach to two-body  $B$  meson decays.

To justify the neglect of FSI, we apply our formalism to the  $B \rightarrow \pi\pi$  and  $B \rightarrow K\pi$  decays without taking into account these long-distance effects. FSI in these decays should be different. Since tree contributions dominate in the  $B \rightarrow \pi\pi$  decays, extra phases from FSI do not change branching ratios very much. This argument applies to the decays  $B^\pm \rightarrow K^0\pi^\pm$  and  $B_d^0 \rightarrow K^0\pi^0$ , where penguin contributions dominate. While the interference between tree and penguin contributions plays an essential role in the  $B_d^0 \rightarrow K^\pm\pi^\mp$  and  $B^\pm \rightarrow K^\pm\pi^0$  decays. Large FSI effects will change the relative phases between tree and penguin amplitudes, and thus branching ratios. If the same formalism without including FSI can be applied to both decays successfully, we believe that these long-distance effects are negligible. The agreement of our results with the data shown in Eq. (70) implies this conclusion.

It has been argued that the  $B \rightarrow KK$  decays are sensitive to FSI effects [58,59]. Without FSI, the  $B_d^0 \rightarrow K^\pm K^\mp$  decays possess very small branching ratios, since they involve only nonfactorizable annihilation amplitudes, and the  $B_d^0 \rightarrow K^0 K^0$  decays do not exhibit  $CP$  asymmetry, since they involve only penguin contributions. We have applied the PQCD formalism to the  $B \rightarrow KK$  decays and predicted the branching ratios and the  $CP$  asymmetries of various modes [60]. The comparison of the predictions with future experimental data will reveal whether FSI effects are important. For more details, refer to [60].

As stated in Sec. IV, large strong phases come from the factorizable annihilation diagrams (phases from nonfactorizable diagrams are small) in the PQCD approach. There has been a widely spread folklore that the annihilation diagrams give negligible contribution due to helicity suppression, the same as in  $\pi \rightarrow e\bar{\nu}$  decay. That is, a left-handed massless electron and a right-handed antineutrino can not fly away back to back because of angular momentum conservation. However, this argument does not apply to  $F_{a6}^P$ . A left-handed quark and a left-handed antiquark, for which helicities are dictated by the  $O_6$  operator, can indeed fly away back to back [61]. These behaviors have been reflected by Eqs. (29) and (30): Eq. (29) vanishes exactly, if the kaon and pion wave functions are identical, while the two terms in Eq. (30) are constructive. The reason the annihilation diagrams from the  $O_6$  operator possess large absorptive parts can be understood in the following way. The cuts on the internal quark lines in Figs. 3(e) and 3(f) correspond to a process

$$B^+ \rightarrow \bar{s}u \rightarrow K^0\pi^+. \quad (85)$$

The intermediate state ( $\bar{s}u$ ) can be regarded as being highly inelastic, if expanded in terms of hadron states. According to Eq. (84), large strong phases are expected.

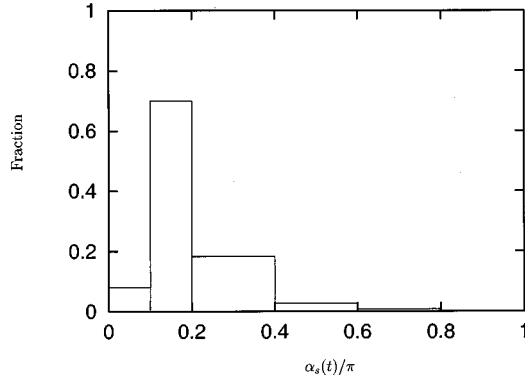
## VII. COMPARISON WITH OTHER ANALYSES

In this section we briefly compare our PQCD approach with the other approaches to exclusive nonleptonic  $B$  meson decays. For more details, refer to [23]. Beneke *et al.* proposed to evaluate nonfactorizable contributions to charmless  $B$  meson decays in the PQCD framework [22]. They argued that factorizable contributions (transition form factors) are not calculable in perturbation theory, but nonfactorizable contributions are. The reasoning is as follows. The internal  $b$  quark in the hard amplitude may go onto mass shell, producing a divergent factor  $x^{-2}$ ,  $x$  being the momentum fraction associated with the pion. The soft divergence from  $x \rightarrow 0$  can not be removed by a pion wave function, if it vanishes like  $x$  as  $x \rightarrow 0$ . Since this divergence is not of the pinched type which is absorbed into a wave function, its appearance implies the breakdown of PQCD factorization theorem. While such a power divergence does not exist in nonfactorizable amplitudes [22].

We argue that the  $x^{-2}$  factor in fact can be easily smeared out by parton transverse momenta  $k_T$  considered in this work or killed by a wave function vanishing faster than  $x$  as  $x \rightarrow 0$ . That is, the conclusion that form factors must be treated as nonperturbative inputs [22] depends on models of the pion wave function one adopts. By including  $k_T$  to regulate the divergence, large logarithmic corrections  $\alpha_s \ln k_T$  appear, and Sudakov resummation is demanded. With the resultant Sudakov suppression, we have explicitly shown that almost 100% of the full contribution to the  $B \rightarrow \pi$  transition form factor arises from the region with the coupling constant  $\alpha_s/\pi < 0.3$ . It indicates that dynamics from hard gluon exchanges indeed dominate in the PQCD calculation. In [22] Sudakov resummation is irrelevant, since all QCD dynamics has been parametrized into models of form factors.

There are other important differences between our approach and [22]. The momentum of the light spectator quark in the  $B$  meson has been ignored in the formalism of [22], such that quark propagators in hard nonfactorizable decay amplitudes always remain time-like. The annihilation diagrams were not included either. With these approximations, leading-order information of strong phases was lost. Strong phases then arise from diagrams of the BSS mechanism, which, as shown below, are small compared to those from annihilation diagrams. On the other hand, Sudakov resummation of large logarithmic corrections was not taken into account. It is then expected that higher-order corrections will be large and spoil the perturbative expansion. It has been shown [62] that the PQCD formalism without including Sudakov suppression is not applicable to exclusive processes for energy scale below 10 GeV.

We show that strong phases from the BSS mechanism are suppressed by the charm mass threshold and by  $O(\alpha_s)$ , since there must be a hard gluon emitted by the spectator as shown in Fig. 10, which turns the soft spectator in the  $B$  meson into a fast spectator in the final-state meson. That is, the contri-

FIG. 10. Feynman diagram for an induced  $c$  ( $u$ )-quark loop.

butions from the penguin diagram have been overestimated. The charm quark loop contributes an imaginary part,

$$C_2(t)\alpha_s(t)\int du u(1-u)\theta(q^2u(1-u)-m_c^2), \quad (86)$$

where  $q^2$  is the invariant mass of the gluon emitted from the penguin contribution. The contribution from the  $u$  quark loop is suppressed by the small CKM factor  $|V_u|$ . Since  $q^2$  is not clearly defined in the FA approach, it is usually chosen as  $q^2=m_b^2$  or  $q^2=m_b^2/2$ , and Eq. (86) gives a substantial amount of imaginary contribution to decay amplitudes [63].

However, the invariant mass  $q^2$  can be defined unambiguously in the PQCD formalism by

$$q^2=(x_2P_2+x_3P_3)^2=x_2x_3M_B^2, \quad (87)$$

since the quark going into the kaon (pion) carries the fractional momentum  $x_2P_2$  ( $x_3P_3$ ). Then,  $q^2=m_b^2$  or  $m_b^2/2$  corresponds to a configuration, in which the two quarks produced from the gluon carry the full momenta of the two final-state mesons. Obviously, this configuration is unlikely because of the strong suppression from the kaon and pion wave functions in the large  $x$  region. Substituting Eq. (87) into Eq. (86), an exact numerical analysis indicates that the BSS mechanism contributes an imaginary part smaller than that from the nonfactorizable and annihilation amplitudes by a factor of 10. Table II shows how the imaginary part of the charm quark loop contribution vanishes with the decrease of  $q^2$ .

On the issue of FSI, Suzuki has argued that strong phases of the  $B\rightarrow K\pi$  amplitudes can not be evaluated in QCD [65].

TABLE II. Real and imaginary parts of the charm quark loop contribution  $G(q^2)=-4\int du u(1-u)\ln[m_c^2-q^2u(1-u)]$  in the BSS mechanism.

$q^2$	Re[ $G$ ]	Im[ $G$ ]
$m_B^2$	-0.760	2.025
$m_B^2/2$	0.139	1.775
$m_B^2/3$	0.912	1.178
$m_B^2/3.5$	1.288	0.392
$m_B^2/4$	1.162	0.000

He pointed out that the invariant masses of the  $\bar{s}u$  and  $\bar{d}u$  pairs in Figs. 3(c) and 3(d), respectively, are of order  $(\Lambda_{\text{QCD}}M_B)^{1/2}\sim 1.2$  GeV. It implies that the  $B\rightarrow K\pi$  decays are located in the resonance region and their strong phases are very complicated. We have computed the average hard scales of the  $B\rightarrow K\pi$  decays, which are about 1.4 GeV, in agreement with the above estimate. However, the outgoing quark pairs possess an invariant mass larger than 1.4 GeV, such that the processes are in fact not so close to the resonance region. We could interpret that the decays occur via a six-fermion operator within space smaller than  $(1/1.4)$  GeV $^{-1}$ . While they are not completely short-distance, the fact that over 90% of contributions come from the  $x$ - $b$  phase space with  $\alpha_s(t)/\pi < 0.3$  allows us to estimate the decay amplitudes reliably. We believe that the strong phases can be computed up to about 20% uncertainties, which result in 30% errors in the predictions for  $CP$  asymmetries.

## VIII. CONCLUSION

In this paper we have analyzed the  $B\rightarrow K\pi$  decays using PQCD factorization theorem. In this approach hadronic matrix elements, including factorizable and nonfactorizable, and real and imaginary contributions can be evaluated explicitly. The strong phases arise from nonpinched singularities of quark and gluon propagators in annihilation and nonfactorizable hard amplitudes. It has been explicitly shown that strong phases from the BSS mechanism are small. The analysis of soft gluon effects and the simultaneous success of the PQCD applications to the  $B\rightarrow K\pi$  and  $B\rightarrow\pi\pi$  decays implied that long-distance FSI effects are negligible. The universal meson wave functions have been determined from the available data of the pion form factor and of the  $B\rightarrow D\pi$  and  $B\rightarrow\pi\pi$  decays. The dependencies of the ratio  $R$  of the neutral  $B$  decay branching ratio to the charged  $B$  decay branching ratio and of the  $CP$  asymmetries on  $\phi_3$  have been derived. Our predictions for all the  $B\rightarrow K\pi$  modes are consistent with the experimental data.

In spite of potential theoretical uncertainties, we have extracted the following features for the  $B\rightarrow K\pi$ ,  $\pi\pi$  decays, which are less ambiguous: (1) Nonfactorizable amplitudes are negligible; (2) annihilation diagrams are not negligible; (3) annihilation diagrams generate large strong phases; (4) more precise data are needed in order to obtain a strong constraint on  $\phi_3$ ; (5)  $R$  is an ideal quantity for the determination of  $\phi_3$ , since it is insensitive to all the Wolfenstein and nonperturbative QCD parameters; (6)  $\phi_3$  is about  $90^\circ$  from fitting our predictions to the central value of the data of  $R$ ; (7) penguin amplitudes are dynamically enhanced, and larger than those employed in the FA approach by 50%; (8) the data of  $B\rightarrow\pi\pi$  decays, i.e., the ratio  $R_\pi$  of the  $B\rightarrow K\pi$  branching ratio to the  $B\rightarrow\pi\pi$  branching ratio can be explained by the smaller angle  $\phi_3\sim 90^\circ$ . That is, the data of  $R_\pi$  do not demand a large  $\phi_3>90^\circ$ .

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### APPENDIX A

In this appendix we supply the details of the Wilson evolution. The matching conditions at  $\mu = M_W$  are given by [32]

$$C_1(M_W) = 0,$$

$$C_2(M_W) = 1,$$

$$C_3(M_W) = -\frac{\alpha_s(M_W)}{24\pi} E_0(x_t) + \frac{\alpha_{em}}{6\pi} \frac{1}{\sin^2 \Theta_W} [2B_0(x_t) + C_0(x_t)],$$

$$C_4(M_W) = \frac{\alpha_s(M_W)}{8\pi} E_0(x_t),$$

$$C_5(M_W) = -\frac{\alpha_s(M_W)}{24\pi} E_0(x_t),$$

$$C_6(M_W) = \frac{\alpha_s(M_W)}{8\pi} E_0(x_t),$$

$$C_7(M_W) = \frac{\alpha_{em}}{6\pi} [4C_0(x_t) + D_0(x_t)],$$

$$C_8(M_W) = 0,$$

$$C_9(M_W) = \frac{\alpha_{em}}{6\pi} \left\{ 4C_0(x_t) + D_0(x_t) + \frac{1}{\sin^2 \Theta_W} [10B_0(x_t) - 4C_0(x_t)] \right\},$$

$$C_{10}(M_W) = 0, \quad (A1)$$

with  $x_t = m_t^2/M_W^2$ ,  $m_t$  being the top quark mass. The functions  $B_0$ ,  $C_0$ ,  $D_0$ , and  $E_0$  are the Inami-Lim functions [64]:

$$B_0(x) = \frac{1}{4} \left[ \frac{x}{1-x} + \frac{x \ln x}{(x-1)^2} \right], \quad (A2)$$

$$C_0(x) = \frac{x}{8} \left[ \frac{x-6}{x-1} + \frac{3x+2}{(x-1)^2} \ln x \right], \quad (A3)$$

$$D_0(x) = -\frac{4}{9} \ln x - \frac{19x^3 - 25x^2}{36(x-1)^3} + \frac{x^2(5x^2 - 2x - 6)}{18(x-1)^4} \ln x, \quad (A4)$$

$$E_0(x) = -\frac{2}{3} \ln x - \frac{x^2(15 - 16x + 4x^2)}{6(x-1)^4} \ln x + \frac{x(18 - 11x - x^2)}{12(1-x)^3}. \quad (A5)$$

We adopt the following parameters:  $m_t = 170$  GeV,  $M_W = 80.2$  GeV,  $\alpha_s(M_W) = 0.118$ ,  $\alpha_{em} = 1/129$ ,  $\sin^2 \Theta_W = 0.23$  and  $\Lambda_{\overline{MS}}^{(4)} = 250$  MeV.

The solution to Eq. (57) is written as

$$\vec{C}(\mu) = U(t, m_b) \vec{C}(m_b). \quad (A6)$$

The evolution function including electroweak penguin diagrams is

$$U(t, m_b, \alpha_{em}) = U_f(t, m_b) + \frac{\alpha_{em}}{4\pi} \int_{\ln m_b}^{\ln t} d \ln \mu' U_f(t, \mu') \times [\hat{\gamma}_e^{(0)T}]_f U_f(\mu', m_b) = U_f(t, M_b) + \frac{\alpha_{em}}{4\pi} R_f(t, m_b), \quad (A7)$$

with

$$U_f(t, m_b) \equiv \exp \left[ \int_{\ln m_b}^{\ln t} d \ln \mu' \frac{\alpha_s(\mu')}{4\pi} [\hat{\gamma}_s^{(0)T}]_f \right]. \quad (A8)$$

For  $\mu = m_b = 4.8$  GeV, the values of  $C_i(m_b)$  are

$$\begin{aligned} C_1(m_b) &= -0.271, & C_2(m_b) &= 1.124, \\ C_3(m_b) &= 1.255 \times 10^{-2}, & C_4(m_b) &= -2.686 \times 10^{-2}, \\ C_5(m_b) &= 7.805 \times 10^{-3}, & C_6(m_b) &= -3.287 \times 10^{-2}, \\ C_7(m_b) &= 3.453 \times 10^{-4}, & C_8(m_b) &= 3.177 \times 10^{-4}, \\ C_9(m_b) &= -9.765 \times 10^{-3}, & C_{10}(m_b) &= 2.240 \times 10^{-3}. \end{aligned} \quad (A9)$$

Values of the Wilson coefficients at different energy scales  $\mu = 1.0$  GeV, 1.5 GeV, 2.0 GeV, 2.5 GeV, 3.0 GeV, and 4.8 GeV are listed in Table III.

### APPENDIX B

Below we summarize the parameters we have adopted in the numerical analysis of this work:

(1) Masses, decay constants, and lifetimes:

$$m_{0\pi} = 1.4 \text{ GeV}, \quad m_{0K} = 1.7 \text{ GeV},$$

$$M_B = 5.28 \text{ GeV}, \quad m_b = 4.8 \text{ GeV},$$

TABLE III. Values of the running coupling constant  $\alpha_s$  and the Wilson coefficients  $C_i$  with  $\Lambda_{\overline{MS}}^{(4)} = 250$  MeV for different energy scales  $\mu = 1.0, 1.5, 2.0, 2.5, 3.0,$  and  $4.8$  GeV.

	$\Lambda_{\overline{MS}}^{(4)} = 250$ MeV					
$\mu$	1.0 GeV	1.5 GeV	2.0 GeV	2.5 GeV	3.0 GeV	4.8 GeV
$\alpha_s(\mu)$	0.5439	0.4208	0.3626	0.3275	0.3034	0.2552
$C_1$	-0.650	-0.510	-0.435	-0.385	-0.349	-0.271
$C_2$	1.362	1.268	1.219	1.189	1.168	1.124
$C_3$	0.036	0.027	0.022	0.019	0.017	0.013
$C_4$	-0.063	-0.050	-0.043	-0.038	-0.035	-0.027
$C_5$	0.015	0.013	0.012	0.011	0.010	0.008
$C_6$	-0.102	-0.074	-0.060	-0.051	-0.045	-0.033
$C_7/\alpha_{em}$	0.040	0.035	0.035	0.036	0.038	0.045
$C_8/\alpha_{em}$	0.128	0.091	0.073	0.062	0.055	0.041
$C_9/\alpha_{em}$	-1.509	-1.416	-1.366	-1.334	-1.311	-1.260
$C_{10}/\alpha_{em}$	0.695	0.546	0.465	0.412	0.373	0.289

$$m_c = 1.5 \text{ GeV}, \quad m_t = 170 \text{ GeV},$$

$$M_W = 80.2 \text{ GeV}, \quad f_B = 190 \text{ MeV},$$

$$f_\pi = 130 \text{ MeV}, \quad f_K = 160 \text{ MeV},$$

$$\tau_{B^0} = 1.55 \text{ ps}, \quad \tau_{B^-} = 1.65 \text{ ps}.$$

(2) QCD and electroweak parameters:

$$G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}, \quad \Lambda_{\overline{MS}}^{(4)} = 250 \text{ MeV},$$

$$\alpha_s(M_Z) = 0.117, \quad \alpha_{em} = 1/129,$$

$$\lambda = 0.2196, \quad A = 0.819,$$

$$R_b = \sqrt{\rho^2 + \eta^2} = 0.38.$$

(3) Meson wave functions:

$$\phi_B(x) = N_B x^2 (1-x)^2 \exp \left[ -\frac{1}{2} \left( \frac{x M_B}{\omega_B} \right)^2 - \frac{\omega_B^2 b^2}{2} \right],$$

$$N_B = 203.664 \text{ GeV}, \quad \omega_B = 0.3 \text{ GeV},$$

$$\phi_\pi(x) = \frac{3}{\sqrt{2N_c}} f_\pi x(1-x) [1 + 0.8(5(1-2x)^2 - 1)],$$

$$\phi'_\pi(x) = \frac{3}{\sqrt{2N_c}} f_\pi x(1-x),$$

$$\phi_K(x) = \frac{3}{\sqrt{2N_c}} f_K x(1-x) [1 + 0.51(1-2x) + 0.3(5(1-2x)^2 - 1)],$$

$$\phi'_K(x) = \frac{3}{\sqrt{2N_c}} f_K x(1-x).$$

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