Remarks on the racetrack scheme: Stabilizing the moduli of string theory

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There are only a small number of ideas for stabilizing the moduli of string theory. One of the most appealing of these is the racetrack mechanism, in which a delicate interplay between two strongly interacting gauge groups fixes the value of the coupling constant. In this paper, we explore this scenario. We find that, quite generally, some number of discrete tunings are required in order that the mechanism yield a small gauge coupling. Even then, there is, in general, no systematic weak coupling approximation. On the other hand, certain holomorphic quantities can be computed, so such a scheme is in principle predictive. Searching for models which realize this mechanism is thus of great interest. We also remark on cosmology in these schemes.

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I. INTRODUCTION

Understanding how the moduli of string or M theory compactifications are fixed is one of the greatest challenges of the subject. For compactifications with more than four supersymmetries ($N>1$ in four dimensions), general considerations suggest that there is an exact moduli space. In the case of four or fewer supersymmetries, generically the flat directions are lifted, with the potential typically tending to zero in any region in which the appropriate couplings tend to zero or the radii tend to ∞.[1]

This argument suggests that the string coupling should be strong and the scales of the theory should be comparable [2]. This in turn raises the question as to why the observed gauge couplings in nature are weak and unified, and why the unification scale seems to differ significantly from the Planck scale. This is the real issue in stabilization of the moduli: given that stabilization must occur, if at all, in regions where no sort of weak coupling approximation can be valid, why should anything be calculable? It is, after all, not hard to imagine schemes to stabilize the moduli, but it is hard to see why the coupling should be small, except as a result of numerical accidents, shrouded in mysterious high energy physics. In this view, none of the parameters of low energy physics would be calculable in any systematic approximation scheme.

These points are illustrated by the various toy models of modulus stabilization which appear in the literature. Most of these focus on a single modulus and postulate superpotentials from one or another source which provides stabilization [3–5]. Generically, however, the couplings turn out to be of order 1 in these proposals, and it is necessary to suppose that

uncontrolled strong coupling dynamics explains why the coupling is small. These models certainly illustrate that the gauge coupling can be small, but they do not predict that any of the parameters of low energy physics should be calculable. It is perhaps worth noting that one *can* construct weakly coupled string models with (at the tree level) as few as a single modulus [6] or in which all moduli or all moduli but one are charged under discrete symmetries. One can also contemplate theories with no moduli at all [7] or in which all moduli transform under unbroken symmetries, but it is unclear whether any controlled approximation might be available.

There are only a small number of proposals for fixing moduli in which some quantities are calculable. One is known as ''Kähler stabilization'' [8]. Here one imagines starting with some weakly coupled limit of M theory (i.e. some limit in which a systematic approximation is available), where one can calculate holomorphic quantities such as the gauge coupling functions and the superpotential. In other words, as one takes some modulus to extreme values, $\mathcal{M} \rightarrow \infty$, one comes to a regime where one can perform systematic calculations in $\mathcal{M}^{-1}$. The superpotential and gauge couplings are holomorphic, and because of discrete shift symmetries, they are functions of $e^{-\mathcal{M}}$. As one increases the couplings one supposes that, in a regime where the exponential is small, there are large corrections to the Kähler potentials of the moduli, such that the potential has a minimum at weak coupling. Any holomorphic quantity which can be computed in the weak coupling limit will be calculable in such a picture.

A second proposal involves the possibility of ''maximally enhanced symmetry'' [9]. Here one argues that the minimum of the full potential might naturally lie at a point where all of the moduli transform under unbroken symmetries. In addition to the fact that such points are automatically stationary points of the effective action, this hypothesis naturally solves the moduli problem of string cosmology. However, generically in such states, $\alpha \approx 1$. One must hope that there are some such states for which the effective low energy couplings happen to be small (and unified). Little is calculable in such a

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[1] In compactifications of non-supersymmetric string theories, the story is slightly different. Generically, in the loop expansion, one finds a non-zero cosmological constant, as a result of which the energy may blow up for large radius, but will still tend to zero for small coupling. An example of an $N=1$ theory where some of the moduli are exact was given in [1].

63 046005-1

picture; however, this hypothesis leads to the prediction that supersymmetry will be broken at low energies and that some sort of gauge mediation will play a crucial role. A third possibility is that there are simply no moduli. Operationally, this hypothesis is similar to that of maximally enhanced symmetry. In the context of large extra dimensions, possibilities involving large topological charges have been proposed [10–12], and in some cases, the size of the extra dimension is correlated with the smallness of the gauge couplings [13]. Finally, we will focus in this note on the ''racetrack mechanism'' [14–17].

The racetrack proposal is in some sense more ambitious than the others we have listed. Here one hopes for a systematic analysis of moduli stabilization in the low energy effective field theory. The basic idea is that competing effects from different low energy gauge groups may give rise to a local minimum for the moduli, in a computable fashion, at weak coupling. One might then hope to compute other quantities relevant to low energy physics.

From the beginning, questions have been raised about calculability in this scenario [18,19]. This question will be a central focus of our investigation. We will see that in some versions of the racetrack scheme, nothing is computable and that most quantities are not likely to be computable in *any* circumstance. But upon more careful consideration, it will become clear that the racetrack scenario has many features in common with the Kähler stabilization and maximally enhanced symmetry hypotheses in that, in some cases, holomorphic quantities such as the gauge couplings and superpotential are computable. To simplify the discussion, we will assume unification, so the standard model gauge couplings are controlled by a single modulus, which we will loosely refer to as the ''dilaton.'' We point out that in order to have any control over low energy physics, it is necessary that the scale of the gauge groups be hierarchically small, i.e. that it be much below the fundamental scale. This requires one (discrete) fine-tuning. Even then, one is unlikely to be able to compute the Kähler potential in a systematic weak coupling approximation; there is, in general, no quantity which can be taken as arbitrarily small in order to justify such a calculation. Holomorphic quantities, however, may be computable, just as in the case of Kähler stabilization. In other words, one can compute holomorphic quantities at weak coupling, and these computations should be reliable at the true minimum. The point is simply that, by symmetries and holomorphy, corrections to holomorphic quantities are controlled by powers of e^{-S}, so that if this quantity is hierarchically small, corrections are similarly small.

This is not the case for the Kahler potential and other non-holomorphic quantities. No simple symmetry argument controls its dependence on S. In some instances, we can exhibit large corrections to the Kähler potential by examining the low energy effective field theory. If we assume that the appropriate cutoff for this theory is that governed by the relations between couplings and scales of the weakly coupled heterotic string, corrections to the Kähler potential, as we will see, are formally of order 1. If we take the scalings suggested by the Horava-Witten picture, the story is somewhat different. In this limit, corrections to the Kahler

potential are suppressed by powers of ρ, the size of the 11th dimension. Corrections to higher derivative operators in the low energy field theory are of order 1. If one assumes that a type I picture is valid, as we will see, corrections to quantities in the low energy theory are suppressed by $g_{eff}^2/8\pi^2$.

Given that one cannot make the gauge groups (and hence the coupling constants) arbitrarily large in these limits, the message we take from these observations is that generally only holomorphic quantities will be calculable. Still, we cannot rule out the possibility that we might be lucky, and that leading order computations might be reliable for other quantities as well.

Apart from these differences, it is perhaps worthwhile to distinguish two cases: supersymmetry unbroken at the minimum of the potential and supersymmetry broken. In the latter case, because of the potential problem of computing the Kahler potential, it is difficult to perform any analysis. If one supposes that the Kähler potential is calculable, then one can compute the cosmological constant; for typical forms of the superpotential, it is unlikely to vanish.

In the case that the potential for S does not break supersymmetry, one has, in principle, more control. Holomorphic quantities are computable. Moreover, as will be described elsewhere, such a situation might be desirable for cosmology. In this case, some other sector of the theory must be responsible for supersymmetry breaking. If gravity is the principle messenger of supersymmetry breaking (as is possible if the symmetries are not enhanced), the noncalculability of the Kähler potential means that one has little control over the low energy dynamics. Soft breakings, in particular, are not computable. The situation is potentially quite different if supersymmetry is broken by low energy dynamics, as in gauge mediated models. The gauge couplings and some Yukawa couplings (i.e. ratios which depend only on holomorphic quantities) would be computable. The soft breakings would be computable in terms of a small number of parameters (some terms in the low energy effective superpotential would depend on uncontrollable Kähler potential corrections). Many of the uncomputable non-holomorphic quantities may be of little relevance to low energy physics.

The racetrack explains how one modulus is fixed, with a large value for its mass. If there are other moduli, they may still pose problems for cosmology. On the other hand, if these moduli all sit at enhanced symmetry points, the cosmological moduli problem [20,21] is solved, and low energy breaking, as in the case of maximally enhanced symmetry, is inevitable. In such a picture, it is quite natural that only one modulus has a large value in fundamental units, and thus is responsible for the observed values of the gauge couplings and their unification. As stressed by the authors of [20], any low energy supersymmetry breaking scheme has a drawback: in order to generate a term in the superpotential of the correct order of magnitude to lead to vanishing cosmological constant, it is necessary to postulate some additional strong dynamics beyond that which breaks supersymmetry. We will make some remarks on this below. It is not clear whether this problem is truly more severe than the extreme fine-tuning required in any case.

This picture is indeed attractive from a cosmological point of view, as discussed in [20]. These authors argued that it might be desirable to fix the mass of some moduli at scales well above the scale of supersymmetry breaking. Any remaining moduli pose potential cosmological problems, unless, as argued in [22], they sit at enhanced symmetry points.

Before investigating these questions, we should introduce our basic assumptions and some terminology. Our focus is on the question as to why the gauge couplings are small and unified. To address this, we will assume, as stated above, that there is one modulus whose value controls the size of the observed gauge couplings (other moduli, with small expectation values, could also couple). We will refer to this modulus as the dilaton, and denote it by $\mathcal{M}$, though we will not assume that this field is to be identified with what is usually called the dilaton in weakly coupled string theory. Second, we need to explain, as in any such discussion, what is meant by the term ''modulus.'' Clearly, since we are discussing the problem of stabilization, we are not supposing that there is an exact moduli space. We have instead in mind the possibility that there are approximate moduli, whose masses at their minima are small compared to the fundamental scale and which become exact moduli in some limit. Finally, we are assuming throughout that there is approximate low energy supersymmetry. Indeed, we will see that it is hard to make sense of the racetrack scheme without it.

One must also note that there are actually several versions of the racetrack idea. Most are tied specifically to gaugino condensation, but this is not necessary [23] and, as we will see, has certain disadvantages. All involve generating a superpotential in the low energy theory. In most versions of the scenario, the dynamics which fixes the moduli does not break supersymmetry. We will argue that this is essential if the gauge couplings are to be calculable. In perhaps the simplest proposal, there are several groups coupled to the dilaton with very large β functions [17,23]. In this case, as we will see, the usual low energy analysis yields a small value for the gauge coupling. However, the scale of the low energy theory is of order the fundamental scale, and the low energy analysis is not valid. One might have hoped that holomorphy and symmetries would allow one to extend the range of validity of these methods. However, the modulus superpotential, even assuming the relevance of the low energy theory, is a much more complicated function than usually assumed. This superpotential is not calculable, and the mass of the modulus is of order 1. Generically, the cosmological constant will be of order 1, though it is possible, in principle, for it to vanish at this level, due to symmetries. So, in this case, little is gained over simply assuming that the theory has a supersymmetry-preserving minimum at some desired value of the coupling.

More promising is a second (discretely) fine-tuned version, in which several groups have very similar, large β functions. In this case, the coupling can be small, and there can be a hierarchy of scales. In the case where gluino condensation is the origin of the superpotential, at least three groups must have nearly equal, large β functions, and special relations must hold among threshold factors. However, following [23], we can consider models with unbroken R sym-

metries (in this reference it was supposed that the symmetries were continuous but discrete symmetries can also accomplish the same objectives; the role of discrete R symmetries in obtaining unbroken supersymmetry was first stressed in [20]). In this case, only one fine-tuning is required, though one also needs many gauge singlet fields and discrete symmetries.

Even in this discretely fine-tuned case, it is unlikely that some sort of weak coupling analysis will be possible. First, there is no small parameter (such as $1/N$) which justifies such an approximation. Second, if one assumes the relations of couplings and scales as in the weakly coupled heterotic string, while the gauge couplings are numerically small, it is easy to exhibit loop corrections, at least for non-holomorphic quantities, which are of order 1. As we have already noted, the situation is different in other string theories, but given that one cannot obtain very large gauge groups in these limits, we view this result as suggestive of a more general difficulty. However, certain holomorphic quantities are under control, and may be susceptible to analysis in the low energy theory. To understand this, one should imagine first passing to the weak coupling limit. In this limit, high energy effects in the superpotential and gauge coupling function go as powers of $e^{-\mathcal{M}}$. The low energy analysis yields a coupling such that $e^{-\mathcal{M}}$ is extremely small. So these corrections should be under control. In a scheme of this sort, one must still understand the breaking of supersymmetry and the fixing of any other moduli. If the breaking is at scales intermediate between the weak and Planck scales (as in ''supergravity'' models), low energy soft breaking is not calculable. If breaking is at low energies, as in gauge mediation, many of the important features of the low energy theory may be calculable.

Different scalings hold in the type I or strongly coupled heterotic string limits and it is conceivable that the Kähler potential is calculable. But even if many quantities are not calculable, one would be left with a rather appealing picture. Allowing one numerical coincidence (some close relation among beta functions), one would hope to develop a complete phenomenology starting from a weakly coupled limit. One would still need to understand the problem of the cosmological constant, of course.

We turn, finally, to the possibility that supersymmetry is broken simultaneously with fixing the moduli. For such scenarios, we note that, in light of our observation that one cannot compute the Kähler potential, very little, if anything, is accessible to analysis. The gauge couplings would not be calculable; nor would soft breaking terms. It is possible that, as in the case of Kahler stabilization, some terms in the superpotential might be. Even if one could use the lowest order Kähler potential (as we will see might conceivably be the case), one has another difficulty: the cosmological constant is calculable and typically non-vanishing at the minimum.

In the following sections, we review the racetrack idea and describe these possibilities in greater detail. We conclude by arguing that indeed the racetrack idea, whether ultimately realized in nature or not, does provide a viable model for understanding the smallness of the gauge couplings in a theory which is inherently strongly coupled. As

an application, we consider the implication of such a picture for scenarios with large internal dimensions.

II. SUSY CONSERVING VERSION OF THE RACETRACK IDEA

Kaplunovsky and Louis [17] have put forth an appealing version of the racetrack idea. They note that studies of F-theory compactifications have yielded classical ground states of the theory with enormous gauge groups. Suppose, now, that one has two gauge groups (without matter—these remarks all readily generalize to cases with matter in which the strong gauge group does not by itself break supersymmetry) with very large β functions, say

$$b_1 = aN, \quad b_2 = bN, \tag{1}$$

where N is a large integer, and a and b are (rational numbers) of order unity. Suppose the gauge couplings of both groups are controlled by a single modulus, $\mathcal{M}$; i.e. the Lagrangian at low energies looks like

$$\int d^2\theta \mathcal{M}(W_1^2 + W_2^2). \tag{2}$$

Then the usual arguments for gluino condensation yield a superpotential for $\mathcal{M}$:

$$W = \alpha b_1 e^{-\mathcal{M}/b_1} - b_2 \beta e^{-\mathcal{M}/b_2}. \tag{3}$$

(If the couplings are not the same at the fundamental scale, this difference can be absorbed into the b_i's.) Here α and β are numbers of order 1 which arise due to threshold corrections. This superpotential has a stationary point at

$$\mathcal{M} = \frac{b_1 b_2}{b_1 - b_2} \ln\left(\frac{\beta}{\alpha}\right). \tag{4}$$

For large N, this behaves as

$$\mathcal{M} \sim N \ln(\beta/\alpha). \tag{5}$$

At the stationary point, however, W is nonzero:

$$W = \alpha(b_1 - b_2)\left(\frac{\beta}{\alpha}\right)^{-b_2/(b_1 - b_2)} \tag{6}$$

(when $b_1 = b_2$, there is no stationary point).

This would appear to be what is needed to stabilize the dilaton at weak coupling, and would even seem to be rather generic. There are at least two difficulties, however, which appear more or less fundamental. The first has to do with the self-consistency of the calculation and the second to do with the problem of the cosmological constant. The usual analysis of gluino condensation assumes that the scale of the low energy gauge theory, Λ, is well below the fundamental scale. But in this scheme, Λ is of order 1. As a result, it is not at all clear why one can look at the gauge group and not consider the full set of massive string (M) theory states.

Still, sometimes holomorphy and symmetries can significantly constrain the form of the superpotential, so we might

hope that the low energy analysis captures some of the truth. To assess this possibility, let us consider the effects of non-renormalizable operators on the low energy analysis. The usual discussion of gluino condensation starts by noting that at the renormalizable level, the effective theory has an R symmetry under which the modulus $\mathcal{M}$ transforms. This R symmetry uniquely determines the dependence of the condensate, $\langle \lambda\lambda \rangle$ on $\mathcal{M}$:

$$\langle \lambda\lambda \rangle = z, \tag{7}$$

where

$$z = e^{-3\mathcal{M}/b_0}. \tag{8}$$

Now suppose that there are terms in the effective action just below the string scale of the form

$$\int d^2\theta \chi W^4 + \zeta W^6 + \cdots. \tag{9}$$

We can think of χ and ζ, etc., as spurions which transform under the R symmetry; χ has R charge -2, ζ charge -4, etc. The same argument which gave the leading term gives

$$\langle \lambda\lambda \rangle = z + a\chi z^2 + b\zeta z^3 + \cdots \tag{10}$$

where a and b are constants of order one.

So we see that the superpotential is a general function of z, even in the case of one modulus. At the stationary point, these corrections are not suppressed. Indeed, even with only one modulus there may now be a stationary point with large $\mathcal{M}$. It is not possible to determine the location of this stationary point, however, without an understanding of the full string theory. The low energy theory is insufficient.

In order that one obtain a supersymmetric vacuum with vanishing cosmological constant, it is necessary that both the superpotential and its first derivative vanish. This is not the case for our simplified treatment of the example above; at the stationary point of W, W is non-zero. It is conceivable that there are models for which W has special properties such that it vanishes at the stationary point. After all, the vanishing of W in weakly coupled string compactifications is a consequence of detailed features such as the Peccei-Quinn symmetry. It is possible that models whose dynamics preserves a discrete R symmetry could naturally yield a vanishing W at the stationary point. We will discuss this possibility below in a different context, but the other difficulties remain.

Finally, note that in these schemes, the modulus itself is massive, with mass of order one. So one might as well suppose that one is studying M theory vacua (in some approximation) with fewer moduli from the start. Logically, there is no problem with this idea, but it leaves unanswered the questions of why the couplings are weak and whether anything is calculable.

III. IMPROVEMENT THROUGH FINE-TUNING

In our example above, if $b_1 \approx b_2$, one can perhaps improve the situation. In this case, z can be small. For example, if the two quantities are within 10% of each other, then z

$\sim 10^{-10}$. The scales are now well separated, and arguably the use of the low energy effective action is self-consistent. The situation with respect to the cosmological constant is also somewhat better. For suppose supersymmetry breaking arises from some more weakly coupled gauge group with beta function $b_3 \ll b_1$. It is possible (for example if the leading order contribution—in z—to the cosmological constant vanishes) that the various terms could cancel with one another.

In the finely tuned case, it is possible to analyze the possible vanishing of W with several gauge groups. With two gauge groups there are still no solutions, but with three or more gauge groups with very similar, large β functions, there are solutions for suitable values for the threshold corrections [15]. One can analyze this problem by considering a superpotential of the form

$$W = \alpha b_1 e^{-\mathcal{M}/b_1} + \beta b_2 e^{-\mathcal{M}/b_2} + \gamma b_3 e^{-\mathcal{M}/b_3}. \quad (11)$$

We want to ask whether there can be solutions of the equations $W' = W = 0$, for some values of α, β and γ. It is a simple algebraic exercise to verify that this is the case. Whether the required values of the threshold corrections actually arise is another question, but it is not perhaps completely implausible, given that corrections to α, β and γ from their one loop form will be exponentially small.

Izawa and Yanagida have proposed a variant on the racetrack scheme which would ameliorate this difficulty [23]. Field theories with quantum moduli spaces, at the level of non-renormalizable terms, leave unbroken R symmetries. If such a theory appears in the low energy limit of a string theory and if the theory has a (discrete) R symmetry, then the superpotential can naturally vanish at the stationary point (the fact that R symmetries can account for unbroken supersymmetry and vanishing cosmological constant was stressed in [20] and is crucial to the cosmological scenario outlined in [24]). These authors give an example with two (discretely tuned) low energy groups. A large number (of order N^2) singlets with suitable couplings to the matter fields of these groups are required to achieve the desired stabilization. Additional discrete symmetries are required in order to obtain the desired patterns of couplings. On the other hand, in brane constructions such large numbers of singlets might be plausible, and elaborate discrete symmetries are familiar in string theory. This type of R-symmetry based scenario, then, seems the most plausible.

Finally, there is the question of what is calculable in this picture. Looking at the explicit solutions, one sees that if the β functions are of order N and their differences of order 1, the gauge couplings are of order $1/N^2$. This is probably enough to ensure that holomorphic quantities are given by their weak coupling values. Corrections to the superpotential and the gauge coupling functions go as $e^{-\mathcal{M}}$, for example, and this is suitably small. However, non-holomorphic quantities are not, in general, described by any weak coupling approximation. In particular, consider corrections to the Kähler potential for $\mathcal{M}$, such as those indicated in Fig. 1. These diagrams come with a factor of $N^2 \Lambda^2/M_p^2$, where the N^2 arises from the N^2 particles propagating in the loop, and
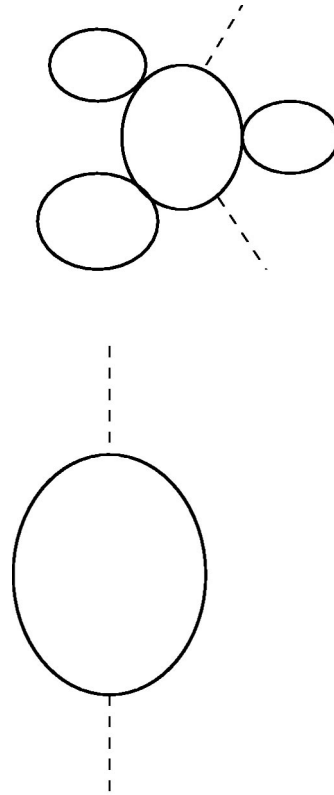

FIG. 1. Contributions to dilaton Kähler potential proportional to N^2.

Λ is a cutoff [25]. In the weakly coupled heterotic string, $\Lambda^2 = g^2 M_p^2$, and so these corrections are of order 1. The resulting one loop supergravity (SUGRA) contributions to the soft masses are of the order $N^2 \Lambda^2/(16\pi^2 M_p^2)$ [26]. Taking into account factors of π in our definition of the dilaton we find that they are of order 1. We should stress here that in some models supergravity contributions may be numerically small. If one examines the scenarios discussed in Ref. [16], one finds that for most of them, $g^2 \Sigma N_i^2/16\pi^2$ is 30% or larger, but there is one where it is as small as 10–15 %. One might hope that the corrections can be reliably computed. However, it will be difficult to establish this fact, since, as we have argued, there is no formal small expansion parameter.

So far we have focused on the weakly coupled heterotic string theory in order to estimate the cutoff. Some hope for optimism is provided by considering other string theories (we are grateful to Antoniadis [36] for a remark which prompted an examination of this question). In the limit of the strongly coupled heterotic string one would expect the cutoff to be given by M_{11}. However, for quantities which are predicted by 10 dimensional supersymmetry, the relevant cutoff will be related to the compactification scale. Support for this comes from studies of the Kähler potential in the Horava-Witten limit, which is easily seen by symmetry arguments to be the same as in the weak coupling limit, up to terms of order $1/\rho$, where ρ is the size of the 11th dimension. This can be traced to the fact that one can pass from one limit to the other in such a way that the theory is always approximately

ten dimensional. So the Kähler potential may be calculable. For example, if the typical size of the M-theory compactification manifold is $R > M_{11}^{-1}$, then the supersymmetry (SUSY) above the compactification scale may provide the relevant cutoff. In such a case corrections could be of order $N^2 R^{-2}/M_p^2 \sim N^2 g^3 \sim 1/N$. Certain higher dimensional operators would obtain non-calculable corrections, but this could be of little relevance to low energy effective theory. In the type I theory, assuming compactification at the string scale, $\Lambda^2 = g^4 M_p^2$, and so again one might hope that these corrections are under control for sufficiently large N (applied to the examples of [16], this gives corrections in some cases as small as 7%, with 20% being more typical). Of course, to argue this formally requires that there exist a set of theories characterized by N such that one can take the limit $N \to \infty$, and this seems unlikely to exist in these limits.[2] F theory suggests that very large gauge groups may exist, but we do not know how to perform the corresponding analysis for F theory, and suspect that one will have similar difficulties to those of the weak coupling heterotic picture. We will adopt the pessimistic view in what follows that one does not expect to be able to compute the Kähler potential. In this view, one does not expect to be able to calculate quantities which depend on the detailed form of the Kähler potential. But it is again important to keep in mind that there may be instances where much more is calculable.

In sum, the finely tuned case is a scenario in which a low energy analysis can in principle provide an explanation of small couplings and large hierarchies. Holomorphic quantities are in principle calculable in such a scheme, but non-holomorphic quantities are probably not. If supersymmetry is broken at an intermediate scale, it will not be possible to say much about the low energy spectrum, since the Kähler potential is not known. On the other hand, if supersymmetry is broken at low energies, as in gauge mediation, many quantities may be calculable. Apart from the gauge couplings themselves, physical quantities which depend holomorphically on terms in the superpotential will be calculable. The soft breakings should be expressible in terms of a small number of parameters.

IV. SUPERSYMMETRY VIOLATING VERSION OF THE RACETRACK SCHEME

When originally proposed, it was hoped that the racetrack scheme would provide a mechanism for fixing some moduli (assumed to be the usual dilaton of weakly coupled heterotic string theory) while simultaneously breaking supersymmetry in a calculable manner, and generating a weak gauge coupling and large hierarchy. This possibility was most thoroughly analyzed in [16], where many interesting examples were developed. Still, we can ask whether such a proposal can truly be analyzed in terms of a low energy effective action.

As in the supersymmetric case, it is necessary, in order that any low energy effective action analysis make sense (and presumably also that one obtain hierarchically small supersymmetry breaking) that one has several (at least two) groups with nearly identical β functions. However, determining whether a minimum of the potential exists and its location (and in particular determining the value of the gauge coupling) requires, in this case, knowledge of the Kähler potential for the modulus. We have argued, however, that this may not be calculable. The problem can be stated more strongly: there is no simple argument, in these cases, that the superpotential is simply a sum of the superpotentials for the different groups. The usual symmetry arguments [27] for the form of the superpotential no longer hold. Moreover, at generic points in the moduli space, the larger condensate induces non-zero—and large—contributions to the other. Examining the appropriate diagrams, one can see that this problem is closely tied to the problem of understanding the Kähler potential.[3]

Indeed, this situation is not so much different than that of ''Kähler stabilization,'' where it is supposed that with a single gauge group, the Kähler potential is such as to give rise to a minimum of the potential at weak coupling. The principle difference is the two, nearly equal, β functions provide a slightly different explanation for the smallness of the gauge coupling than the accident proposed in [8].

As we have remarked above, the analysis of [16] is consistent with these remarks. In most cases, a rough estimate of the corrections yields a large value for the effective expansion parameter, but in one of their examples it is about 10%. Whether this is good enough in string theory is, of course, an open question.

Operationally, it is not clear that there is much difference between the two hypotheses. In particular, in both cases, some quantities protected by holomorphy, i.e. the superpotential and gauge coupling functions, are accessible. Quantities which are not, such as the soft breaking masses, are unpredictable. Recently, in [28], it has been shown that a combination of Kähler stabilization and multiple condensates provides an interesting model for stabilization. A quite specific and plausible picture for the origin of the Kähler potential corrections is presented, though again control of non-holomorphic quantities, in the sense of there being a systematic, weak coupling approximation scheme, is limited.

It should be noted that if the Kähler potential is given by its weak coupling (or strong heterotic coupling) form, the cosmological constant can be calculated at the level of the

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[2]The same, of course, is true for holomorphic quantities, but given their exponential dependence, this seems more plausible.

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[3]Similar issues were raised in [34], where it was noted that in certain grand unified theories, one obtained inconsistencies if one assumed the superpotential was a simple sum of this type. In [35], it is asserted that the superpotential is a sum. However, this analysis treats the dilaton superfield as a non-dynamical background and ignores the fact that one condensate induces corrections to the other. It is conceivable that the correct form is a sum and that these other effects can be absorbed into corrections to the Kähler potential, but this is by no means obvious and is a question worthy of further investigation.

effective action. The dilaton potential is (now calling the dilaton S, as appropriate to the weak coupling limit)

$$V(S,S^\dagger) = \frac{1}{S+S^\dagger}\left[\left|\left(\frac{\partial W}{\partial S} + \frac{1}{S+S^\dagger}\right)W\right|^2(S+S^\dagger)^2 - 3|W|^2\right].$$
(12)

It is not easy to find systems whose W gives minima with $V=0$ [14].

V. OTHER ALTERNATIVES TO THE RACETRACK SCHEME

We have stressed in the preceding sections that the problem in string theory is not to explain how moduli can be stabilized, but rather how they can be stabilized in such a way that the gauge couplings are weak and supersymmetry is hierarchically broken, and such that anything is computable. We have argued that the finely tuned version of the racetrack scheme does provide a picture in which the gauge couplings could be fixed at small values and a hierarchy explained in a calculable fashion. Of course, we do not have a detailed string model which realizes these ideas, but at least the scenario permits us to frame the discussion.

The literature contains discussion of other proposals for stabilizing the moduli, which are alternatives to the racetrack scheme. They all suffer, however, from difficulties similar to those which have been discussed here. We will not attempt a complete review, but mention a few examples.

Reference [4] focuses specifically on the heterotic string dilaton, though similar arguments can be applied to other moduli. It is argued that modifications of the gauge coupling function along with gaugino condensation can lead to stabilization. SL(2,Z) duality is used to significantly constrain the form of this function, as well as the form of the Kähler potential. Not surprisingly, however, this leads to stabilization at values of the coupling (α) of order 1. It is argued, there, that uncomputable corrections might give a phenomenologically acceptable value for the coupling. However, this means precisely that nothing is computable in such a picture. The smallness of the low energy gauge couplings is an accident; no aspect of string (M) theory dynamics is accessible to a systematic, weak coupling analysis.

In Ref. [5], another low energy mechanism for stabilizing the moduli is proposed. In this scheme, the low energy theory, in a certain approximation, has a quantum-modified moduli space. The authors only consider the global limit; their models do not yield supersymmetry-conserving minima with vanishing cosmological constant when coupled to gravity. Ignoring this issue, as these authors note, the generic value of the gauge coupling is of order 1. It is possible to obtain smaller couplings if the models have large discrete symmetries. These models are not sufficiently developed to decide whether a weak coupling analysis is applicable, but we would argue it is unlikely that any sort of perturbative treatment of non-holomorphic quantities is possible. If free of anomalies, large discrete R symmetries require large gauge groups (or large matter content). These lead to problems of calculability identical to those we have described

above. Even if anomalies are canceled by a Green-Schwarz mechanism, it is difficult to avoid such large groups.

VI. CONCLUSIONS

Conceptually, the difficult issue in understanding how moduli are stabilized in string theory is understanding why couplings are weak and unified, and there are large hierarchies. It is certainly not hard to imagine that moduli are stabilized in such a way that couplings and dimensionless ratios are of order 1. How large pure numbers arise in a theory without small parameters is distinctly more puzzling. The racetrack scheme and its variants which have been reviewed here are probably the most concrete proposals for how moduli are stabilized at weak gauge coupling. We have seen that in order that one obtain small couplings in a controllable approximation, some degree of fine-tuning is required: if gaugino condensation is the origin of the moduli superpotential, one must have at least three gauge groups with closely related β functions; in theories with an unbroken discrete R symmetry, one needs two groups and an elaborate field and symmetry structure. In these cases, not everything is calculable, but holomorphic quantities such as the superpotential and the gauge coupling functions may be. If supersymmetry is broken at intermediate energy scales, many quantities will not be computable. If supersymmetry is broken at low energies, it is likely that many quantities relevant to low energy physics could be.

Comparing with other proposals for modulus stabilization in string theory, the racetrack model has a certain appeal. While the fine-tuning is unattractive and we do not have explicit examples which provide a complete realization, the scenario is quite concrete. As we have described here, it offers the hope of computing the gauge couplings and the superpotential. If supersymmetry is broken at low energies, many quantities relevant to low energy physics might be computable. Kähler stabilization, by contrast, invokes uncontrollable and unknown corrections to the Kähler potential. As in the racetrack scenario, certain holomorphic quantities are calculable, but not the gauge couplings. It is hard to reconcile this mechanism with low energy supersymmetry breaking. Maximally enhanced symmetry, while possessing a certain economy, requires that through some mysterious mechanism the gauge couplings are quite small, and while it does suggest low energy supersymmetry breaking, is not likely to offer the hope of computing even holomorphic quantities.

There has been much interest recently in the possibility of large internal dimensions. The first well-developed proposal of this type appeared in [29], where it was assumed that the strong coupling limit of the heterotic string, with compactification on a Calabi-Yau space was appropriate. The difficulty with this idea, noted in [31], is that at large radius, the supersymmetry of the higher dimensional theory ensures that the potential vanishes. Clearly, however, something like the racetrack picture could operate here as well [30]. In the scheme of [29], in particular, one has two walls, and supersymmetry breaking arises from dynamics in the walls. Two competing groups could give rise to a potential for the

moduli, with a minimum for some large value of some of the radii. In this regime, again, the Kähler potential would not be calculable, but the gauge couplings and other holomorphic quantities would be. This would presumably mean that one could not take the geometric picture too literally; at best, it would only be qualitatively correct. Similar remarks apply to other scenarios in which stabilization for large values of geometrical moduli is required, e.g. [32].

It is also interesting to consider the possibility that the dilaton of this picture is an inflaton (these remarks are inspired by the recent work of [24] as well as earlier work of [33]). In the models here, the dilaton is fixed, with a mass large compared to the expected scale of supersymmetry breaking. This is perhaps promising, since for inflation one wants a rather large scale. Moreover, the potential, as a result of the discrete fine-tuning, is rather flat. Determining whether or not inflation occurs, and the values of the relevant scales, requires an understanding of the Kähler potential, which we have argued is not calculable in these schemes. In any case, the scale of the potential can quite naturally be two or three orders of magnitude below the Planck scale, perhaps leading to a suitable spectrum of density fluctuations. Determining the number of *e*-foldings also requires knowledge of the Kähler potential, but it is plausible that this number might be large. Because the potential is a function of S/N, taking the Kähler potential to be of order $(N)^0$, a simple scaling argument gives $N_e \sim N^2$. In other words, the fine-tuning required to obtain a weak gauge coupling might be the same as the fine-tuning required to obtain adequate inflation.

Inflation in this picture requires explicit, if plausible, assumptions about the Kähler potential. Still, the connection of the tuning required to obtain small gauge couplings in this picture and that required to obtain inflation raises the possibility that the two are truly correlated; perhaps the explanation of the smallness of the gauge couplings is that only regions with weak gauge couplings inflate.

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