Abrikosov string in $\mathcal{N}=2$ supersymmetric QED

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We study the Abrikosov-Nielsen-Olesen string in $\mathcal{N}=2$ supersymmetric QED with $(\mathcal{N}=2)$-preserving superpotential, in which case the Abrikosov string is found to be 1/2-BPS saturated. Adding a quadratic small perturbation in the superpotential breaks $\mathcal{N}=2$ supersymmetry to $\mathcal{N}=1$ supersymmetry. Then the Abrikosov string is no longer BPS saturated. The difference between the string tensions for the non-BPS and BPS saturated situation is found to be negative to first order of the perturbation parameter.

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I. INTRODUCTION

The simplest theory where saturated strings exist in the weak coupling regime is supersymmetric electrodynamics with the Fayet-Iliopoulos term [1]. Topologically stable solutions in this model and its modifications were considered more than once in the past [1–4]. If we consider the $\mathcal{N}=2$ supersymmetric Yang-Mills theory softly broken near the monopole or dyon singularities, Abrikosov strings develop [5]. They have been discussed in the literature [6–9] previously. We can see them in the effective Lagrangians near the singularities [3], where the superpotential for the monopoles (or dyons) can be written as

$$\mathcal{W}=\mu u(a_D)+\sqrt{2}\tilde{M}a_D M, \tag{1}$$

where M is the monopole field. Minimization of the potential yields the monopole condensation; as a result, the standard Abrikosov-Nielsen-Olesen string (ANO string) appears with a tension proportional to the mass of the $\mathcal{N}=1$ chiral field μ.

We can expand the $u(a_D)$ term in Eq. (1) as

$$\mu u(a_D)=-\mu a_D+\eta a_D^2+\cdots,$$

where the zeroth-order approximation corresponds to the linear term in a_D which preserves $\mathcal{N}=2$ supersymmetry [7]. Our task is to find the correction due to the quadratic term in a_D. In Sec. II we investigate supersymmetric electrodynamics (SQED) with only a linear term in a_D. The Abrikosov strings in this case are found to be 1/2 Bogomol'nyi-Prasad-Sommerfield (BPS) saturated. In Sec. III, we consider the additional quadratic term in the superpotential which destroys the BPS property. And the correction in the string tension in this case is calculated numerically to the first order of the perturbation parameter.

II. $\frac{1}{2}$-BPS SATURATED ABRIKOSOV STRINGS

Consider $\mathcal{N}=2$ supersymmetric electrodynamics [5]. The "photon" A_μ is accompanied by its $\mathcal{N}=2$ superpartners (photinos)—two neutral Weyl spinors λ and ψ, and a complex neutral scalar a. They form an irreducible $\mathcal{N}=2$ representation that can be decomposed as a sum of two $\mathcal{N}=1$ representations: a and ψ are in a chiral representation Φ, while A_μ and λ are in a vector representation W_α. The matter sector consists of two $\mathcal{N}=1$ chiral multiplets M and $\tilde{M}$ with opposite electric charge. In summary, the field content of each superfield is as follows:

$$\begin{array}{llll} \Phi: & a, & \psi, & F \\ W_\alpha: & A_\mu, & \lambda, & D \\ M: & M, & \psi_M, & F_M \\ \tilde{M}: & \tilde{M}, & \psi_{\tilde{M}}, & F_{\tilde{M}}. \end{array}$$

The renormalizable $\mathcal{N}=2$ invariant Lagrangian is described in an $\mathcal{N}=1$ language by canonical kinetic terms and minimal gauge couplings for all the fields as well as a superpotential

$$\mathcal{W}=\sqrt{2}\Phi M\tilde{M}-\mu\Phi; \tag{2}$$

here we replace the a_D in Eq. (1) by Φ for simplicity.

The Lagrangian in component fields is given by

$$\begin{aligned}\mathcal{L}=&-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\partial^\mu\bar{a}\partial_\mu a+i\lambda\sigma^\mu\partial_\mu\bar{\lambda}+i\psi\sigma^\mu\partial_\mu\bar{\psi}\\ &+D_\mu\bar{M}D^\mu M+D_\mu\bar{\tilde{M}}D^\mu\tilde{M}+i\psi_M\sigma^\mu D_\mu\bar{\psi}_M\\ &+i\psi_{\tilde{M}}\sigma^\mu D_\mu\bar{\psi}_{\tilde{M}}+(\sqrt{2}i\psi_M\lambda\bar{M}+\text{H.c.})\\ &+(-\sqrt{2}i\psi_{\tilde{M}}\lambda\bar{\tilde{M}}+\text{H.c.})+(\sqrt{2}aMF_{\tilde{M}}-\sqrt{2}a\psi_M\psi_{\tilde{M}}\\ &+\sqrt{2}aF_M\tilde{M}-\sqrt{2}\psi\psi_{\tilde{M}}M-\sqrt{2}\psi\psi_M\tilde{M}+\sqrt{2}FM\tilde{M}\\ &-\mu F+\text{H.c.})+F\bar{F}+\frac{1}{2}D^2+D(\bar{M}M-\bar{\tilde{M}}\tilde{M})\\ &+F_M\bar{F}_M+F_{\tilde{M}}\bar{F}_{\tilde{M}}.\end{aligned} \tag{3}$$

Here we set the coupling constant $e=1$ for the convenience, which is not important in our later results. We stick to this convention in what follows (see the discussion in Sec. IV). Here F, F_M, $F_{\tilde{M}}$, and D are the auxiliary fields which are given by

$$\begin{aligned}F&=\mu-\sqrt{2}\bar{M}\bar{\tilde{M}},\\ F_M&=-\sqrt{2}\bar{a}\bar{\tilde{M}},\\ F_{\tilde{M}}&=-\sqrt{2}\overline{aM},\\ D&=-(\bar{M}M-\bar{\tilde{M}}\tilde{M}).\end{aligned} \tag{4}$$

Here $M,\tilde{M}$ are the lowest components of the corresponding superfields, respectively, with the electric charges ± 1, e.g.,

$$D_\mu M = \partial_\mu M - iA_\mu M, \quad D_\mu \tilde{M} = \partial_\mu \tilde{M} + iA_\mu \tilde{M}.$$

The scalar potential is minimized at

$$F = F_M = F_{\tilde{M}} = 0, \quad D = 0, \tag{5}$$

which occurs when

$$a = 0, \quad M\tilde{M} = \frac{\mu}{\sqrt{2}}, \quad \text{and} \quad |M| = |\tilde{M}|. \tag{6}$$

Then the supersymmetric transformations which preserve $\mathcal{N}=2$ supersymmetry are given by

$$\begin{aligned}
\delta a &= \sqrt{2}\xi\lambda + \sqrt{2}\varepsilon\psi,\\
\delta\psi &= i\xi D - i\xi\sigma^{\mu\nu}F_{\mu\nu} + \sqrt{2}\varepsilon F + \sqrt{2}i\sigma^\mu\overline{\varepsilon}\partial_\mu a,\\
\delta F &= -\sqrt{2}i\xi\sigma^\mu\partial_\mu\overline{\lambda} + \sqrt{2}i\partial_\mu\psi\sigma^\mu\overline{\varepsilon},\\
\delta A_\mu &= -i\xi\sigma_\mu\overline{\psi} + i\psi\sigma_\mu\overline{\xi} - i\varepsilon\sigma_\mu\overline{\lambda} + i\lambda\sigma_\mu\overline{\varepsilon},\\
\delta\lambda &= \sqrt{2}\xi\overline{F} + \sqrt{2}i\sigma^\mu\overline{\xi}\partial_\mu a + iD\varepsilon + i\sigma^{\mu\nu}F_{\mu\nu}\varepsilon,\\
\delta D &= \partial_\mu\psi\sigma^\mu\overline{\xi} + \xi\sigma^\mu\partial_\mu\overline{\psi} + \partial_\mu\lambda\sigma^\mu\overline{\varepsilon} + \varepsilon\sigma^\mu\partial_\mu\overline{\lambda},\\
\delta M &= \sqrt{2}\varepsilon\psi_M + \sqrt{2}\overline{\xi\psi_{\tilde{M}}},\\
\delta\tilde{M} &= \sqrt{2}\overline{\xi\psi_M} + \sqrt{2}\varepsilon\psi_{\tilde{M}},\\
\delta\psi_M &= \sqrt{2}\varepsilon F_M + \sqrt{2}i\sigma^\mu\overline{\varepsilon}D_\mu M + \sqrt{2}\sigma^\mu\overline{\xi}D_\mu\overline{\tilde{M}} - 2i\overline{a}\xi M,\\
\delta\psi_{\tilde{M}} &= \sqrt{2}\sigma^\mu\overline{\xi}D_\mu\overline{M} + \sqrt{2}i\sigma^\mu\overline{\varepsilon}D_\mu\tilde{M} + \sqrt{2}\varepsilon F_{\tilde{M}} - 2i\overline{a}\xi\tilde{M},\\
\delta F_M &= \sqrt{2}iD_\mu\psi_M\sigma^\mu\overline{\varepsilon} - 2\overline{\xi\lambda\tilde{M}} - 2i\overline{a}\xi\psi_M,\\
\delta F_{\tilde{M}} &= \sqrt{2}iD_\mu\psi_{\tilde{M}}\sigma^\mu\overline{\varepsilon} - 2\overline{\xi\lambda M} - 2i\overline{a}\xi\psi_{\tilde{M}},
\end{aligned} \tag{7}$$

where the spinorial indices are suppressed.

Without loss of generality, we can assume that the Abrikosov string axis lies along the z axis, while the string profile depends only on x,y. Then we obtain the saturation equations by requiring the fermionic field transformations in Eqs. (7) to vanish as follows:

$$\begin{aligned}
F_{12} &= \sqrt{2}(\sqrt{2}\overline{M}\overline{\tilde{M}} - \mu),\\
(D_1 + iD_2)M &= 0,\\
(D_1 - iD_2)\tilde{M} &= 0,
\end{aligned} \tag{8}$$

with the constraint determining the parameter of the residual supersymmetry,

$$i\tau_3\xi = \varepsilon, \tag{9}$$

which reduces the number of supersymmetries from 8 to 4. The Abrikosov-Nielsen-Olesen string is 1/2-BPS saturated.

The ansatz which goes through Eq. (8) is

$$\begin{aligned}
M &= \left(\frac{\mu}{\sqrt{2}}\right)^{1/2} e^{i\phi} f(r),\\
\tilde{M} &= \left(\frac{\mu}{\sqrt{2}}\right)^{1/2} e^{-i\phi} f(r),\\
A_\phi &= -2\frac{g(r)}{r},
\end{aligned} \tag{10}$$

with the boundary conditions

$$\begin{aligned}
&f(0) = g(0) = 0,\\
&\lim_{r\to\infty} f(r) = 1,\\
&\lim_{r\to\infty} g(r) = -\frac{1}{2}.
\end{aligned}$$

The profile functions $f(r)$ and $g(r)$ satisfy the first-order differential equations

$$\begin{aligned}
f' &= \frac{f}{r}(1+2g),\\
g' &= \frac{1}{2}r(1-f^2)\sqrt{2}\mu,
\end{aligned} \tag{11}$$

where a prime denotes differentiation over r.

One can calculate the string tension as follows:

$$\begin{aligned}
\mathcal{T} = \int d^2x \Big\{ & \frac{1}{2}F_{12}^2 + D_1\overline{M}D_1M + D_2\overline{M}D_2M + D_1\overline{\tilde{M}}D_1\tilde{M}\\
& + D_2\overline{\tilde{M}}D_2\tilde{M} + (\mu - \sqrt{2}\overline{M}\overline{\tilde{M}})(\mu - \sqrt{2}M\tilde{M})\\
& + \frac{1}{2}(\overline{M}M - \overline{\tilde{M}}\tilde{M})^2 \Big\}\\
= \int d^2x \Big\{ & \left| \frac{1}{\sqrt{2}}F_{12} - (\sqrt{2}\overline{M}\overline{\tilde{M}} - \mu) \right|^2 + (D_1 + iD_2)\\
& \times \overline{M}(D_1 + iD_2)M + (D_1 - iD_2)\overline{\tilde{M}}(D_1 - iD_2)\tilde{M}\\
& + \frac{1}{2}(\overline{M}M - \overline{\tilde{M}}\tilde{M})^2 - \sqrt{2}\mu F_{12} - i[\partial_1(\overline{M}D_2M)\\
& - \partial_2(\overline{M}D_1M)] + i[\partial_1(\overline{\tilde{M}}D_2\tilde{M}) - \partial_2(\overline{\tilde{M}}D_1\tilde{M})] \Big\}.
\end{aligned} \tag{12}$$

Applying Eq. (8) and neglecting the total derivative terms, we get

$$\mathcal{T}=-\sqrt{2}\mu\int d^2x\, F_{12}. \tag{13}$$

III. SMALL PERTURBATION IN THE SUPERPOTENTIAL

We can add a small perturbation in the superpotential, Eq. (2),

$$\mathcal{W}=\sqrt{2}\Phi M\tilde{M}-\mu\Phi+\eta\Phi^2, \tag{14}$$

where η is a real small perturbation parameter.

Then we can go over the analysis in Sec. II in a similar way. But the small perturbation will break $\mathcal{N}=2$ supersymmetry and the resultant Abrikosov string is no longer BPS saturated which can be seen clearly in the string tension

$$\begin{aligned}\mathcal{T}=&\int d^2x\bigg\{\frac{1}{2}F_{12}^2+D_1\bar{M}D_1M+D_2\bar{M}D_2M+D_1\bar{\tilde{M}}D_1\tilde{M}+D_2\bar{\tilde{M}}D_2\tilde{M}+(\mu-\sqrt{2}\bar{M}\bar{\tilde{M}}-2\eta\bar{a})(\mu-\sqrt{2}M\tilde{M}-2\eta a)\\&+\frac{1}{2}(\bar{M}M-\bar{\tilde{M}}\tilde{M})^2+\partial_1\bar{a}\partial_1a+\partial_2\bar{a}\partial_2a+2a\bar{a}M\bar{M}+2a\bar{a}\tilde{M}\bar{\tilde{M}}\bigg\}\\=&\int d^2x\bigg\{\left|\frac{1}{\sqrt{2}}F_{12}-(\sqrt{2}\bar{M}\bar{\tilde{M}}-\mu)\right|^2+(D_1+iD_2)\bar{M}(D_1+iD_2)M+(D_1-iD_2)\bar{\tilde{M}}(D_1-iD_2)\tilde{M}\\&+\frac{1}{2}(\bar{M}M-\bar{\tilde{M}}\tilde{M})^2-\sqrt{2}\mu F_{12}-i[\partial_1(\bar{M}D_2M)-\partial_2(\bar{M}D_1M)]+i[\partial_1(\bar{\tilde{M}}D_2\tilde{M})-\partial_2(\bar{\tilde{M}}D_1\tilde{M})]+\partial_1(\bar{a}\partial_1a)\\&+\partial_2(\bar{a}\partial_2a)-2\eta a(\mu-\sqrt{2}\bar{M}\bar{\tilde{M}})\bigg\},\end{aligned} \tag{15}$$

where we have used the equation of motion for field a:

$$-\partial_1^2a-\partial_2^2a=2\eta(\mu-\sqrt{2}M\tilde{M}-2\eta a)-2aM\bar{M}-2a\tilde{M}\bar{\tilde{M}}. \tag{16}$$

To the first order of η, we can still use the ansatz (10) for the fields M, $\tilde{M}$, and A_μ. After applying ansatz (10), Eq. (16) becomes, to first order in η,

$$\partial_1^2a+\partial_2^2a=-2\eta\mu(1-f^2)+2\sqrt{2}\mu af^2. \tag{17}$$

Then the string tension turns out to be

$$\mathcal{T}=\int d^2x\{(-\sqrt{2}\mu F_{12})-2\eta a(\mu-\sqrt{2}\bar{M}\bar{\tilde{M}})\}. \tag{18}$$

Then from Eqs. (13),(18), we can find the difference of the string tensions between non-BPS and BPS saturated situation to be

$$\begin{aligned}\Delta\mathcal{T}&=\int d^2x\{-2\eta a(\mu-\sqrt{2}\bar{M}\bar{\tilde{M}})\}\\&=-2\eta\mu\int d^2x\, a(1-f^2),\end{aligned} \tag{19}$$

where we have used the ansatz (10).

To see Eq. (19) more clearly, one can switch to dimensionless quantities

$$\begin{aligned}x&\to\frac{1}{(\sqrt{2}\mu)^{1/2}}x,\\y&\to\frac{1}{(\sqrt{2}\mu)^{1/2}}y,\\a&\to\eta a.\end{aligned} \tag{20}$$

Then we get

$$\Delta\mathcal{T}=-\sqrt{2}\eta^2\int a(1-f^2)d^2x, \tag{21}$$

where a, f, and x here are dimensionless.

We can solve Eqs. (11),(17), and calculate $\Delta\mathcal{T}$ in Eq. (21). The result is

$$\Delta\mathcal{T}=-2\sqrt{2}\pi\eta^2 0.68<0. \tag{22}$$

IV. CONCLUSIONS

We investigated the Abrikosov-Nielsen-Olesen string solution in $\mathcal{N}=2$ supersymmetric electrodynamics with some $(\mathcal{N}=2)$-preserving superpotential. The string solution is due to the superpotential rather than due to the Fayet-Iliopoulos term. The Abrikosov string was found to be 1/2-BPS saturated which follows directly from the $\mathcal{N}=2$ supersymmetric transformations. After the $\mathcal{N}=2$ supersymmetry is broken to $\mathcal{N}=1$ by the perturbation in the superpotential, the Abrikosov string is no longer BPS saturated. And the string tension in this case was found to be less than that of the BPS case. Here, we only investigate the ANO string with winding number n equal to 1. For winding number $n>1$, the whole process is almost the same just with the ansatz (10) modified by

$$M=\left(\frac{\mu}{\sqrt{2}}\right)^{1/2} e^{in\phi} f(r),$$

$$\tilde{M}=\left(\frac{\mu}{\sqrt{2}}\right)^{1/2} e^{-in\phi} f(r),$$

$$A_{\phi}=-2n\frac{g(r)}{r}.$$

For example, for $n=2$, we get finally

$$\Delta T_{n=2}=-2\sqrt{2}\pi\eta^2\times 7.35. \tag{23}$$

Note that we have

$$T_{n=2}<2T_{n=1}. \tag{24}$$

In fact, this relationship holds for winding number $n>1$; just replace the number 2 above with n. Actually, in this case, the Higgs boson mass in the effective Abelian Higgs model is less than the photon mass [10], which means that the theory has type I superconductivity. In a type I superconductor, the tension of the vortex with winding number n is less than the sum of the tensions of n vortices with winding number $n=1$. Therefore, the vortices with $n>1$ are stable.

As we mentioned before, we have set the coupling constant $e=1$ for convenience, which is not important for our result (22). However, if we increase the perturbation parameter η, the strings with winding number $n>1$ can be broken by W-boson production at large η. Furthermore, at large η, the $N=2$ supersymmetric QED at hand enters the strong coupling regime and is no longer under control. In this case, the monopoles which play the role of matter fields can hardly be considered as local degrees of freedom and the effective $N=2$ supersymmetric QED description breaks down [10].

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