Superstring action in $AdS_5\times S^5$: κ-symmetry light cone gauge

R. R. Metsaev*
Department of Physics, The Ohio State University, Columbus, Ohio 43210-1106
and Department of Theoretical Physics, P.N. Lebedev Physical Institute, Leninsky prospect 53, Moscow 117924, Russia

A. A. Tseytlin†
Department of Physics, The Ohio State University, Columbus, Ohio 43210-1106
(Received 18 July 2000; published 18 January 2001)

As part of a program to quantize superstrings in $AdS_5\times S^5$ in a light-cone gauge we find the explicit form of the corresponding Green-Schwarz action in the fermionic light-cone κ-symmetry gauge. The resulting action contains terms quadratic and quartic in fermions. In the flat space limit it reduces to the standard light-cone GS action, while for $\alpha'\rightarrow 0$ it has the correct $AdS_5\times S^5$ light-cone superparticle limit. We discuss fixing the bosonic light-cone gauge and a reformulation of the action in terms of 2D Dirac spinors.

DOI: 10.1103/PhysRevD.63.046002 PACS number(s): 11.25.Mj

I. INTRODUCTION AND SUMMARY

A. Motivations for light-cone gauge approach

The two maximally supersymmetric backgrounds of type-IIB superstring theory are flat Minkowski space $R^{1,9}$ and $AdS_5\times S^5$. The manifestly supersymmetric superstring action in flat space—the Green-Schwarz (GS) action—is well known [1], and its $AdS_5\times S^5$ analogue was constructed in Ref. [2] (see also Refs. [3,4]).

Progress in understanding the AdS conformal field theory (CFT) duality [5], i.e., in solving (the large N) supersymmetric $\mathcal{N}=4$ Yang-Mills (YM) theory in terms of (first-quantized) superstring in $AdS_5\times S^5$ depends on developing its GS description and making it more practical. Some advances in this direction with application to ''long'' strings ending at the boundary of AdS_5 were discussed in Refs. [6–8].

While the Neveu-Schwarz-Ramond (NSR) string action in curved NS-NS backgrounds has well-defined kinetic terms and is at most quartic in fermions, the GS action in curved $AdS_5\times S^5$ background with R-R flux looks, in general, very nonlinear [2–4]. Its fermion structure simplifies in some special κ-symmetry gauges [9–11,6], but, as in flat space, one may still face the question of dependence of the fermion kinetic term on a choice of bosonic string background, i.e. of its potential degeneracy [6].

String configurations in $AdS_5\times S^5$ include ''short'' closed strings and ''long'' stretched strings that may end at the boundary. The GS action is well suited for description of small fluctuations near long string backgrounds (for which fermion kinetic term is well defined). However, to be able to determine the fundamental closed string spectrum in $AdS_5\times S^5$ one is to learn how to quantize the $AdS_5\times S^5$ string action in the ''short string'' sector, i.e., without explicitly expanding near a nontrivial bosonic string configuration.

It is well known how this is achieved for the flat space GS action—by choosing a light-cone gauge [12,1]. The superstring light-cone gauge fixing consists of the two steps: (I) fermionic light-cone gauge choice, i.e., fixing the κ symmetry by $\Gamma^+\theta^I=0$; (II) bosonic light-cone gauge choice, i.e., using the conformal gauge[1] $\sqrt{g}g^{\mu\nu}=\eta^{\mu\nu}$ and fixing the residual conformal diffeomorphism symmetry by $x^+(\tau,\sigma)=p^+\tau$.

Fixing the fermionic light-cone gauge already produces a substantial simplification of the flat-space GS action: it becomes quadratic in θ. Choosing the bosonic light-cone gauge, i.e., using an explicit choice of x^+, may not always be necessary (see Refs. [13,14]), but it makes derivation of the physical string spectrum straightforward.

Our eventual aim is to develop a systematic light-cone gauge framework for the GS strings in $AdS_5\times S^5$. In this paper we shall concentrate on the first and crucial step of fixing the fermionic light-cone gauge, i.e., imposing an analog of $\Gamma^+\theta^I=0$ condition.[2] The idea is to get a simple gauge-fixed form of the action where the nondegeneracy of the kinetic term for the fermions will not depend on a choice of a specific string background in transverse directions, i.e., as in flat space, the fermion kinetic term will have the structure $\partial x^+\bar{\theta}\partial\theta$.

There are other motivations for studying $AdS_5\times S^5$strings in the light-cone gauge.

(i) One of the prime goals is to clarify the relation between the string theory and $\mathcal{N}=4$ super YM (SYM) theory at the boundary. The SYM theory does not admit a manifestly $\mathcal{N}=4$ supersymmetric Lorentz-covariant description, but has a simple superspace description in the light-cone gauge A^+

*Email address: metsaev@lpi.ru, metsaev@pacific.mps.ohio-state.edu

†Also at Blackett Laboratory, Imperial College, London and Lebedev Physics Institute, Moscow. Email address: tseytlin@mps.ohio-state.edu

[1]We use Minkowski signature 2D world sheet metric $g_{\mu\nu}$ with $g\equiv -\det g_{\mu\nu}$.

[2]A previous work in this direction was reported in Ref. [15], but the κ-symmetry light-cone gauge used there is different from ours and we do not understand the relation of the action presented in Ref. [15] to our light-cone gauge fixed action.

$=0$ [16]. It is based on a single chiral superfield $\Phi(x,\theta)=A(x)+\theta^i\psi_i(x)+\cdots$, where $A=A_1+iA_2$ represents the transverse components of the gauge field and ψ_i its fermionic partner which transforms under the fundamental representation of R-symmetry group SU(4). In addition to the standard light-cone supersymmetry (shifts of θ), the light-cone superspace SYM action $S[\Phi]$ has also a nonlinear superconformal symmetry. This suggests that it may be possible to formulate the bulk string theory in a way which is naturally related to the light-cone form of the boundary SYM theory. In particular, it may be useful to split the corresponding fermionic string coordinates into the two parts with manifest SU(4) $\simeq$SO(6) transformation properties which will be the counterparts of the linearly realized Poincaré supersymmetry supercharges and the nonlinearly realized conformal supersymmetry supercharges of the SYM theory.

(ii) As was shown in Refs. [17–19], field theories in AdS space (in particular, type-IIB supergravity) admit a simple light-cone description. There exists a light-cone action for a superparticle in $AdS_5\times S^5$ which was used to formulate AdS/CFT correspondence in the light-cone gauge. This suggests that the full superstring theory in $AdS_5\times S^5$ should also have a natural light-cone gauge formulation, which should be useful in the context of the AdS/CFT correspondence.

B. Structure of the light-cone gauge string action

Our fermionic κ-symmetry light-cone gauge (which is different from the naive $\Gamma^+\theta^I=0$ but is related to it in the flat space limit) will reduce the 32 fermionic coordinates θ^I_α (two left Majorana-Weyl 10D spinors) to 16 physical Grassmann variables: ''linear'' θ^i and ''nonlinear'' η^i and their Hermitian conjugates θ_i and η_i $(i=1,2,3,4)$, which transform according to the fundamental representations of SU(4). The superconformal algebra psu(2,2|4) dictates that these variables should be related to the Poincaré and the conformal supersymmetry in the light-cone gauge description of the boundary theory. The action and symmetry generators will have simple (quadratic) dependence on θ^i, but complicated (quartic) dependence on η^i.[3]

We shall split the 10 bosonic coordinates of $AdS_5\times S^5$ as follows. The 4 isometric coordinates along the boundary directions will be

$$x^a=(x^+,x^-,x,\bar{x}),\quad x^\pm\equiv\frac{1}{\sqrt{2}}(x^3\pm x^0),$$

$$x,\bar{x}=\frac{1}{\sqrt{2}}(x^1\pm \mathrm{i}x^2),\tag{1.1}$$

the radial direction of AdS_5 will be ϕ, and the S^5 coordinates will be denoted as $y^{A'}$ $(A'=1,2,3,4,5)$.

[3]These coordinates are direct counterparts of the Grassmann coordinates in the light-cone action for a superparticle in $AdS_5\times S^5$in Refs. [18,19].

Choosing a light-cone gauge in the parametrization of the supercoset PSU(2,2|4)/[SO(4,1)×SO(5)] described below, the $AdS_5\times S^5$ superstring Lagrangian of Ref. [2] can be written as[4]

$$\mathcal{L}=\mathcal{L}_B+\mathcal{L}_F^{(2)}+\mathcal{L}_F^{(4)}.\tag{1.2}$$

Here $\mathcal{L}_B=-\frac{1}{2}\sqrt{g}g^{\mu\nu}G_{MN}(X)\partial_\mu X^M\partial_\nu X^N$ is the standard bosonic sigma model with $AdS_5\times S^5$ as target space[5]

$$\mathcal{L}_B=-\sqrt{g}g^{\mu\nu}\bigg[e^{2\phi}(\partial_\mu x^+\partial_\nu x^-+\partial_\mu x\partial_\nu\bar{x})+\frac{1}{2}\partial_\mu\phi\partial_\nu\phi+\frac{1}{2}e^{A'}_\mu e^{A'}_\nu\bigg].\tag{1.3}$$

$e^{A'}_\mu$ is the projection of the vielbein of S^5 which in the special parametrization we will be using is given by

$$e^{A'}_\mu=-\frac{i}{2}\mathrm{Tr}(\gamma^{A'}\partial_\mu UU^{-1}),\quad U^i{}_j\equiv(e^y)^i{}_j,\quad U^\dagger U=I,\tag{1.4}$$

where Tr is over i,j. The matrix $U\in SU(4)$ depends on 5 independent coordinates $y^{A'}$

$$y^i{}_j\equiv\frac{i}{2}y^{A'}(\gamma^{A'})^i{}_j,\quad (y^i{}_j)^*=-y^j{}_i,\quad y^i{}_i=0,\tag{1.5}$$

and $\gamma^{A'}$ are SO(5) Dirac matrices. $\mathcal{L}_F^{(2)}$ is the quadratic part of the fermionic action

$$\mathcal{L}_F^{(2)}=e^{2\phi}\partial_\mu x^+\bigg[\frac{i}{2}\sqrt{g}g^{\mu\nu}(-\theta_i\mathcal{D}_\nu\theta^i-\eta_i\mathcal{D}_\nu\eta^i+i\,\eta_i e^i_{\nu j}\eta^j)+\epsilon^{\mu\nu}\eta^i C'_{ij}(\mathcal{D}_\nu\theta^j-i\sqrt{2}e^\phi\eta^j\partial_\nu x)\bigg]+\mathrm{H.c.}\tag{1.6}$$

The P-odd $\epsilon^{\mu\nu}$ dependent term in Eq. (1.6) came from the WZ term in the original supercoset GS action [2].

Here we used the following notation:

$$\mathcal{D}\theta^i=d\theta^i-\Omega^i{}_j\theta^j,\quad \mathcal{D}\theta_i=d\theta_i+\theta_j\Omega^j{}_i,\quad e^i{}_j\equiv(\gamma^{A'})^i{}_j e^{A'},\tag{1.7}$$

and $\mathcal{D}=\mathcal{D}_\mu d\sigma^\mu, e^i{}_j=e^i_{\mu j}d\sigma^\mu$, where $\sigma^\mu=(\tau,\sigma)$ are 2D coordinates. $\mathcal{D}$ is the generalized spinor derivative on S^5. It

[4]The light-cone gauge action can be found in two related forms. One of them corresponds to the Wess-Zumino type gauge in 10D superspace while another is based on the Killing gauge (see Refs. [3,10]). These ''gauges'' (better to be called ''parametrizations'') do not reduce the number of fermionic degrees of freedom but only specialize a choice of fermionic coordinates. The action given in this section corresponds to the WZ parametrization, while the action in the Killing parametrization will be discussed in Section VI.

[5]Our index notation differs from Ref. [2]: here we use $\mu,\nu=0,1$ for 2D indices, i,j for SU(4) indices, $A=0,1,\ldots,4$ for AdS_5 and $A'=1,\ldots,5$ for S^5 tangent space indices (repeated indices are contracted with flat metric). We use $\epsilon^{01}=1$.

has the general representation $\mathcal{D}=d+\Omega^i{}_j J^j{}_i$ and satisfies the relation $\mathcal{D}^2=0$. $\Omega^i{}_j$ is given by

$$\Omega=dU\,U^{-1},\quad d\Omega-\Omega\wedge\Omega=0, \tag{1.8}$$

and can be written in terms of the S^5 spin connection $\omega^{A'B'}$ and the 5-bein $e^{A'}$ as follows:

$$\Omega^i{}_j=-\frac{1}{4}(\gamma^{A'B'})^i{}_j\omega^{A'B'}+\frac{i}{2}(\gamma^{A'})^i{}_j e^{A'}. \tag{1.9}$$

C'_{ij} is the constant charge conjugation matrix of the SO(5) Dirac matrix algebra $C'^{\dagger}C'=I, C'^{T}=-C'$. The Hermitean conjugation rules are $\theta_i^\dagger=\theta^i$, $\eta_i^\dagger=\eta^i$. The quartic fermionic term in Eq. (1.2) depends only η but not on θ

$$\mathcal{L}_F^{(4)}=\frac{1}{2}\sqrt{g}g^{\mu\nu}e^{4\phi}\partial_\mu x^+\partial_\nu x^+[(\eta^i\eta_i)^2-(\eta_i\gamma^{A'i}{}_j\eta^j)^2]. \tag{1.10}$$

C. Some properties of the action

The action (1.2),(1.3),(1.6),(1.10) has several important properties.

(a) The dependence on x^- is only linear—through the bosonic $\partial x^+\partial x^-$ term in Eq. (1.3).

(b) The bosonic factor in the fermion kinetic term is simply $e^{2\phi}\partial x^+$. It is the crucial property of this light-cone κ-symmetry gauge fixed form of the action that the fermion kinetic term involves the derivative of only *one* space-time direction—x^+, i.e., its (non)degeneracy does not depend on transverse string profile.[6]

(c) The fact that the action has only quadratic and quartic fermionic terms has to do with special symmetries of the $\mathrm{AdS}_5\times\mathrm{S}^5$ background (covariantly constant curvature and 5-form field strength). The presence of the η^4 term (1.10) reflects the curvature of the background.[7] As follows from the discussion in Ref. [2], the ''extra'' terms in Eq. (1.6) such as $\eta_i e^i{}_j\eta^j$ and $\eta C'\eta\partial x$ should have the interpretation of the couplings to the R-R 5-form background.[8]

(d) The gauge we considered treats the AdS_5 and S^5 factors asymmetrically. In particular, the action contains only SO(5) but not SO(4,1) gamma matrices, and θ_i and η_i are not spinors under SO(4,1).[9]

(e) The $\mathrm{AdS}_5\times\mathrm{S}^5$ superstring action depends on two parameters: the scale (equal radii) R of $\mathrm{AdS}_5\times\mathrm{S}^5$ and the inverse string tension or α'. Restoring the dependence on R set equal to 1 in Eq. (1.2) one finds that in the flat space limit $R\to\infty$ the quartic term (1.10) goes away, while the kinetic term (1.6) reduces to the standard one with $\mathcal{D}_\mu\to\partial_\mu$. The resulting action is equivalent to the flat space light-cone GS action [1] after representing each of the two SO(8) spinors in terms of the two SU(4) spinors. The action takes ''diagonal'' form in terms of the combinations $\psi^i_{1,2}$ of our two fermionic variables [see Eq. (1.22) below].

(f) For $\alpha'\to 0$ the action has the correct particle limit, i.e., it reduces to the light-cone gauge superparticle action in $\mathrm{AdS}_5\times\mathrm{S}^5$ [18].

(g) A special feature of this action is that SU(4) $\simeq$SO(6) symmetry is realized linearly on fermions, but not on bosons, i.e., is not manifest. This is a consequence of the factor $\mathrm{SO}(4,1)\times\mathrm{SO}(5)$ in the underlying supercoset $\mathrm{PSU}(2,2|4)/[\mathrm{SO}(4,1)\times\mathrm{SO}(5)]$ being purely bosonic. The $\mathrm{S}^5=\mathrm{SO}(6)/\mathrm{SO}(5)$ part of the bosonic action can be represented as a special case of the 2D G/H coset sigma model $L=\mathrm{Tr}(\partial_\mu\mathcal{U}\mathcal{U}^{-1}+A_\mu)^2$, $\mathcal{U}\in G=\mathrm{SO}(6)$, with the 2D gauge field A_μ being in the algebra of $H=\mathrm{SO}(5)$. This action does not have manifest SO(6) symmetry after A_μ is integrated out and $\mathcal{U}$ is restricted to belong to the coset as a gauge choice.

(h) The action is symmetric under shifting $\theta\to\theta+\epsilon$ supplemented by an appropriate transformation of x^-. Here ϵ is a Killing spinor on S^5, satisfying the equation $\mathcal{D}\epsilon^i=0$. It is this symmetry that is responsible for the fact that the

---

[6]The action thus has similar structure to that of the light-cone gauge action for the GS string in curved magnetic R-R background constructed in Ref. [20].

[7]Note that the light-cone gauge GS action in a curved space of the form $R^{1,1}\times M^8$ with generic NS-NS and R-R backgrounds [21] (reconstructed from the light-cone flat space GS vertex operators [22]) contains, in general, higher than quartic fermionic terms, multiplied by higher derivatives of the background fields. This light-cone GS action has quartic fermionic term [23,21] involving the curvature tensor

$$R\cdots\partial x^+\partial x^+(\bar\theta\Gamma^-\cdots\theta)(\bar\theta\Gamma^-\cdots\theta)\sim R\cdots(p^+)^2$$
$$\times(\bar\theta\Gamma^-\cdots\theta)(\bar\theta\Gamma^-\cdots\theta)$$

which is similar to the one present in the NSR string action (i.e., in the standard 2D supersymmetric sigma model).

[8]The part of the action in Ref. [2] quadratic in θ^I is a direct generalization of the quadratic term in the flat-space GS action (before κ symmetry gauge fixing) $S_F^{(2)}=(i/2\pi\alpha')\int d^2\sigma(\sqrt{g}g^{\mu\nu}\delta^{IJ}-\epsilon^{\mu\nu}s^{IJ})\overline{\theta^I}\rho_\mu D_\nu\theta^J$. Here ρ_μ are projections of the 10D Dirac matrices $\rho_\mu\equiv\Gamma_{\hat m}E_M^{\hat m}\partial_\mu X^M=(\Gamma_A E_M^A+\Gamma_{A'}E_M^{A'})\partial_\mu X^M$, and $E_M^{\hat m}$ is the vielbein of the 10D target space metric. The covariant derivative D_μ is the projection of the 10D derivative $D_M=\partial_M+\frac{1}{4}\omega_M^{\hat m\hat n}\Gamma_{\hat m\hat n}-(1/8\times 5!)\Gamma^{M_1\cdots M_5}\Gamma_M e^\Phi F_{M_1\cdots M_5}$ which appears in the Killing spinor equation of type-IIB supergravity. It has the following explicit form: $D_\mu\theta^I\equiv[\delta^{IJ}D_\mu-(i/2)\epsilon^{IJ}\tilde\rho_\mu]\theta^J$, $D_\mu=\partial_\mu+\frac{1}{4}\partial_\mu x^M\omega_M^{\hat m\hat n}\Gamma_{\hat m\hat n}$, where the term with $\tilde\rho_\mu\equiv(\Gamma_A E_M^A+i\Gamma_{A'}E_M^{A'})\partial_\mu X^M$ originates from the coupling to the R-R 5-form field strength.

[9]θ and η are not scalars with respect to SO(4,1). Combined together with fermions eliminated by κ-symmetry gauge they transform in spinor representation of $\mathrm{SO}(4,1)\times\mathrm{SO}(5)$. But after gauge fixing which is based on γ matrices from AdS_5 part ($\gamma^+\theta=0$), the SO(4,1) group, with the exception of its SO(2) subgroup generated by J^{xx} [17] (which is part of little group for the AdS_5 case) becomes realized nonlinearly. Thus [modulo subtleties of nonlinear realization of su(4) on bosons] the algebra $\mathrm{so}(2)\oplus\mathrm{su}(4)$ is a counterpart of the algebra so(8) in flat case.

theory is linear in θ, i.e., that there are no quartic interactions in θ.

D. Bosonic light-cone gauge fixing and elimination of x^+

To proceed further to quantization of the theory one would like, as in the flat case, to eliminate the ∂x^+ factors from the fermion kinetic terms in Eq. (1.6). In flat space this was possible by choosing the bosonic light-cone gauge. In the Brink–Di Vecchia–Howe–Polyakov (BDHP) formulation [24,25] which we are using this may be done by fixing the conformal gauge

$$\gamma^{\mu\nu}=\eta^{\mu\nu}, \quad \gamma^{\mu\nu}\equiv\sqrt{g}g^{\mu\nu}, \quad \det\gamma^{\mu\nu}=-1, \tag{1.11}$$

and then noting that since the resulting equation $\partial^2 x^+=0$ has the general solution $x^+(\tau,\sigma)=f(\tau-\sigma)+h(\tau+\sigma)$ one can fix the residual conformal diffeomorphism symmetry on the plane by choosing $x^+(\tau,\sigma)=p^+\tau$. An alternative (equivalent) approach is to use the original Goddard-Goldstone-Rebbi-Thorn (GGRT) [26] formulation based on writing the Nambu action in the canonical first order form (with constraints added with Lagrange multipliers) and fixing the diffeomorphisms by 2 conditions—on one coordinate and one canonical momentum: $x^+=p^+\tau, P^+=\text{const.}$[10]

The first approach based on the conformal gauge does not in general apply in curved spaces with null Killing vectors which are not of the direct product form $R^{1,1}\times M^8$ (the gauge conditions will not in general be consistent with classical equations of motion). It does apply, however, if the null Killing vector is covariantly constant [29]. There is no need, in principle, to insist on fixing the standard conformal gauge (1.11). Instead, one may fix the diffeomorphism gauge by imposing the two conditions $\gamma^{00}=-1$, $x^+=p^+\tau$. This choice is consistent provided the background metric satisfies $G_{+-}=1$, $G_{--}=G_{-i}=0$, $\partial_- G_{MN}=0$ [30]. This approach is essentially equivalent to the GGRT approach applied to the curved space case.

The above conditions do not apply in the AdS case: the null Killing vectors are not covariantly constant and $G_{+-}=e^{2\phi}\neq 1$.[11] It is easy to see, however, that a slight modification of the above conditions on γ^{00},x^+ represents a consistent gauge choice

$$e^{2\phi}\gamma^{00}=-1, \quad x^+=p^+\tau. \tag{1.12}$$

Indeed, the equation for x^+

$$\partial_\mu(e^{2\phi}\sqrt{g}g^{\mu\nu}\partial_\nu x^+)=0 \tag{1.13}$$

is then satisfied. The coordinate space BDHP approach based on Eq. (1.12) is equivalent to the phase space GGRT approach based on fixing the diffeomorhisms by $x^+=p^+\tau$, $P^+=\text{const}$. The possibility to fix the light-cone gauge for the bosonic string in AdS space using the latter GGRT approach was originally suggested by Thorn [32].

A complication in the case of fixing the diffeomorphisms by the conditions on γ^{00} and x^+ (or on P^+ and x^+ in the phase space approach) compared to the cases where one can fix the 2D metric completely by choosing the conformal gauge is that here one is still to integrate over the remaining independent component of the 2D metric (or γ^{01}) and to solve the resulting constraint. One may try to avoid this by fixing instead a modification of the conformal gauge (1.11) suggested by Polyakov [33]

$$\gamma^{\mu\nu}=\text{diag}(-e^{-2\phi},e^{2\phi}), \tag{1.14}$$

such that Eq. (1.13) still has $x^+=p^+\tau$ as its solution. This is just a particular classical solution, and it may seem that in contrast with the flat space case here one is unable to argue that $x^+=p^+\tau$ represents a gauge fixing condition for some residual symmetry. However, this ansatz may indeed be justified *a posteriori* as being the outcome of a systematic procedure based on fixing x^+ and one condition on a 2D metric such as Eq. (1.12) and then integrating over x^- (assuming it has no sources).

In this paper we shall not discuss in detail the consequences of fixing the bosonic light-cone gauge (1.12) in the superstring action (1.2) (or the equivalent light-cone gauge fixing in the phase space GGRT approach [34]) and follow a simplified approach based on using a particular classical solution.

Let us first not make any explicit gauge choice and consider the superstring path integral assuming that there is no sources for x^-. The linear dependence of the action (1.2),(1.3) on x^- allows us to integrate over x^- explicitly. This produces the δ-function constraint imposing the equation of motion (1.13) for x^+, which is formally solved by setting

$$\sqrt{g}g^{\mu\nu}e^{2\phi}\partial_\nu x^+=\epsilon^{\mu\nu}\partial_\nu f, \tag{1.15}$$

where $f(\tau,\sigma)$ is an *arbitrary* function. Since our action (1.2) depends only on x^+ only through $e^{2\phi}\partial x^+$, we are then able to integrate over x^+ as well, eliminating it in favor of the function f. The action will contain the fermionic terms (1.6),(1.10) with

$$e^{2\phi}\partial_\mu x^+\to f_\mu\equiv g_{\mu\nu}\frac{\epsilon^{\nu\lambda}}{\sqrt{g}}\partial_\lambda f. \tag{1.16}$$

[10]Yet another approach is to fix $g_{--}=0$, $x^+=h(\tau,\sigma)$ where h is determined by external sources [27]. For a discussion of various ways of fixing the light-cone gauge in the case of flat target space and their relations see, e.g., Ref. [28].

[11]In fact, there is no globally well-defined null Killing vector in AdS space as its norm proportional to $e^{2\phi}$ vanishes at the horizon $\phi=-\infty$ (this point and a possibility to fix a global diffeomorphism gauge for AdS string was discussed in Ref. [31]). In this paper we shall use a formal approach to this issue: since the boundary SYM theory in $R^{1,3}$ space has a well-defined light-cone description, it should be possible to fix some analogue of a light-cone gauge for the dual string as well (assuming it is defined on the Poincare patch of the AdS space). A potential problem of that approach which will be reflected in the degeneracy of the resulting light-conegauge fixed action near the horizon region should then be addressed at a later stage.

The resulting fermion kinetic term is then nondegenerate (for a properly chosen f), and may be interpreted as an action of 2D fermions in curved 2D geometry determined by f and $g_{\mu\nu}$ (see Refs. [8,37,36]).

We may then simplify the action further by making a special choice of f and fixing a diffeomorphism gauge on $g_{\mu\nu}$ in a consistent way. One possibility is to choose the gauge (1.14) and $f \sim \sigma$ which implies according to Eq. (1.15) that $x^+ \sim \tau$, i.e.,[12]

$$f=\sigma, \quad x^+=\tau, \quad \sqrt{g}g^{\mu\nu}=\mathrm{diag}(-e^{-2\phi},e^{2\phi}). \tag{1.17}$$

E. "2D spinor" form of the action

Like in the flat space case [1] and in the "long string" cases discussed in Ref. [8] the resulting action can then be put into the "2D spinor" form. Indeed, the 8+8 fermionic degrees of freedom can be organized into 4 Dirac 2D spinors, defined in *curved* 2D geometry. Using (1.17) we can write the kinetic term Eq. (1.6) as

$$\mathcal{L}_F^{(2)}=\frac{i}{2}(\theta_i\mathcal{D}_0\theta^i+\eta_i\mathcal{D}_0\eta^i-i\eta_i e^i_{0\,j}\eta^j) + e^{2\phi}\eta^i C'_{ij}(\mathcal{D}_1\theta^j-i\sqrt{2}e^{\phi}\eta^j\partial_1 x)+\mathrm{H.c.} \tag{1.18}$$

Introducing a 2D zweibein corresponding to the metric in Eq. (1.17)

$$e^m_\mu=\mathrm{diag}(e^{2\phi},1), \quad g_{\mu\nu}=-e^0_\mu e^0_\nu+e^1_\mu e^1_\nu, \tag{1.19}$$

we can put Eq. (1.18) in the 2D form as follows:

$$e^{-1}\mathcal{L}_F^{(2)}=-\frac{i}{2}\bar{\psi}\varrho^m e^\mu_m\mathcal{D}_\mu\psi+\frac{i}{2}\bar{\psi}\psi\partial_1\phi-\frac{1}{\sqrt{2}}\bar{\psi}_i e^i_{0\,j}\varrho^-\psi^j + i\sqrt{2}e^{\phi}(\psi^i)^T\pi^- C'_{ij}\psi^j\partial_1\bar{x}+\mathrm{H.c.} \tag{1.20}$$

Here ϱ^m are 2D Dirac matrices

$$\varrho^0=i\sigma_2, \quad \varrho^1=\sigma_1, \quad \varrho^3=\varrho^0\varrho^1=\sigma_3,$$

$$\varrho^\pm\equiv\frac{1}{\sqrt{2}}(\varrho^3\pm\varrho^0), \quad \pi^-\equiv\frac{1}{2}(1-\varrho^1), \tag{1.21}$$

$\bar{\psi}_i=(\psi^i)^\dagger\varrho^0$, $\bar{\psi}\psi$ stands for $\bar{\psi}_i\psi^i$, ψ^T denotes the transposition of 2D spinor and ψ's are related to the original (2D scalar) fermionic variables θ's and η's by[13]

$$\psi^i=\begin{pmatrix}\psi_1^i\\ \psi_2^i\end{pmatrix}, \quad \psi_1^i=\frac{1}{\sqrt{2}}[\theta^i-i(C'^{-1})^{ij}\eta_j],$$

$$\psi_2^i=\frac{1}{\sqrt{2}}[\theta^i+i(C'^{-1})^{ij}\eta_j]. \tag{1.22}$$

The quartic interaction term (1.10) takes the following form:

$$e^{-1}\mathcal{L}_F^{(4)}=\frac{1}{4}[(\bar{\psi}_i\gamma^{A'i}{}_j\varrho^-\psi^j)^2-(\bar{\psi}_i\varrho^-\psi^i)^2]. \tag{1.23}$$

The total action is thus a kind of G/H bosonic sigma model coupled to a Thirring-type 2D fermionic model in curved 2D geometry (1.19) (determined by the profile of the radial function of the AdS space), and coupled to some 2D vector fields. The interactions are such that they ensure the quantum 2D conformal invariance of the total model [2].

Properties of the resulting action and whether it can be put into simpler and useful form remain to be studied. It is clear of course that the action has a rather complicated structure and is not solvable in terms of free fields in any obvious way. A hope is that the light-cone form of the action we have found (or its first order phase space analog) may suggest a choice of more adequate variables which may allow further progress.

We finish this discussion with few remarks.

(i) The mass term $\bar{\psi}\psi\partial_1\phi$ in Eq. (1.20) is similar to the one in Ref. [8] (where the background string configuration was nonconstant only in the radial ϕ direction) and has its origin in the $\epsilon^{\mu\nu}e^{2\phi}\partial_\mu x^+\partial_\nu\phi\eta^i C'_{ij}\theta^j$ term appearing after $\eta\leftrightarrow\theta$ symmetrization of the $\epsilon^{\mu\nu}$ term in Eq. (1.6) (its "noncovariance" is thus a consequence of the choice $x^+=\tau$).

(ii) The action is symmetric under shifting $\psi^i\to\psi^i+\varrho^-\epsilon^i$, where ϵ^i is the 2D Killing spinor. This symmetry reflects the fact that our original action is symmetric under shifting θ^i by a Killing spinor on S^5.

(iii) The 2D Lorentz invariance is preserved by the fermionic light-cone gauge (original GS fermions θ are 2D scalars) but is broken by our special choice of the bosonic gauge (1.17). The special form of $g_{\mu\nu}$ in Eq. (1.17) implies "noncovariant" dependence on ϕ in the bosonic part of the action: the action (1.3) for the 3 fields $\phi,x,\bar{x}$ and the 5-sphere coordinates $y^{A'}$ has the form

$$\mathcal{L}_B=\partial_0 x\partial_0\bar{x}-e^{4\phi}\partial_1 x\partial_1\bar{x}+\frac{1}{2}e^{-2\phi}\partial_0\phi\partial_0\phi-\frac{1}{2}e^{2\phi}\partial_1\phi\partial_1\phi + \frac{1}{2}G_{\mathcal{AB}}(y)\left(e^{-2\phi}\partial_0 y^{\mathcal{A}}\partial_0 y^{\mathcal{B}}-e^{2\phi}\partial_1 y^{\mathcal{A}}\partial_1 y^{\mathcal{B}}\right), \tag{1.24}$$

where $G_{\mathcal{AB}}$ is the metric of 5-sphere.[14] A peculiarity of the $g_{\mu\nu}$ gauge choice in Eq. (1.17) compared to the usual con-

[12]Note that the standard conformal gauge $\sqrt{g}g^{\mu\nu}=\mathrm{diag}(-1,1)$ leads to inconsistency for generic ϕ if one insists on the simplest $f=\sigma$ choice. Consistency for generic ϕ is achieved only if f (and x^+) are nontrivial. But then the structure of the resulting action is complicated.

[13]In our notation $i\bar{\psi}\varrho^m\nabla_m\psi=-i\psi_1^\dagger(\nabla_0-\nabla_1)\psi_1-i\psi_2^\dagger(\nabla_0+\nabla_1)\psi_2$, $\nabla_m=e^\mu_m\partial_\mu$.

[14]here we renamed the (tangent space) indices A',B' into the coordinate space ones $\mathcal{A},\mathcal{B}$ for consistency with the notation used later in Sec. VI ($y^{\mathcal{A}}\equiv y^{A'}$).

formal gauge is that here the S^5 part of the action is no longer decoupled from the radial AdS_5 direction ϕ.

(iv) The form of the quadratic fermionic part of the $AdS_5\times S^5$ superstring action expanded near a straight long string configuration along ϕ direction of AdS_5 was discussed in Ref. [8] using the ''covariant'' κ-symmetry gauge condition $\theta^1=\theta^2$ (equivalent result was found also in the $\theta^1=i\gamma_4\theta^2$ gauge used in Refs. [11,6]). It is easy to show that an equivalent fermionic action is found also in the present light-cone κ-symmetry gauge. Expanding near the configuration $x^0=\tau,\phi=\sigma,\ x=0,\ y=0$ (it is easy to check that this is a classical string solution) and choosing the bosonic gauge so that the 2D metric $g_{\mu\nu}$ is equal to the induced (AdS_2) metric $ds^2=(1/\sigma^2)(-d\tau^2+d\sigma^2)$ we find that the corresponding function f in Eq. (1.15) is then equal to σ^{-2}. The quadratic part of the fermion action (1.20) becomes [we redefine the η fermions by the constant unitary matrix C' in Eq. (1.18)]

$$\int d\tau\, d\sigma\, \sigma^{-2}(\theta\partial_0\theta+\eta\partial_0\eta-\eta\partial_1\theta). \tag{1.25}$$

Rescaling the fields $\theta=\sigma\theta',\eta=\sigma\eta'$ (so that they have σ-independent normalization, $\int d\tau\, d\sigma\sqrt{g}\,\theta\theta=\int d\tau\, d\sigma\, \theta'\theta'$) and integrating by parts we find

$$\int d\tau\, d\sigma(\theta'\partial_0\theta'+\eta'\partial_0\eta'-\eta'\partial_1\theta'-\sigma^{-1}\eta'\theta'). \tag{1.26}$$

The first three terms here are as in the flat GS action, while the last term represents the AdS_2 fermion mass term which is the same as found in Ref. [8]. Indeed, diagonalizing the action as in Eq. (1.22) we get

$$\int d\tau\, d\sigma(\psi_+\partial_+\psi_++\psi_-\partial_-\psi_--\sigma^{-1}\psi_+\psi_-), \tag{1.27}$$

which is the special case of the general form of the quadratic action (1.20) with $\partial_\sigma\phi$ in the mass term computed for $\phi=\ln\sigma$.

F. Contents of the rest of the paper

The rest of the paper contains derivation of the action (1.2) and related explanations and technical details. In Sec. II we start with the case of the flat space GS action and illustrate on this simplest example the procedure of light-cone gauge fixing we shall use in the $AdS_5\times S^5$ case. We present a particular light-cone form of the GS action to which our $AdS_5\times S^5$ light-cone gauge fixed action will reduce in the flat space limit.

In Sec. III we discuss the basic superalgebra psu(2,2|4) and write down its (anti) commutation relations in the light-cone basis, corresponding to the light-cone decomposition [see Eq. (1.1)] of the so(4,2) generators.[15]

In Sec. IV we adapt the original $AdS_5\times S^5$ GS action of Ref. [2] to the case of the light-cone basis of psu(2,2|4). The resulting κ-symmetric action is written entirely in terms of Cartan 1-forms corresponding to the light-cone basis and in an arbitrary (e.g., Wess-Zumino or Killing) parametrization of the supercoset space.

In Sec. V we fix the light-cone κ-symmetry gauge and find the corresponding Cartan 1-forms. These light-cone gauge 1-forms are given in the Killing parametrization of the original superspace.

In Sec. VI we find the fermionic light-cone gauge fixed form of the action of Sec. IV. We present the action in the Killing parametrization, discuss some of its properties, and also transform it into the ''4+6'' manifestly SU(4) invariant form [see Eqs. (6.13),(6.14) and (6.22),(6.23)]. We then explain the transformation of the action into the Wess-Zumino parametrization form which was presented above in Eqs. (1.2),(1.6),(1.10). We also mention that our results for $AdS_5\times S^5$ case can be easily generalized to the $AdS_3\times S^3$ case.

In Appendix A we discuss the relations between the so(4,1)⊕so(5) (or ''5+5'') basis[16] of the psu(2,2|4) superalgebra used in Ref. [2] in the construction of the GS action in $AdS_5\times S^5$ and the more familiar so(3,1)⊕su(4) ≃sl(2,C)⊕su(4) (or ''4+6'') basis (naturally appearing in the discussion of $\mathcal{N}=4,d=4$ superconformal symmetry of SYM theory). We use the later basis to identify the generators of the algebra in the light-cone [or so(1,1)⊕u(1) ⊕so(2)⊕su(4)] basis. The knowledge of the explicit relations between the generators in the three bases is useful in order to find normalizations in the forms of the string action corresponding to the so(3,1)⊕su(4) and the light-cone bases.

In Appendix B we explain the transformation of the $AdS_5\times S^5$ string action from its original form in the so(4,1)⊕so(5) basis [2] to the so(3,1)⊕su(4) basis and then to the light-cone basis. We also discuss some details of derivation of the light-cone gauge fixed action given in Sec. VI.

In Appendix C we present another version of the $AdS_5\times S^5$ superstring action using the ''S gauge'' to fix the κ symmetry (S refers to the conformal supersymmetry generator). In this gauge all of the superconformal η fermions are gauged away.

II. LIGHT CONE κ-SYMMETRY GAUGE FIXING IN FLAT SPACE

It is useful first to discuss the case of light-cone gauge fixing in the standard flat space GS action. This allows to explain in the simplest setting the procedure of light-cone

[15]We shall use the following terminology: ''light cone basis'' (or ''light cone frame'') will refer to the decomposition of superalgebra generators, while the ''light cone gauge'' will refer to the choice of the κ-symmetry gauge.

[16]We label the basis by the symmetry algebras under which supercoordinates are transforming in a linear way.

gauge fixing we are going to follow in the case of $AdS_5\times S^5$. In particular, we shall discuss the split of supercoordinates which is closely related to the one we will use in the $AdS_5\times S^5$ case, and obtain the form of the GS action to which our $AdS_5\times S^5$ light-cone action will reduce in the flat space limit.

We start with the flat GS action [1] in the form [35]

$$I_0=-\frac{1}{2}\int_{\partial M_3} d^2\sigma\sqrt{g}g^{\mu\nu}L^{\hat{A}}_\mu L^{\hat{A}}_\nu+i\int_{M_3} s^{IJ}L^{\hat{A}}\wedge(\overline{L^I}\Gamma^{\hat{A}}\wedge L^J), \tag{2.1}$$

where $s^{IJ}\equiv\mathrm{diag}(1,-1)$ $(I,J=1,2)$ and $2\pi\alpha'=1$. The 2D metric $g_{\mu\nu}$ $(\mu,\nu=0,1)$ has signature $(-+)$, and $g\equiv -\det g_{\mu\nu}$.

The left-invariant Cartan 1-forms are defined on the type-IIB coset superspace defined as [10D super Poincare group]/[SO(9,1) Lorentz group]

$$G^{-1}dG=L^{\hat{A}}P_{\hat{A}}+L^IQ_I,\quad L^{\hat{A}}=dX^ML^{\hat{A}}_M,\quad X^M=(x,\theta), \tag{2.2}$$

where $G=G(x,\theta)$ is an appropriate coset representative. A specific choice of $G(x,\theta)$ commonly used is

$$G(x,\theta)=\exp(x^{\hat{A}}P_{\hat{A}}+\theta^IQ_I),$$

$$[P_{\hat{A}},P_{\hat{B}}]=0,\quad \{Q_I,Q_J\}=-2i\delta_{IJ}(\mathcal{C}\Gamma^{\hat{A}})P_{\hat{A}}, \tag{2.3}$$

and thus the coset space vielbeins defined by Eq. (2.2) are given by

$$L^{\hat{A}}=dx^{\hat{A}}-i\bar{\theta}^I\Gamma^{\hat{A}}d\theta^I,\quad L^I=d\theta^I. \tag{2.4}$$

θ^I are two left Majorana-Weyl 10D spinors. The explicit 2D form of the GS action [1]

$$\begin{aligned}I_0=\int d^2\sigma\mathcal{L}_0=\int d^2\sigma\Big[&-\frac{1}{2}\sqrt{g}g^{\mu\nu}(\partial_\mu x^{\hat{A}}-i\bar{\theta}^I\Gamma^{\hat{A}}\partial_\mu\theta^I)\\ &\times(\partial_\nu x^{\hat{A}}-i\bar{\theta}^J\Gamma^{\hat{A}}\partial_\nu\theta^J)-i\epsilon^{\mu\nu}s^{IJ}\bar{\theta}^I\Gamma^{\hat{A}}\partial_\nu\theta^J\\ &\times\left(\partial_\mu x^{\hat{A}}-\frac{1}{2}i\bar{\theta}^K\Gamma^{\hat{A}}\partial_\mu\theta^K\right)\Big].\end{aligned} \tag{2.5}$$

One usually imposes the κ-symmetry light-cone gauge by starting with the component form of action given by Eq. (2.5). It turns out to be cumbersome to generalize this procedure to the case of strings in $AdS_5\times S^5$. It is more convenient to first impose the light-cone gauge at the level of the Cartan forms $L^{\hat{A}}$, L^I and then use them in the action taken in its general form (2.1). The light-cone gauge form of $L^{\hat{A}}$ is

$$L^+=dx^+,\quad L^-=dx^--i\bar{\theta}^I\Gamma^-d\theta^I,$$

$$L^N=dx^N,\quad N=1,\dots,8, \tag{2.6}$$

where θ^I are subject to the light-cone gauge condition $\Gamma^+\theta^I=0$.[17] Inserting these expressions into action (2.1) we get

$$\mathcal{L}=\mathcal{L}_{\rm kin}+\mathcal{L}_{\rm WZ}, \tag{2.7}$$

$$\begin{aligned}\mathcal{L}_{\rm kin}=\sqrt{g}g^{\mu\nu}\Big(&-\partial_\mu x^+\partial_\nu x^--\frac{1}{2}\partial_\mu x^N\partial_\nu x^N\\ &+i\partial_\mu x^+\bar{\theta}^I\Gamma^-\partial_\nu\theta^I\Big),\end{aligned} \tag{2.8}$$

$$\mathcal{L}_{\rm WZ}=-i\epsilon^{\mu\nu}s^{IJ}\partial_\mu x^+\bar{\theta}^I\Gamma^-\partial_\nu\theta^J. \tag{2.9}$$

Next, let us do the "5+5" splitting of the 10D Clifford algebra generators, the charge conjugation matrix $\mathcal{C}$ and the supercoordinates

$$\Gamma^A=\gamma^A\times I\times\sigma_1,\quad \Gamma^{A'}=I\times\gamma^{A'}\times\sigma_2,$$

$$\mathcal{C}=C\times C'\times i\sigma_2,\quad \theta^I=\begin{pmatrix}\theta^{I\alpha i}\\ 0\end{pmatrix}, \tag{2.10}$$

where I is 4×4 unit matrix, σ_n are Pauli matrices, $\alpha=1,2,3,4$, and $i=1,2,3,4$. Let us also introduce the complex coordinates

$$\theta^q\equiv\frac{1}{\sqrt{2}}(\theta^1+i\theta^2), \tag{2.11}$$

and use the parametrization

$$\theta^{q\alpha i}=\frac{v}{2}\begin{pmatrix}\eta^{-i}\\ \eta^{+i}\\ -i\theta^{+i}\\ i\theta^{-i}\end{pmatrix},\quad v\equiv2^{1/4}. \tag{2.12}$$

Decompositions of so(4,1) γ matrices we use may be found in Appendix A [see Eq. (A18)]. The light-cone gauge

$$\Gamma^+\theta^I=0,\quad \Gamma^\pm\equiv\frac{1}{\sqrt{2}}(\Gamma^3\pm\Gamma^0), \tag{2.13}$$

leads to

$$\theta^{+i}=\eta^{+i}=0. \tag{2.14}$$

Changing sign $x^{\hat{A}}\to -x^{\hat{A}}$, using the notation

$$\theta^i\equiv\theta^{-i},\quad \eta^i\equiv\eta^{-i},\quad \theta_i=(\theta^i)^\dagger,\quad \eta_i=(\eta^i)^\dagger, \tag{2.15}$$

[17]The transverse bosonic Cartan forms L^N in Eq. (2.6) should not be confused with fermionic ones L^I.

and inserting the above decomposition into the action (2.7) we finally get the following expressions for the kinetic and WZ parts of the light-cone gauge flat space GS Lagrangian

$$\mathcal{L}_{\rm kin}=\sqrt{g}g^{\mu\nu}\Big[-\partial_\mu x^+\partial_\nu x^- -\frac{1}{2}\partial_\mu x^N\partial_\nu x^N -\partial_\mu x^+\Big(\frac{i}{2}\theta_i\partial_\nu\theta^i+\frac{i}{2}\eta_i\partial_\nu\eta^i+{\rm H.c.}\Big)\Big], \quad (2.16)$$

$$\mathcal{L}_{\rm WZ}=\epsilon^{\mu\nu}\partial_\mu x^+\eta^i C'_{ij}\partial_\nu\theta^j+{\rm H.c.} \quad (2.17)$$

It is to this form of the flat GS action that our light-cone $AdS_5\times S^5$ action will reduce in the flat space limit. A characteristic feature of this parametrization is that while the kinetic term is diagonal in θ's and η's, they are mixed in the Wess-Zumino (WZ) term.

III. psu(2,2|4) SUPERALGEBRA IN THE LIGHT CONE BASIS

The superalgebra psu(2,2|4) which is the algebra of isometry transformations of $AdS_5\times S^5$ superspace plays the central role in the construction of the GS action in $AdS_5\times S^5$ [2]. In this section we shall present the form of this algebra which will be used in the present paper. The even part of psu(2,2|4) is the sum of the algebra so(4,2) which is the isometry algebra of AdS_5 and the algebra so(6) which is the isometry algebra of S^5. The odd part consists of 32 supercharges corresponding to 32 Killing spinors in $AdS_5\times S^5$ vacuum [38] of type-IIB supergravity (see Refs. [39–41]; for recent developments in representation theory see Ref. [42]).

We shall use the form of the basis of so(4,2) subalgebra implied by its interpretation as conformal algebra in 4 dimensions. The generators are then called translations P^a, conformal boosts K^a, dilatation D, and Lorentz rotations J^{ab} and satisfy the standard commutation relations

$$[P^a,J^{bc}]=\eta^{ab}P^c-\eta^{ac}P^b,$$

$$[K^a,J^{bc}]=\eta^{ab}K^c-\eta^{ac}K^b, \quad [P^a,K^b]=\eta^{ab}D-J^{ab}, \quad (3.1)$$

$$[D,P^a]=-P^a, \quad [D,K^a]=K^a,$$

$$[J^{ab},J^{cd}]=\eta^{bc}J^{ad}+3 \ \text{ terms}, \quad (3.2)$$

where $\eta^{ab}=(-,+,+,+)$ and $a,b,c,d=0,1,2,3$. In the light cone basis (1.1) we have the following generators:

$$J^{+-},\quad J^{\pm x},\quad J^{\pm\bar{x}},\quad J^{x\bar{x}},$$

$$P^{\pm},\quad P^x,\quad P^{\bar{x}},\quad K^{\pm},\quad K^x,\quad K^{\bar{x}}. \quad (3.3)$$

To simplify the notation we shall set

$$P\equiv P^x,\quad \bar{P}=P^{\bar{x}},\quad K\equiv K^x,\quad \bar{K}=K^{\bar{x}}. \quad (3.4)$$

The light cone form of so(4,2) algebra commutation relations can be obtained from Eq. (3.1) using that the light cone metric has the following elements $\eta^{+-}=\eta^{-+}=1$, $\eta^{x\bar{x}}=\eta^{\bar{x}x}=1$.

In this paper the so(6) algebra will be interpreted as su(4) one $(i,j,k,l=1,2,3,4)$

$$[J^i{}_j,J^k{}_n]=\delta^k_j J^i{}_n-\delta^i_n J^k{}_j\,. \quad (3.5)$$

To describe the odd part of psu(2,2|4) superalgebra we introduce 32 supercharges $Q^{\pm i}$, $Q^\pm_i$, $S^{\pm i}$, $S^\pm_i$. They carry the D, J^{+-}, and $J^{x\bar{x}}$ charges, as follows from the structure of the algebra. The commutation relations of the supercharges with the dilatation D

$$[D,Q^{\pm i}]=-\frac{1}{2}Q^{\pm i}, \quad [D,Q^\pm_i]=-\frac{1}{2}Q^\pm_i\,,$$

$$[D,S^{\pm i}]=\frac{1}{2}S^{\pm i}, \quad [D,S^\pm_i]=\frac{1}{2}S^\pm_i\,, \quad (3.6)$$

allow to interpret Q's as the standard supercharges of the super Poincaré subalgebra and S's as the conformal supercharges. The supercharges with superscript $+$ $(-)$ have positive (negative) J^{+-} charge

$$[J^{+-},Q^{\pm i}]=\pm\frac{1}{2}Q^{\pm i}, \quad [J^{+-},Q^\pm_i]=\pm\frac{1}{2}Q^\pm_i\,,$$

$$[J^{+-},S^{\pm i}]=\pm\frac{1}{2}S^{\pm i}, \quad [J^{+-},S^\pm_i]=\pm\frac{1}{2}S^\pm_i\,.$$

The $J^{x\bar{x}}$ charges are fixed by the commutation relations

$$[J^{x\bar{x}},Q^{\pm i}]=\pm\frac{1}{2}Q^{\pm i}, \quad [J^{x\bar{x}},Q^\pm_i]=\mp\frac{1}{2}Q^\pm_i\,, \quad (3.7)$$

$$[J^{x\bar{x}},S^\pm_i]=\pm\frac{1}{2}S^\pm_i\,, \quad [J^{x\bar{x}},S^{\pm i}]=\mp\frac{1}{2}S^{\pm i}. \quad (3.8)$$

The transformation properties of the Q supercharges with respect to su(4) subalgebra are determined by

$$[Q^\pm_i,J^j{}_k]=\delta^j_i Q^\pm_k-\frac{1}{4}\delta^j_k Q^\pm_i\,,$$

$$[Q^{\pm i},J^j{}_k]=-\delta^i_k Q^{\pm j}+\frac{1}{4}\delta^j_k Q^{\pm i},$$

with the same relations for the S supercharges. Anticommutation relations between the supercharges are

$$\{Q^{\pm i},Q^\pm_j\}=\mp iP^\pm\delta^i_j\,, \quad \{Q^{+i},Q^-_j\}=-iP\delta^i_j\,, \quad (3.9)$$

$$\{S^{\pm i},S^\pm_j\}=\pm iK^\pm\delta^i_j\,, \quad \{S^{-i},S^+_j\}=iK\delta^i_j\,, \quad (3.10)$$

$$\{Q^{+i},S_j^+\}=-J^{+x}\delta_j^i,\quad \{Q^{-i},S_j^-\}=-J^{-\bar{x}}\delta_j^i,$$

$$\{Q^{\pm i},S_j^{\mp}\}=\frac{1}{2}(J^{+-}+J^{x\bar{x}}\mp D)\delta_j^i\mp J^i{}_j.$$

The remaining commutation relations between odd and even generators have the following form:

$$[Q^{-i},J^{+x}]=-Q^{+i},\quad [S^{-i},J^{+\bar{x}}]=-S^{+i},$$

$$[Q^{+i},J^{-\bar{x}}]=Q^{-i},\quad [S^{+i},J^{-x}]=S^{-i},$$

$$[S_i^{\mp},P^{\pm}]=iQ_i^{\pm},\quad [S_i^-,P]=iQ_i^-,$$

$$[S_i^+,\bar{P}]=-iQ_i^+,\quad [Q^{\mp i},K^{\pm}]=-iS^{\pm i},$$

$$[Q^{-i},K]=-iS^{-i},\quad [Q^{+i},\bar{K}]=iS^{+i}.$$

The generators are subject to the following Hermitean conjugation conditions:

$$(P^{\pm})^\dagger=-P^{\pm},\quad P^\dagger=-\bar{P},\quad (K^{\pm})^\dagger=-K^{\pm},\quad K^\dagger=-\bar{K},$$

$$(J^{\pm x})^\dagger=-J^{\pm\bar{x}},\quad (J^{+-})^\dagger=-J^{+-},$$

$$(J^{x\bar{x}})^\dagger=J^{x\bar{x}},\quad D^\dagger=-D,\quad (J^i{}_j)^\dagger=J^j{}_i,$$

$$(Q^{\pm i})^\dagger=Q_i^{\pm},\quad (S^{\pm i})^\dagger=S_i^{\pm},\tag{3.11}$$

All the remaining nontrivial (anti)commutation relations of $psu(2,2|4)$ superalgebra may be obtained by using these Hermitean conjugation rules and the (anti)commutation relations given above.

IV. LIGHT CONE BASIS FORM OF $AdS_5\times S^5$ STRING ACTION

Superstring action in $AdS_5\times S^5$ [2] has the same structure as the flat space GS action (2.1)

$$I=\int_{\partial M_3}\mathcal{L}_{\rm kin}+\int_{M_3}i\mathcal{H}.\tag{4.1}$$

In Ref. [2] the Cartan forms in terms of which the action is written were given in the $so(4,1)\oplus so(5)$ basis of $psu(2,2|4)$. This is the basis that allows to present the $AdS_5\times S^5$ GS action in the form similar to the one in the flat space. Our present goal is to rewrite the action in the light-cone basis discussed in the previous section and then (in the next section) to impose a κ-symmetry light-cone gauge. We shall use the conformal algebra and light-cone frame notation.

The Cartan 1-forms in the light-cone basis are defined by

$$\begin{aligned}G^{-1}dG=&L_P^aP^a+L_K^aK^a+L_DD+\frac{1}{2}L^{ab}J^{ab}+L^i{}_jJ^j{}_i+L_Q^{-i}Q_i^+\\&+L_{Qi}^-Q^{+i}+L_Q^{+i}Q_i^-+L_{Qi}^+Q^{-i}+L_S^{-i}S_i^++L_{Si}^-S^{+i}\\&+L_S^{+i}S_i^-+L_{Si}^+S^{-i},\end{aligned}\tag{4.2}$$

where G is a coset representative in $PSU(2,2|4)$. Let us define also the following combinations:

$$\hat{L}^a\equiv L_P^a-\frac{1}{2}L_K^a,\quad L^{A'}\equiv-\frac{i}{2}(\gamma^{A'})^i{}_jL^j{}_i,$$

$$(C'L)_{ij}\equiv C'_{ik}L^k{}_j.\tag{4.3}$$

Then the kinetic term in Eq. (4.1) takes the form

$$\mathcal{L}_{\rm kin}=-\frac{1}{2}\sqrt{g}g^{\mu\nu}(\hat{L}_\mu^a\hat{L}_\nu^a+L_{D\mu}L_{D\nu}+L_\mu^{A'}L_\nu^{A'}),\tag{4.4}$$

while the 3-form $\mathcal{H}$ in the WZ term can be written as (we suppress the signs of exterior products of 1-forms)

$$\mathcal{H}=\mathcal{H}^q_{AdS_5}+\mathcal{H}^q_{S^5}-\text{H.c.},\tag{4.5}$$

$$\begin{aligned}\mathcal{H}^q_{AdS_5}=&-\frac{i}{\sqrt{2}}(\hat{L}^+L_S^{-i}C'_{ij}L_Q^{-j}+\hat{L}^-L_Q^{+i}C'_{ij}L_S^{+j}\\&+\hat{L}^xL_S^{-i}C'_{ij}L_Q^{+j}+\hat{L}^{\bar{x}}L_S^{+i}C'_{ij}L_Q^{-j})\\&+\frac{1}{\sqrt{2}}L_D\bigg(\frac{1}{2}L_S^{+i}C'_{ij}L_S^{-j}\\&+L_Q^{-i}C'_{ij}L_Q^{+j}\bigg),\end{aligned}\tag{4.6}$$

$$\begin{aligned}\mathcal{H}^q_{S^5}=&\frac{1}{2\sqrt{2}}[L_S^{+i}(C'L)_{ij}L_S^{-j}\\&-L_S^{-i}(C'L)_{ij}L_S^{+j}]\\&+\frac{1}{\sqrt{2}}[L_Q^{+i}(C'L)_{ij}L_Q^{-j}\\&-L_Q^{-i}(C'L)_{ij}L_Q^{+j}].\end{aligned}$$

Derivation of these expressions from the original ones given in Ref. [2] may be found in Appendix B.

V. COORDINATE PARAMETRIZATION OF CARTAN FORMS AND FIXING THE LIGHT-CONE κ-SYMMETRY GAUGE

To represent the Cartan 1-forms in terms of the even and odd coordinate fields we shall start with the following supercoset representative [see (2.3)]:

$$G=g_{x,\theta}g_\eta g_yg_\phi,\tag{5.1}$$

where

$$g_{x,\theta}=\exp(x^aP^a+\theta^{-i}Q_i^++\theta_i^-Q^{+i}+\theta^{+i}Q_i^-+\theta_i^+Q^{-i}),\tag{5.2}$$

$$g_\eta=\exp(\eta^{-i}S_i^++\eta_i^-S^{+i}+\eta^{+i}S_i^-+\eta_i^+S^{-i}),\tag{5.3}$$

and g_ϕ and g_y depend on the radial AdS_5 coordinate ϕ and S^5 coordinates $y^{A'}$, respectively.

$$g_\phi \equiv \exp(\phi D), \tag{5.4}$$

$$g_y \equiv \exp(y^i{}_j J^j{}_i), \quad y^i{}_j \equiv \frac{i}{2}(\gamma^{A'})^i{}_j y^{A'}. \tag{5.5}$$

Choosing the parametrization of the coset representative in the form (5.1) corresponds to what is usually referred to as ''Killing gauge'' in superspace.

Since the supercharges transform in the fundamental representation of su(4) the corresponding fermionic coordinates θ's and η's also transform in the fundamental representation of su(4). The above expressions provide the definition of the Cartan forms in the light-cone basis. Let us further specify these expressions by setting to zero some of the fermionic coordinates which corresponds to fixing a particular κ-symmetry gauge. Namely, we shall fix the κ symmetry by putting to zero all the Grassmann coordinates which carry positive J^{+-} charge [see (2.14)]:

$$\theta^{+i} = \theta_i^+ = \eta^{+i} = \eta_i^+ = 0. \tag{5.6}$$

To simplify the notation we shall set in what follows:

$$\theta^i \equiv \theta^{-i}, \quad \theta_i \equiv \theta_i^-, \quad \eta^i \equiv \eta^{-i}, \quad \eta_i \equiv \eta_i^-. \tag{5.7}$$

As a result, the κ-symmetry fixed form of the coset representative (5.1) is

$$G_{\text{g.f.}} = (g_{x,\theta})_{\text{g.f.}} (g_\eta)_{\text{g.f.}} g_y g_\phi, \tag{5.8}$$

$$(g_{x,\theta})_{\text{g.f.}} = \exp(x^a P^a + \theta^i Q_i^+ + \theta_i Q^{+i}), \tag{5.9}$$

$$(g_\eta)_{\text{g.f.}} = \exp(\eta^i S_i^+ + \eta_i S^{+i}). \tag{5.10}$$

Plugging this $G_{\text{g.f.}}$ into (4.2) we get the κ-symmetry gauge fixed expressions for the Cartan 1-forms

$$L_P^+ = e^\phi dx^+, \quad L_P^- = e^\phi\left(dx^- - \frac{i}{2}\tilde\theta^i \, \tilde d\theta_i - \frac{i}{2}\tilde\theta_i \, \tilde d\theta^i\right), \tag{5.11}$$

$$L_P^x = e^\phi dx, \quad L_P^{\bar x} = e^\phi d\bar x, \tag{5.12}$$

$$L_K^- = e^{-\phi}\left[\frac{1}{4}(\tilde\eta^2)^2 dx^+ + \frac{i}{2}\tilde\eta^i \, \tilde d\eta_i + \frac{i}{2}\tilde\eta_i \, \tilde d\eta^i\right], \tag{5.13}$$

$$L_D = d\phi, \tag{5.14}$$

$$L^i{}_j = (dU\, U^{-1})^i{}_j + i\left(\tilde\eta^i\tilde\eta_j - \frac{1}{4}\tilde\eta^2\delta^i_j\right)dx^+, \tag{5.15}$$

$$L_Q^{-i} = e^{\phi/2}(\tilde d\theta^i + i\tilde\eta^i \, dx), \quad L_{Qi}^- = e^{\phi/2}(\tilde d\theta_i - i\tilde\eta_i \, d\bar x), \tag{5.16}$$

$$L_Q^{+i} = -ie^{\phi/2}\tilde\eta^i \, dx^+, \quad L_{Qi}^+ = ie^{\phi/2}\tilde\eta_i \, dx^+, \tag{5.17}$$

$$L_S^{-i} = e^{-\phi/2}\left(\widetilde{d\eta}^i + \frac{i}{2}\tilde\eta^2\tilde\eta^i dx^+\right),$$

$$L_{Si}^- = e^{-\phi/2}\left(\tilde d\eta_i - \frac{i}{2}\tilde\eta^2\tilde\eta_i \, dx^+\right), \tag{5.18}$$

where $\tilde\eta^2 \equiv \tilde\eta^i\tilde\eta_i$. All the remaining forms are equal to zero. We have introduced the notation

$$\tilde\theta^i \equiv U^i{}_j\theta^j, \quad \tilde\theta_i \equiv \theta_j(U^{-1})^j{}_i, \tag{5.19}$$

$$\tilde d\theta^i \equiv U^i{}_j \, d\theta^j, \quad \tilde d\theta_i \equiv d\theta_j(U^{-1})^j{}_i, \tag{5.20}$$

and similar ones for η. Note that $\tilde\theta^2 = \theta^2$ and $\tilde\theta\tilde d\theta = \theta \, d\theta$. The matrix $U \in SU(4)$ is defined in terms of the S^5 coordinates y_j^i or $y^{A'}$ by Eqs. (1.4),(1.5). It can be written explicitly as

$$U = \cos\frac{|y|}{2} + i\gamma^{A'}n^{A'}\sin\frac{|y|}{2}, \quad |y| \equiv \sqrt{y^{A'}y^{A'}},$$

$$n^{A'} \equiv \frac{y^{A'}}{|y|}. \tag{5.21}$$

VI. $AdS_5 \times S^5$ STRING ACTION IN THE LIGHT-CONE GAUGE

Plugging the above expressions (5.11)–(5.18) into the action (4.1) we get the following result for the light-cone gauge fixed superstring Lagrangian in terms of the light cone supercoset coordinates

$$\begin{aligned}\mathcal{L}_{kin} = \sqrt{g}g^{\mu\nu}\Bigg(& -e^{2\phi}(\partial_\mu x^+\partial_\nu x^- + \partial_\mu x\partial_\nu \bar x) - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi \\ & - \frac{1}{2}e_\mu^{A'}e_\nu^{A'} + \partial_\mu x^+\bigg[\frac{i}{2}e^{2\phi}(\theta^i\partial_\nu\theta_i + \theta_i\partial_\nu\theta^i) \\ & + \frac{i}{4}(\eta^i\partial_\nu\eta_i + \eta_i\partial_\nu\eta^i) + \frac{1}{2}\tilde\eta_i e^i_{\nu j}\tilde\eta^j\bigg] \\ & + \frac{1}{8}\partial_\mu x^+\partial_\nu x^+\{(\eta^2)^2 - [\tilde\eta_i(\gamma^{A'})^i{}_j\tilde\eta^j]^2\}\Bigg),\end{aligned} \tag{6.1}$$

$$\mathcal{L}_{\text{WZ}} = -\frac{\epsilon^{\mu\nu}}{\sqrt{2}}e^\phi\partial_\mu x^+\tilde\eta^i C'_{ij}(\widetilde{\partial_\nu\theta}^j + i\tilde\eta^j\partial_\nu x) + \text{H.c.} \tag{6.2}$$

The kinetic terms are obtained in a straightforward way. Details of derivation of the WZ part are given in Appendix B.

A few remarks are in order.

(i) In the flat space limit this action [after an appropriate rescaling of fermionic variables given in Eq. (6.3)] reduces to the GS light-cone κ-symmetry gauge fixed action represented in the form (2.16),(2.17). In the particle theory limit $\alpha' \to 0$ (corresponding to keeping only the τ dependence of the fields and omitting the WZ term) this action reduces (after an appropriate bosonic light-cone gauge fixing and rescaling of some fermionic variables) to the light-cone action of a superparticle propagating in $AdS_5\times S^5$ [18].[18]

(ii) The kinetic terms for the fermionic coordinates have manifest linear su(4) invariance. In the remaining terms this symmetry is not manifest and is not realized linearly.

(iii) Since the WZ term depends on θ through its derivative, it is invariant under a shift of θ. To maintain this invariance in the kinetic terms the shifting of θ should be supplemented, as usual, by an appropriate transformation of x^-. The action is invariant under shifts of the bosonic coordinates x^a along the boundary directions.

(iv) As in the superparticle case [18,19] this action is quadratic in half of the fermionic coordinates (θ) but of higher order (quartic) in the another half (η). It was the desire to split the fermionic variables in such θ's and η's that motivated our choice of the supercoset parametrization in Eq. (5.8).

(v) The action contains the terms like $(\eta^2)^2$ and $\eta_i e^i{}_j \eta^j$ which in the superparticle case played important role [18] in establishing the AdS/CFT correspondence. These terms should also play a similar important role in formulating the AdS/CFT correspondence at the string theory level.

The fermionic variables θ and η as defined above in Eq. (5.1) have opposite conformal dimensions. It is convenient, however, to use the variables with the same conformal dimensions.[19] To achieve this we rescale η as follows:

$$\eta^i \to \sqrt{2} e^{\phi}\eta^i, \qquad \eta_i \to \sqrt{2} e^{\phi}\eta_i . \tag{6.3}$$

To get convenient sign in front of kinetic terms of fermions we change sign $x^a \to -x^a$. Then the Lagrangian (6.1),(6.2) may be written as

$$\begin{aligned}\mathcal{L}_{\rm kin} = {} & \sqrt{g} g^{\mu\nu}\Big[-e^{2\phi}(\partial_\mu x^+ \partial_\nu x^- + \partial_\mu x \partial_\nu \bar{x}) - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi \\ & - \frac{1}{2} G_{\mathcal{AB}}(y) D_\mu y^{\mathcal{A}} D_\nu y^{\mathcal{B}}\Big] - \frac{i}{2}\sqrt{g} g^{\mu\nu} e^{2\phi}\partial_\mu x^+ [\theta^i \partial_\nu \theta_i \\ & + \theta_i \partial_\nu \theta^i + \eta^i \partial_\nu \eta_i + \eta_i \partial_\nu \eta^i + i e^{2\phi}\partial_\nu x^+ (\eta^2)^2],\end{aligned} \tag{6.4}$$

$$\mathcal{L}_{\rm WZ} = \epsilon^{\mu\nu} e^{2\phi}\partial_\mu x^+ \eta^i C^U_{ij}(\partial_\nu \theta^j - i\sqrt{2} e^{\phi}\eta^j \partial_\nu x) + \text{H.c.} \tag{6.5}$$

Here $G_{\mathcal{AB}}$ is the metric of the 5 sphere.[20] The matrix C^U_{ij} and the differential $D_\mu y^{\mathcal{A}}$ are defined by

$$D_\mu y^{\mathcal{A}} = \partial_\mu y^{\mathcal{A}} - 2i\eta_i (V^{\mathcal{A}})^i{}_j \eta^j e^{2\phi}\partial_\mu x^+ \tag{6.6}$$

$$C^U_{ij} \equiv U^l{}_i C'_{lk} U^k{}_j ,$$

$$C^U_{ij} = C'_{ij}\cos|y| + i(C'\gamma^{A'})_{ij} n^{A'} \sin|y|, \tag{6.7}$$

where $(V^{\mathcal{A}})^i{}_j$ are the components of the Killing vectors $(V^{\mathcal{A}})^i{}_j \partial_{y^{\mathcal{A}}}$ of S^5 ($\partial_{y^{\mathcal{A}}} = \partial/\partial y^{\mathcal{A}}$).

Note that x^+ enters the action only through the combination $e^{2\phi}\partial_\mu x^+$. An attractive feature of this representation is that the terms in Eq. (6.1) involving $\tilde{\eta}_i (\gamma^{A'})^i{}_j \tilde{\eta}^j$ are now collected in the second term in the derivative (6.6) and thus have a natural geometrical interpretation, multiplying the Killing vectors.

The Killing vectors $(V^{\mathcal{A}})^i{}_j \partial_{y^{\mathcal{A}}}$ satisfy the so(6)$\simeq$su(4) commutation relations (3.5) and may be written as

$$(V^{\mathcal{A}})^i{}_j \partial_{y^{\mathcal{A}}} = \frac{1}{4}(\gamma^{A'B'})^i{}_j V^{A'B'} + \frac{i}{2}(\gamma^{A'})^i{}_j V^{A'}, \tag{6.8}$$

where $V^{A'}$ and $V^{A'B'}$ correspond to the 5 translations and SO(5) rotations, respectively, and are given by [see Eq. (5.21)]

$$V^{A'} = [\,|y|\cot|y|(\delta^{A'\mathcal{A}} - n^{A'} n^{\mathcal{A}}) + n^{A'} n^{\mathcal{A}}]\partial_{y^{\mathcal{A}}}, \tag{6.9}$$

$$V^{A'B'} = y^{A'}\partial_{y^{B'}} - y^{B'}\partial_{y^{A'}} . \tag{6.10}$$

Here $\delta^{A'\mathcal{A}}$ is Kronecker delta symbol and we use the conventions $y^{\mathcal{A}} = \delta^{\mathcal{A}}_{A'} y^{A'}$, $n^{\mathcal{A}} = \delta^{\mathcal{A}}_{A'} n^{A'}$, $n^{\mathcal{A}} = n_{\mathcal{A}}$. In these coordinates the S^5 metric tensor has the form

$$G_{\mathcal{AB}} = e^{A'}_{\mathcal{A}} e^{A'}_{\mathcal{B}}, \qquad e^{A'}_{\mathcal{A}} = \frac{\sin|y|}{|y|}(\delta^{A'}_{\mathcal{A}} - n_{\mathcal{A}} n^{A'}) + n_{\mathcal{A}} n^{A'} . \tag{6.11}$$

Note that while deriving Eq. (6.4) we use the relation $(U^\dagger \gamma^{A'} U)^i{}_j = -2i e^{A'}_{\mathcal{A}} (V^{\mathcal{A}})^i{}_j$.

The Lagrangian (6.4),(6.5) can be put into the manifestly SU(4) invariant form by changing the coordinates from $\phi, y^{A'}$ to the Cartesian coordinates Y^M ($M = 1, \ldots, 6$):

$$Y^{A'} = e^{\phi}\sin|y| n^{A'}, \qquad Y^6 = e^{\phi}\cos|y|,$$

$$Y^2 = Y^M Y^M = |Y|^2 = e^{2\phi}. \tag{6.12}$$

In terms of the new coordinates the superstring Lagrangian $\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm WZ}$ takes then the following more transparent manifestly SU(4) invariant form

---

[18]Reference [18] found the Hamiltonian for the superparticle in $AdS_5\times S^5$ [see Eq. (12) there]. The action is obtained from the Hamiltonian in the usual way.

[19]The light-cone formulation of the superparticle in $AdS_5\times S^5$ [18,19] used similar Grassmann variables with the same conformal dimensions

[20]We introduced the coordinate S^5 indices $\mathcal{A},\mathcal{B} = 1, \ldots, 5$ (to be distinguished from the tangent space indices A', B') and set $y^{\mathcal{A}} = \delta^{\mathcal{A}}_{A'} y^{A'}$.

$$\mathcal{L}_{\rm kin} = -\sqrt{g}g^{\mu\nu}\bigg[Y^2(\partial_\mu x^+ \partial_\nu x^- + \partial_\mu x \partial_\nu \bar{x}) + \frac{1}{2Y^2}D_\mu Y^M D_\nu Y^M\bigg] - \frac{i}{2}\sqrt{g}g^{\mu\nu}Y^2\partial_\mu x^+[\theta^i\partial_\nu\theta_i + \theta_i\partial_\nu\theta^i + \eta^i\partial_\nu\eta_i + \eta_i\partial_\nu\eta^i + {\rm i}Y^2\partial_\nu x^+(\eta^2)^2], \tag{6.13}$$

$$\mathcal{L}_{\rm WZ} = \epsilon^{\mu\nu}|Y|\partial_\mu x^+ \eta^i Y^M \rho^M_{ij}(\partial_\nu\theta^j - i\sqrt{2}|Y|\eta^j\partial_\nu x) + {\rm H.c.}, \tag{6.14}$$

where

$$DY^M = dY^M - 2i\eta_i(R^M)^i{}_j\eta^j Y^2\, dx^+. \tag{6.15}$$

The 6 matrices ρ^M_{ij} are the SO(6) γ matrices in the chiral representation. The usual SO(6) Dirac matrices can be expressed in terms of ρ^M_{ij} as follows:

$$\gamma^M = \begin{pmatrix} 0 & (\rho^M)^{ij} \\ \rho^M_{ij} & 0 \end{pmatrix},$$

$$(\rho^M)^{il}\rho^N_{lj} + (\rho^N)^{il}\rho^M_{lj} = 2\delta^{MN}\delta^i_j, \quad \rho^M_{ij} = -\rho^M_{ji}, \tag{6.16}$$

where $(\rho^M)^{ij} = -(\rho^M_{ij})^*$. In deriving Eq. (6.14) we used the following representation for C' and SO(5) γ matrices in terms of the ρ matrices

$$(\gamma^{A'})^i{}_j = i(\rho^{A'})^{il}\rho^6_{lj}, \quad C'_{ij} = \rho^6_{ij}, \tag{6.17}$$

implying an interesting relation

$$e^{\phi}C^U_{ij} = \rho^M_{ij}Y^M. \tag{6.18}$$

The matrices $(R^M)^i{}_j$ in the covariant derivative (6.15) are defined as follows ($M,N,K,L = 1,\dots,6$)

$$(R^M)^i{}_j = \frac{1}{4}(\rho^{KL})^i{}_j(R^M)^{KL}, \tag{6.19}$$

where

$$(R^M)^{KL} = Y^K\delta^{LM} - Y^L\delta^{KM},$$

$$(\rho^{KL})^i{}_j \equiv \frac{1}{2}(\rho^K)^{il}\rho^L_{lj} - (K\leftrightarrow L). \tag{6.20}$$

Note that $(R^M)^i{}_j\partial_{Y^M}$ satisfy the so(6)$\simeq$su(4) commutation relations (3.5). In contrast to $V^{\mathcal{A}}$ which are complicated functions of $y^{\mathcal{A}}$ the matrices R^M take simpler form.

Note that in terms of the 6 Cartesian coordinates Y^M the metric of $AdS_5\times S^5$ takes the "4+6" form

$$ds^2 = Y^2\, dx^a\, dx^a + Y^{-2}\, dY^M\, dY^M.$$

Similar choice of the bosonic part of superstring coordinates was used, e.g., in Refs. [6,15]. The advantage of the resulting action is a more transparent structure of the WZ term (6.14).

The above action (6.13),(6.14) can be transformed into the equivalent form corresponding to the choice of the conformally flat coordinates in $AdS_5\times S^5$, i.e., ($Y^M\to Z^M/Z^2$)

$$ds^2 = \frac{1}{Z^2}(dx^a\, dx^a + dZ^M\, dZ^M).$$

If we start again with Eqs. (6.4),(6.5) and introduce [see Eq. (6.12)]

$$Z^{A'} = e^{-\phi}\sin|y|n^{A'}, \quad Z^6 = e^{-\phi}\cos|y|,$$

$$Z^2 = Z^M Z^M = |Z|^2 = e^{-2\phi}, \tag{6.21}$$

then we finish with [see Eqs. (6.13),(6.14)]

$$\mathcal{L}_{\rm kin} = -\sqrt{g}g^{\mu\nu}Z^{-2}\bigg[\partial_\mu x^+\partial_\nu x^- + \partial_\mu x\partial_\nu\bar{x} + \frac{1}{2}D_\mu Z^M D_\nu Z^M\bigg] - \frac{i}{2}\sqrt{g}g^{\mu\nu}Z^{-2}\partial_\mu x^+[\theta^i\partial_\nu\theta_i + \theta_i\partial_\nu\theta^i + \eta^i\partial_\nu\eta_i + \eta_i\partial_\nu\eta^i + iZ^{-2}\partial_\nu x^+(\eta^2)^2], \tag{6.22}$$

$$\mathcal{L}_{\rm WZ} = \epsilon^{\mu\nu}|Z|^{-3}\partial_\mu x^+\eta^i\rho^M_{ij}Z^M(\partial_\nu\theta^j - i\sqrt{2}|Z|^{-1}\eta^j\partial_\nu x) + {\rm H.c.}, \tag{6.23}$$

where $Z^{-2}\equiv(Z^2)^{-1}$ and [see Eqs. (6.15),(6.19)]

$$DZ^M = dZ^M - 2i\eta_i(R^M)^i{}_j\eta^j Z^{-2}\, dx^+, \quad R^M = -\frac{1}{2}\rho^{ML}Z^L. \tag{6.24}$$

All other notations are the same as above. One can obtain Eqs. (6.22), (6.23) directly from Eqs. (6.13),(6.14) by making the inversion $Y^M\to Z^M/Z^2$ and taking into account the relation $R^M Z^M = 0$.

In this section we have discussed the light-cone action in the Killing parametrization of superspace. In order to get the light-cone gauge action in the Wess-Zumino parametrization one needs to make the following redefinitions in Eqs. (6.1), (6.2) [see Eqs. (5.19), (5.20)]

$$\theta^i\to(U^{-1})^i{}_j\theta^i, \qquad \theta_i\to\theta_j U^j{}_i, \tag{6.25}$$

$$\eta^i\to\sqrt{2}e^{\phi}(U^{-1})^i{}_j\eta^j, \quad \eta_i\to\sqrt{2}e^{\phi}\eta_j U^j{}_i. \tag{6.26}$$

In addition we change sign of 4D coordinates $x^a\to -x^a$. The fermionic derivatives ∂_μ will then get the generalized connection $\Omega_\mu = \partial_\mu U U^{-1}$ (1.8) contributions, i.e., become the covariant derivatives $\mathcal{D}_\mu$ [see Eq. (1.7)]. The action in terms of these new variables was presented in Eqs. (1.6),(1.10) in the Introduction.

Finally, let us note that our results for the $AdS_5\times S^5$ space can be generalized to the case of $AdS_3\times S^3$ in a rather straightforward way. To get the light-cone gauge action for this case one could use the κ invariant action of Ref. [43] and then apply the same procedure of light-cone splitting and gauge fixing as developed in this paper. However, our light-cone gauge action is already written in the form which allows a straightforward generalization to the case of $AdS_3\times S^3$: one is just to do a dimensional reduction. Let us discuss the $AdS_3\times S^3$ Lagrangian using for definiteness the WZ parametrization where the action has the form given by Eq. (1.2). To get the $\mathcal{L}_B$ and $\mathcal{L}_F^{(2)}$ terms in the $AdS_3\times S^3$ case we are to set $x=\bar{x}=0$ in Eqs. (1.3) and (1.6) and also to assume that the fermionic variables θ and η now transform in the fundamental representation of SU(2) (i.e., the indices i,j take values 1,2). The matrix C'_{ij} is then given by $C'=h\sigma_2$, $|h|=1$. The matrices $(\gamma^{A'})^i{}_j$, $A'=1,2,3$ are now SO(3) Dirac gamma matrices and the matrix $U(y)$ takes the same form as in Eqs. (1.4), (5.21). The quartic part of the Lagrangian $\mathcal{L}_F^{(4)}$ in Eq. (1.10) simplifies to[21]

$$\mathcal{L}_F^{(4)}=2\sqrt{g}g^{\mu\nu}e^{4\phi}\partial_\mu x^+\partial_\nu x^+(\eta^i\eta_i)^2. \tag{6.27}$$

ACKNOWLEDGMENTS

We are grateful to A. M. Polyakov and C. Thorn for stimulating discussions and suggestions, and to S. Frolov for useful comments. This work was supported in part by the U.S. DOE Grant No. DOE/ER/01545 and the INTAS Project No. 991590. R.R.M. is also supported by the RFBR Grant No 99-02-17916. A.A.T. would like to acknowledge also the support of the EC TMR Grant No. ERBFMRX-CT96-0045 and PPARC SPG grant No. PPA/G/S/1998/00613.

APPENDIX A: PSU(2,2|4) SUPERALGEBRA: $so(4,1)\oplus so(5)$, $so(3,1)\oplus su(4)$ AND LIGHT-CONE BASES

Commutation relations of psu(2,2|4) superalgebra in $so(4,1)\oplus so(5)$ basis were given in Ref. [2]. This basis is most adequate for finding the covariant action in $AdS_5\times S^5$ space [2] which is the direct analogue of the GS action in flat space. To develop the light-cone formulation it is convenient, however, to make a transformation to the basis in which the supercharges are diagonal with respect to the generators J^{+-}, D, $J^{x\bar{x}}$ [see Eq. (3.3)] and belong to the fundamental representation of su(4). This basis we shall call light-cone basis.

We shall find the transformation to the light-cone basis at the level of the algebra, and this will allow us to find the Cartan 1-forms and the action in the form corresponding to the light-cone basis. It is convenient to first make the transformation to the intermediate $so(3,1)\oplus su(4)$ basis and only then to the light-cone basis. A bonus of this procedure is that this intermediate form will allow us to find as a by-product another interesting version of the κ-symmetry gauge fixed action (see Appendix C).

[21]To transform Eq. (1.10) to this form we use the completeness relation for SO(3) gamma matrices $(\gamma^{A'})^i{}_j(\gamma^{A'})^k{}_l=-\delta^i_j\delta^k_l+2\delta^i_l\delta^k_j$.

We start with the commutation relations of psu(2,2|4) superalgebra in $so(4,1)\oplus so(5)$ basis given in [2]

$$[\hat{P}_A,\hat{P}_B]=\hat{J}_{AB},\quad [P_{A'},P_{B'}]=-J_{A'B'}, \tag{A1}$$

$$[\hat{J}^{AB},\hat{J}^{CE}]=\eta^{BC}\hat{J}^{AE}+3\,\text{terms},$$

$$[J^{A'B'},J^{C'E'}]=\eta^{B'C'}J^{A'E'}+3\ \text{terms}, \tag{A2}$$

$$[Q_I,\hat{P}_A]=-\frac{i}{2}\epsilon_{IJ}Q_J\gamma_A,$$

$$[Q_I,\hat{J}_{AB}]=-\frac{1}{2}Q_I\gamma_{AB}, \tag{A3}$$

$$[Q_I,P_{A'}]=\frac{1}{2}\epsilon_{IJ}Q_J\gamma_{A'},$$

$$[Q_I,J_{A'B'}]=-\frac{1}{2}Q_I\gamma_{A'B'}, \tag{A4}$$

$$\begin{aligned}\{Q_{\alpha iI},Q_{\beta jJ}\}=\delta_{IJ}[&-2iC'_{ij}(C\gamma^A)_{\alpha\beta}\hat{P}_A\\ &+2C_{\alpha\beta}(C'\gamma^{A'})_{ij}P_{A'}]\\ &+\epsilon_{IJ}[C'_{ij}(C\gamma^{AB})_{\alpha\beta}\hat{J}_{AB}\\ &-C_{\alpha\beta}(C'\gamma^{A'B'})_{ij}J_{A'B'}].\end{aligned} \tag{A5}$$

Unless otherwise specified, we use the notation Q^I for $Q^{I\alpha i}$ and Q_I for $Q_{I\alpha i}$, where $Q_{I\alpha i}\equiv Q^{J\beta j}\delta_{JI}C_{\beta\alpha}C'_{ji}$. Hermitean conjugation rules in this basis are

$$\hat{P}_A^\dagger=-\hat{P}_A,\quad P_{A'}^\dagger=-P_{A'},$$

$$\hat{J}_{AB}^\dagger=-\hat{J}_{AB},\quad J_{A'B'}^\dagger=-J_{A'B'}, \tag{A6}$$

$$(Q^{I\beta i})^\dagger(\gamma^0)_\alpha^\beta=-Q^{I\beta j}C_{\beta\alpha}C'_{ji}. \tag{A7}$$

Let us first transform the bosonic generators into the conformal algebra basis. To this end we introduce the Poincaré translations P^a, the conformal boosts K^a and the dilatation D by

$$P^a=\hat{P}^a+\hat{J}^{4a},\quad K^a=\frac{1}{2}(-\hat{P}^a+\hat{J}^{4a}),\quad D=-\hat{P}^4. \tag{A8}$$

Making use of the commutation relations (A1),(A2) it is easy to check that these generators satisfy the commutation relations given in Eqs. (3.1),(3.2).

Next, we introduce the new "charged" supergenerators

$$Q^q\equiv\frac{1}{\sqrt{2}}(Q^1+iQ^2),\quad Q^{\bar{q}}\equiv\frac{1}{\sqrt{2}}(Q^1-iQ^2). \tag{A9}$$

We shall use the simplified notation

$$Q^{\alpha i}\equiv -Q^{q\alpha i}, \quad Q_{\alpha i}\equiv Q_{q\alpha i}. \tag{A10}$$

Then the nonvanishing values of δ_{IJ} (ϵ_{IJ}, $\epsilon_{12}=1$) become replaced by $\delta_{q\bar{q}}=1$ ($\epsilon_{q\bar{q}}=i$) and the Majorana condition takes the form $(Q^{\beta i})^{\dagger}(\gamma^0)_{\alpha}^{\beta}=Q_{\alpha i}$. The commutators have the form

$$[Q^{\alpha i},\hat{P}^A]=-\frac{1}{2}(\gamma^A Q)^{\alpha i}, \quad [Q^{\alpha i},\hat{J}^{AB}]=\frac{1}{2}(\gamma^{AB}Q)^{\alpha i}, \tag{A11}$$

$$[Q_{\alpha i},\hat{P}^A]=\frac{1}{2}(Q\gamma^A)_{\alpha i}, \quad [Q_{\alpha i},\hat{J}^{AB}]=-\frac{1}{2}(Q\gamma^{AB})_{\alpha i}, \tag{A12}$$

$$[Q^{\alpha i},P^{A'}]=-\frac{i}{2}(\gamma^{A'}Q)^{\alpha i},$$

$$[Q^{\alpha i},J^{A'\mathrm{B}'}]=\frac{1}{2}(\gamma^{A'\mathrm{B}'}Q)^{\alpha i}, \tag{A13}$$

$$[Q_{\alpha i},P^{A'}]=\frac{i}{2}(Q\gamma^{A'})_{\alpha i},$$

$$[Q_{\alpha i},J^{A'\mathrm{B}'}]=-\frac{1}{2}(Q\gamma^{A'\mathrm{B}'})_{\alpha i}, \tag{A14}$$

while the anticommutators transform into the form

$$\{Q^{\alpha i},Q_{\beta j}\}=[2i(\gamma^A)^{\alpha}_{\beta}\hat{P}^A+(\gamma^{AB})^{\alpha}_{\beta}\hat{J}^{AB}]\delta^i_j-4i\,\delta^{\alpha}_{\beta}J^i{}_j, \tag{A15}$$

where we use the notation

$$J^i{}_j\equiv-\frac{i}{2}(\gamma^{A'})^i{}_jP^{A'}+\frac{1}{4}(\gamma^{A'\mathrm{B}'})^i{}_jJ^{A'\mathrm{B}'}. \tag{A16}$$

Starting with the commutation relations for $P^{A'}$ and $J^{A'\mathrm{B}'}$ and applying various Fierz identities one proves that $J^i{}_j$ ($J^{i\,\dagger}_{\ j}=J^j{}_i$) satisfy the commutation relations of su(4) algebra.

Using the commutators (A13), (A14), and (A16) and completeness relation for Dirac matrices one proves that

$$[Q_{\alpha i},J^j{}_k]=\delta^j_iQ_{\alpha k}-\frac{1}{4}\delta^j_kQ_{\alpha i},$$

$$[Q^{\alpha i},J^j{}_k]=-\delta^i_kQ^{\alpha j}+\frac{1}{4}\delta^j_kQ^{\alpha i}. \tag{A17}$$

This demonstrates that supercharges transform in the fundamental representations of su(4).

In what follows we will use the following decomposition of so(4,1) Dirac and charge conjugation matrices in the sl(2) basis

$$(\gamma^a)^{\alpha}{}_{\beta}=\begin{pmatrix}0 & (\sigma^a)^{\mathrm{a}\dot{\mathrm{b}}}\\ \bar{\sigma}^a_{\dot{\mathrm{a}}\mathrm{b}} & 0\end{pmatrix}, \quad \gamma^4=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},$$

$$C_{\alpha\beta}=\begin{pmatrix}\epsilon_{\mathrm{ab}} & 0\\ 0 & \epsilon^{\dot{\mathrm{a}}\dot{\mathrm{b}}}\end{pmatrix}, \tag{A18}$$

where the matrices $(\sigma^a)^{\mathrm{a}\dot{\mathrm{a}}}$, $(\bar{\sigma}^a)_{\dot{\mathrm{a}}\mathrm{a}}$ are related to Pauli matrices in the standard way

$$\sigma^a=(1,\sigma^1,\sigma^2,\sigma^3), \quad \bar{\sigma}^a=(-1,\sigma^1,\sigma^2,\sigma^3). \tag{A19}$$

Note that $\sigma^a_{\mathrm{a}\dot{\mathrm{a}}}=\bar{\sigma}^a_{\dot{\mathrm{a}}\mathrm{a}}$, $\bar{\sigma}^a_{\dot{\mathrm{a}}\mathrm{b}}=\sigma^{a*}_{\mathrm{a}\dot{\mathrm{b}}}$ where $\sigma^a_{\mathrm{a}\dot{\mathrm{a}}}\equiv(\sigma^a)^{\mathrm{b}\dot{\mathrm{b}}}\epsilon_{\mathrm{ba}}\epsilon_{\dot{\mathrm{b}}\dot{\mathrm{a}}}$. We use the following conventions for the sl(2) indices: $\epsilon_{12}=\epsilon^{12}=-\epsilon_{\dot{1}\dot{2}}=-\epsilon^{\dot{1}\dot{2}}=1$,

$$\psi^{\mathrm{a}}=\epsilon^{\mathrm{ab}}\psi_{\mathrm{b}}, \quad \psi_{\mathrm{a}}=\psi^{\mathrm{b}}\epsilon_{\mathrm{ba}}, \quad \psi^{\dot{\mathrm{a}}}=\epsilon^{\dot{\mathrm{a}}\dot{\mathrm{b}}}\psi_{\dot{\mathrm{b}}}, \quad \psi_{\dot{\mathrm{a}}}=\psi^{\dot{\mathrm{b}}}\epsilon_{\dot{\mathrm{b}}\dot{\mathrm{a}}}. \tag{A20}$$

We then decompose the supercharges in the sl(2)⊕su(4) basis

$$Q^{\alpha i}=\begin{pmatrix}2iv^{-1}Q^{\mathrm{a}i}\\ 2vS^i_{\dot{\mathrm{a}}}\end{pmatrix}, \quad Q_{\alpha i}=(2vS_{\mathrm{a}i},-2iv^{-1}Q^{\dot{\mathrm{a}}}_i),$$

$$v\equiv2^{1/4}. \tag{A21}$$

In terms of these new supercharges the commutation relations take the form

$$[D,Q^{\mathrm{a}i}]=-\frac{1}{2}Q^{\mathrm{a}i}, \quad [D,S^{\mathrm{a}}_i]=\frac{1}{2}S^{\mathrm{a}}_i, \tag{A22}$$

$$[S^{\mathrm{a}}_i,P^a]=\frac{i}{\sqrt{2}}(\sigma^a)^{\mathrm{a}\dot{\mathrm{a}}}Q_{\dot{\mathrm{a}}i}, \quad [S^i_{\dot{\mathrm{a}}},P^a]=-\frac{i}{\sqrt{2}}(\bar{\sigma}^a)_{\dot{\mathrm{a}}\mathrm{a}}Q^{\mathrm{a}i}, \tag{A23}$$

$$[Q^{\mathrm{a}i},K^a]=-\frac{i}{\sqrt{2}}(\sigma^a)^{\mathrm{a}\dot{\mathrm{a}}}S^i_{\dot{\mathrm{a}}}, \quad [Q_{\dot{\mathrm{a}}i},K^a]=\frac{i}{\sqrt{2}}(\bar{\sigma}^a)_{\dot{\mathrm{a}}\mathrm{a}}S^{\mathrm{a}}_i, \tag{A24}$$

$$\{Q^{\mathrm{a}i},Q^{\dot{\mathrm{b}}}_j\}=\frac{i}{\sqrt{2}}\sigma^{\mathrm{a}\dot{\mathrm{b}}}_aP^a\delta^i_j, \quad \{S^{\mathrm{a}}_j,S^{\dot{\mathrm{b}}i}\}=-\frac{i}{\sqrt{2}}\sigma^{\mathrm{a}\dot{\mathrm{b}}}_aK^a\delta^i_j, \tag{A25}$$

$$[Q^{\mathrm{a}i},J^{ab}]=\frac{1}{2}(\sigma^{ab})^{\mathrm{a}}{}_{\mathrm{b}}Q^{\mathrm{b}i}, \tag{A26}$$

$$\{Q^{\mathrm{a}i},S^{\mathrm{b}}_j\}=\left(\frac{1}{2}\epsilon^{\mathrm{ab}}D+\frac{1}{4}\sigma^{\mathrm{ab}}_{ab}J^{ab}\right)\delta^i_j+\epsilon^{\mathrm{ab}}J^i{}_j, \tag{A27}$$

where $(\sigma^{ab})^{\mathrm{ab}}=\epsilon^{\mathrm{bc}}(\sigma^{ab})^{\mathrm{a}}{}_{\mathrm{c}}$, $(\sigma^{ab})^{\mathrm{a}}{}_{\mathrm{b}}\equiv\frac{1}{2}(\sigma^a)^{\mathrm{a}\dot{\mathrm{c}}}(\bar{\sigma}^b)_{\dot{\mathrm{c}}\mathrm{b}}-(a\leftrightarrow b)$. Hermitean conjugation rules of the supercharges are

$$Q^{i\mathrm{a}\dagger}=Q^{\dot{\mathrm{a}}}_i, \quad Q^{i\dagger}_{\mathrm{a}}=-Q_{\dot{\mathrm{a}}i}, \tag{A28}$$

and the same for S supercharges. The spinor sl(2) indices a,b are raised and lowered as in Eq. (A20). From these commutation relations we learn that $Q^{\mathsf{a}i}$, $Q_i^{\dot{\mathsf{a}}}$ may be interpreted as the supercharges of the super Poincaré subalgebra while $S_i^{\dot{\mathsf{a}}}$, $S^{\mathsf{a}i}$ are the conformal supercharges.

This finishes the description of the $so(3,1)\oplus su(4)$ basis. We are now ready to introduce the light-cone basis. The transformation of the bosonic generators is implied by the light-cone decomposition of the coordinates (1.1) and is given by (3.3). The transformation of supercharges amounts to attaching the signs $+$ and $-$ which will show explicitly their J^{+-} charges. The corresponding supercharges are defined by

$$Q^{1i}\equiv -Q^{-i},\quad Q^{2i}\equiv Q^{+i},\quad Q_i^{\dot{1}}\equiv -Q_i^-,\quad Q_i^{\dot{2}}\equiv Q_i^+, \tag{A29}$$

$$S_i^1\equiv S_i^-,\quad S_i^2\equiv -S_i^+,\quad S^{\dot{1}i}\equiv S^{-i},\quad S^{\dot{2}i}\equiv -S^{+i}. \tag{A30}$$

Choice of signs in these definitions is a matter of convention. Hermitean conjugation rules (A28) lead then to the conjugation rules given in Eq. (3.11).

APPENDIX B: CARTAN FORMS IN $so(3,1)\oplus su(4)$ AND LIGHT CONE BASES

The kinetic term of the $\mathrm{AdS}_5\times \mathrm{S}^5$ GS action and the 3-form in its WZ term have the following form in the $so(4,1)\oplus so(5)$ basis [2]:

$$\mathcal{L}_{\mathrm{kin}}=-\frac{1}{2}\sqrt{g}g^{\mu\nu}(\hat{L}_\mu^A\hat{L}_\nu^A+L_\mu^{A'}L_\nu^{A'}), \tag{B1}$$

$$\mathcal{H}=s^{IJ}\hat{L}^A\overline{L^I}\gamma^A L^J+is^{IJ}L^{A'}\overline{L^J}\gamma^{A'}L^J. \tag{B2}$$

They are expressed in terms of the Cartan 1-forms defined in the $so(4,1)\oplus so(5)$ basis by

$$G^{-1}\,dG=(G^{-1}\,dG)_{\mathrm{bos}}+L^{I\alpha i}Q_{I\alpha i}, \tag{B3}$$

where the restriction to the bosonic part is

$$(G^{-1}\,dG)_{\mathrm{bos}}=\hat{L}^A\hat{P}^A+\frac{1}{2}\hat{L}^{AB}\hat{J}^{AB}+L^{A'}P^{A'}+\frac{1}{2}L^{A'B'}J^{A'B'}. \tag{B4}$$

The transformation of the psu(2,2|4) algebra into the light-cone basis described in Appendix A allows us to find the corresponding Cartan 1-forms and thus to write down the GS action in the light-cone basis.

We first consider the $so(3,1)\oplus su(4)$ basis and define the bosonic (even) Cartan forms by

$$(G^{-1}\,dG)_{\mathrm{bos}}=L_P^aP^a+L_K^aK^a+L_DD+\frac{1}{2}L^{ab}J^{ab}+L^i{}_jJ^j{}_i. \tag{B5}$$

Comparing this with Eq. (B4) and using Eqs. (A8), (A16) we get

$$\hat{L}^a=L_P^a-\frac{1}{2}L_K^a,\quad \hat{L}^{4a}=L_P^a+\frac{1}{2}L_K^a,\quad \hat{L}^4=-L_D, \tag{B6}$$

$$L^i{}_j=\frac{i}{2}(\gamma^{A'})^i{}_jL^{A'}-\frac{1}{4}(\gamma^{A'B'})^i{}_jL^{A'B'}. \tag{B7}$$

Using these relations in the expression for the kinetic term (B1) gives the action (4.4).

Now let us consider the fermionic 1-forms. They satisfy Hermitean conjugation rule

$$(L^{I\beta i})^\dagger(\gamma^0)_\alpha^\beta=L^{I\beta j}C_{\beta\alpha}C'_{ji} \tag{B8}$$

and we use the notation $L_{I\alpha i}\equiv L^{J\beta j}\delta_{JI}C_{\beta\alpha}C'_{ji}$. Let us define

$$L^q\equiv\frac{1}{\sqrt{2}}(L^1+iL^2),\quad L^{\bar{q}}\equiv\frac{1}{\sqrt{2}}(L^1-iL^2), \tag{B9}$$

introduce the notation $L^{\alpha i}=L^{q\alpha i}$, $L_{\alpha i}=L_{q\alpha i}$ and use the following decomposition into $sl(2)\oplus su(4)$ Cartan 1-forms:

$$L^{\alpha i}=\frac{1}{2}\begin{pmatrix} v^{-1}L_S^{\mathsf{a}i}\\ ivL_{Q\dot{\mathsf{a}}}^{i}\end{pmatrix},\quad L_{\alpha i}=\frac{1}{2}(-ivL_{Q\mathsf{a}i},v^{-1}L_{Si}^{\dot{\mathsf{a}}}). \tag{B10}$$

Hermitean conjugation rules for the new Cartan 1-forms then take the same form as in Eq. (A28). The light-cone frame Cartan 1-forms are defined by

$$L_{Qi}^1=-L_{Qi}^-,\quad L_{Qi}^2=-L_{Qi}^+,\quad L_Q^{\dot{1}i}=-L_Q^{-i},\quad L_Q^{\dot{2}i}=-L_Q^{+i}, \tag{B11}$$

$$L_S^{1i}=L_S^{-i},\quad L_S^{2i}=L_S^{+i},\quad L_{Si}^{\dot{1}}=L_{Si}^-,\quad L_{Si}^{\dot{2}}=L_{Si}^+. \tag{B12}$$

These relations imply

$$L^{I\alpha i}Q_{I\alpha i}=L^{\alpha i}Q_{\alpha i}-L_{\alpha i}Q^{\alpha i} \tag{B13}$$

$$=L_{Qi}^{\mathsf{a}}Q_{\mathsf{a}}^i-L_Q^{\dot{\mathsf{a}}i}Q_{\dot{\mathsf{a}}i}+L_S^{\mathsf{a}i}S_{\mathsf{a}i}-L_{Si}^{\dot{\mathsf{a}}}S_{\dot{\mathsf{a}}}^i \tag{B14}$$

$$=L_Q^{+i}Q_i^-+L_Q^{-i}Q_i^++L_{Qi}^+Q^{-i}+L_{Qi}^-Q^{+i}$$
$$+L_S^{-i}S_i^++L_S^{+i}S_i^-+L_{Si}^-S^{+i}+L_{Si}^+S^{-i}. \tag{B15}$$

The representation (B14) corresponds to the $sl(2)\oplus su(4)$ basis while Eq. (B15) corresponds to the light-cone basis.

Using the relation between the Cartan 1-forms in Eqs. (B9)–(B12) we are ready to consider the decomposition of the WZ 3-form (B2). We start with the AdS_5 contribution which is given by the first term in right-hand side of Eq. (B2). Taking into account that $\overline{L^I}=L^ICC'$ and Eq. (B9) we can rewrite the AdS_5 contribution in terms of the "charged" Cartan forms L^q, $L^{\bar{q}}$

$$\mathcal{H}_{\mathrm{AdS}_5}=\mathcal{H}_{\mathrm{AdS}_5}^q+\mathcal{H}_{\mathrm{AdS}_5}^{\bar{q}}, \tag{B16}$$

$$\mathcal{H}^{q}_{\mathrm{AdS}_5}\equiv\hat{L}^{A}L^{q\alpha i}(C\gamma^{A})_{\alpha\beta}C'_{ij}L^{q\beta j},$$

$$\mathcal{H}^{\bar{q}}_{\mathrm{AdS}_5}\equiv\hat{L}^{A}L^{\bar{q}\alpha i}(C\gamma^{A})_{\alpha\beta}C'_{ij}L^{\bar{q}\beta j}. \tag{B17}$$

Since $i\mathcal{H}^{\bar{q}}_{\mathrm{AdS}_5}$ is Hermitean conjugate to $i\mathcal{H}^{q}_{\mathrm{AdS}_5}$ we restrict our attention to decomposition of the first term. We get

$$\mathcal{H}^{q}_{\mathrm{AdS}_5}=\hat{L}^{a}L^{q\alpha i}(C\gamma^{a})_{\alpha\beta}C'_{ij}L^{q\beta j}-L_{D}L^{q\alpha i}(C\gamma^{4})_{\alpha\beta}C'_{ij}L^{q\beta j} \tag{B18}$$

$$=\frac{i}{2}\hat{L}^{a}L^{i}_{S\mathrm{a}}C'_{ij}(\sigma^{a})^{\mathrm{a}\dot{\mathrm{b}}}L^{j}_{Q\dot{\mathrm{b}}}+\frac{1}{4}L_{D}\left(\frac{1}{\sqrt{2}}L^{\mathrm{a}i}_{S}C'_{ij}L^{j}_{S\mathrm{a}}+\sqrt{2}L^{\dot{\mathrm{a}}i}_{Q}C'_{ij}L^{j}_{Q\dot{\mathrm{a}}}\right)$$

$$=-\frac{i}{\sqrt{2}}(\hat{L}^{+}L^{-i}_{S}C'_{ij}L^{-j}_{Q}+\hat{L}^{-}L^{+i}_{Q}C'_{ij}L^{+j}_{S}+\hat{L}^{x}L^{-i}_{S}C'_{ij}L^{+j}_{Q}+\hat{L}^{\bar{x}}L^{+i}_{S}C'_{ij}L^{-j}_{Q})+\frac{1}{\sqrt{2}}L_{D}\left(\frac{1}{2}L^{+i}_{S}C'_{ij}L^{-j}_{S}+L^{-i}_{Q}C'_{ij}L^{+j}_{Q}\right). \tag{B19}$$

Equation (B19) provides representation of the AdS_5 part of the 3-form in the $sl(2)\oplus su(4)$ basis, while Eq. (B19) represents the light-cone basis.

Let us now consider the S^5 part of the WZ 3-form in Eq. (B2), i.e., $is^{IJ}L^{A'}\overline{L^{I}}\gamma^{A'}L^{J}$. Representing it in terms of the charged Cartan forms as in Eq. (B16), $\mathcal{H}_{S^5}=\mathcal{H}^{q}_{S^5}+\mathcal{H}^{\bar{q}}_{S^5}$, we get

$$\mathcal{H}^{q}_{S^5}=iL^{A'}L^{q\alpha i}C_{\alpha\beta}(C'\gamma^{A'})_{ij}L^{q\beta j}=-2L^{q\alpha i}C_{\alpha\beta}C'_{ik}L^{k}{}_{j}L^{q\beta j} \tag{B20}$$

$$=\frac{1}{2\sqrt{2}}L^{\mathrm{a}i}_{S}(C'L)_{ij}L^{j}_{S\mathrm{a}}-\frac{1}{\sqrt{2}}L^{\dot{\mathrm{a}}i}_{Q}(C'L)_{ij}L^{j}_{Q\dot{\mathrm{a}}} \tag{B21}$$

$$=\frac{1}{2\sqrt{2}}[L^{+i}_{S}(C'L)_{ij}L^{-j}_{S}-L^{-i}_{S}(C'L)_{ij}L^{+j}_{S}]+\frac{1}{\sqrt{2}}[L^{+i}_{Q}(C'L)_{ij}L^{-j}_{Q}-L^{-i}_{Q}(C'L)_{ij}L^{+j}_{Q}]. \tag{B22}$$

Note that in Eq. (B20) we exploited the relation (B7) and used the fact that $(C'\gamma^{A'\mathrm{B}'})_{ij}$ is symmetric in i,j, the charge conjugation matrix $C_{\alpha\beta}$ is antisymmetric in α,β and the fermionic Cartan 1-forms L^{q} are commuting with each other. Equation (B21) provides representation of S^5 part of the WZ 3-form in the $sl(2)\oplus su(4)$ basis, while Eq. (B22) represents the light-cone basis.

Next, let us outline the procedure of derivation of the WZ term in the light-cone κ-symmetry gauge. Taking into account that $L^{+i}_{S}=0$, $L^{+}_{Si}=0$, $L^{+}_{K}=0$, $L^{x}_{K}=0$, $L^{\bar{x}}_{K}=0$ and plugging the Cartan 1-forms given by Eqs. (5.11)–(5.18) into the above expressions we get $\mathcal{H}^{q}_{\mathrm{AdS}_5}=\mathcal{H}^{q(1)}_{\mathrm{AdS}_5}+\mathcal{H}^{q(2)}_{\mathrm{AdS}_5}$, where [see Eqs. (5.19),(5.20)]

$$\mathcal{H}^{q(1)}_{\mathrm{AdS}_5}=-\frac{i}{\sqrt{2}}e^{\phi}\,dx^{+}\,\tilde{d}\eta^{i}\,C'_{ij}\tilde{d}\theta^{j}-\frac{i}{\sqrt{2}}e^{\phi}\,d\phi\,\tilde{d}\theta^{i}C'_{ij}\tilde{\eta}^{j}dx^{+}\approx d\left(\frac{i}{\sqrt{2}}e^{\phi}\,dx^{+}\,\tilde{\eta}^{i}C'_{ij}\tilde{d}\theta^{j}\right), \tag{B23}$$

$$\mathcal{H}^{q(2)}_{\mathrm{AdS}_5}=-\sqrt{2}e^{\phi}\,dx^{+}\,dx\,\tilde{d}\eta^{i}\,C'_{ij}\tilde{\eta}^{j}-\frac{1}{\sqrt{2}}e^{\phi}\,d\phi\,dx^{+}\,dx\,\tilde{\eta}^{i}C'_{ij}\tilde{\eta}^{j}\approx d\left(-\frac{1}{\sqrt{2}}e^{\phi}\,dx^{+}dx\,\tilde{\eta}^{i}C'_{ij}\tilde{\eta}^{j}\right). \tag{B24}$$

The signs $\approx$ indicate that these relations are valid modulo terms which are obtained by acting by differential d on the matrix $U^{i}{}_{j}$ which enters in the definition of $\tilde{\eta}$, $\tilde{d}\theta$. Such $dU^{i}{}_{j}$ terms are canceled by contributions coming from the S^5 part of WZ 3-form which in the light-cone gauge takes the form

$$\mathcal{H}^{q}_{S^5}=\frac{1}{\sqrt{2}}[L^{+i}_{Q}(C'L)_{ij}L^{-j}_{Q}-L^{-i}_{Q}(C'L)_{ij}L^{+j}_{Q}]. \tag{B25}$$

To summarize, one gets the following exact relation:

$$\mathcal{H}^{q}_{\mathrm{AdS}_5}+\mathcal{H}^{q}_{S^5}=d\left[\frac{i}{\sqrt{2}}e^{\phi}\,dx^{+}\,\tilde{\eta}^{i}C'_{ij}(\tilde{d}\theta^{j}+i\tilde{\eta}^{j}dx)\right]. \tag{B26}$$

Multiplying this expression by i, adding the Hermitean conjugate and going from the 3D to the 2D representation of the WZ term gives the WZ part of the string Lagrangian $\mathcal{L}_{\mathrm{WZ}}$ in Eq. (6.2).

APPENDIX C: $\mathrm{AdS}_5\times\mathrm{S}^5$ ACTION IN S GAUGE

The results for the Cartan forms in the $sl(2)\oplus su(4)$ basis described in Appendix B allow us to find another version of the κ-symmetry gauge fixed action of superstring in $\mathrm{AdS}_5\times S^5$. Let us start with the supercoset representative [see Eqs. (5.1)–(5.8)]

$$G=g_{x,\theta}g_{\eta}g_{y}g_{\phi}, \tag{C1}$$

$$g_{x,\theta}=\exp(x^{a}P^{a}+\theta^{\mathrm{a}}_{i}Q^{i}_{\mathrm{a}}-\theta^{\dot{\mathrm{a}}i}Q_{\dot{\mathrm{a}}i}), \tag{C2}$$

$$g_{\eta}=\exp(\eta^{\mathrm{a}i}S_{\mathrm{a}i}-\eta^{\dot{\mathrm{a}}}_{i}S^{i}_{\dot{\mathrm{a}}}), \tag{C3}$$

and impose the κ-symmetry gauge by

$$\eta^{\mathrm{a}i}=\eta_i^{\dot{\mathrm{a}}}=0, \tag{C4}$$

i.e.,

$$G_{\mathrm{g.f.}}=g_{x,\theta}g_y g_\phi. \tag{C5}$$

Since we have set to zero the fermionic coordinates η which correspond to the conformal supercharges S we shall call this the S gauge.[22] The resulting gauge fixed expressions for the Cartan 1-forms are given by

$$L_P^a=e^\phi\left[dx^a-\frac{i}{2\sqrt{2}}(\theta_{i\mathrm{a}}\sigma^{a\mathrm{a}\dot{\mathrm{b}}}d\theta^i_{\dot{\mathrm{b}}}+\theta^{i\dot{\mathrm{a}}}\bar{\sigma}^a_{\dot{\mathrm{a}}\mathrm{b}}d\theta^{\mathrm{b}}_i)\right], \tag{C6}$$

$$L_{Qi}^{\mathrm{a}}=e^{\phi/2}\tilde{d}\theta_i^{\mathrm{a}},\quad L_Q^{\dot{\mathrm{a}}i}=e^{\phi/2}\tilde{d}\theta^{\dot{\mathrm{a}}i}, \tag{C7}$$

$$L_D=d\phi,\quad L^i{}_j=(dU\,U^{-1})^i{}_j, \tag{C8}$$

where $\tilde{d}\theta$ is defined as in Eq. (5.20) while the matrix U is defined by Eqs. (1.4),(1.5). All the remaining Cartan 1-forms are equal to zero.

Using that now $L_S=0$ we get from Eq. (B19) the following expressions for the AdS_5 part of the 3-form $\mathcal{H}$:

$$\mathcal{H}^q_{\mathrm{AdS}_5}=\frac{1}{2\sqrt{2}}d\phi\, e^\phi\tilde{d}\theta^{\dot{\mathrm{a}}i}C'_{ij}\tilde{d}\theta^j_{\mathrm{a}}, \tag{C9}$$

while Eq. (B21) gives

$$\mathcal{H}^q_{S^5}=-\frac{1}{\sqrt{2}}e^\phi\tilde{d}\theta^{\dot{\mathrm{a}}i}(C'L)_{ij}\tilde{d}\theta^j_{\mathrm{a}}. \tag{C10}$$

Thus we conclude that

$$\mathcal{H}^q_{\mathrm{AdS}_5}+\mathcal{H}^q_{S^5}=d\left(\frac{1}{2\sqrt{2}}e^\phi\tilde{d}\theta^{\dot{\mathrm{a}}i}C'_{ij}\tilde{d}\theta^j_{\mathrm{a}}\right), \tag{C11}$$

which allows us to find the 2D form of the WZ term.

Using the above relations and Eqs. (4.4),(4.3) and taking into account that $L_K^a=0$ we finally get the following kinetic and WZ parts of the $AdS_5\times S^5$ string Lagrangian [see Eqs. (6.1),(6.2)]

$$\mathcal{L}_{\mathrm{kin}}=-\frac{1}{2}\sqrt{g}g^{\mu\nu}(L^a_{P\mu}L^a_{P\nu}+\partial_\mu\phi\partial_\nu\phi+e^{A'}_\mu e^{A'}_\nu), \tag{C12}$$

$$\mathcal{L}_{\mathrm{WZ}}=\frac{i}{2\sqrt{2}}\epsilon^{\mu\nu}e^\phi\partial_\mu\theta^{\dot{\mathrm{a}}i}C^U_{ij}\partial_\nu\theta^j_{\mathrm{a}}+\mathrm{H.c.}, \tag{C13}$$

where $L^a_{P\mu}$ is given by Eq. (C6) and C^U_{ij} as in Eq. (6.7). Note that in this S gauge the 1-form $L^{A'}$ which is given in terms of $L^i{}_j$ as in Eqs. (4.3) is equal simply to the S^5 1-form $e^{A'}$. The reason is that, in contrast to what happens in the light-cone gauge (5.15), here the Cartan form $L^i{}_j$ does not contain fermionic contributions [see Eqs. (C8)]. Making use of formula (6.18) we get the following manifestly SU(4) invariant representation for WZ part

$$\mathcal{L}_{\mathrm{WZ}}=\frac{i}{2\sqrt{2}}\epsilon^{\mu\nu}\partial_\mu\theta^{\dot{\mathrm{a}}i}\rho^M_{ij}Y^M\partial_\nu\theta^j_{\mathrm{a}}+\mathrm{H.c.} \tag{C14}$$

This form of WZ action by using usual SO(6) γ matrices (6.16) can be cast into the form similar to the one given in Refs. [9,11] (see also Ref. [6]). The kinetic term (C12) can be transformed into SU(4) manifestly invariant form in a standard way. Our presentation gives self-contained derivation of SU(4) manifestly invariant action from the original $5+5$ form of action given in Ref. [2].

[22]The ‘‘S gauge’’ and ‘‘Q gauge’’ terminology was introduced in Ref. [44], but our S gauge is different from the one used in Ref. [44].

[1] M. B. Green and J. H. Schwarz, Phys. Lett. **136B**, 367 (1984); Nucl. Phys. **B243**, 285 (1984).

[2] R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. **B533**, 109 (1998).

[3] R. Kallosh, J. Rahmfeld, and A. Rajaraman, J. High Energy Phys. **09**, 002 (1998).

[4] R. R. Metsaev and A. A. Tseytlin, Phys. Lett. B **436**, 281 (1998).

[5] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).

[6] R. Kallosh and A. A. Tseytlin, J. High Energy Phys. **10**, 016 (1998).

[7] S. Forste, D. Ghoshal, and S. Theisen, J. High Energy Phys. **08**, 013 (1999).

[8] N. Drukker, D. J. Gross, and A. A. Tseytlin, J. High Energy Phys. **04**, 021 (2000).

[9] I. Pesando, J. High Energy Phys. **11**, 002 (1998); Mod. Phys. Lett. A **14**, 343 (1999).

[10] R. Kallosh, ‘‘Superconformal actions in Killing gauge,’’ hep-th/9807206.

[11] R. Kallosh and J. Rahmfeld, Phys. Lett. B **443**, 143 (1998).

[12] M. B. Green and J. H. Schwarz, Phys. Lett. **109B**, 444 (1982).

[13] S. Carlip, Nucl. Phys. **B284**, 365 (1987); R. E. Kallosh, Phys. Lett. B **195**, 369 (1987).

[14] R. Kallosh and A. Yu. Morozov, Int. J. Mod. Phys. A **3**, 1943 (1988).

[15] I. Pesando, Phys. Lett. B **485**, 246 (2000).

[16] L. Brink, O. Lindgren, and B. E. Nilsson, Nucl. Phys. **B212**, 401 (1983); Phys. Lett. **123B**, 323 (1983); S. Mandelstam, Nucl. Phys. **B213**, 149 (1983); L. Brink, M. B. Green, and J.

H. Schwarz, *ibid.* **B223**, 125 (1983).
[17] R. R. Metsaev, Nucl. Phys. **B563**, 295 (1999).
[18] R. R. Metsaev, Phys. Lett. B **468**, 65 (1999).
[19] R. R. Metsaev, ''IIB supergravity and various aspects of light-cone formalism in AdS space-time,'' hep-th/0002008.
[20] J. Russo and A. A. Tseytlin, J. High Energy Phys. **04**, 014 (1998).
[21] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. **160B**, 69 (1985).
[22] M. B. Green and J. H. Schwarz, Nucl. Phys. **B198**, 252 (1982); J. H. Schwarz, Phys. Rep. **8**, 269 (1973).
[23] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985).
[24] L. Brink, P. Di Vecchia, and P. Howe, Phys. Lett. **65B**, 471 (1976).
[25] A. M. Polyakov, Phys. Lett. **103B**, 207 (1981).
[26] P. Goddard, J. Goldstone, C. Rebbi, and C. B. Thorn, Nucl. Phys. **B56**, 109 (1973).
[27] J. L. Gervais and B. Sakita, Phys. Rev. Lett. **30**, 716 (1973).
[28] R. Tzani, Phys. Rev. D **38**, 3112 (1988); C. Arvanitis and R. Tzani, ''Equivalence between light-cone and conformal gauge in the two-dimensional string,'' hep-th/9602109; E. Smith, Nucl. Phys. **B382**, 229 (1992).
[29] G. T. Horowitz and A. R. Steif, Phys. Rev. D **42**, 1950 (1990).
[30] R. E. Rudd, Nucl. Phys. **B427**, 81 (1994).
[31] A. Rajaraman and M. Rozali, Phys. Lett. B **468**, 58 (1999).
[32] C. Thorn (private communication).
[33] A. M. Polyakov (private communication).
[34] R. R. Metsaev, C. Thorn, and A. A. Tseytlin (unpublished).
[35] M. Henneaux and L. Mezincescu, Phys. Lett. **152B**, 340 (1985).
[36] P. B. Wiegmann, Nucl. Phys. **B323**, 330 (1989).
[37] A. R. Kavalov, I. K. Kostov, and A. G. Sedrakian, Phys. Lett. B **175**, 331 (1986); A. G. Sedrakian and R. Stora, *ibid.* **188**, 442 (1987); D. R. Karakhanian and A. G. Sedrakian (unpublished); D. R. Karakhanian (unpublished).
[38] J. H. Schwarz, Nucl. Phys. **B226**, 269 (1983).
[39] M. Günaydin and N. Marcus, Class. Quantum Grav. **2**, L19 (1985).
[40] M. Günaydin, L. J. Romans, and N. P. Warner, Phys. Lett. **154B**, 268 (1985); M. Pernici, K. Pilch, and P. van Nieuwenhuizen, Nucl. Phys. **B259**, 460 (1985).
[41] R. Haag, J. T. Lopuszanski, and M. Sohnius, Nucl. Phys. **B88**, 257 (1975).
[42] M. Günaydin, D. Minic, and M. Zagermann, Nucl. Phys. **B544**, 737 (1999); **B534**, 96 (1998).
[43] I. Pesando, J. High Energy Phys. **02**, 007 (1999); J. Rahmfeld and A. Rajaraman, Phys. Rev. D **60**, 064014 (1999); J. Park and S. Rey, J. High Energy Phys. **01**, 001 (1999).
[44] R. Kallosh, ''Black holes, branes and superconformal symmetry,'' hep-th/9901095.