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Notion of potential in quantum gravity

Kirill A. Kazakov*
Physics Faculty, Department of Theoretical Physics, Moscow State University, 117234 Moscow, Russian Federation
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The problem of a consistent definition of the quantum corrected gravitational field is considered in the framework of the *S*-matrix method. The gauge dependence of the one-particle-reducible part of the two-scalar-particle scattering amplitude, with the help of which the potential is usually defined, is investigated at the one-loop approximation. The $1/r^2$ terms in the potential, which are of zero order in the Planck constant $\hbar$, are shown to be independent of the gauge parameter weighting the gauge condition in the action. However, the $1/r^3$ terms, proportional to $\hbar$, describing the first proper quantum correction, are proved to be gauge dependent. With the help of the Slavnov identities, their dependence on the weighting parameter is calculated explicitly. The reason for the gauge dependence is briefly discussed.

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I. INTRODUCTION

Quantization of the general theory of relativity is conventionally performed along the formal lines of quantization of ordinary Yang-Mills theories. Apart from complications introduced by gauge invariance, both are carried out on the basis of Bohr's correspondence principle that gives certain prescriptions as to construction of the operators for physical field quantities. It implies, in particular, that the noncommutativity of these operators becomes negligible when the occupation numbers of physical states get large, and so the quantum equations of motion of free fields become effectively classical. Switching on the interaction results in both the classical nonlinear and quantum radiative corrections to these equations. The property of being classical, however, should be retained by the largely occupied states even in the presence of the interaction, at least in the case of small coupling constants (or small time intervals the states are observed in). The radiative corrections to these states are thus supposed to be measurable in the classical sense, since it is the filling of states, rather than the relative value of the corrections, that determines the system property of being classical.

As is well known, the above immediate interpretation of the effective fields runs into the problem of their gauge dependence. One is prompted therefore to seek an indirect interpretation based on the use of explicitly gauge-independent means.

In many cases, a gauge-independent definition of the potential can be given with the help of the *S* matrix whose gauge independence is insured by the well-known equivalence theorem [1,2]. In the case of spinor electrodynamics, for instance, the potential can be defined with the help of the two-particle scattering amplitude Fourier transformed with respect to the momentum transfer between the particles. Incidentally, with the help of the potential so defined one usually formulates the physical renormalization conditions which are nothing but the classical definitions of the charges and masses of the particles.

There is, however, an obstacle in direct application of the equivalence theorem to the potential. The point is that the latter cannot be defined directly through the two-particle scattering amplitude, since the set of Feynman graphs describing the given scattering process contains diagrams irreducible with respect to the gauge field as well as reducible ones. Only after the reducible part is separated out of the whole set of diagrams can the notion of the potential be introduced by a straightforward generalization of the usual definition used in electrodynamics. This is exactly the way followed in Ref. [3] in investigation of the post-Newtonian classical and quantum corrections to the gravitational potential.

The purpose of this paper is to investigate consistency of the above mentioned separation in the case of quantum gravity. As will be explained in Sec. II, actually there is no intrinsic reason underlying the division of diagrams according to the property of reducibility in this case, threatening thereby the validity of the equivalence theorem as applied to the reducible subset of diagrams. That the potential defined with the help of this subset does depend on the gauge, losing thereby any significance as a means for description of particle interactions, is shown in Sec. IV by an explicit calculation. Section III contains an account of the method used in evaluation of the gauge-dependence of the one-loop logarithmic radiative corrections. The results of the work are discussed in Sec. V. Some formulas needed in calculation of the Feynman integrals are obtained in the Appendix.

The highly condensed notations of DeWitt [1] are employed throughout this paper. Also left derivatives with respect to anticommuting variables are used. The dimensional regularization of all divergent quantities is supposed.

II. DEFINITION OF THE POTENTIAL IN QUANTUM GRAVITY

It was mentioned in the Introduction that the notion of potential makes sense only if one is justified to disregard the set of Feynman graphs irreducible with respect to the gauge field. Before we proceed to actual calculations, let us consider this point in more detail.

Note, first of all, that the potential must be defined in

*Email address: kirill@theor.phys.msu.su

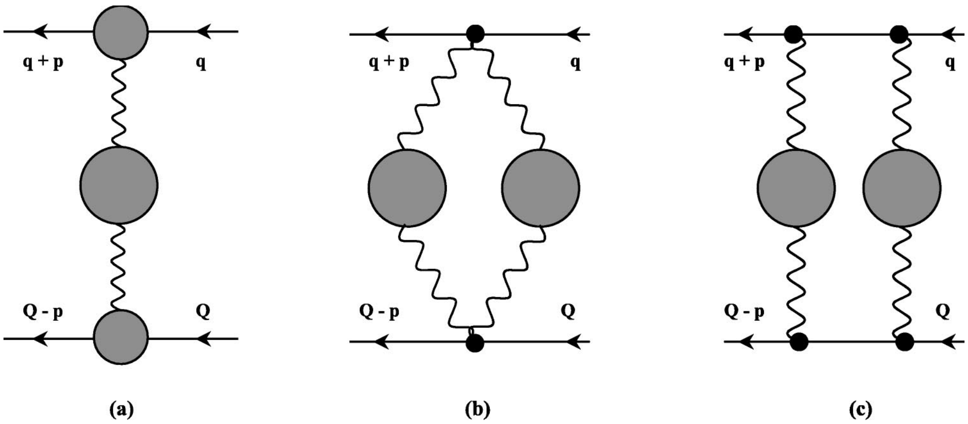

FIG. 1. Feynman graphs representing general structure of various contributions to the two-particle scattering amplitude. (a) The one-particle-reducible part. (b) Contributions occurring when the gauge-field–matter interaction is nonlinear in the gauge field. (c) The irreducible contribution to the gravitational scattering amplitude, remaining finite in the limit $m\to\infty$. Wavy lines represent gravitons, solid lines matter fields.

terms characterizing motion of interacting particles, simply because only in this case would the definition be relevant to an experiment. For this purpose, the scattering matrix approach can be used, in which case the potential is conventionally defined as the Fourier transform (with respect to the momentum transfer from one particle to the other) of the suitably normalized[1] two-particle scattering amplitude. By itself this definition is not of great value unless one is able to separate the whole scattering process as follows: interaction of the first particle with the gauge field → propagation of the gauge field → interaction of the gauge field with the second particle. Only if such a separation is possible can one introduce a self-contained notion of the potential. In terms of the Feynman diagrams, one would say in this case that the diagrams describing the scattering process are one-particle reducible with respect to the gauge field.

In general, the complete set of Feynman graphs corresponding to a given scattering process includes irreducible diagrams as well as reducible.[2] It is important, however, that in many cases a subset of diagrams, consisting of only reducible ones, can be extracted from the complete set, which contains contributions remaining finite in the limit $m\to\infty$, m denoting the masses of the scattering particles. In electrodynamics and Yang-Mills theories, for instance, this is the case for the spin-$\frac{1}{2}$ particles, the subset containing all diagrams without internal lines of the scattering particles [see Fig. 1(a)], but not for the spin-0 particles, in which case one also has diagrams of the type shown in Fig. 1(b). In the case of quantum gravity, furthermore, things are even more complicated. In addition to the diagrams of Fig. 1(b), one has also diagrams pictured in Fig. 1(c), which do not disappear in the limit $m\to\infty$, since m multiplies the vertices of gravitational interactions of the particles, i.e., turns out to be not only in the denominators, but also in the numerators of the Feynman integrals.

We see that the definition of potential via scattering amplitudes is hardly justified in cases when the gauge-field–matter interaction is nonlinear in the gauge field. The requirement of *one*-particle reducibility, underlying this definition, seems to be adequate only for *linear* interactions.

Definition of the potential through the scattering amplitudes is not the only possible way to introduce an independent notion of the gauge field. It *is*, however, if one is interested in giving a *gauge-independent* definition, i.e., the one that would give values for the gauge field, which are independent of the choice of gauge conditions needed to fix gauge invariance of the theory.[3] Actually, it was recently proposed that, in the case of quantum gravity, such a definition can be given beyond the *S*-matrix approach through the introduction of classical point particle moving in the given gravitational field and playing the role of a measuring device [4]. In particular, it was shown that the one-loop effective equations of motion of the point particle (the effective geodesic equation), calculated in the weak field approximation in the nonrelativistic limit, turn out to be independent of the gauge conditions fixing the general covariance [4]. Although this result, undoubtedly, is of considerable importance on its own, it lies out of the main line of our concern here, since it is based on the introduction of the classical point particle into the functional integral ''by hands,'' which certainly cannot be justified using consistent limiting procedure of transition from the underlying quantum field theory to the classical theory. On the other hand, as was shown in Ref. [5], introduction of the classical *field* matter (scalar field) instead of the pointlike still leads to the gauge-dependent values for the gravitational field.[4]

Turning back to the problem of definition of the gravitational potential through the scattering amplitudes, we see that since irreducible diagrams to be dropped out do not disappear even in the limit $m\to\infty$, validity of the most attractive property of the potential defined through the scattering amplitudes is jeopardized by the fact that the equivalence theo-

[1]The normalization is fixed by the requirement that the potential takes the Newtonian form at the tree level.

[2]Here and below in this section, the term ''reducible'' is used with respect to the gauge field only.

[3]One also has to require independence of the choice of a set of dynamical variables in terms of which the theory is quantized. This last condition is particularly important in the case of gravity, where one is free to take any tensor density as a dynamical parametrization of the metric field.

[4]It seems that in the case of ordinary Yang-Mills theories, inclusion of the classical field matter does solve the gauge-dependence problem, at least in the low-energy limit, see Ref. [6].

rem asserting the gauge independence of the S matrix is applicable only to the whole set of diagrams, containing irreducible as well as reducible Feynman graphs describing given scattering process [1,2]. As will be shown below, the gravitational potential constructed in Ref. [3] (i.e., using only reducible Feynman diagrams) does depend on the gauge, losing thereby any significance as a means for description of particle interactions.

III. GENERATING FUNCTIONALS AND SLAVNOV IDENTITIES

As in Ref. [3], we consider the gravitational scattering of two scalar particles with masses m_1, m_2. Dynamics of their quantum fields denoted by ϕ_1, ϕ_2, respectively, is described by the action

$$S_\phi = \frac{1}{2}\int d^4x\sqrt{-g}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2),$$

$$\phi = \phi_{1,2}, \quad m = m_{1,2},$$

while the action for the gravitational field[5]

$$S = -\frac{1}{k^2}\int d^4x\sqrt{-g}R,$$

k being the gravitational constant.[6]

The action $S + S_{\phi_1} + S_{\phi_2}$ is invariant under the following (infinitesimal) gauge transformations[7]

$$\delta h_{\mu\nu} = \xi^\alpha\partial_\alpha h_{\mu\nu} + (\eta_{\mu\alpha} + h_{\mu\alpha})\partial_\nu\xi^\alpha + (\eta_{\nu\alpha} + h_{\nu\alpha})\partial_\mu\xi^\alpha \equiv D^\alpha_{\mu\nu}(h)\xi_\alpha,$$

$$\delta\phi = \xi^\alpha\partial_\alpha\phi \equiv \tilde{D}^\alpha(\phi)\xi_\alpha,$$

where ξ^α are the (infinitesimal) gauge functions. The generators $D, \tilde{D}$ span the closed algebra

$$D^{\alpha,\sigma\lambda}_{\mu\nu}D^\beta_{\sigma\lambda} - D^{\beta,\sigma\lambda}_{\mu\nu}D^\alpha_{\sigma\lambda} = f^{\alpha\beta}{}_\gamma D^\gamma_{\mu\nu},$$

$$\tilde{D}^\alpha_1\tilde{D}^\beta - \tilde{D}^\beta_1\tilde{D}^\alpha = f_\gamma{}^{\alpha\beta}\tilde{D}^\gamma,$$

the "structure constants" $f_\gamma{}^{\alpha\beta}$ being defined by

$$f_\gamma{}^{\alpha\beta}\xi_\alpha\eta_\beta = \xi_\alpha\partial^\alpha\eta_\gamma - \eta_\alpha\partial^\alpha\xi_\gamma.$$

Let the gauge invariance be fixed by the term

$$S_{gf} = \frac{1}{2\xi}\eta^{\alpha\beta}F_\alpha F_\beta,$$

$$F_\alpha = \partial^\mu h_{\mu\alpha} - \frac{1}{2}\partial_\alpha h, \quad h \equiv \eta^{\mu\nu}h_{\mu\nu}.$$

Next, introducing the Faddeev-Popov ghost fields $C_\alpha, \overline{C}^\alpha$ we write the Faddeev-Popov quantum action [7]

$$S_{\rm FP} = S + S_{\phi_1} + S_{\phi_2} + S_{gf} + \overline{C}^\beta F^{,\mu\nu}_\beta D^\alpha_{\mu\nu}C_\alpha.$$

$S_{\rm FP}$ is still invariant under the following Becchi-Rouet-Stora-Tyutin (BRST) transformations [8]

$$\delta h_{\mu\nu} = D^\alpha_{\mu\nu}(h)C_\alpha\lambda,$$

$$\delta\phi = \tilde{D}^\alpha(\phi)C_\alpha\lambda,$$

$$\delta C_\gamma = -\frac{1}{2}f_\gamma{}^{\alpha\beta}C_\alpha C_\beta\lambda,$$

$$\delta\overline{C}^\alpha = \frac{1}{\xi}F^\alpha\lambda, \tag{1}$$

λ being a constant anticommuting parameter.

The generating functional of Green functions[8]

$$Z[T,J,\overline{\beta},\beta,K,\tilde{K},L] = \int dh\,d\phi\,dC\,d\overline{C}\exp\{i(\Sigma + \overline{\beta}^\alpha C_\alpha + \overline{C}^\alpha\beta_\alpha + T^{\mu\nu}h_{\mu\nu} + J\phi)\},$$

where $J = \{J_{1,2}\}$, $d\phi \equiv d\phi_1 d\phi_2$, $J\phi \equiv J_1\phi_1 + J_2\phi_2$, and

$$\Sigma = S_{\rm FP} + K^{\mu\nu}D^\alpha_{\mu\nu}C_\alpha + \tilde{K}\tilde{D}^\alpha C_\alpha + L^\gamma\frac{1}{2}f_\gamma{}^{\alpha\beta}C_\alpha C_\beta,$$

$K^{\mu\nu}(x)$, $\tilde{K}(x)$ (anticommuting), $L^\alpha(x)$ (commuting) being the BRST transformation sources [9].

To determine the dependence of field-theoretical quantities on the gauge parameter ξ, we modify the quantum action adding the term

$$YF_\alpha\overline{C}^\alpha,$$

Y being a constant anticommuting parameter [10]. Thus we write the generating functional of Green functions as

$$Z[T,J,\overline{\beta},\beta,K,\tilde{K},L,Y] = \int dh\,d\phi\,dC\,d\overline{C}\exp\{i(\Sigma + YF_\alpha\overline{C}^\alpha + \overline{\beta}^\alpha C_\alpha + \overline{C}^\alpha\beta_\alpha + T^{\mu\nu}h_{\mu\nu} + J\phi)\}. \tag{2}$$

[5]Our notation is $R_{\mu\nu} \equiv R^\alpha{}_{\mu\alpha\nu} = \partial_\alpha\Gamma^\alpha_{\mu\nu} - \cdots$, $R \equiv R_{\mu\nu}g^{\mu\nu}$, $g \equiv \det g_{\mu\nu}$, $g_{\mu\nu} = \mathrm{sgn}(+,-,-,-)$. Dynamical variables of the gravitational field $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$, $\eta_{\mu\nu} = \mathrm{diag}\{+1,-1,-1,-1\}$.

[6]We choose units in which $c = \hbar = k = 1$ from now on.

[7]Indices of the functions F, ξ, as well as of the ghost fields below, are raised and lowered, if convenient, with the help of Minkowski metric $\eta_{\mu\nu}$.

[8]For brevity, the product symbol, as well as tensor indices of the fields $h_{\mu\nu}, C_\alpha, \overline{C}^\alpha$, is omitted in the path integral measure.

Finally, we introduce the generating functional of connected Green functions

$$W[T,J,\bar{\beta},\beta,K,\tilde{K},L,Y]=-i\ln Z[T,J,\bar{\beta},\beta,K,\tilde{K},L,Y],$$

and then define the effective action Γ in the usual way as the Legendre transform of W with respect to the mean fields

$$h_{\mu\nu}=\frac{\delta W}{\delta T^{\mu\nu}},\quad \phi=\frac{\delta W}{\delta J},\quad C_{\alpha}=\frac{\delta W}{\delta\bar{\beta}^{\alpha}},\quad \bar{C}^{\alpha}=-\frac{\delta W}{\delta\beta_{\alpha}}, \tag{3}$$

(denoted by the same symbols as the corresponding field operators):

$$\Gamma[h,\phi,C,\bar{C},K,\tilde{K},L,Y]=W[T,J,\bar{\beta},\beta,K,\tilde{K},L,Y]-\bar{\beta}^{\alpha}C_{\alpha}-\bar{C}^{\alpha}\beta_{\alpha}-T^{\mu\nu}h_{\mu\nu}-J\phi.$$

Evaluation of derivatives of diagrams with respect to the gauge parameters is an easier task than their direct calculation in arbitrary gauge.[9] This is because these derivatives can be expressed through another set of diagrams with more simple structure. The rules for such a transformation of diagrams are conveniently summarized in the Slavnov identities corresponding to the generating functional (2). Since these identities are widely used in what follows, their derivation will be briefly described below [10].

First of all, we perform a BRST shift (1) of integration variables in the path integral (2). Equating the variation to zero we obtain the following identity:

$$\int dh d\phi dC d\bar{C}\left[iY\bar{C}^{\alpha}F_{\alpha}^{,\mu\nu}D_{\mu\nu}^{\beta}C_{\beta}+i\frac{Y}{\xi}F_{\alpha}^{2}+T^{\mu\nu}\frac{\delta}{\delta K^{\mu\nu}}+J\frac{\delta}{\delta\tilde{K}}-\bar{\beta}^{\alpha}\frac{\delta}{\delta L^{\alpha}}-i\beta_{\alpha}\frac{F^{\alpha}}{\xi}\right]\exp\{i(\Sigma+YF_{\alpha}\bar{C}^{\alpha}+\bar{\beta}^{\alpha}C_{\alpha}+\bar{C}^{\alpha}\beta_{\alpha}+T^{\mu\nu}h_{\mu\nu}+J\phi)\}=0. \tag{4}$$

Next, the first term in the square brackets in Eq. (4) can be transformed with the help of the quantum ghost equation of motion, obtained by performing a shift $\bar{C}\rightarrow\bar{C}+\delta\bar{C}$ of integration variables in the functional integral (2):

$$\int dh d\phi dC d\bar{C}[F_{\gamma}^{,\mu\nu}D_{\mu\nu}^{\alpha}C_{\alpha}-YF_{\gamma}+\beta_{\gamma}]\exp\{\cdots\}=0,$$

from which it follows that

$$Y\int dh d\phi dC d\bar{C}\left[i\bar{C}^{\gamma}F_{\gamma}^{,\mu\nu}D_{\mu\nu}^{\alpha}C_{\alpha}+\beta_{\gamma}\frac{\delta}{\delta\beta_{\gamma}}\right]\exp\{\cdots\}=0,$$

where we used the property $Y^{2}=0$, and omitted the expression $\delta\beta_{\gamma}/\delta\beta_{\gamma}\sim\delta(0)$. Putting this all together, we rewrite Eq. (4)

$$\left(T^{\mu\nu}\frac{\delta}{\delta K^{\mu\nu}}+J\frac{\delta}{\delta\tilde{K}}-\bar{\beta}^{\alpha}\frac{\delta}{\delta L^{\alpha}}-\frac{1}{\xi}\beta_{\alpha}F^{\alpha,\mu\nu}\frac{\delta}{\delta T^{\mu\nu}}-Y\beta_{\gamma}\frac{\delta}{\delta\beta_{\gamma}}-2Y\xi\frac{\partial}{\partial\xi}\right)Z=0.$$

This is the Slavnov identity for the generating functional of Green functions we are looking for. In terms of the generating functional of connected Green functions, it takes the form

$$T^{\mu\nu}\frac{\delta W}{\delta K^{\mu\nu}}+J\frac{\delta W}{\delta\tilde{K}}-\bar{\beta}^{\alpha}\frac{\delta W}{\delta L^{\alpha}}-\frac{1}{\xi}\beta_{\alpha}F^{\alpha,\mu\nu}\frac{\delta W}{\delta T^{\mu\nu}}-Y\beta_{\gamma}\frac{\delta W}{\delta\beta_{\gamma}}-2Y\xi\frac{\partial W}{\partial\xi}=0. \tag{5}$$

It can be transformed further into an identity for the generating functional of proper vertices: with the help of equations

$$T^{\mu\nu}=-\frac{\delta\Gamma}{\delta h_{\mu\nu}},\quad J=-\frac{\delta\Gamma}{\delta\phi},$$

$$\bar{\beta}^{\alpha}=\frac{\delta\Gamma}{\delta C_{\alpha}},\quad \beta_{\alpha}=-\frac{\delta\Gamma}{\delta\bar{C}^{\alpha}}, \tag{6}$$

which are the inverse of Eqs. (3), and the relations

$$\frac{\delta W}{\delta K^{\mu\nu}}=\frac{\delta\Gamma}{\delta K^{\mu\nu}},\quad \frac{\partial W}{\partial\xi}=\frac{\partial\Gamma}{\partial\xi},\ \text{etc.},$$

we rewrite Eq. (5)

$$\frac{\delta\Gamma}{\delta h_{\mu\nu}}\frac{\delta\Gamma}{\delta K^{\mu\nu}}+\frac{\delta\Gamma}{\delta\phi}\frac{\delta\Gamma}{\delta\tilde{K}}+\frac{\delta\Gamma}{\delta C_{\alpha}}\frac{\delta\Gamma}{\delta L^{\alpha}}-\frac{F^{\alpha}}{\xi}\frac{\delta\Gamma}{\delta\bar{C}^{\alpha}}+Y\frac{\delta\Gamma}{\delta\bar{C}^{\alpha}}\bar{C}^{\alpha}+2Y\xi\frac{\partial\Gamma}{\partial\xi}=0.$$

Written down via the reduced functional

$$\Gamma_{0}=\Gamma-\frac{1}{2\xi}F_{\alpha}F^{\alpha}-YF_{\sigma}\bar{C}^{\sigma},$$

the latter equation takes particularly simple form

$$\frac{\delta\Gamma_{0}}{\delta h_{\mu\nu}}\frac{\delta\Gamma_{0}}{\delta K^{\mu\nu}}+\frac{\delta\Gamma_{0}}{\delta\phi}\frac{\delta\Gamma_{0}}{\delta\tilde{K}}+\frac{\delta\Gamma_{0}}{\delta C_{\sigma}}\frac{\delta\Gamma_{0}}{\delta L^{\sigma}}+2Y\xi\frac{\partial\Gamma_{0}}{\partial\xi}=0. \tag{7}$$

[9] In actual quantum gravity calculations, this fact was first used in Ref. [11] to evaluate divergences of the Einstein gravity in arbitrary gauge off the mass shell.

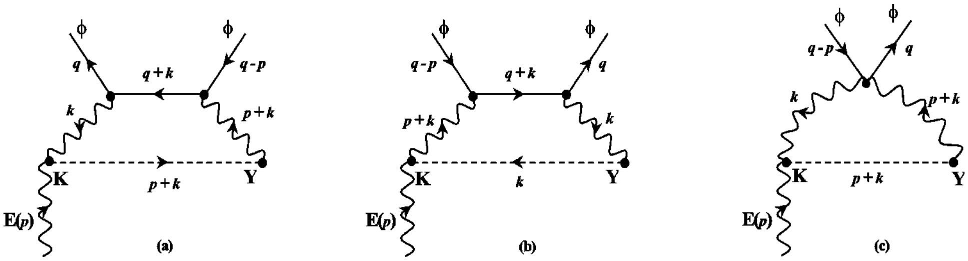

FIG. 2. Diagrams with two scalar and one graviton external lines, responsible for the nonvanishing of the ξ-dependent contribution to the one-particle-reducible gravitational potential. Solid lines represent scalar particles, dashed lines ghosts.

IV. GAUGE DEPENDENCE OF THE ONE-PARTICLE-REDUCIBLE GRAVITATIONAL POTENTIAL

Let us now turn to the explicit evaluation of the ξ dependence of the one-loop contribution to the potential. Its general structure is shown in Fig. 1(a). In view of the assumed reducibility, corrections to the vertices and graviton propagator, which are the building blocks for the potential, can be considered separately. Let us note first of all that the (tree) graviton propagators, with respect to which the potential is reducible, can be considered gauge independent. Indeed, at the one-loop level, each of these propagators has one of its ends attached to the tree ϕ-h-ϕ vertex with the ϕ lines on the mass shell. This combination is gauge independent on the same grounds as is the S matrix at the tree level. Thus, we have to consider only the proper ϕ-h-ϕ vertex and the graviton self-energy. To evaluate the ξ derivative of these quantities, we use the Slavnov identity (7). Extracting terms proportional to the source Y, we get

$$2\xi\frac{\partial\Gamma_1}{\partial\xi}=\frac{\delta\Gamma_1}{\delta h_{\mu\nu}}\frac{\delta\Gamma_2}{\delta K^{\mu\nu}}+\frac{\delta\Gamma_1}{\delta\phi}\frac{\delta\Gamma_2}{\delta\tilde{K}}, \qquad (8)$$

where $\Gamma_{1,2}$ are defined by

$$\Gamma_1=\Gamma_0|_{Y=0}, \qquad \Gamma_2=\frac{\partial\Gamma_0}{\partial Y}.$$

At the one-loop level, Eq. (8) is just[10]

$$2\xi\frac{\partial\Gamma_1^{(1)}}{\partial\xi}=\frac{\delta\Gamma_1^{(0)}}{\delta h_{\mu\nu}}\frac{\delta\Gamma_2^{(1)}}{\delta K^{\mu\nu}}, \qquad (9)$$

since the external scalar lines are on the mass shell

$$\frac{\delta S_\phi}{\delta\phi}=0.$$

Graphs representing the ξ derivatives of the form factors according to the right hand side of Eq. (9), are shown in Figs. 2,3.

Diagrams of Fig. 3 need not be calculated explicitly. It is easy to see that they just cancel the ξ-dependent contribution to the graviton self-energy when the potential is being constructed. Indeed, according to Eq. (9), this contribution is given by the diagrams of Fig. 4. In the course of construction of the potential, the two h lines of the graviton self-energy are connected to the ϕ-h-ϕ vertices by the graviton propagators. When these propagators are attached to the left most vertices in Figs. 4(a), 4(b), we get exactly the diagrams of Figs. 3(a), 3(b), respectively, but with the opposite sign, because it follows from Eqs. (3),(6) that

$$\frac{\delta^2 S}{\delta h_{\mu\nu}\delta h_{\alpha\beta}}\frac{\delta^2 W^{(0)}}{\delta T^{\alpha\beta}\delta T^{\gamma\delta}}=-\delta^{\mu\nu}_{\gamma\delta}.$$

Thus, explicit calculation of diagrams of Fig. 2 is needed only. Their analytic expressions

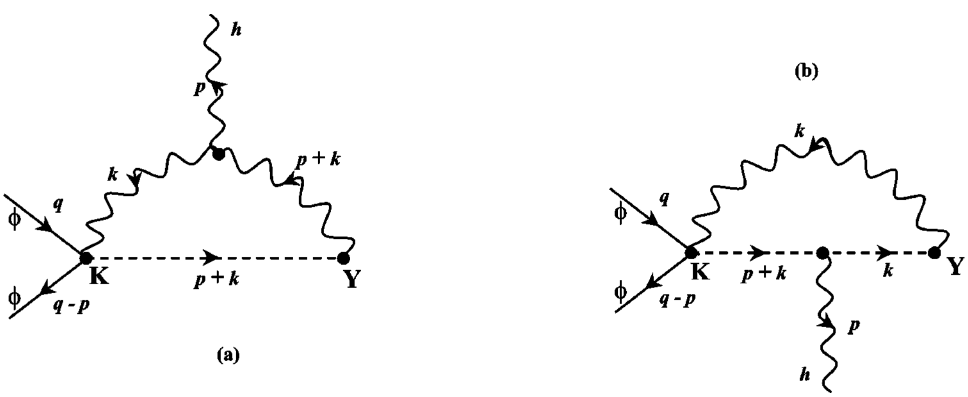

FIG. 3. Diagrams representing the part of the ξ-dependent contribution to the gravitational form factors of scalar particles, that cancels the corresponding contribution coming from the graviton self-energy (see Fig. 4) in the course of construction of the potential.

[10]Enclosed in the parentheses is the number of loops in a diagram representing given term.

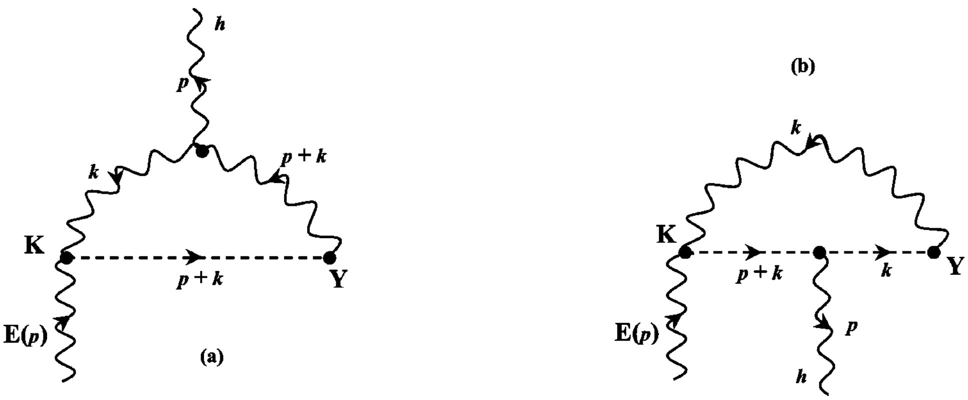

FIG. 4. Diagrams representing the ξ-dependent contribution to the graviton self energy.

$$
\begin{aligned}
I_{2(a)}(p,q) = {} & \frac{-iE^{\mu\nu}(p)}{2\sqrt{\varepsilon_q\varepsilon_{q-p}}}\mu^{\epsilon}\int\frac{d^{4-\epsilon}k}{(2\pi)^4}\left\{\frac{1}{2}W^{\alpha\beta\gamma\delta}q_{\gamma}(k_{\delta}+q_{\delta})\right.\\
& \left. -m^2\frac{\eta^{\alpha\beta}}{2}\right\}G_{\phi}\left\{\frac{1}{2}W^{\rho\tau\sigma\lambda}(q_{\sigma}-p_{\sigma})(k_{\lambda}+q_{\lambda})\right.\\
& \left. -m^2\frac{\eta^{\rho\tau}}{2}\right\}\xi D^{(0)\eta}_{\rho\tau}\tilde{G}^{\xi}_{\eta}(k+p)\\
& \times\tilde{G}^{\zeta}_{\xi}(p+k)\{k_{\zeta}\delta^{\chi\theta}_{\mu\nu}-\delta^{\chi\theta}_{\zeta\mu}(k_{\nu}+p_{\nu})\\
& -\delta^{\chi\theta}_{\zeta\nu}(k_{\mu}+p_{\mu})\}G_{\chi\theta\alpha\beta}(k),
\end{aligned}
\tag{10}
$$

$$
I_{2(b)}(p,q) = I_{2(a)}(p,p-q),
$$

$$
\begin{aligned}
I_{2(c)}(p,q) = {} & \frac{-iE^{\mu\nu}(p)}{2\sqrt{\varepsilon_q\varepsilon_{q-p}}}\mu^{\epsilon}\int\frac{d^{4-\epsilon}k}{(2\pi)^4}\left\{\left[-\frac{1}{2}(\delta^{\sigma\lambda}_{\zeta\xi}\eta^{\tau\rho}\right.\right.\\
& +\delta^{\tau\rho}_{\zeta\xi}\eta^{\sigma\lambda})+(\delta^{\tau\rho}_{\zeta\omega}\delta^{\sigma\lambda}_{\xi\omega}+\delta^{\tau\rho}_{\xi\omega}\delta^{\sigma\lambda}_{\zeta\omega})\\
& \left.+\frac{\eta_{\zeta\xi}}{4}W^{\tau\rho\sigma\lambda}\right]q^{\xi}(q^{\zeta}-p^{\zeta})\\
& \left.-\frac{m^2}{4}W^{\tau\rho\sigma\lambda}\right\}G_{\tau\rho\chi\theta}(k)\{k_{\alpha}\delta^{\chi\theta}_{\mu\nu}-\delta^{\chi\theta}_{\mu\alpha}(k_{\nu}+p_{\nu})\\
& -\delta^{\chi\theta}_{\nu\alpha}(k_{\mu}+p_{\mu})\}\tilde{G}^{\alpha}_{\beta}(p+k)\,\xi D^{(0)\gamma}_{\sigma\lambda}\tilde{G}^{\beta}_{\gamma}(k+p),
\end{aligned}
\tag{11}
$$

where the following notation is introduced:

$$
W^{\alpha\beta\gamma\delta} = \eta^{\alpha\beta}\eta^{\gamma\delta}-\eta^{\alpha\gamma}\eta^{\beta\delta}-\eta^{\alpha\delta}\eta^{\beta\gamma},
$$

$$
\delta^{\mu\nu}_{\alpha\beta} = \frac{1}{2}(\delta^{\mu}_{\alpha}\delta^{\nu}_{\beta}+\delta^{\nu}_{\alpha}\delta^{\mu}_{\beta}),
$$

$G_{\mu\nu\sigma\lambda}$ is the graviton propagator defined by

$$
\frac{\delta^2 S}{\delta h_{\rho\tau}\delta h_{\mu\nu}}G_{\mu\nu\sigma\lambda} = -\delta^{\rho\tau}_{\sigma\lambda},
$$

$$
\begin{aligned}
G_{\mu\nu\sigma\lambda} = {} & -W_{\mu\nu\sigma\lambda}\frac{1}{\Box}+(\xi-1)(\eta_{\mu\sigma}\partial_{\nu}\partial_{\lambda}\\
& +\eta_{\mu\lambda}\partial_{\nu}\partial_{\sigma}+\eta_{\nu\sigma}\partial_{\mu}\partial_{\lambda}+\eta_{\nu\lambda}\partial_{\mu}\partial_{\sigma})\frac{1}{\Box^2},
\end{aligned}
$$

$\tilde{G}^{\alpha}_{\beta}$ is the ghost propagator

$$
\tilde{G}^{\alpha}_{\beta} = -\frac{\delta^{\alpha}_{\beta}}{\Box},
$$

satisfying

$$
F^{,\mu\nu}_{\alpha}D^{(0)\beta}_{\mu\nu}\tilde{G}^{\gamma}_{\beta} = -\delta^{\gamma}_{\alpha}, \quad D^{(0)\alpha}_{\mu\nu}\equiv D^{\alpha}_{\mu\nu}(h=0),
$$

G_{ϕ} is the scalar particle propagator

$$
G_{\phi} = \frac{1}{\Box+m^2},
$$

$E^{\mu\nu}$ stands for the linearized Einstein tensor

$$
E^{\mu\nu} = R^{\mu\nu}-\frac{1}{2}\eta^{\mu\nu}R_{\alpha\beta}\eta^{\alpha\beta},
$$

$$
R_{\mu\nu} = \frac{1}{2}(\partial^{\alpha}\partial_{\mu}h_{\alpha\nu}+\partial^{\alpha}\partial_{\nu}h_{\alpha\mu}-\Box h_{\mu\nu}-\partial_{\mu}\partial_{\nu}h),
$$

μ is the arbitrary mass scale, $\varepsilon_q=\sqrt{q^2+m^2}$, and $\epsilon=4-d$, d being the dimensionality of space time. To simplify the tensor structure of diagrams Fig. 2, the use has been made of the identity

$$
\frac{1}{\xi}F^{\alpha,\mu\nu}G_{\mu\nu\sigma\lambda}(x) = -D^{(0)\beta}_{\sigma\lambda}\tilde{G}^{\alpha}_{\beta}(x),
$$

which is nothing but the well-known *first* Slavnov identity at the tree level; it is easily obtained differentiating Eq. (5) twice with respect to β_α and $T^{\mu\nu}$, and setting all the sources equal to zero.

Let us begin with evaluation of the diagram of Fig. 2(a). This takes most of the effort.

The tensor multiplication in Eq. (10) is conveniently performed with the help of the new tensor package for the REDUCE system [12]

$$I_{2(a)}(p,q)=\frac{-iE^{\mu\nu}(p)}{2\sqrt{\varepsilon_q\varepsilon_{q-p}}}\mu^\epsilon\int\frac{d^{4-\epsilon}k}{(2\pi)^4}\frac{1}{k^4}\frac{1}{(k+p)^4}\frac{1}{m^2-(k+q)^2}\xi[\eta_{\mu\nu}k^2m^2\{(kp)-(kq)\}\{k^2+2(kq)\}$$
$$+k_\mu k_\nu k^4\xi(p^2-2m^2)+2k_\mu k_\nu k^2(kq)(2\xi-1)(p^2-2m^2)+4k_\mu(k_\nu+p_\nu)(kq)^2(\xi-1)(p^2-2m^2)$$
$$+k_\mu p_\nu k^4(-2\xi m^2+\xi p^2-2m^2)+2k_\mu p_\nu k^2(kq)(-4\xi m^2+2\xi p^2-p^2)+2k_\mu q_\nu k^4(p^2-m^2)$$
$$+4k_\mu q_\nu k^2(kq)(p^2-m^2)-2p_\mu p_\nu k^2m^2\{k^2+2(kq)\}+2p_\mu q_\nu k^2\xi\{(kp)-(kq)\}\{k^2+4(kq)\}$$
$$+4p_\mu q_\nu k^2(kq)\{p^2-m^2+(kq)-(kp)\}+8p_\mu q_\nu(kq)^2(\xi-1)\{(kp)-(kq)\}$$
$$+2p_\mu q_\nu k^4(p^2-m^2)+2q_\mu q_\nu k^4\{(kq)-(kp)\}+4q_\mu q_\nu k^2(kq)\{(kq)-(kp)\}]. \tag{12}$$

Evaluation of the loop integrals can be automatized to a considerable extent if the Schwinger parametrization of denominators in Eq. (12) is used

$$\frac{1}{k^4}=\int_0^\infty dy\,y\,\exp\{yk^2\},$$

$$\frac{1}{(k+p)^4}=\int_0^\infty dx\,x\,\exp\{x(k+p)^2\},$$

$$\frac{1}{k^2+2(kq)}=-\int_0^\infty dz\,\exp\{z[k^2+2(kq)]\}.$$

It is convenient to apply these formulas as they stand, i.e., eluding cancellation of the k^2 factors in Eq. (12). The k integrals are then evaluated using

$$\int d^dk\,\exp\{k^2(x+y+z)+2k^\mu(xp_\mu+zq_\mu)\}$$
$$=i\left(\frac{\pi}{x+y+z}\right)^{d/2}\exp\left\{\frac{p^2xy-m^2z^2}{x+y+z}\right\},$$

$$\int d^dk\,k_\alpha\exp\{k^2(x+y+z)+2k^\mu(xp_\mu+zq_\mu)\}$$
$$=i\left(\frac{\pi}{x+y+z}\right)^{d/2}\exp\left\{\frac{p^2xy-m^2z^2}{x+y+z}\right\}\left[-\frac{xp_\alpha+zq_\alpha}{x+y+z}\right],$$

etc., up to six k factors in the integrand.

From now on, all formulas will be written out for the sum

$$\tilde{I}_2\equiv I_{2(a)}(p,q)+I_{2(b)}(p,q).$$

Changing the integration variables (x,y,z) to (t,u,v) via

$$x=\frac{t(1+t+u)v^2}{m^2(1+\alpha tu)},\quad y=\frac{u(1+t+u)v^2}{m^2(1+\alpha tu)},$$

$$z=\frac{(1+t+u)v^2}{m^2(1+\alpha tu)},\quad \alpha\equiv-\frac{p^2}{m^2},$$

integrating v out, subtracting the ultraviolet divergence[11]

$$\tilde{I}_2^{\rm div}=\frac{1}{32\pi^2\epsilon}\left(\frac{\mu}{m}\right)^\epsilon E^{\mu\nu}(p)\,\eta_{\mu\nu}\xi^2(p^2-2m^2),$$

setting $\epsilon=0$, and retaining only the terms giving rise to the roots and logarithms of p^2/m^2, leading at $p^2\to0$, we obtain

$$(\tilde{I}_2-\tilde{I}_2^{\rm div})_{\epsilon\to0}=\frac{E^{\mu\nu}(p)\xi}{32\pi^2\sqrt{\varepsilon_q\varepsilon_{q-p}}}\int_0^\infty\int_0^\infty dudt$$
$$\times\left\{\frac{8m^2(\xi-1)}{p^2DN^3}\left(q_\mu q_\nu-\frac{m^2}{p^2}p_\mu p_\nu\right)\right.$$
$$\times\left(6-\frac{9}{D}+\frac{4}{D^2}\right)+\frac{\eta_{\mu\nu}m^2}{DN}\left(1-\frac{5}{D}+\frac{4}{D^2}\right)$$
$$+\frac{4\eta_{\mu\nu}m^4\xi}{DN^3p^2}\left(3-\frac{2}{D}\right)+\frac{8\xi m^2}{DN^2}$$
$$\times\left[\frac{p_\mu p_\nu}{p^2}\left(\xi-\frac{\xi}{D}-\frac{1}{D}+\frac{1}{D^2}\right)+\frac{p_\mu q_\nu}{p^2}\right.$$

[11]Since we are interested only in the nonanalytic at $p^2=0$ terms responsible for the long-range quantum corrections, particularities of the subtraction scheme are immaterial.

$$\times\left(-3\xi+1+\frac{7\xi}{D}-\frac{3}{D}-\frac{4\xi}{D^2}+\frac{2}{D^2}\right)\Bigg]\Bigg\},$$

$$D\equiv 1+\alpha ut, \quad N\equiv 1+u+t. \tag{13}$$

Equation (13) is written out in such a form that the leading roots come only from the first three terms in the curly brackets. The remaining (u,t) integrals are evaluated in the Appendix. Using Eqs. (A3) one readily sees that the terms proportional to $\sqrt{-p^2}$ in Eq. (13) cancel. As explained elsewhere (see Ref. [13]), this fact allows one to give a physical interpretation to the root contributions to the form factors directly in the framework of the effective action method, as describing quantum deviations of the space-time metric from classical solutions of the Einstein equations.

It is easy to see also that the diagrams of Fig. 2 are the only that give rise to the root singularities in the potential defined according to Ref. [3], so the found cancellation proves the gauge independence of the $1/r^2$ terms in this potential as well (r being the distance from the source particle). Let us, therefore, push our calculations further and turn to the $1/r^3$ terms, i.e., to the leading logarithms. With the help of Eqs. (A4) of the Appendix, we get from Eq. (13)

$$(\tilde{I}_2-\tilde{I}_2^{\rm div})^{\rm log}_{\epsilon\to 0}\equiv\tilde{I}_2^{\rm ren}=-\ln\alpha\,\frac{E^{\mu\nu}(p)\,\eta_{\mu\nu}m^2\xi^2}{32\pi^2\sqrt{\varepsilon_q\varepsilon_{q-p}}}. \tag{14}$$

It remains only to calculate the diagram of Fig. 2(c). This is a much easier task than the above calculation, since the loop does not contain scalar lines. On dimensional grounds, $I_{2(c)}(p,q)$ has the following structure:

$$I_{2(c)}(p,q)=\frac{E^{\mu\nu}(p)P_{\mu\nu}(p,q)}{\sqrt{\varepsilon_q\varepsilon_{q-p}}}\left(\frac{\mu^2}{-p^2}\right)^{\epsilon/2}\left[\frac{1}{\epsilon}+c\right]$$
$$=\frac{E^{\mu\nu}(p)P_{\mu\nu}(p,q)}{\sqrt{\varepsilon_q\varepsilon_{q-p}}}\left[\frac{1}{\epsilon}-\frac{1}{2}\ln\left(\frac{-p^2}{\mu^2}\right)\right.$$
$$\left.+c+O(\epsilon)\right], \tag{15}$$

where c is some number, and $P_{\mu\nu}(p,q)$ polynomials in p_μ,q_μ. It follows from Eq. (15) that one can obtain the logarithmic contribution from divergent one substituting

$$\frac{1}{\epsilon}\to-\frac{1}{2}\ln\left(\frac{-p^2}{\mu^2}\right).$$

$I_{2(c)}$ is ultraviolet divergent. It is important, on the other hand, that it is free of infrared divergences. Indeed, the integrand in Eq. (11) is the sum of products of powers $(p+k)^n$ and k^l, times a polynomial in p_μ,q_μ. Since the diagram is logarithmically divergent, we have $n+l=-4$. On the other hand, infrared divergences appear only if $n\leqslant-4$, or $l\leqslant-4$, and, therefore, we have $l\geqslant 0$, or $n\geqslant 0$. In either case the dimensionally regularized loop integrals turn into zero.

Now, the calculation is straightforward. To find the ultraviolet divergences, one sets $p+k\to k$ in the propagators and the vertex factors (since the degree of divergence is zero), averages over angles (in k space), and retains only $1/k^4$ terms in the integrand, changing them to $2\pi^2 i/\epsilon$ afterwards. The tensor multiplication as well as integration over angles in the momentum space is again performed with the help of the tensor package of Ref. [12]. Subtracting the $1/\epsilon$ divergence and setting $\epsilon=0$, one obtains the following result:

$$I_{2(c)}^{\rm ren}(p,q)\big|_{\rm log}=\frac{\xi\ln\alpha}{96\pi^2\sqrt{\varepsilon_q\varepsilon_{q-p}}}E^{\mu\nu}(p)\{\eta_{\mu\nu}m^2(-5\xi+2)$$
$$-q_\mu q_\nu(4\xi+8)\}.$$

The total logarithmic contribution of diagrams of Fig. 2 is

$$I_2^{\rm ren}\equiv I_{2(c)}^{\rm ren}(p,q)+\tilde{I}_2^{\rm ren}$$
$$=-\frac{\xi\ln\alpha}{48\pi^2\sqrt{\varepsilon_q\varepsilon_{q-p}}}E^{\mu\nu}(p)$$
$$\times\{\eta_{\mu\nu}m^2(4\xi-1)+q_\mu q_\nu(2\xi+4)\}. \tag{16}$$

Finally, multiplying Eq. (16) with $m=m_1$ by the graviton propagator and the tree vertex factor corresponding to the second particle with $m=m_2$, and adding the result of this calculation with m_1,m_2 interchanged (and $p\to-p$), we have for the ξ derivative of the one-loop contribution to the one-particle-reducible part of the two-particle scattering amplitude, in the case $|\mathbf{q}_1|\ll m_1$, $|\mathbf{q}_2|\ll m_2$,

$$\frac{\partial A^{(1)}_{\rm 1PR}}{\partial\xi}=-\ln(-p^2)\,\frac{m_1m_2(2\xi+1)}{64\pi^2}. \tag{17}$$

This completes exposition of the main result of the work.

V. CONCLUSION

The $1/r^3$ terms in the one-particle-reducible gravitational potential are thus shown to be ξ dependent, the form of this dependence being given by the Fourier transform of Eq. (17). The formal reason for the occurrence of gauge dependence should be clear from the considerations of Sec. IV. The gauge invariance of the classical action is crucial for the proof of the gauge independence of the S matrix [1,2]. Being inhomogeneous in the field $h_{\mu\nu}$, the generators of the gauge transformations mix vertices with different number of h lines. The gauge invariance of the scattering amplitude is therefore preserved only if every combination of vertices, contributing at a given loop order, is taken into account. Omission of the irreducible part of the two-particle scattering amplitude inevitably violates the latter condition, the result being only the partial cancellation of the gauge-dependent contributions, found in Sec. IV. Thus, the one-particle-reducible gravitational potential is irrelevant to the issue of interpretation of the quantum corrections to the classical metric.

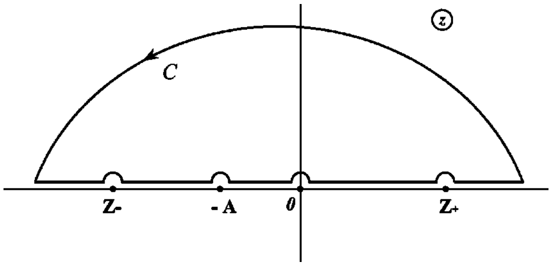

FIG. 5. Contour of integration in Eq. (A1).

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APPENDIX

The integrals

$$J_{nm} \equiv \int_0^\infty \int_0^\infty \frac{du\,dt}{(1+t+u)^n (1+\alpha t u)^m},$$

encountered in Sec. IV, can be evaluated as follows. Consider the auxiliary quantity

$$J(A,B) = \int_0^\infty \int_0^\infty \frac{du\,dt}{(A+t+u)(B+\alpha t u)},$$

where $A,B>0$ are some numbers eventually set equal to 1. Performing an elementary integration over u, we get

$$J(A,B) = \int_0^\infty dt \frac{\ln B - \ln\{\alpha t(A+t)\}}{B-\alpha t(A+t)}.$$

Now consider the integral

$$\tilde{J}(A,B) = \int_C dz\, f(z,A,B),$$

$$f(z,A,B) = \frac{\ln B - \ln\{\alpha z(A+z)\}}{B-\alpha z(A+z)}, \tag{A1}$$

taken over the contour C shown in Fig. 5. $\tilde{J}(A,B)$ is zero identically. On the other hand,

$$\tilde{J}(A,B) = \int_{-\infty}^{-A} dw \frac{\ln B - \ln\{\alpha w(A+w)\}}{B-\alpha w(A+w)} + \int_{-A}^{0} dw \frac{\ln B - \ln\{-\alpha w(A+w)\} + i\pi}{B-\alpha w(A+w)} + \int_0^{+\infty} dw \frac{\ln B - \ln\{\alpha w(A+w)\} + 2i\pi}{B-\alpha w(A+w)} - i\pi \sum \operatorname{Res} f(z,A,B).$$

Thus, changing $w \to -A-w$ in the first integral and $w \to -w$ in the second, we have

$$J(A,B) = \frac{\pi^2}{2\sqrt{\alpha}} B^{-1/2} \left(1 + \frac{\alpha A^2}{4B}\right)^{-1/2} - \frac{1}{2}\int_0^A dt \frac{\ln B - \ln\{\alpha t(A-t)\}}{B+\alpha t(A-t)}. \tag{A2}$$

The roots are contained entirely in the first term on the right of Eq. (A2), while the logarithms in the second. The integrals J_{nm} are found by repeated differentiation of Eq. (A2) with respect to A,B. Expanding the square root $(1+\alpha A^2/4B)^{1/2}$ in powers of α, we find the leading roots needed in Sec. IV

$$J_{11}^{\rm root} = \frac{\pi^2}{2\sqrt{\alpha}}, \quad J_{12}^{\rm root} = \frac{\pi^2}{4\sqrt{\alpha}}, \quad J_{13}^{\rm root} = \frac{3\pi^2}{16\sqrt{\alpha}}, \tag{A3}$$

$$J_{31}^{\rm root} = -\frac{\pi^2}{16}\sqrt{\alpha}, \quad J_{32}^{\rm root} = -\frac{3\pi^2}{32}\sqrt{\alpha}, \quad J_{33}^{\rm root} = -\frac{15\pi^2}{128}\sqrt{\alpha}.$$

Next, expanding the integrand in the second term of Eq. (A2), we get the leading logarithms

$$J_{11}^{\rm log} = J_{12}^{\rm log} = J_{13}^{\rm log} = -J_{21}^{\rm log} = -J_{22}^{\rm log} = -J_{23}^{\rm log} = \frac{1}{2}\ln\alpha,$$

$$J_{31}^{\rm log} = -\frac{\alpha}{4}\ln\alpha, \quad J_{32}^{\rm log} = -\frac{\alpha}{2}\ln\alpha, \quad J_{33}^{\rm log} = -\frac{3\alpha}{4}\ln\alpha,$$

$$J_{41}^{\rm log} = \frac{\alpha}{12}\ln\alpha, \quad J_{42}^{\rm log} = \frac{\alpha}{6}\ln\alpha, \quad J_{43}^{\rm log} = \frac{\alpha}{4}\ln\alpha. \tag{A4}$$

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