Note on solitons in brane worlds

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We obtain the zero-mode effective action for gravitating objects in the bulk of dilatonic domain walls. Without additional fields included in the bulk action, the zero-mode effective action reproduces the action in one lower dimension obtained through ordinary Kaluza-Klein (KK) compactification, only when the transverse (to the domain wall) component of the bulk metric does not have a nontrivial term depending on the domain wall world volume coordinates and the tension of the domain wall is positive. With additional fields included in the bulk action, a nontrivial dependence of the transverse metric component on the domain wall longitudinal coordinates appears to be essential in reproducing the lower-dimensional action obtained via ordinary KK compactification. We find, in particular, that the effective action for the charged $(p+1)$-brane in the domain wall bulk reproduces the action for the p-brane in one lower dimension.

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I. INTRODUCTION

Recently, phenomenologists have actively considered the possibility that the existence of additional compact spatial dimensions may account for the large hierarchy between the electroweak scale and the Planck scale, the so-called hierarchy problem in particle physics. In this scenario, our four-dimensional world is confined within the world volume of a three-brane, within which the fields of the standard model are contained. The earlier proposal [1,2] relies on a large enough volume of the compact extra space for solving the hierarchy problem. A more compelling scenario proposed by Randall and Sundrum (RS) [3–5] assumes that the spacetime is nonfactorizable, in contrast with the conventional view of Kaluza-Klein (KK) theory that the spacetime is the direct product of the four-dimensional spacetime and the compact extra space. Such a point of view of spacetime was also previously taken [6–13] as an alternative to compact compactification—namely, as a mechanism to trap matter within the four-dimensional hypersurface without having to assume that the extra space is compact. However, what is new in the RS model is that it is not just matter but also gravity that is localized within the hypersurface. The exponential falloff (as one moves away from the brane) of the metric warp factor accounts for the large hierarchy between the electroweak and the Planck scales in our four-dimensional world, which is assumed to be located away from the wall.

Since gravity is shown to be effectively compactified to one lower dimension (even when the extra spatial dimension is infinite) in the bulk of the RS domain walls [4], it is of interest to study various gravitating objects in such a background. (Some of the previous works on a related subject are Refs. [14–18].) In our previous works [19–21], we attempted to understand charged branes in the bulk of the RS-type domain wall. It turns out that the domain wall bulk background is so restrictive about the possible gravitating objects that nondilatonic domain wall bulk in general does not allow charged branes. One of the ways to get around this difficulty is to allow the bulk cosmological constant term to have the dilaton factor. In fact, unlike the nondilatonic domain wall of RS, dilatonic domain walls can be readily realized within string theories. It is observed [19] that even the dilatonic domain wall effectively compactifies gravity. The warp factor of the dilatonic domain wall that traps gravity also decreases within the finite allowed coordinate interval around the wall as a powerlaw, instead of exponentially within the infinite allowed coordinate interval just like the nondilatonic domain wall of the RS model [3–5], and becomes zero at the end of the allowed finite coordinate interval. So anyway one can also use such dilatonic domain walls for tackling the hierarchy problem.

It is realized [20] that charged p-branes, as observed in one lower dimension, should rather be regarded as charged $(p+1)$-branes in the bulk of domain walls (if one wishes to regard charged branes in the brane world as being originated from 10 or 11 dimensions of string theories), because charged p-branes in the bulk of domain wall backgrounds are not effectively compactified to the charged p-branes in one lower dimension on the hypersurface of the wall. We studied [21] the dynamics of probes in the background of such charged branes. In this paper, we check whether such charged $(p+1)$-branes in the domain wall bulk effectively describe the corresponding charged p-branes in one lower dimension or describe different physics in one lower dimension by obtaining the effective action for such charged $(p+1)$-branes in the bulk of the domain walls. The study of the effective action of the brane world scenario can generically provide one with the physical implications of the RS model such as any possible deviations of the effective lower-dimensional theory of the RS model from ordinary Einstein gravity. In fact, for the first RS model [3] with negative tension domain wall included (with nontrivial radion field), it was pointed out [22] that the lower-dimensional effective theory is described by Brans-Dicke (BD) theory [23], rather than by Einstein's general relativity. In the case of the second RS model [4] with only a positive tension domain wall and its dilatonic generalization, we find that the zero-mode gravity effective action in one lower dimension is that of

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Einstein's general relativity. Also, we find that the effective action for the bulk $(p+1)$-brane has exactly the same form as the action for the p-brane in one lower dimension that is obtained from the action for the $(p+1)$-brane through ordinary KK compactification on S^1. Therefore, for the gravitating configurations under consideration in this paper, we find no deviation from Einstein's general relativity and from ordinary KK compactification. Our result on the bulk charged $(p+1)$-brane case implies that when there are additional bulk fields the RS gauge of metric perturbations should be modified to include the transverse (to the domain wall) metric perturbation, and yet gravity can be localized around the domain wall.

II. DILATONIC DOMAIN WALL SOLUTION

We begin by discussing the D-dimensional extreme dilatonic domain wall solution studied in Ref. [19]. The total action for the domain wall is the sum of the bulk action

$$S_{\rm bulk}=\frac{1}{2\kappa_D^2}\int d^Dx\sqrt{-G}\left[\mathcal{R}_G-\frac{4}{D-2}\partial_M\phi\partial^M\phi+e^{-2a\phi}\Lambda\right], \tag{1}$$

and the following world volume action:

$$S_{\rm DW}=-\sigma_{\rm DW}\int d^{D-1}x\sqrt{-\gamma}e^{-a\phi}. \tag{2}$$

Here, Λ is the bulk cosmological constant, $\sigma_{\rm DW}$ is the domain wall tension, and γ is the determinant of the induced metric $\gamma_{\mu\nu}=\partial_\mu X^M\partial_\nu X^N G_{MN}$ on the domain wall world volume, where $M,N=0,1,\dots,D-1$ and $\mu,\nu=0,1,\dots,D-2$.

In this paper, we consider the domain wall solution in the conformally flat form as follows:

$$G_{MN}dx^Mdx^N=\mathcal{C}[\eta_{\mu\nu}dx^\mu dx^\nu+dz^2],\quad e^{2\phi}=\mathcal{C}^{(D-2)^2a/4}, \tag{3}$$

where the conformal factor $\mathcal{C}$ is given by

$$\mathcal{C}(z)=\left(1+\frac{\Delta+2}{\Delta}Q|z|\right)^{4/(D-2)(\Delta+2)},$$

$$\Delta=\frac{(D-2)a^2}{2}-\frac{2(D-1)}{D-2}. \tag{4}$$

Through the boundary condition at $z=0$ and the equations of motion, one can relate Λ and σ_{DW} to the parameter Q of the solution in the following way:

$$\Lambda=-\frac{2Q^2}{\Delta},\quad \sigma_{\rm DW}=-\frac{4}{\Delta}\frac{Q}{\kappa_D^2}. \tag{5}$$

In this paper, we assume that $Q>0$, so that the tension $\sigma_{\rm DW}$ of the wall is positive when $\Delta<0$. In particular, the domain wall solution of the RS model [3–5] corresponds to the $(D,a)=(5,0)$ case of the above general solution.

III. ZERO-MODE EFFECTIVE ACTION

In this section, we study the $(D-1)$-dimensional effective action obtained from the total action by integrating over the extra spatial coordinate. We shall consider only the zero modes of the fields, because the gravitating configurations under consideration in this paper are independent of the extra spatial coordinate.

We consider the following form of the bulk metric:

$$G_{MN}dx^Mdx^N=\mathcal{C}[g_{\mu\nu}dx^\mu dx^\nu+h^2dz^2], \tag{6}$$

where $\mathcal{C}$ is given by Eq. (4), and $g_{\mu\nu}$ and h depend on x^μ, only. When the bulk action has no additional fields, D-dimensional equations of motion are guaranteed to be satisfied when $h=1$ and $g_{\mu\nu}$ is Ricci flat (cf. Ref. [24]). When there are additional bulk fields, h can have a nontrivial dependence on x^μ, as can be seen from examples of the solutions for charged branes in the bulk of dilatonic domain wall.

First, we consider the case of no additional bulk fields. The total action (1) + (2) reduces to the following form:

$$\begin{aligned}S&=\frac{1}{2\kappa_D^2}\int d^{D-1}xdz\sqrt{-g}\varpi^{2/(\Delta+2)}\\&\quad\times\left[\mathcal{R}_g-8\frac{D-1}{D-2}\frac{Q}{\Delta}\left\{\delta(z)-\frac{DQ}{4(D-1)}\varpi^{-2}\right\}-\frac{2Q^2}{\Delta}\varpi^{-2}\right]\\&\quad+\frac{4}{\Delta}\frac{Q}{\kappa_D^2}\int d^{D-1}x\sqrt{-g}\\&=\frac{1}{2\kappa_D^2}\int d^{D-1}x\sqrt{-g}\left[-\frac{2\Delta Q^{-1}}{\Delta+4}\mathcal{R}_g-\frac{4Q}{\Delta}(1+1-2)\right],\end{aligned} \tag{7}$$

where $\varpi\equiv1+[(\Delta+2)/\Delta]Q|z|$. Since the domain wall is located at $z=0$, the fields in the world volume action take the forms $e^{-a\phi}=1$ and $\gamma_{\mu\nu}=g_{\mu\nu}$ (in the static gauge). Note that, in the last equality, we integrated over all possible values of z: Namely,

$$\frac{\Delta}{\Delta+2}Q^{-1}\leqslant z\leqslant-\frac{\Delta}{\Delta+2}Q^{-1}$$

for the $-2<\Delta<0$ case, and $-\infty<z<\infty$ for the $\Delta<-2$ case [cf. Eq. (4)]. (The $\Delta<-2$ case includes the $a=0$ case, i.e., the nondilatonic domain wall of the original RS model.) When $\Delta>0$ [in which case $\sigma_{\rm DW}$ given in Eq. (5) is negative if $Q>0$], integration over all the possible values of z, i.e., $-\infty<z<\infty$, will make the Einstein term diverge; i.e., the $(D-1)$-dimensional gravitational constant is zero and the gravity is not effectively compactified. So when $\sigma_{DW}>0$ (with $Q>0$), the effective action (7) reduces to the action for $(D-1)$-dimensional general relativity with the gravitational constant given by

$$\kappa_{D-1}^2=-\frac{\Delta+4}{2\Delta}Q\kappa_D^2, \tag{8}$$

as can be seen from the last line of Eq. (7). Note that Δ is always greater than -4 for $D>4$, so $\kappa^2_{D-1}>0$ if $\Delta<0$ and $Q>0$. Note that it is essential that $\sigma_{DW}>0$, in order for Einstein's gravity in one lower dimension to be reproduced. Namely, if $\sigma_{DW}<0$, the Einstein term would have diverged[1] (unless the extra spatial dimension is truncated) and no cancellation of the extra terms in the action (7) would have occurred (even if the extra spatial dimension is truncated). This is in accordance with the result of our previous work [19], that gravity cannot be localized around the domain wall if Δ and Q are positive. So any D-dimensional gravitating objects in the dilatonic (as well as nondilatonic) domain wall background with the bulk action given by Eq. (1) and the metric given by Eq. (6) with $h=1$ and Ricci-flat $g_{\mu\nu}$ effectively describe the corresponding configurations in general relativity in one lower dimension, as long as $\sigma_{DW}>0$, e.g., $\Delta<0$ with $Q>0$. This confirms that the RS model can be extended to the dilatonic domain wall case.

Now, we consider the case when the domain wall bulk spacetime contains a charged $(p+1)$-brane, where one of the longitudinal directions of the brane is along the transverse direction of the domain wall, whose explicit solution is studied in our previous work [21]. The total action S for this case is give by the sum of the bulk action

$$S_{\rm bulk}=\frac{1}{2\kappa_D^2}\int d^Dx\sqrt{-G}\bigg[\mathcal{R}_G-\frac{4}{D-2}(\partial\phi)^2+e^{-2a\phi}\Lambda - \frac{1}{2\cdot(p+3)!}e^{2a_{p+1}\phi}F^2_{p+3}\bigg], \tag{9}$$

the domain wall world volume action (2), and the following additional world volume action for the charged $(p+1)$-brane:

$$S_{p+1}=-T_{p+1}\int d^{p+2}\xi\bigg[e^{-a_{p+1}\phi}\sqrt{-\det\,\partial_aX^M\partial_bX^NG_{MN}} + \frac{\sqrt{\Delta_{p+1}}}{2}\frac{1}{(p+2)!}\epsilon^{a_1\cdots a_{p+2}}\partial_{a_1}X^{M_1}\cdots\partial_{a_{p+2}} \times X^{M_{p+2}}A_{M_1\cdots M_{p+2}}\bigg],$$

$$\Delta_{p+1}=\frac{(D-2)a^2_{p+1}}{2}+\frac{2(p+2)(D-p-4)}{(D-2)}. \tag{10}$$

The consistency of the equations of motion requires that a and a_{p+1} satisfy the following constraint:

$$aa_{p+1}=-\frac{4(D-p-4)}{(D-2)^2}. \tag{11}$$

Guided by the explicit solution presented in Ref. [21], we take Eq. (6) as the D-dimensional metric *Ansatz* and the following as the remaining field *Ansätze*:

$$e^{2\phi}=\mathcal{C}^{(D-2)^2a/4}e^{2\tilde\phi},\quad A_{p+2}=\mathcal{C}^{(D-2)/2}\tilde A_{p+2}, \tag{12}$$

where the tilded fields [describing the parts of fields associated with the bulk $(p+1)$-brane] depend on x^μ, only, and $\mathcal{C}$ is given by Eq. (4). Substituting the above *Ansätze* for the fields into the total action S and integrating over z, we obtain the following effective action:

$$\begin{aligned}S=&\frac{1}{2\kappa_D^2}\int d^{D-1}xdz\sqrt{-g}\varpi^{\frac{2}{\Delta+2}}\bigg[h\mathcal{R}_g-\frac{4}{D-2}h(\partial\tilde\phi)^2\\&-\frac{1}{2(p+2)!}h^{-1}e^{2a_{p+1}\tilde\phi}\tilde F^2_{p+2}-8\frac{D-1}{D-2}\frac{Q}{\Delta}h^{-1}\\&\times\bigg\{\delta(z)-\frac{DQ}{4(D-1)}\varpi^{-2}\bigg\}-\frac{2Q^2}{\Delta}\varpi^{-2}he^{-2a\tilde\phi}\bigg]\\&+\frac{4}{\Delta}\frac{Q}{\kappa_D^2}\int d^{D-1}x\sqrt{-g}e^{-a\tilde\phi}+S_{p+1}\\=&\frac{1}{2\kappa^2_{D-1}}\int d^{D-1}x\sqrt{-g}\bigg[h\mathcal{R}_g-\frac{4}{D-2}h(\partial\tilde\phi)^2\\&-\frac{1}{2(p+2)!}h^{-1}e^{2a_{p+1}\tilde\phi}\tilde F^2_{p+2}\\&+2\frac{(\Delta+4)Q^2}{\Delta^2}(h^{-1}+he^{-2a\tilde\phi}-2e^{-a\tilde\phi})\bigg]+S_{p+1},\end{aligned} \tag{13}$$

where $\tilde F_{p+2}=d\tilde A_{p+1}$ with $(\tilde A_{p+1})_{\mu_1\cdots\mu_{p+1}}\equiv(\tilde A_{p+2})_{\mu_1\cdots\mu_{p+1}z}$, κ^2_{D-1} is given by Eq. (8), and we let $\Delta<0$ and made use of the constraint (11) in the form potential kinetic term. Note that the explicit expressions for h and $e^{\tilde\phi}$ are given in terms of the harmonic function H_{p+1} for the D-dimensional $(p+1)$-brane as [21]

$$h=H_{p+1}^{-2(D-p-4)/(D-2)\Delta_{p+1}},\quad e^{\tilde\phi}=H_{p+1}^{(D-2)a_{p+1}/2\Delta_{p+1}}. \tag{14}$$

Note that $\tilde A_{p+2}$ and $g_{\mu\nu}$ are also given in terms of H_{p+1} but we choose to leave it in the general form in the action, just assuming such specific forms to ensure consistency with the D-dimensional equations of motion. Just as in the previous case, $g_{\mu\nu}$ can contain a Ricci-flat metric without violating the equations of motion. So the scalar potential term of the $(D-1)$-dimensional action in Eq. (13) becomes zero. The $(D-1)$-dimensional effective action in Eq. (13), therefore, becomes of the form of the bulk action for the dilatonic p-brane in $D-1$ dimensions, obtained from the bulk action for the D-dimensional dilatonic $(p+1)$-brane through ordinary KK compactification on S^1 along one of its longitudinal

[1]The Einstein term in the effective action is finite; i.e., the effective gravitational constant κ_{D-1} is nonzero if and only if the graviton KK zero mode is normalizable, i.e., if gravity is trapped on the domain wall. Note, however, that the effective action may have undesirable terms such as a cosmological constant term in some cases even if the KK graviton zero mode is normalizable.

directions. Next, we consider the world volume action S_{p+1} for the dilatonic $(p+1)$-brane. In the static gauge with constant transverse [to the $(p+1)$-brane] target space coordinates, the $(p+1)$-brane world volume action (10) takes the following form:

$$S_{p+1}=-T_{p+1}\int d^{p+2}x\left[e^{-a_{p+1}\phi}\sqrt{-\det G_{ab}}+\frac{\sqrt{\Delta_{p+1}}}{2}A_{tx_1\cdots x_p z}\right], \tag{15}$$

where $a,b=t,x_1,\ldots,x_p,z$. After substituting the *Ansätze* for the fields in the above and integrating over z, we obtain the following effective action:

$$\begin{aligned}S_{p+1}&=-T_{p+1}\int d^{p+1}x dz\, \varpi^{2/(\Delta+2)}\\&\quad\times\left[e^{-a_{p+1}\tilde{\phi}}h\sqrt{-\det g_{\tilde{a}\tilde{b}}}+\frac{\sqrt{\Delta_{p+1}}}{2}\tilde{A}_{tx_1\cdots x_p z}\right]\\&=-T_p\int d^{p+1}x\left[e^{-a_{p+1}\tilde{\phi}}h\sqrt{-\det g_{\tilde{a}\tilde{b}}}+\frac{\sqrt{\Delta_p}}{2}\tilde{A}_{tx_1\cdots x_p}\right],\end{aligned} \tag{16}$$

where $T_p\equiv[-2\Delta/(\Delta+4)Q]T_{p+1}$ and $\tilde{a},\tilde{b}=t,x_1,\ldots,x_p$. This is of the form of the world volume action for the $(D-1)$-dimensional p-brane obtained from the world volume action for the D-dimensional $(p+1)$-brane through ordinary KK compactification on S^1 along one of its longitudinal directions.

This result indicates that when there are additional fields in the bulk action gravity can be trapped within the domain wall even if the domain wall metric has a nontrivial perturbation along the transverse (to the domain wall) direction. The zero mode of this transverse perturbation is identified as a scalar in one lower dimension. As we have seen from the dilatonic $(p+1)$-brane solution in the domain wall bulk, such a zero mode of the transverse metric perturbation conspires with the zero mode of the dilaton perturbation in such a way that the possible scalar potential term or the cosmological term in one lower dimension is eliminated. Thereby, a configuration in the domain wall bulk effectively describes the corresponding configuration in *an asymptotically flat spacetime* in one lower dimension.

On the other hand, the domain wall bulk spacetime is very restrictive about the possible gravitating configurations. The gravitating configurations with nontrivial transverse (to the wall) metric component are not preferred if there are no additional fields in the bulk action, as it was shown [25–27] that the zero mode of the transverse perturbation has to vanish. Also, without additional fields in the bulk action, the zero-mode effective action with nontrivial h in Eq. (6) yields the potential term whose minimum is at $h=1$, implying that a solution having metric (6) with $h=1$ is the classically preferred stable configuration. And only the dilatonic charged brane with the dilaton coupling parameter satisfying the constraint (11) is allowed in the bulk of a dilatonic domain wall. Because of this restriction, a nondilatonic domain wall bulk background cannot admit charged branes (with asymptotically flat spacetime in one lower dimension) and the current RS-type models (dilatonic and nondilatonic) cannot admit, for example, Reissner-Nordström black holes in one lower dimension.

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