Generation of high frequency gravitational waves

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An electric charge shaken in an homogeneous stationary magnetic field produces both electromagnetic and gravitational waves (Gertsenshtein waves). We first study this process in a homogeneous medium with a refractive index n, expressing the gravitational wave as a simple functional of the electromagnetic field. Then, an inhomogeneous dielectric is considered: under certain conditions, the diffraction of a plane electromagnetic wave by a small dielectric sphere produces a promising gravitational energy flux.

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I. INTRODUCTION

Some time ago Gertsenshtein [1] discussed the phenomenon known as the resonance of light and gravitational waves. A monochromatic electromagnetic wave, of frequency ω, propagating through a stationary and uniform magnetic field produces a gravitational wave with the same frequency ω and with an amplitude proportional to the distance traveled in the magnetic field. The source of this gravitational field is the mixed stationary-radiative term of the energy momentum tensor of the total e.m. field. This term is formed by the static and radiative magnetic fields. A brief review of this process, and the inverse one, may be found in the book by Zel'dovich and Novikov [2].

The process was considered in cosmological and astrophysical contexts, leading to unobservable predictions. It was also considered as a method for laboratory generation of high frequency gravitational waves: synchroton radiation by ultrarelativistic particles moving in a constant magnetic field [3], or in the Coulomb field of a huge charge [4]. Expressions for the gravitational energy flux were derived in both cases, and the Gertsenshtein contributions to the total energy flux were bigger than the mass contribution when the γ relativistic factor was great enough.

In Sec. III, we revise the classical calculation of the gravitational waves (Gertsenshtein waves) emitted by variable electric currents in a static magnetic field, considering a homogeneous medium with electrical permittivity ϵ, instead of the vacuum. We express the result as a simple functional of the radiative magnetic field. As expected, the results are compatible with those obtained in previous work [1,3,4]. Section III A is devoted to the emission of Gertsenshtein waves by a dielectric sphere irradiated by a plane e.m. wave, obtaining estimates which seem promising for generation of detectable high frequency gravitational waves in the laboratory.

II. THE BASIC EQUATIONS

Let us denote by h the magnetic field radiated by a variable current in a medium of refractive index n covered by a static magnetic field H. The Maxwell's tensor T_f of the total e.m. field splits in three terms: $T_f = T_1 + T_2 + T_{12}$. The first one corresponds to the radiated electromagnetic field, the second to the static field, and the third to the mixed term depending on the radiative and static fields. The total energy momentum tensor (matter and field) $T_m + T_f$ produces a weak gravitational field. Using harmonic coordinates, the space components of the linearized Einstein's equations may be written in the form: $\Box_1 v^{ij} = 16\pi G(T_m^{ij} + T_f^{ij})$, where $\Box_1 = -\partial_t^2 + \Delta$ is the d'Alembertian operator in the vacuum, and $v^{ij}, i,j = 1,2,3$ is the first order correction to the space components of the metric density tensor $\sqrt{-g}g^{ij} = \delta^{ij} + v^{ij}$. We are interested here in the contribution of the mixed term: $T_{12}^{ij} = (1/4\pi)(H \cdot h\,\delta^{ij} - H^i h^j - h^i H^j)$ (the only one in T_f that produces a gravitational wave) , i.e., we shall study the solutions to

$$\Box_1 v^{ij} = -4G(H^i h^j + h^i H^j - H \cdot h\,\delta^{ij}). \tag{1}$$

The solution to the wave equation, with null initial conditions, is given by Kirchhoff's formula, in the form of a retarded integral. In this section we shall use a different method, which allows the solution to be expressed more explicitly. The method is based on the fact that the e.m. field h satisfies a wave equation too:

$$\Box_n h = -4\pi\epsilon \,\mathrm{rot}\left(\frac{j}{\epsilon}\right) - \epsilon \,\mathrm{rot}\, h \wedge \mathrm{grad}(\epsilon^{-1}) \tag{2}$$

and the supplementary condition $\mathrm{div}(h) = 0$. We have introduced the operator

$$\Box_n = -n^2 \partial_t^2 + \Delta, \quad n = \sqrt{\epsilon}. \tag{3}$$

It is convenient to consider the second time derivatives of the metric, so let us define $R_{ij} = \partial_t^2 v_{ij}$. Then from Eq. (1) we get

$$\Box_1 R = -4G(H \tilde{\otimes} \partial_t^2 h - H \cdot \partial_t^2 h I) \tag{4}$$

where we have left out the index notation. The symbol $\tilde{\otimes}$ means symmetrized tensorial product $A \tilde{\otimes} B = A \otimes B + B \otimes A$, and I is the identity tridimensional tensor.

To calculate the energy transported by the gravitational waves, we shall follow Papapetrou's expressions [5]. Defining the tensor Ω by

$$\partial_t v^{ij} = \frac{4G}{r}\Omega^{ij} + O\left(\frac{1}{r^2}\right) \tag{5}$$

one constructs the transverse traceless component $\omega^{ij} = \Omega_{TT}^{ij}$

$$\Omega_{TT}^{ij} = \Omega_T^{ij} - \frac{1}{2} W^{ij} \Omega_T^{mm} \tag{6}$$

$$\Omega_T^{ij} = W^{im} W^{jn} \Omega^{mn} \tag{7}$$

where $W^{ij} = \delta^{ij} - \nu^i \nu^j$ is the projector on to the space orthogonal to the unit vector ν in the propagation direction. Then, the scalar $Q = \omega^{ij}\omega^{ij}$ gives the gravitational flux, $\mathcal{F}_G$, at distance r from the source

$$\mathcal{F}_G = \frac{GQ}{2\pi r^2}. \tag{8}$$

In Sec. IV A we shall use monocrhomatic fields, represented by the real part of complex fields. The matrix ω_{ij} is in this case of the form $\omega_{ij} = e^{-i\omega t} s_{ij}$. Then, averaging the gravitational flux over a wave period, one gets

$$F_G = \frac{G}{4\pi r^2} s_{ij} \overline{s^{ij}} \tag{9}$$

where the overbar denotes complex conjugate.

III. THE GERTSENSHTEIN EFFECT IN A HOMOGENEOUS DIELECTRIC

In this section we begin with the simplest case, that is when both the static magnetic field H and the electric permittivity ϵ are uniform. Let us consider an electric current j stationary for $t \leqslant 0$ that starts radiating e.m. waves at $t = 0$. We want to estimate to what extent the mixed radiative-static component of the total energy-momentum tensor contributes to the induced gravitational waves.

Taking into account the identity

$$\Box_1 = \Box_n + (n^2 - 1)\partial_t^2 \tag{10}$$

the following result is easily obtained:

Proposition 1. If the static magnetic field H and the electric permittivity ϵ are uniform, the field

$$R = -\frac{4G}{n^2 - 1}(H \tilde{\otimes} (h - \hat{h}) - H \cdot (h - \hat{h}) I) \tag{11}$$

is the solution to Eq. (4) with null initial conditions, provided the fields h and $\hat{h}$ are solutions, with null initial conditions, of the wave equations $\Box_n h = -4\pi \,\mathrm{rot}\, j$ and $\Box_1 \hat{h} = -4\pi \,\mathrm{rot}\, j$, respectively.

Equation (11) generalizes in a compact form, to the case $\epsilon \neq 1$, the results obtained by Pustovoit and Gertsenshtein for an accelerated charge in the vacuum [3].

Henceforth, we shall call Gertsenshtein waves the gravitational waves emitted by a variable electric current in presence of a uniform and static magnetic field.

A. Plane Gertsenshtein waves

Let (a_x, a_y, a_z) be Cartesian unit vector basis. To obtain the expression for a plane monochromatic Gertsenshtein wave, propagating in the direction of the unit 3-vector a_z, we substitute in Eq. (11)

$$h = A a_y e^{i\omega(t - nz)}, \quad \hat{h} = A e^{-i\beta_o} a_y e^{i\omega(t - z)} \tag{12}$$

where the unit vector a_y is the polarization of a plane e.m. wave of amplitude A and frequency ω. We have denoted by β_o the phase difference, in the plane $z = 0$, between the waves h and $\hat{h}$.

After a straightforward calculation one puts the Gertsenshtein wave R into the form

$$R = B e^{i\omega(t - nz - \beta)}(H \tilde{\otimes} a_y - H \cdot a_y I) \tag{13}$$

$$B = \frac{8\pi G A}{n^2 - 1} \sin\left(\frac{\omega(n-1)z - \beta_o}{2}\right) \tag{14}$$

$$\tan\beta = \frac{\sin(\omega(n-1)z - \beta_o)}{1 - \cos(\omega(n-1)z - \beta_o)}. \tag{15}$$

The amplitude of the wave is a periodic function of distance. If, for the sake of simplicity we choose $\beta_o = 0$, the amplitude is proportional to $\sin(\pi(n-1)z/\lambda)$, where λ is the wavelength. Therefore it reaches a maximum, equal to $8\pi GA/(n^2 - 1)$, after travelling a distance $z = \lambda/2(n-1)$. Gertsenshtein, in his first paper on this subject [1], considered the vacuum instead of a medium with refractive index $n \neq 1$. He obtained an amplitude growing linearly with distance. As it must be, this is in agreement with our result (13), if the distance travelled is small, as compared to $z = \lambda/2(n-1)$.

B. Detection of high frequency gravitational waves

The type of detector envisaged for these high frequency gravitational waves is based in the inverse Gertsenshtein effect. We shall summarize here the main idea concerning these detectors (see Ref. [10] for more details). Let us assume that inside an electromagnetic resonator there exist a strong static magnetic field. Under these conditions, a gravitational wave of wavelength λ and amplitude v excites the natural electromagnetic modes of the resonator $\lambda_1 = 2l > \lambda_2 > \lambda_3 \ldots$, where l is the linear dimension of the cavity. The effect is larger when the gravitational wavelength coincides with the first mode of the cavity, i.e, $\lambda = 2l$. The induced electromagnetic energy in the cavity does not continue to increase indefinitely with time, because the absorption process taking place in the walls is proportional to the electromagnetic energy already present in the cavity. It has been estimated [7] that after a time $\tau \approx \mathcal{Q}/\omega$, where $\mathcal{Q}$ is the quality factor of the cavity, the energy density of the induced radiation reaches the limit value $(v\mathcal{Q}H)^2/8\pi$. One assumes that the background is properly controlled so as to be able to distinguish the photons induced by the gravitational wave. Then, multiplying the limit energy density by the volume of the cavity $(\lambda/2)^3$ and dividing by $\hbar\omega$ one gets the maximun number of photons induced in the cavity:

$$N=\frac{v^2\mathcal{Q}^2H^2\lambda^4}{128\pi^2\hbar c}. \tag{16}$$

The authors of Ref. [10] considered an electromagnetic resonator as the generator of a high frequency gravitational waves. In this case the amplitude of the wave is proportional to length of the resonator, as well as to the static magnetic field. Part of this wave is presumed to be converted into photons inside another electromagnetic resonator (the antenna). They conclude, using the best feasible parameters, that after $\tau\approx 1000$ sec the number of photons generated in the antenna (10^{-5}) is many orders below the threshold (one photon).

IV. GRAVITATIONAL WAVES EMITTED BY A DIELECTRIC INHOMOGENEITY

We shall show that the efficiency of the Gertsenshtein effect can be dramatically improved when the dielectric is not homogeneous. The following discussion is valid for any compact inhomogenity but, as the calculation in the next subsection corresponds to a dielectric ball, we prefer here to make reference to a dielectric ball.

Let us consider the case where a plane electromagnetic wave propagating along the z-axis is incident on a dielectric ball of radius a and refractive index n_{II} embedded in a medium with index $n_I<n_{II}$, all covered by a uniform and static magnetic field H.

To obtain the gravitational waves originated in this way, we shall follow the method used in the previous section. So we start by considering the equation (2) for the e.m. field h. As the electric permittivity is now a discontinous function we shall write $\epsilon=n^2=n_I^2+\delta n^2 Y$, with $\delta n^2=n_{II}^2-n_I^2$, and Y is the characteristic function of the ball of radius a, i.e., $Y=1$ inside the ball and $Y=0$ elsewhere. Substituting this expression into Eq. (2) one gets

$$\Box_{n_I}h=F(h)$$

$$F(h)=-\delta n^2\partial_t e\wedge\mu\,\delta_S+\delta n^2 Y\partial_t^2 h \tag{17}$$

where μ is the sphere's unit normal vector directed towards the exterior, and δ_S is the Dirac distribution with support on the surface of the sphere.

We consider as initial instant the moment at which the plane wave front hits the dielectric ball. For $t\gg a/c$ all the transients disappears and the electromagnetic field tends to a well known result [6]. For $t<0$, we have, in according to Eqs. (11) and (12), a plane monochromatic Gertsenhstein wave:

$$R_o=-\frac{4G}{n_I^2-1}(H\tilde{\otimes}(h_o-\hat{h}_o)-H\cdot(h_o-\hat{h}_o)I) \tag{18}$$

with $h_o=Aa_ye^{i\omega(t-n_Iz)}$ and $\hat{h}_o=Ae^{-i\beta_o}a_ye^{i\omega(t-z)}$. Then, proceeding as in Sec. II one derives immediately the following:

Proposition 2. The field

$$R=-\frac{4G}{n_I^2-1}(H\tilde{\otimes}(h-\hat{h}_o-\hat{h}_s)-H\cdot(h-\hat{h}_o-\hat{h}_s)I) \tag{19}$$

where $\hat{h}_s$ is the solution of the equation

$$\Box_1\hat{h}_s=F(h)$$

$$F(h)=-\delta n^2\partial_t e\wedge\mu\,\delta_S+\delta n^2 Y\partial_t^2 h \tag{20}$$

with null initial conditions, is the unique solution of Einstein's equations (4) representing an incident Gertsenshtein's wave, R_o given by Eq. (18), on a dielectric sphere.

The proof is a direct consequence of the relations $\Box_1\hat{h}_o=0$ and $\Box_1=\Box_{n_I}+(n_I^2-1)\partial_t^2$.

So, as in the previous section, we have given the gravitational field as a functional of the electromagnetic field h. Then, it remains to determine the auxiliary field $\hat{h}_s$ for a given electromagnetic field h. For $t\gg a/c$, the field $\hat{h}_s$ will be given by the retarded integral

$$\hat{h}_s(t,x)=-\frac{\delta n^2}{4\pi}\int_B\frac{\partial_t^2 h(t-|x-x'|,x')}{|x-x'|}dV+\frac{\delta n^2}{4\pi}e^{-i\omega t}\int_S\frac{\partial_t e(t-|x-x'|,x')\wedge\mu}{|x-x'|}dS \tag{21}$$

where B is a ball of radius a, S its surface, and the electomagnetic fields h and e do not contain the transient modes.

In a recent meeting [8] we gave a wrong estimate of these integrals, but the conclusion obtained in this paper will be even more optimistic than it was then.

First of all, we prove that we can write Eq. (21) as a unique volume integral of the interior electric field. Without losing generality we shall consider monochromatic fields, of the form $h(t,x)=e^{-i\omega t}h(x)$, $e(t,x)=e^{-i\omega t}e(x)$. Let us introduce the function $\Theta(x-x')=e^{i\omega|x-x'|}/|x-x'|$, and write Eq. (21) in the form

$$\hat{h}_s(t,x)=\frac{\omega^2\delta n^2}{4\pi}e^{-i\omega t}\int_B\Theta(x-x')h(x')dV-\frac{i\omega\delta n^2}{4\pi}e^{-i\omega t}\int_S\Theta(x-x')e(x')\wedge\mu dS. \tag{22}$$

Using Gauss theorem, we convert the surface integral into a volume integral:

$$\int_S \Theta(x-x')e(x')\wedge\mu dS=\int_B e(x')\wedge\nabla'\Theta(x-x')dV - \int_B \Theta(x-x')\nabla'\wedge e(x')dV \tag{23}$$

and taking into account the Maxwell equation $\nabla\wedge e(x) = i\omega h(x)$ one realizes that the second term of this equation is just cancelled with the first term of Eq. (22), resulting

$$\hat{h}_s=-i\omega\frac{\delta n^2}{4\pi}e^{-i\omega t}\int_B e(x')\wedge\nabla'\Theta(x-x')dV \tag{24}$$

as we wanted to prove.

To facilitate the calculation, in the next section, of the flux of gravitational energy, we shall get the asymptotic expression for the auxiliary field $\hat{h}_s$. At great distance from the source, $r\gg a$, one has $|x-x'|=r-\nu\cdot x'$, where ν is the unit vector $\nu=x/r$. Substituting this into Eq. (24) one gets

$$\hat{h}_s=-\omega^2\frac{\delta n^2}{4\pi}\frac{e^{-i\omega(t-r)}}{r}\mathcal{F}e\{-\omega\nu\}\wedge\nu+O(1/r^2) \tag{25}$$

where $\mathcal{F}e\{-\omega\nu\}$ stands for the Fourier transform of the internal electric field

$$\mathcal{F}e\{-\omega\nu\}=\int_B e^{-i\omega\nu\cdot x'}e(x')dV. \tag{26}$$

A. Gravitational waves emitted by a small sphere

The diffraction of a plane electromagnetic wave by a dielectric ball is described in many books. We shall follow the notation and results contained in [9]. Here, as in Sec. II A, we shall denote a Cartesian unit basis by (a_x,a_y,a_z). Let us consider a plane e.m. wave propagating along the z-axis: $e_o(t,x)=e_o a_x e^{-i(\omega t-kz)}$, $h_o(t,x)=h_o a_y e^{-i(\omega t-kz)}$ with e_o and h_o the amplitudes of the electric and magnetic field respectively. The electric field inside the ball, denoted by $e(t,x)$, is given in terms of the vector spherical wave functions of first kind $m^{(1)}_{o1n}, n^{(1)}_{e1n}$:

$$e(t,x)=e_o e^{-i\omega t}\sum_{n=1}^{\infty} i^n\frac{2n+1}{n(n+1)}(a^t_n m^{(1)}_{o1n}-ib^t_n n^{(1)}_{e1n}) \tag{27}$$

where a^t_n and b^t_n are coefficients depending on the ball's refractive index n_2 and on the parameter $q=n_I ka =2\pi n_I a/\lambda$.

In a similar way, the diffracted magnetic field, denoted by $h_s(t,x)$, is given in terms of the vector spherical wave functions of third kind $m^{(3)}_{e1n}, n^{(3)}_{o1n}$:

$$h_s(t,x)=h_o e^{-i\omega t}\sum_{n=1}^{\infty} i^n\frac{2n+1}{n(n+1)}(b^r_n m^{(3)}_{e1n}+ia^r_n n^{(3)}_{o1n}). \tag{28}$$

The Fourier transform, Eq. (26), of the interior electric field may be written as follows:

$$\mathcal{F}e\{-\omega\nu\}=e_o e^{-i\omega t}\sum_{n=1}^{\infty} i^n\frac{2n+1}{n(n+1)}(a^t_n M_n-ib^t_n N_n) \tag{29}$$

where the vector functions M_n and N_n are the Fourier transform of the vector spherical wave functions of first kind

$$M_n=\int_B e^{-i\omega\nu\cdot x'}m^{(1)}_{o1n}(x')dV' \tag{30}$$

$$N_n=\int_B e^{-i\omega\nu\cdot x'}n^{(1)}_{e1n}(x')dV'. \tag{31}$$

Then, substituting into Eq. (25) we get the asymptotic expression, for $r\gg a$, of the auxiliary field $\hat{h}_s$

$$\hat{h}_s=-\omega^2 h_o\frac{\delta n^2}{4\pi}\frac{e^{-i\omega(t-r)}}{r}\times\sum_{n=1}^{\infty} i^n\frac{2n+1}{n(n+1)}(a^t_n M_n\wedge\nu-ib^t_n N_n\wedge\nu). \tag{32}$$

Henceforth, we shall consider the interesting case where n_I is very close to unity, which may be accomplished placing the dielectric ball inside a vacuum chamber. In the Appendix we give a summary of the calculation of the first summand of this series, i.e., the first partial wave. We shall denote it by $\hat{h}_{s1}$ and we obtain for it

$$\hat{h}_{s1}=-\frac{3ih_o}{2}\frac{e^{-i\omega(t-r)}}{kr}(\hat{a}^r_1(a_y\wedge\nu)\wedge\nu+\hat{b}^r_1 a_x\wedge\nu) \tag{33}$$

where the coefficients $\hat{a}^r_1$ and $\hat{b}^r_1$ are given by

$$\hat{a}^r_1=ia^t_1 q^2(n_2 j_2(n_2 q)j_1(q)-j_2(q)j_1(n_2 q)) \tag{34}$$

$$\hat{b}^r_1=ib^t_1\frac{\delta n^2}{n_2}j_1(n_2 q)q\sin q+ib^t_1\frac{q^2}{3}\times(n_2 j_3(n_2 q)j_2(q)-j_3(q)j_2(n_2 q))+ib^t_1\frac{q^2}{3}\times\left(3j_1(q)j_2(n_2 q)-\frac{3}{n_2}j_1(n_2 q)j_2(q)\right)+ib^t_1\frac{q^2}{3}\times(-n_2 j_1(n_2 q)j_o(q)+j_1(q)j_o(n_2 q)) \tag{35}$$

where $k=\omega/c$ stands for the wave number, $q=ka$ and j_n are the spherical Bessel functions of the first kind.

That is all about the auxiliary field $\hat{h}_s$. Now, we need the first partial wave of the diffracted magnetic field h_s, which will be denoted by h_{s1}. From Eq. (28) we get its asymptotic expression, valid for $r\gg a$:

$$h_{s1}=-\frac{3ih_o}{2}\frac{e^{-i\omega(t-r)}}{kr}(a_1^r(a_y\wedge\nu)\wedge\nu)+b_1^r a_x\wedge\nu). \tag{36}$$

Next we calculate the difference $h_{s1}-\hat{h}_{s1}$:

$$h_{s1}-\hat{h}_{s1}=-\frac{3ih_o}{2}\frac{e^{-i\omega(t-r)}}{kr}d \tag{37}$$

where we have introduced the vector field d

$$d=(a_1^r-\hat{a}_1^r)(a_y\wedge\nu)\wedge\nu+(b_1^r-\hat{b}_1^r)a_x\wedge\nu. \tag{38}$$

In order to obtain R , given by Eq. (19), outside the dielectric ball, we substitute $h=h_o+h_s$ and one gets R_o+R_s, where R_o is a plane monochromatic Gertsenshtein wave as described by Eq. (18) and R_s is given by

$$R_s=-\frac{4G}{n_I^2-1}(H\tilde{\otimes}(h_s-\hat{h}_s)-H\cdot(h_s-\hat{h}_s)I). \tag{39}$$

The amplitude of R_o is negligible as compared with the one of R_s, if the refractive index n_I is close to unity. Henceforth, we will omit the incident Gertsenshtein wave. Substituting Eq. (37) into Eq. (39) and integrating twice with respect the time-like coordinate (remember that $R=\partial_t^2 v$), we get the first partial wave contribution, v_1, to the linearized gravitational field:

$$v_1=-\frac{6iGh_o e^{-i(\omega t-kr)}}{c^4(n_I^2-1)k^3r}(H\tilde{\otimes}d-d\cdot HI) \tag{40}$$

where $c.g.s$ unities have been restored.

The corresponding flux of energy, averaged over a period of the gravitational wave, is obtained from Eq. (9)

$$F_G=\frac{9GH_T^2d^2F_E}{c^4(n^2-1)^2k^4r^2} \tag{41}$$

where F_E is the incident electromagnetic flux and H_T is the projection of the 3-vector H on to the subspace orthogonal to ν, i.e., $H_T=H-(H\cdot\nu)\nu$. The angular dependence is contained in the product $H_T^2\ d^2$.

Let us draw about the diffracting sphere a concentric spherical surface of radius r. Assuming the static magnetic field orthogonal to the z-axis and bisecting the coordinate plane (a_x,a_y), we integrate the gravitational flux over the sphere and we get the gravitational luminosity (erg/sec).

$$L_G=\frac{84\pi GH^2\Delta F_E}{5c^4(n_I^2-1)^2k^4} \tag{42}$$

where the dimensionless parameter Δ has been introduced

$$\Delta=|a_1^r-\hat{a}_1^r|^2+|b_1^r-\hat{b}_1^r|^2. \tag{43}$$

This parameter depends on the refractive index of the ball as well as on the dimensionless parameter $q=ka$. Admitting complex values for the refractive index one can extend these results to nontransparent mediums [6].

TABLE I. The first maximum of the function $\Delta_{n_2}(q)$ for different refractive indexes.

n_2	1.25	10	3+3.45i	10+4i
q	5	0.41	1.2	1.8
Δ	0.6	17	14	900

B. On the possibility of observing these waves

In order to make order of magnitude estimates in a detector, we associate the gravitational luminosity L_G produced by the irradiated ball with a plane gravitational wave with a flux of energy equal to $L_G/4\pi r^2$, being r the distance from the detector to the ball. The amplitude of this wave is

$$v=\left(\frac{4GL_G}{c^3\omega^2r^2}\right)^{1/2}. \tag{44}$$

Hence, according to Eq. (16), the number of photons generated in the cavity is

$$N=2.72\times10^{-6}\frac{G^2\mathcal{Q}^2\lambda^{10}H^4F_E\Delta}{\hbar c^{10}(n_I^2-1)^2r^2}. \tag{45}$$

Taking optimistic values for the involved parameters we get for the number of photons generated in the cavity

$$N=0.13\Delta\left(\frac{\lambda}{22\ \ \text{cm}}\right)^{10}\left(\frac{H}{10^5\ \ \text{g}}\right)^4\left(\frac{100\ \ \text{cm}}{r}\right)^2\left(\frac{10^{-17}}{n_I-1}\right)^2 \times\left(\frac{\mathcal{Q}}{10^{12}}\right)^2\frac{F_E}{1\ \ \text{KW/cm}^2}. \tag{46}$$

Let us justify why we expect for n_I-1 so a small value as 10^{-17}. The refractive index of the air above earth surface is very close to unity. For microwaves one uses the formula [11]

$$n-1=10^{-6}p\left(\frac{77.6}{T}+600\frac{S}{T^2}\right) \tag{47}$$

with p in milibars, T in Kelvin degrees and S the specific humidity in grammes of water per Kg of air. For standard conditions one gets $n-1=0.0003$. Now, it seems possible to get a vacuum of 10^{-10} Tors, and that amounts to $n-1\approx3\times10^{-17}$.

We have studied the function $\Delta_{n_2}(q)=\Delta(n_2,q)$ for different values of n_2. A sequence of maxima appears at $q=q_1<q_2<q_3\ldots$. The position q_1 of the first maximum and the corresponding value of Δ is shown in Table I.

By way of illustration we consider one case. A material with refractive index of the order of ten (for example Titania D-100), inside a void chamber at 10^{-10} Torrs, irradiated with an e.m. flux of 10 KW/cm^2 and $\lambda=22$ cm, will get the detectability threshold. As, for this value of n_2, the maximum of Δ appears at $q=0.41$, the radius of the dielectric ball ($a=q\lambda/2\pi$) is not too large: 1.4 cm.

We shall not dwell more on this subject because we think, and we are working on this now, that one can improve the efficiency of the detection process doing otherwise.

APPENDIX

As it is shown in Eq. (36), the first partial wave, h_{s1}, of the diffracted electromagnetic field depends on the coefficients a_1^r and b_1^r. Similarly, the first partial wave contribution to the gravitational wave has been written in terms of the coefficients $\hat{a}_1^r$, $\hat{b}_1^r$ [see Eqs. (38) and (40)]. Let us summarize how these ones may be obtained. In Eq. (32) we have given the development in partial waves of the auxiliary field $\hat{h}_s$. Denoting the first summand as $\hat{h}_{s1}$ we obtain

$$\hat{h}_{s1}=-3i\omega^2 h_o \frac{\delta n^2}{8\pi}\frac{e^{-i\omega(t-r)}}{r}(a_1^t M_1\wedge\nu - i b_1^t N_1\wedge\nu). \tag{A1}$$

The 3-vector functions M_1, N_1, according to Eqs. (30) and (31), are the Fourier transform of the first kind spherical waves, and these ones may be obtained from [9]:

$$m_{011}^1=-j_1(k_2 r)a_y\wedge\nu \tag{A2}$$

$$n_{e11}^1=-j_1(k_2 r)a_x\cdot\nu\nu-\frac{[k_2 r j_1(k_2 r)]'}{k_2 r}a_x \tag{A3}$$

where $k_2=n_2 k=n_2\omega/c$, and the prime denotes differentiation with respect the argument $k_2 r$. Multiplying its Fourier transforms by the unit vector ν one gets

$$M_1\wedge\nu=-J_1(a_y\wedge\nu)\wedge\nu \tag{A4}$$

$$N_1\wedge\nu=-(J_o+J_2)a_x\wedge\nu \tag{A5}$$

where J_a, $a=0,1,2$ are the following multiple integrals:

$$J_o=\int e^{-i\omega|\xi|\nu\cdot\nu_\xi}\frac{[k_2|\xi| j_1(k_2|\xi|)]'}{k_2|\xi|}d^3\xi, \tag{A6}$$

$$J_1=\int e^{-i\omega|\xi|\nu\cdot\nu_\xi} j_1(k_2|\xi|)\nu\cdot\nu_\xi d^3\xi, \tag{A7}$$

$$J_2=\frac{1}{2}\int e^{-i\omega|\xi|\nu\cdot\nu_\xi} j_2(k_2|\xi|)(1-(\nu\cdot\nu_\xi)^2)d^3\xi. \tag{A8}$$

This allows us to write

$$\hat{h}_{s1}=-3i\omega^2 h_o\frac{\delta n^2}{8\pi}\frac{e^{-i\omega(t-r)}}{r}\times(-a_1^t J_1(a_y\wedge\nu)\wedge\nu + i b_1^t(J_0+J_2)a_x\wedge\nu). \tag{A9}$$

In the last section we wrote the auxiliary field $\hat{h}_{s1}$ in the form

$$\hat{h}_{s1}=-\frac{3ih_o}{2}\frac{e^{-i\omega(t-r)}}{kr}(\hat{a}_1^r(a_y\wedge\nu)\wedge\nu+\hat{b}_1^r a_x\wedge\nu). \tag{A10}$$

So, comparing with the previous equation we get the coefficients

$$\hat{a}_1^r=-\frac{\omega^3\delta n^2}{4\pi}a_1^t J_1 \tag{A11}$$

$$\hat{b}_1^r=i b_1^t\frac{\omega^3\delta n^2}{4\pi}(J_o+J_2). \tag{A12}$$

It is remarkable that the multiple integrals J_a can be expressed in terms of Bessel's functions. A straightforward calculation reduces them first to unidimensional integrals of products of Bessel's functions

$$J_1=-4\pi i I_1 \tag{A13}$$

$$J_o+J_2=\frac{4\pi a}{kk_2}j_1(k_2 a)\sin ka-\frac{4\pi}{3}I_o+\frac{4\pi}{3}I_2+\frac{4\pi}{n_2}I_1 \tag{A14}$$

$$I_n=\int_0^a r^2 j_n(k_2 r)j_n(kr)dr,\quad n=0,1,2 \tag{A15}$$

and using well known properties of integrals of Bessel's functions one writes I_n, $n=0,1,2$, in terms of products of Bessel's functions:

$$I_n=\frac{a^2}{k_2^2-k^2}(k_2 j_{n+1}(k_2 a)j_n(ka)-k j_{n+1}(ka)j_n(k_2 a)). \tag{A16}$$

Substituting into Eqs. (A11) and (A12), one gets the expressions (34) and (35) for the coefficients $\hat{a}_1^r$ and $\hat{b}_1^r$.

[1] M. E. Gertsenshtein, Zh. Éksp. Teor. Fiz. **41**, 113 (1961) [Sov. Phys. JETP **14**, 84 (1962)].

[2] Ya. B. Zel'dovich and I. D. Novikov, *Relativistic Astrophysics* (The University of Chicago Press, Chicago, 1983), Vol. 2.

[3] V. I. Pustovoit and M. E. Gertsenshtein, Zh. Éksp. Teor. Fiz. **42**, 499 (1962) [Sov. Phys. JETP **15**, 116 (1962)].

[4] O. P. Sushkov and I. B. Khriplovich, Zh. Éksp. Teor. Fiz. **66**, 3 (1974) [Sov. Phys. JETP **39**, 1 (1974)].

[5] A. Papapetrou, *Lectures on General Relativity* (Reidel, Dordrecht, 1974).

[6] M. Born and E. Wolf, *Principles of Optics* (Pergamon, New York, 1991).

[7] L. P. Grishchuk and A. G. Polnarev, in *General Relativity and Gravitation*, edited by A. Held (Plenum Press, New York, 1980).
[8] M. Portilla and R. Lapiedra, *Proceedings of the Spanish Relativity Meeting in Honour of Lluís Bel* (World Scientific, Singapore, 1999).
[9] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941).
[10] V. B. Braginski, L. P. Grishchuk, A. G. Doroshkievich, Ya. B. Zel'dovich, I. D. Novikov, and M. V. Sazhin, in *Gravitational Radiation and Gravitational Collapse*, International Astronomical Union, Symposium No. 64 (Reidel, Dordrecht, 1974).
[11] L. S. Brandford and M. H. Carpentier, *The Microwave Engineering Handbook* (Chapman and Hall, London, 1993), Vol. 2.