

Superstrings and superbranes with a modified measure

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For superstrings, the consequences of replacing the measure of integration $\sqrt{-\gamma}d^2x$ in the Polyakov's action by Φd^2x , where Φ is a density built out of degrees of freedom independent of the metric γ_{ab} defined in the string, are studied. As in the Siegel reformulation of the Green-Schwarz formalism the Wess-Zumino term is the square of supersymmetric currents. As opposed to the Siegel case, the compensating fields needed for this do not enter into the action just as in a total derivative. They instead play a crucial role in making up a consistent dynamics. The string tension appears as an integration constant of the equations of motion. The generalization to higher dimensional extended objects is also studied using in this case the Bergshoeff and Sezgin formalism with the associated additional fields, which again are dynamically relevant unlike the standard formulation. Also unlike the standard formulation, there is no need for a cosmological term on the world brane.

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I. INTRODUCTION

There are reasons to consider changing the way we think and formulate generally covariant theories. If these are generally covariant theories of gravity, one of the reasons is the cosmological constant problem [1], which is a consequence of the fact that in gravitational theory, as usually formulated, the origin of the energy density is important.

This is very much related to the fact that the action for generally covariant theories, which is usually written as

$$S = \int d^d x \sqrt{-\gamma} L, \quad (1)$$

where γ is the determinant of the metric and L is a scalar, is not invariant under the shift $L \rightarrow L + \text{const}$.

If in Eq. (1) we were to change the measure of integration $d^d x \sqrt{-\gamma}$ by $d^d x \Phi$, where Φ is a total derivative, then the shift $L \rightarrow L + \text{const}$ will indeed be a symmetry.

This possibility was studied in the context of gravitational theories which can handle the cosmological constant problem [2] and as a tool for the construction of new types of scale invariant theories consistent with nontrivial masses and potentials which are of the form required by inflation [3–6]. In the models of Refs. [3–6], no fundamental dimensionful parameter really appears in the fundamental Lagrangian (one can indeed introduce such parameters, but they can be reabsorbed, for example, by a rescaling of the fields that define the measure Φ).

It is very interesting that the issues raised above in the case of gravitational theories have their analogues in string and brane theories, even before we attempt to use these theories as theories of gravity; that is, we are talking here of string or brane world sheet analogues of the issues raised above.

String and brane theories [7] have appeared as candidates for unifying all interactions of nature. One aspect of string and brane theories seems to many not quite appealing however: this is the introduction from the beginning of a fundamental scale, the string or brane tension. The idea that the

fundamental theory of nature whatever that may be, should not contain any fundamental scale has attracted a lot of attention. According to this point of view, whatever scale appears in nature, must not appear in the fundamental Lagrangian of physics. Rather, the appearance of these scales must be spontaneous, for example, due to boundary conditions in a classical context or a process of dimensional transmutation, to give an example of such effect in the context of quantum field theory. The issue, that is, the spontaneous appearance of scale in scale invariant theories which use a modified measure, was addressed (in a four dimensional context) and satisfactory solved in Refs. [3–6].

Interestingly enough, the cosmological constant of gravitational theories has its analogue in brane theory. It is well known that the generalization of the Polyakov formalism to branes must incorporate an explicit world brane cosmological term, unlike the string case, where such a term is forced to vanish. A definite asymmetry between string and higher branes gets established this way.

Here we will see what we obtain from string and brane theories with a modified measure. As we will see, string theories or more generally brane theories without a fundamental scale are possible if the extended objects do not have boundaries (i.e., they are closed). Although it will not be studied in detail here, the case of extended objects with boundaries is also possible in this modified measure formalism. In this case, a scale can be introduced by the coupling to boundary charges (that is, a scale is induced by a boundary effect, which fits in the general context we are talking about: scales arise either from boundary effects or initial conditions). These charges are needed so that the extended object finishes at the boundary, as has been shown in the bosonic case [8]. This should also hold for the supersymmetric case, but these generalizations and further complications of the theory will be ignored here for simplicity, since the basic points can be made in the case of closed strings and branes.

Also, for higher dimensional branes no explicit world brane cosmological constant needs to be included, therefore restoring the symmetry between strings and branes. How the ‘‘brane cosmological constant problem’’ is related to the

cosmological constant problem of the gravitational low energy theory is not known, but that connection may very well exist.

Some related works where the string or brane tension appears as an integration constant also have been done. Comparisons with the approach taken here (which can be made only after a presentation of the basic ideas of our approach) and relevant references can be found at the end of Sec. II.

The situation for bosonic strings and branes with a modified measure was already studied in a previous work [8]. Here we review this and then proceed to generalize this to the supersymmetric case.

II. BOSONIC STRING AND BRANE THEORIES WITH A MODIFIED MEASURE

In this section we review the previous work [8] on bosonic extendons with a modified measure before going into the supersymmetric case. The Polyakov action for the bosonic string is [9]

$$S_P[X, \gamma_{ab}] = -T \int d\tau d\sigma \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}. \quad (2)$$

Here γ_{ab} is the metric defined in the 1+1 world sheet of the string and $\gamma = \det(\gamma_{ab})$. $g_{\mu\nu}$ is the metric of the embedding space. T is here the string tension, a dimensionful quantity introduced into the theory, which defines a scale.

We recognize the measure of integration $d\tau d\sigma \sqrt{-\gamma}$ and as we anticipated before, we want to replace this measure of integration by another one which does not depend on γ_{ab} .

If we introduce two scalars (both from the point of view of the 1+1 world sheet of the string and from the embedding D -dimensional universe) φ_i , $i=1,2$, we can construct the world sheet density

$$\Phi = \varepsilon^{ab} \varepsilon_{ij} \partial_a \varphi_i \partial_b \varphi_j \quad (3)$$

where ε^{ab} is given by $\varepsilon^{01} = -\varepsilon^{10} = 1$, $\varepsilon^{00} = \varepsilon^{11} = 0$ and ε_{ij} is defined by $\varepsilon_{12} = -\varepsilon_{21} = 1$, $\varepsilon_{11} = \varepsilon_{22} = 0$.

It is interesting to notice that $d\tau d\sigma \Phi = 2d\varphi_1 d\varphi_2$, that is, the measure of integration $d\tau d\sigma \Phi$ corresponds to integrating in the space of the scalar fields φ_1, φ_2 .

We proceed now with the construction of an action that uses $d\tau d\sigma \Phi$ instead of $d\tau d\sigma \sqrt{-\gamma}$. When considering the types of actions we can have under these circumstances, the first one that comes to mind (a straightforward generalization of the Polyakov action) is

$$S_1 = - \int d\tau d\sigma \Phi \gamma^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}. \quad (4)$$

Notice that multiplying S_1 by a constant, before boundary or initial conditions are specified is a meaningless operation, since such a constant can be absorbed in a redefinition of the measure fields φ_1, φ_2 that appear in Φ .

The form (4) however is not a satisfactory action, because the variation of S_1 with respect to γ^{ab} leads to the rather strong condition

$$\Phi \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} = 0. \quad (5)$$

If $\Phi \neq 0$, it means that $\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} = 0$, which means that the metric induced on the string vanishes, clearly not an acceptable dynamics. Alternatively, if $\Phi = 0$, no further information is available, also a nondesirable situation.

To make further progress, it is important to note that terms considered as contributions to L in

$$S = \int d\tau d\sigma \sqrt{-\gamma} L \quad (6)$$

which do not contribute to the equations of motion, i.e., such that $\sqrt{-\gamma} L$ is a total derivative, may contribute when we consider the same L in a contribution to the action of the form

$$S = \int d\tau d\sigma \Phi L. \quad (7)$$

This is so because if $\sqrt{-\gamma} L$ is a total divergence, ΦL in general is not.

This fact is indeed crucial and if we consider an Abelian gauge field A_a defined in the world sheet of the string, in addition to the measure fields φ_1, φ_2 that appear in Φ , the metric γ^{ab} , and the string coordinates X^μ , we can then construct the nontrivial contribution to the action of the form

$$S_{gauge} = \int d\tau d\sigma \Phi \frac{\varepsilon^{ab}}{\sqrt{-\gamma}} F_{ab} \quad (8)$$

where

$$F_{ab} = \partial_a A_b - \partial_b A_a. \quad (9)$$

The total action to be considered is now

$$S = S_1 + S_{gauge} \quad (10)$$

with S_1 defined as in Eq. (4) and S_{gauge} defined by Eqs. (8) and (9).

The action (10) is invariant under a set of diffeomorphisms in the space of the measure fields combined with a conformal transformation of the metric γ_{ab} ,

$$\varphi_i \rightarrow \varphi'_i = \varphi'_i(\varphi_j), \quad (11)$$

so that

$$\Phi \rightarrow \Phi' = J\Phi, \quad (12)$$

where J is the Jacobian of the transformation (11) and

$$\gamma_{ab} \rightarrow \gamma'_{ab} = J\gamma_{ab}. \quad (13)$$

The combination $(\varepsilon^{ab}/\sqrt{-\gamma})F_{ab}$ is a genuine scalar. In two dimensions, it is proportional to $\sqrt{F_{ab}F^{ab}}$.

Working with Eq. (10), we obtain the following equations of motion: From the variation of the action with respect to φ_j

$$\varepsilon^{ab} \partial_b \varphi_j \partial_a \left(-\gamma^{cd} \partial_c X^\mu \partial_d X^\nu g_{\mu\nu} + \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd} \right) = 0. \quad (14)$$

If $\det(\varepsilon^{ab}\partial_b\varphi_j) \neq 0$, which means $\Phi \neq 0$, then we must have all the derivatives of the quantity inside the parenthesis in Eq. (14) vanish, that is, such a quantity must equal a constant which will be determined later, but which we will call M in the mean time,

$$-\gamma^{cd}\partial_c X^\mu\partial_d X^\nu g_{\mu\nu} + \frac{\varepsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = M. \quad (15)$$

The equation of motion of the gauge field A_a tells us about how the string tension appears as an integration constant. Indeed this equation is

$$\varepsilon^{ab}\partial_b\left(\frac{\Phi}{\sqrt{-\gamma}}\right) = 0, \quad (16)$$

which can be integrated to give

$$\Phi = c\sqrt{-\gamma}. \quad (17)$$

Notice that Eq. (17) is perfectly consistent with the conformal symmetry equations (11), (12), and (13). Equation (15) on the other hand is consistent with such a symmetry only if $M=0$. Indeed, we will check that the equations of motion indeed imply that $M=0$. In the case of higher dimensional branes, the equations of motion will imply that M is nonvanishing.

If the Abelian gauge field couples to a point particle living in the string, we would have $\Phi = c_1\sqrt{-\gamma}$ on one side of the string and $\Phi = c_2\sqrt{-\gamma}$ on the other side, $c_1 - c_2$ being the charge of the point particle. Notice that this gives us the possibility of having $c_2=0$. This is therefore the formulation of an open string in the modified measure approach. Noting the possibility of formulating also extended objects with boundaries, we will leave the subject and in what follows focus only on theories of closed strings and branes.

By calculating the Hamiltonian, after dropping boundary terms (this is totally justified in the case of closed strings) and (only at the end of the process) using Eq. (17), we find that c equals the string tension.

Now let us turn our attention to the equation of motion derived from the variation of Eq. (10) with respect to γ^{ab} . We get, then,

$$-\Phi\left(\partial_a X^\mu\partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\frac{\varepsilon^{cd}}{\sqrt{-\gamma}}F_{cd}\right) = 0. \quad (18)$$

From the constraint (15), we can solve $(\varepsilon^{cd}/\sqrt{-\gamma})F_{cd}$ and insert back into Eq. (18), obtaining (if $\Phi \neq 0$)

$$\partial_a X^\mu\partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu\partial_d X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}M = 0. \quad (19)$$

Multiplying the above equation by γ^{ab} and summing over a, b , we obtain $M=0$, that is, the equations are exactly those of the Polyakov action. After Eq. (16) is used, the equation obtained from the variation of X^μ is seen to be exactly the same as the one obtained from the Polyakov action as well.

Let us now consider a $d+1$ extended object, described [generalizing the action (9)] as,

$$S = S_d + S_{d-gauge}, \quad (20)$$

where

$$S_d = - \int d^{d+1}x \Phi \gamma^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}, \quad (21)$$

$$S_{d-gauge} = \int d^{d+1}x \Phi \frac{\varepsilon^{a_1 a_2 \dots a_{d+1}}}{\sqrt{-\gamma}} \partial_{[a_1} A_{a_2 \dots a_{d+1}]}, \quad (22)$$

and

$$\Phi = \varepsilon^{a_1 a_2 \dots a_{d+1}} \varepsilon_{j_1 j_2 \dots j_{d+1}} \partial_{a_1} \varphi_{j_1} \dots \partial_{a_{d+1}} \varphi_{j_{d+1}}. \quad (23)$$

This model does not have a symmetry which involves an arbitrary diffeomorphism in the space of the measure fields coupled with a conformal transformation of the metric, except if $d=1$ [Eqs. (11), (12), and (13)]. For $d \neq 1$, there is still a global scaling symmetry where the metric transforms as (θ being a constant)

$$\gamma_{ab} \rightarrow e^\theta \gamma_{ab}. \quad (24)$$

The φ_j are transformed according to

$$\varphi_j \rightarrow \lambda_j \varphi_j \quad (25)$$

(no sum on j) which means $\Phi \rightarrow (\prod_j \lambda_j) \Phi \equiv \lambda \Phi$.

Finally, we must demand that $\lambda = e^\theta$ and that the transformation of $A_{a_2 \dots a_{d+1}}$ be defined as

$$A_{a_2 \dots a_{d+1}} \rightarrow \lambda^{(d-1)/2} A_{a_2 \dots a_{d+1}}. \quad (26)$$

Then we have a symmetry. Also, no scale is introduced into the theory from the beginning. This is apparent from the fact that any constants multiplying the separate contributions to the action (21) or (22) is meaningless if no boundary or initial conditions are specified, because then such factors can be absorbed by a redefinition of the measure fields and of the gauge fields. Notice that the existence of a symmetry alone is not enough to guarantee that no fundamental scale appears in the action. For example, string theory, as usually formulated has conformal symmetry, but the string tension is still a fundamental scale in the theory.

Another interesting symmetry of the action (up to the integral of a total divergence) consists of the infinite dimensional set of transformations $\varphi_j \rightarrow \varphi_j + f_j(L)$, where $f_j(L)$ are arbitrary functions of

$$L = -\gamma^{cd}\partial_c X^\mu\partial_d X^\nu g_{\mu\nu} + \frac{\varepsilon^{a_1 a_2 \dots a_{d+1}}}{\sqrt{-\gamma}} \partial_{[a_1} A_{a_2 \dots a_{d+1]}}. \quad (27)$$

This symmetry does depend on the explicit form of the Lagrangian density L , but only the fact that L is φ_a independent.

Now we go through the same steps we went through in the case of the string. The variation with respect to the measure field φ_j gives

$$K_j^a \partial_a \left(-\gamma^{cd} \partial_c X^\mu \partial_d X^\nu g_{\mu\nu} + \frac{\varepsilon^{a_1 a_2 \dots a_{d+1}}}{\sqrt{-\gamma}} \partial_{[a_1} A_{a_2 \dots a_{d+1}]} \right) = 0 \quad (28)$$

where

$$K_j^a = \varepsilon^{a a_2 \dots a_{d+1}} \varepsilon_{j j_2 \dots j_{d+1}} \partial_{a_2} \varphi_{j_2} \dots \partial_{a_{d+1}} \varphi_{j_{d+1}}. \quad (29)$$

Since

$$\det(K_j^a) = \frac{(d+1)^{-(d+1)}}{(d+1)!} \Phi^d,$$

it therefore follows that for $\Phi \neq 0$, $\det(K_j^a) \neq 0$ and

$$-\gamma^{cd} \partial_c X^\mu \partial_d X^\nu g_{\mu\nu} + \frac{\varepsilon^{a_1 a_2 \dots a_{d+1}}}{\sqrt{-\gamma}} \partial_{[a_1} A_{a_2 \dots a_{d+1}]} = M, \quad (30)$$

where M is some constant of integration. If $d \neq 1$, then $M \neq 0$, as we will see. Furthermore, under a scale transformation (24), (25), and (26), M does change from one constant value to another.

The variation with respect to the gauge field $A_{a_2 \dots a_{d+1}}$ leads to the equation

$$\varepsilon^{a_1 a_2 \dots a_{d+1}} \partial_{a_1} \frac{\Phi}{\sqrt{-\gamma}} = 0 \quad (31)$$

which means

$$\Phi = c \sqrt{-\gamma} \quad (32)$$

once again. As in the case of the string the brane tension has been generated spontaneously instead of appearing as a parameter of the fundamental Lagrangian. Again, using a simple calculation of the Hamiltonian and then the above equation, we obtain that c is proportional to the brane tension.

The variation of the action with respect to γ^{ab} leads to

$$-\Phi \left(\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2} \gamma_{ab} \frac{\varepsilon^{a_1 a_2 \dots a_{d+1}}}{\sqrt{-\gamma}} \partial_{[a_1} A_{a_2 \dots a_{d+1}]} \right) = 0. \quad (33)$$

We can now solve for

$$\frac{\varepsilon^{a_1 a_2 \dots a_{d+1}}}{\sqrt{-\gamma}} \partial_{[a_1} A_{a_2 \dots a_{d+1]}}$$

from Eq. (30) and then reinsert the result in the above equation, obtaining

$$\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} = \frac{1}{2} \gamma_{ab} (\gamma^{cd} \partial_c X^\mu \partial_d X^\nu g_{\mu\nu} + M). \quad (34)$$

This is the same equation that we would have obtained from a Polyakov-type action augmented by a cosmological term.

As in the case of the string, M can be found by contracting both sides of the equation. For $d \neq 1$, M is nonzero and equal to

$$M = \frac{\gamma^{cd} \partial_c X^\mu \partial_d X^\nu g_{\mu\nu} (1-d)}{1+d}. \quad (35)$$

We can also solve for $\gamma^{cd} \partial_c X^\mu \partial_d X^\nu g_{\mu\nu}$ in terms of M from Eq. (35) and insert the result in the right-hand side of Eq. (34), obtaining

$$\gamma_{ab} = \frac{1-d}{M} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}. \quad (36)$$

Which means γ_{ab} is up to the constant factor $(1-d)/M$, equal to the induced metric. Since there is the scale invariance (24), (25), and (26), an overall constant factor in the evolution of γ_{ab} cannot be determined. The same scale invariance means however that there is a field redefinition which does not affect any parameter of the Lagrangian and which allows us to set γ_{ab} equal to the induced metric (at least if we start from any negative value of M), that is,

$$\gamma_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}. \quad (37)$$

In such a case M is consistently given [inserting Eq. (37) into Eqs. (36) or (35)]

$$M = 1 - d. \quad (38)$$

Notice that in contrast with the standard approach for Polyakov-type actions in the case of higher dimensional branes [10], here we do not have to fine tune a parameter of the Lagrangian, the brane ‘‘cosmological constant,’’ so as to force Eq. (37) to be satisfied. Here instead, it is an integration constant that appears from an action without an original cosmological term, which can be set to the value given by Eq. (38) by means of a scale transformation. Such a choice then ensures that Eq. (38) is satisfied [and therefore Eq. (37)]. Furthermore, it appears that this treatment is more appealing if one thinks of all branes on similar footing, since in the approach of this paper they can all be described by a similar looking Lagrangian, unlike in the usual approach which discriminates in a radical way between strings, these having no cosmological constant associated to them, and the higher dimensional branes, which require a fine tuned cosmological constant.

If we do not make the choice (38), the constant c is not directly the brane tension, which is instead

$$c \left(\frac{1-d}{M} \right)^{(d-1)/2},$$

which can be checked as a scale invariant combination, since under a scale transformation $M \rightarrow e^\theta M$ and $c \rightarrow e^{\theta(d-1)/2} c$. If the choice $M = 1 - d$ is made, the brane tension is simply c . The principle remains the same as in the case of the string and the constant c is still responsible for the spontaneous generation of a brane tension.

One should notice that other authors have also constructed actions for branes that do not contain a brane-cosmological term [11]. Such formulations, unlike what has been developed here, depend on the dimensionality, in particular, whether this is even or odd, so that it is clear that those formulations do not have much relation with what has been done here. Yet other approaches [12] to an action without a brane cosmological involve Lagrangians with nonlinear dependence on the invariant $\gamma^{cd} \partial_c X^\alpha \partial_d X^\beta g_{\alpha\beta}$, also a rather different path to the one followed here. For an interesting analysis of different possible Lagrangians for extendons see [13].

An approach that has some common features to the one developed here is that of Refs. [14] and [15] where the tension of the brane is found as an integration constant. Here also gauge fields are introduced, but they appear in a quadratic form rather than in a linear form. Also, scale invariance is discussed in [14], but it is a target space scale invariance since no metric defined in the brane is studied there, i.e., no connection to a Polyakov-type action, which is known to be more useful in the quantum theory, is made. For the case of superstrings and superbranes we will follow a procedure that keeps the basic structure found in the bosonic case, except that the gauge fields introduced here in order to obtain a consistent dynamics turn out to be composites of other fields, a rather different approach to that of Ref. [14]. The linearity of the Lagrangian on the (in this case composite) gauge fields will be maintained, also unlike Ref. [14] (Ref. [15] does not discuss the supersymmetric case).

III. SUPERSTRINGS WITH A MODIFIED MEASURE

The general structure that we have found for the bosonic strings and branes suggest the way to follow in the case of superstrings and supermembranes.

In fact, the additional term with the gauge fields defined by Eqs. (8) and (9), being associated with the alternating symbol in two dimensions, appears very much related to the Wess-Zumino term in the Green-Schwarz formulation of the superstring [16].

It is important to notice that in the Green-Schwarz formulation, the Wess-Zumino term is not invariant under supersymmetry, but only invariant up to a total divergence. And since we have already discussed, in our formulation total derivatives have to be handled with care, since if $\sqrt{-\gamma} L$ is a total divergence, ΦL in general is not.

Under these circumstances, Siegel [17] reformulation of the Green-Schwarz superstring, where the Wess-Zumino term is manifestly supersymmetric becomes of special interest from our point of view.

In the Siegel [17] reformulation, the action of the superstring in a flat embedding spacetime (with metric $\eta_{\mu\nu}$) is written as

$$S = T \int L \sqrt{-\gamma} d^2 x, \quad (39)$$

where the scalar L is given by

$$L = -\frac{1}{2} \gamma^{ab} J_a^\mu J_b^\nu \eta_{\mu\nu} - i \frac{\epsilon^{ab}}{\sqrt{-\gamma}} J_a^\alpha J_{ab}, \quad (40)$$

where the currents J_a^μ , J_a^α , and J_{ab} are defined as

$$J_a^\alpha = \partial_a \theta^\alpha, \quad (41)$$

$$J_a^\mu = \partial_a X^\mu - i \partial_a \theta^\alpha \Gamma_{\alpha\beta}^\mu \theta^\beta, \quad (42)$$

$$J_{\alpha a} = \partial_a \phi_\alpha - 2i (\partial_a X^\mu) \Gamma_{\mu\alpha\beta} \theta^\beta - \frac{2}{3} (\partial_a \theta^\beta) \Gamma_{\beta\delta}^\mu \theta^\delta \Gamma_{\mu\alpha\epsilon} \theta^\epsilon. \quad (43)$$

Then, there is the following symmetry of the Lagrangian (exact, not up to total divergence)

$$\delta \theta^\alpha = \epsilon^\alpha, \quad \delta X^\mu = -i \epsilon^\alpha \Gamma_{\alpha\beta}^\mu \theta^\beta, \quad (44)$$

and

$$\delta \phi_\alpha = 2i \epsilon^\beta \Gamma_{\mu\alpha\beta} X^\mu + \frac{2}{3} (\epsilon^\beta \Gamma_{\beta\epsilon}^\mu \theta^\epsilon) \Gamma_{\mu\alpha\kappa} \theta^\kappa, \quad (45)$$

provided the Dirac $\Gamma_{\alpha\beta}^\mu$ matrices satisfy a condition which requires the target space dimensionality to be 3, 4, 6, or 10. In this formalism, the fields ϕ_α are not determined at all by the equations of motion, since their dependence enters only as a total divergence.

The situation changes if in Eq. (39), $T \sqrt{-\gamma} d^2 x \rightarrow \Phi d^2 x$ and we now consider

$$S = \int L \Phi d^2 x, \quad (46)$$

with L still given by Eqs. (40), (41), (42), and (43).

It is now crucial to recognize that the Abelian gauge field defined in the world sheet of the string, which was introduced in order to obtain sensible equations of motion in the case of the bosonic string, appears here induced by the additional fields introduced by Siegel.

The identification of the Abelian gauge field proceeds according to the equation

$$-i \epsilon^{ab} \partial_a \theta^\alpha \partial_b \phi_\alpha \equiv \epsilon^{ab} \partial_a A_b \quad (47)$$

which can be solved by the composite gauge field construction

$$A_b \equiv -i \theta^\alpha \partial_b \phi_\alpha. \quad (48)$$

Such composite gauge field construction is indeed very closely related to the ones studied by Guendelman, Nissimov, and Pacheva and also by Castro in Ref. [2].

One can then see that if no singularities or degenerate situations are present, then all of the equations, except one, are the same as those obtained in the standard Siegel formulation of the Green-Schwarz superstring [17].

The difference is due to the fact that the Abelian gauge field A_b , and therefore the ϕ_α fields, play a dynamical role, unlike the case of the Siegel formalism. And unlike the Siegel formalism, there is an equation that tells us something about the ϕ_α fields. This is the equation obtained from the variation of the measure fields,

$$\varepsilon^{ab}\partial_b\varphi_j\partial_aL=0, \quad (49)$$

which means, if $\Phi \neq 0$,

$$L=M=\text{const.} \quad (50)$$

The variation with respect to ϕ_α gives the equation

$$\varepsilon^{ab}\partial_a\theta^\alpha\partial_b\left(\frac{\Phi}{\sqrt{-\gamma}}\right)=0. \quad (51)$$

If we have a nondegenerate situation, that is, for enough linearly independent nonvanishing components of $\partial_a\theta^\alpha$, it follows that

$$\frac{\Phi}{\sqrt{-\gamma}}=c=\text{const} \quad (52)$$

and as in the bosonic case, the integration constant c is the string tension.

Following the steps of Sec. II, we can once again find, by combining Eq. (50) and the equation obtained from the variation with respect to the world sheet metric that

$$M=0. \quad (53)$$

As anticipated, once Eq. (52) is used, all the resulting equations are exactly those found in Ref. [17], except for Eq. (50) with $M=0$, i.e., the vanishing on the mass shell of the Lagrangian. Such condition imposes a constraint on the ϕ_α fields, which in [17] are totally undetermined.

The interesting role of the new fields ϕ_α in obtaining a perfect balance, so as to ensure that the Lagrangian is exactly zero, may very well be connected to a resolution of the cosmological constant, is the effective low energy gravitational theory. Recall that the ideas of using a modified measure were motivated in the first place in this context [2].

IV. SUPERBRANES WITH A MODIFIED MEASURE AND WITHOUT A COSMOLOGICAL TERM

For higher dimensional superbranes Bergshoeff and Sezgin [18] have generalized the auxiliary field formalism of Siegel. As we saw in the bosonic case, the new feature that appears when considering higher dimensional branes, instead of strings, is that in the usual Polyakov-type formalism, a world brane cosmological term must be included, but when the modified measure is used, no explicit cosmological term is required. Instead, when the equations of motion are considered, we are forced to consider a nonvanishing value of the constant of integration M . These features are maintained when we formulate the supermembrane generalization of the above.

We begin our discussion of the higher dimensional branes with the consideration of the 2 + 1 dimensional brane, which will be treated in some detail. Once this is understood, the higher dimensional cases follow more or less in a similar fashion, provided the results of Ref. [18] are properly applied.

Once again, as in the superstring case, we want to write the Lagrangian as the sum of products of invariant supercurrents. For this to be achieved, we need to introduce, in the case of the 2 + 1 brane, the additional fields $\phi_{\mu\nu}$ (field with two target space indices), $\phi_{\mu\alpha}$ (field with one target space index and one spinor index) and also $\phi_{\alpha\beta}$ (field with two spinor indices), in addition to the original θ^α and X^μ fields of the brane.

Then we are in a position to define the currents (where an abbreviated notation is used in what follows, like $\bar{\theta}\Gamma^\mu\partial_b\theta$ being a short cut for $\theta^\alpha\Gamma^\mu_{\alpha\beta}\partial_b\theta^\beta$, etc.; also in this section we follow normalizations of the θ fields and other conventions of Ref. [18] rather than those of Ref. [17]):

$$L_a^\alpha=\partial_a\theta^\alpha, \quad (54)$$

$$L_a^\mu=\partial_aX^\mu+\frac{1}{2}\bar{\theta}\Gamma^\mu\partial_a\theta, \quad (55)$$

$$L_{a\mu\nu}=\partial_a\phi_{\mu\nu}+\frac{1}{2}\bar{\theta}\Gamma_{\mu\nu}\partial_a\theta, \quad (56)$$

$$L_{a\mu\alpha}=\partial_a\phi_{\mu\alpha}+\partial_a\phi_{\mu\nu}(\Gamma^\nu\theta)_\alpha+\partial_aX^\nu(\Gamma_{\mu\nu}\theta)_\alpha \\ +\frac{1}{6}(\Gamma_{\mu\nu}\theta)_\alpha\bar{\theta}\Gamma^\nu\partial_a\theta+\frac{1}{6}(\Gamma^\nu\theta)_\alpha\bar{\theta}\Gamma_{\mu\nu}\partial_a\theta, \quad (57)$$

$$L_{\alpha\alpha\beta}=\partial_a\phi_{\alpha\beta}-\frac{1}{2}X^\mu\partial_a\phi_{\mu\nu}(\Gamma^\nu)_{\alpha\beta}+\partial_a\phi_{\mu\nu}(\Gamma^\mu\theta)_{(\alpha}(\Gamma^\nu\theta)_{\beta)} \\ +\frac{1}{4}(\bar{\theta}\partial_a\phi_\mu)(\Gamma^\mu)_{\alpha\beta}+2(\Gamma^\mu\theta)_{(\alpha}\partial_a\phi_{\mu\beta)} \\ -\frac{1}{2}X^\mu\partial_aX^\nu(\Gamma_{\mu\nu})_{\alpha\beta}-(\Gamma^\nu\theta)_{(\alpha}(\Gamma_{\mu\nu}\theta)_{\beta)}\partial_aX^\mu \\ -\frac{1}{12}(\Gamma_\nu\theta)_{(\alpha}(\Gamma^{\mu\nu}\theta)_{\beta)}(\bar{\theta}\Gamma_\mu\partial_a\theta) \\ -\frac{1}{12}(\Gamma_\nu\theta)_{(\alpha}(\Gamma_\mu\theta)_{\beta)}(\bar{\theta}\Gamma^{\mu\nu}\partial_a\theta). \quad (58)$$

And the supersymmetry under which the above currents are invariant is

$$\delta\theta^\alpha=\varepsilon^\alpha, \quad (59)$$

$$\delta X^\mu=-\frac{1}{2}\bar{\varepsilon}\Gamma_\mu\theta, \quad (60)$$

$$\delta\phi_{\mu\nu}=-\frac{1}{2}\bar{\varepsilon}\Gamma_{\mu\nu}\theta, \quad (61)$$

$$\delta\phi_{\mu\alpha}=-X^\nu(\Gamma_{\mu\nu}\varepsilon)_\alpha-\phi_{\mu\nu}(\Gamma^\nu\varepsilon)_\alpha+\frac{1}{6}(\bar{\varepsilon}\Gamma_{\mu\nu}\theta)(\Gamma^\nu\theta)_\alpha \\ +\frac{1}{6}(\bar{\varepsilon}\Gamma^\nu\theta)(\Gamma_{\mu\nu}\theta)_\alpha, \quad (62)$$

$$\delta\phi_{\alpha\beta}=-\frac{1}{4}(\Gamma^\mu)_{\alpha\beta}\bar{\varepsilon}\phi_\mu-2(\Gamma^\mu\varepsilon)_{(\alpha}\phi_{\mu\beta)}-\frac{1}{4}X^\mu(\bar{\varepsilon}\Gamma_{\mu\nu}\theta) \\ \times(\Gamma^\nu)_{\alpha\beta}-\frac{1}{4}X^\mu(\bar{\varepsilon}\Gamma^\nu\theta)(\Gamma_{\mu\nu})_{\alpha\beta} \\ -\frac{1}{12}\bar{\varepsilon}\Gamma_{\mu\nu}\theta(\Gamma^\mu\theta)_{(\alpha}(\Gamma^\nu\theta)_{\beta)} \\ -\frac{1}{12}\bar{\varepsilon}\Gamma_\mu\theta(\Gamma^{\mu\nu}\theta)_{(\alpha}(\Gamma_\nu\theta)_{\beta)}. \quad (63)$$

Such transformations are indeed a symmetry if the gamma matrices satisfy a condition which requires the dimensionality of the target space to be 4, 5, 7, and 11.

Given those supersymmetric currents, Bergshoeff and Sezgin construct the invariant action

$$S = T \int d^3x \sqrt{-\gamma} \left[-\frac{1}{2} \gamma^{ab} L_a^\mu L_{b\mu} + \frac{1}{2} - \frac{\varepsilon^{abc}}{\sqrt{-\gamma}} (L_a^\mu L_b^\nu L_{c\mu\nu} + \frac{9}{10} L_a^\mu L_b^\alpha L_{c\mu\alpha} - \frac{1}{5} L_a^\alpha L_b^\beta L_{c\alpha\beta}) \right]. \quad (64)$$

The coefficients of the three last terms, which are cubic in the currents, and which are contracted to ε^{abc} are chosen so that all the dependence on the additional fields $\phi_{\mu\nu}$, $\phi_{\mu\alpha}$, and $\phi_{\alpha\beta}$ is through a total divergence. This is exactly the total divergence by means of which we can once again define a composite gauge field analogous to the one used in the bosonic case, as it was done in the case of the superstring.

We now consider the 2 + 1 brane action with a modified measure. For this we first eliminate the cosmological term and second consider the change $T\sqrt{-\gamma}d^3x \rightarrow \Phi d^3x$, where

$$\Phi \equiv \varepsilon^{abc} \varepsilon_{ijk} \partial_a \varphi_i \partial_b \varphi_j \partial_c \varphi_k, \quad (65)$$

that is, we consider the action,

$$S = T \int d^3x \Phi \left[-\frac{1}{2} \gamma^{ab} L_a^\mu L_{b\mu} - \frac{\varepsilon^{abc}}{\sqrt{-\gamma}} (L_a^\mu L_b^\nu L_{c\mu\nu} + \frac{9}{10} L_a^\mu L_b^\alpha L_{c\mu\alpha} - \frac{1}{5} L_a^\alpha L_b^\beta L_{c\alpha\beta}) \right]. \quad (66)$$

In spite of the higher complexity, the basic structure of the theory and the way the equations of motion work is the same as that of the superstring, explained in Sec. III.

As in any case, the variation with respect to the measure fields φ_i imposes the constraint that the Lagrangian equals a constant, if $\Phi \neq 0$, that is

$$L = -\frac{1}{2} \gamma^{ab} L_a^\mu L_{b\mu} - \frac{\varepsilon^{abc}}{\sqrt{-\gamma}} \left(L_a^\mu L_b^\nu L_{c\mu\nu} + \frac{9}{10} L_a^\mu L_b^\alpha L_{c\mu\alpha} - \frac{1}{5} L_a^\alpha L_b^\beta L_{c\alpha\beta} \right) = M = \text{const.} \quad (67)$$

Second, all the conditions obtained from extremizing with respect to variations of the fields $\phi_{\mu\nu}$, $\phi_{\mu\alpha}$, and $\phi_{\alpha\beta}$, are satisfied if

$$\Phi = c \sqrt{-\gamma}, \quad (68)$$

where c is a constant. From here we once again obtain the brane tension as an integration constant.

The consideration of the equations obtained from the variation with respect to the world brane metric follow the same general structure to the one discussed in the bosonic case.

Once this is realized, it is clear that, except for the existence of the constraint (67), all of the equations are the same as the ones we obtain in the Bergshoeff-Sezgin case [18] after an appropriate rescaling of the metric γ_{ab} , which is equivalent to making the choice

$$M = 1 - d = -1 \quad (69)$$

(as discussed in Sec. II, M is not invariant under scaling transformations and through the use of scalings it can be changed continuously).

The constraint (67) however is totally absent in the case of Ref. [18], where the fields $\phi_{\mu\nu}$, $\phi_{\mu\alpha}$, and $\phi_{\alpha\beta}$, although playing an interesting group theoretical role are totally irrelevant dynamically and therefore totally undetermined.

V. THE CASE OF HIGHER BRANES WITH A MODIFIED MEASURE

It is clear that for higher branes, once the Bergshoeff-Sezgin construction is known [18], the two operations quoted in the case of the 2 + 1 superbrane could also apply, that is, take the Bergshoeff-Sezgin Lagrangian, first eliminate the cosmological term and second, modify the integration measure (in a way that generalizes straightforwardly from what we have done in the string and in the 2 + 1 brane) by making the replacement $T\sqrt{-\gamma}d^{d+1}x \rightarrow \Phi d^{d+1}x$, with Φ given as in Eq. (23).

Then the gauge fields, which we had to introduce in the bosonic case in order to have a consistent dynamics, are provided by the extra fields required by the Bergshoeff-Sezgin formalism, who got to these constructions from a group theoretic point of view [18].

VI. DISCUSSION AND CONCLUSIONS

In this paper, we have seen that a formulation of superstrings and superbranes with a modified measure is possible.

Due to the construction of this measure as $\Phi d^{d+1}x$, to the Lagrangian that multiplies this structure, we can add an arbitrary constant since Φ is a total derivative. In this sense, the origin of the vacuum energy density need not be specified in the theory. It may appear through the initial conditions.

In these theories, the tension of the string or brane appears as an integration constant. Furthermore, such a formulation appears to give a dynamical role and not just a group theoretical role to the extra fields introduced by Siegel [17] and Bergshoeff and Sezgin [18].

This may be important in the quantization of the theory and may also be important in the consequences for the low energy gravitational theory that follows from these kind of brane theories. Recall that the original motivation for introducing a modified measure was in this context [2].

Finally, a very interesting phenomena takes place in the formalism studied here, which is the fact that what we used

to think was a total divergence becomes dynamically relevant, even at the classical level and beyond purely topological effects. This is of course due to the use of the modified measure. Such observation raises new possibilities concerning the study and resolution of fundamental questions concerning the dynamical role of total divergences like in the strong CP problem. Some observations concerning a possible resolution of the strong CP problem by use of the

composite scalar field structures have been made in the last paper of Ref. [2].

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