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Influence of the $U(1)_A$ anomaly on the QCD phase transition

Jonathan T. Lenaghan[*]
Department of Physics, Yale University, New Haven, Connecticut 06520-8124

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The $SU(3)_r \times SU(3)_l$ linear sigma model is used to study the chiral symmetry restoring phase transition of QCD at nonzero temperature. The line of second order phase transitions separating the first order and smooth crossover regions is located in the plane of the strange and nonstrange quark masses. It is found that if the $U(1)_A$ symmetry is explicitly broken by the $U(1)_A$ anomaly, then there is a smooth crossover to the chirally symmetric phase for physical values of the quark masses. However, if the $U(1)_A$ anomaly is absent, the region of first order phase transitions is significantly enlarged and it is found that there is a phase transition for physical values of the quark masses provided that the σ meson mass is at least 600 MeV. In both cases, the region of first order phase transitions in the quark mass plane is enlarged as the mass of the σ meson is increased.

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The ultimate goal of relativistic heavy ion experiments is to probe the phase diagram of quantum chromodynamics (QCD). General theoretical considerations indicate that at sufficiently high temperatures there should be a transition from ordinary hadronic matter to a chirally symmetric plasma of quarks and gluons [1]. The order parameter for this phase transition is the quark-antiquark condensate. Results from lattice gauge theory predict the temperature of this transition to be about 150 MeV [2]. The order of the phase transition, however, seems to depend very much on the number of quark flavors and their masses [3].

Classically, the matter part of the QCD Lagrangian with N_f flavors is invariant under the symmetry group $SU(N_f)_r \times SU(N_f)_l \times U(1)_A$. The axial $U(1)_A$ symmetry is broken to $Z(N_f)_A$ by a nonvanishing topological susceptibility [4] and the $SU(N_f)_r \times SU(N_f)_l$ symmetry is spontaneously broken to the diagonal group of vector transformations, $SU(N_f)_{r+l} = SU(N_f)_V$, by a nonvanishing expectation value for the quark-antiquark condensate. The $SU(N_f)_r \times SU(N_f)_l \times U(1)_A$ group is also explicitly broken by the effects of nonzero quark masses. It was shown by Pisarski and Wilczek that for three or more massless flavors, the phase transition for the restoration of the $SU(N_f)_r \times SU(N_f)_l$ symmetry is first order, while for two massless flavors the phase transition is second order [1].

The $U(1)_A$ symmetry may also be restored, if only partially, since instanton effects are Debye screened at high temperatures [5,6]. There are now two possibilities: either the $U(1)_A$ symmetry is restored at a temperature much greater than the $SU(N_f)_r \times SU(N_f)_l$ symmetry or the two symmetries are restored at (approximately) the same temperature [7]. Recent lattice gauge theory computations have demonstrated a rapid decrease in the topological susceptibility at T_c [8] and random matrix models also indicate that the two symmetries are restored simultaneously [9]. Perhaps more dramatically, it was also shown that the topological susceptibility vanishes at T_c in the large-N_c limit [10]. On the other hand, the fate of the $U(1)_A$ anomaly in nature is not completely clear since instanton liquid model calculations indicate that the topological susceptibility is essentially unchanged at T_c [11]. Additionally, other lattice computations which measure the chiral susceptibility find that the $U(1)_A$ symmetry restoration is at or below the 15% level [12,13].

Unlike the idealized massless quark limit, there are no general theoretical arguments which require that a phase transition exists for massive quarks. Indeed, some lattice simulations indicate that for physical quark masses, no phase transition occurs [3,14,15]. The general consensus from lattice computations is that in the plane of light quark masses (see Fig. 1) there is a first order region bounded by a line of second order transitions. Outside this region, there is no phase transition, but rather a crossover characterized by a rapid but smooth and continuous decrease of the quark-antiquark condensate. Given the present difficulties with performing lattice computations with realistic quark masses and a large number of sites, it is useful to complement the

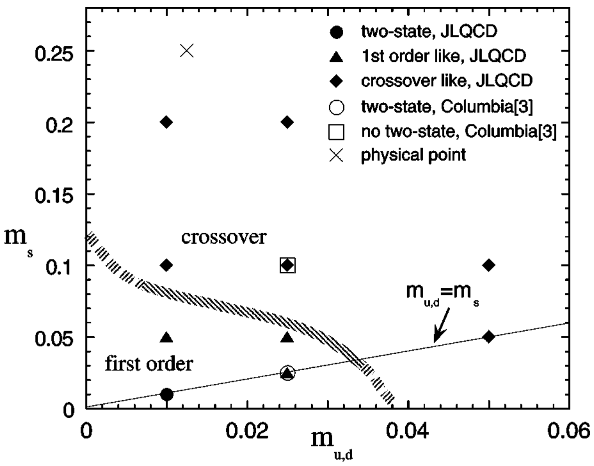

FIG. 1. The phase diagram on the $(m_{u,d}, m_s)$ plane as obtained from lattice computations. These results are a compilation of data from the JLQCD and the Columbia groups taken from Refs. [3] and [14]. The plot is from Ref. [14].

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[*] Current address: The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark.

present lattice results with effective models that capture some of the relevant dynamics of QCD. Some work for three flavors has been done in this direction [16–19]. In these works, the $SU(3)_r \times SU(3)_l$ linear sigma model was used to study the order of the chiral symmetry restoring phase transition as a function of the current quark masses with the ratio of the up-down to strange quark masses held fixed. In Refs. [16,17,19], a loop expansion is used to compute the effective potential and in Ref. [18] a mean-field analysis of this model was presented. The effects of the restoration of the $U(1)_A$ symmetry on the spectrum of hadronic observables in heavy ion collisions were addressed within the context of this model in Ref. [20].

In this paper, I present results concerning the order of the chiral symmetry restoring phase transition as a function of the current quark masses using the $SU(3)_r \times SU(3)_l$ linear sigma model without fixing the ratio of the masses. In addition, the effects of the $U(1)_A$ anomaly on the order of the QCD phase transition are investigated. Here, the Cornwall-Jackiw-Tomboulis (CJT) [21] formalism is used to derive gap equations for the condensates and the tadpole-resummed scalar and pseudoscalar nonet meson masses at nonzero temperature. The derivation of and the solutions to this set of equations in a variety of limits have been presented elsewhere [22]. The results agree qualitatively with earlier studies on a lattice [3,14,15] and with other studies using the $SU(3)_r \times SU(3)_l$ linear sigma model [16–19]. In the presence of an explicit $U(1)_A$ symmetry breaking term, I find that for physical values of the strange and nonstrange current quark masses, there is no phase transition but rather a smooth crossover. For smaller values of the masses, the phase transition is first order with a line of second order transitions separating the first order and the crossover regions. In the absence of an explicit $U(1)_A$ symmetry breaking term, the region of phase transitions is greatly enlarged. In particular, if the σ meson mass is greater than 600 MeV, then the transition is driven to first order for physical values of the quark masses. In both cases, the region of first order phase transitions is enlarged as the mass of the σ meson is increased.

The most general renormalizable theory compatible with the flavor symmetries of QCD is the $SU(3)_r \times SU(3)_l$ linear sigma model. While this model cannot account for the full dynamics of QCD, on the line of second order phase transitions the only relevant dynamics are determined by the symmetries of the theory. So, in the vicinity of this line, the use of the $SU(3)_r \times SU(3)_l$ linear sigma model is appropriate. Its Lagrangian is given by

$$\begin{aligned}\mathcal{L}(\Phi) = {} & \mathrm{Tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2\, \Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 \\ & - \lambda_2 \mathrm{Tr}(\Phi^\dagger \Phi)^2 + c[\mathrm{Det}(\Phi) + \mathrm{Det}(\Phi^\dagger)] \\ & + \mathrm{Tr}[H(\Phi + \Phi^\dagger)]. \end{aligned} \tag{1}$$

Here, Φ is a $U(3)$ matrix defined by $\Phi = T_a(\sigma_a + i\pi_a)$. The $T_a = \hat{\lambda}_a/2$ are the generators of $U(3)$ where $\hat{\lambda}_a$ are the Gell-Mann matrices with $\lambda_0 = \sqrt{2/3}\, I$. The T_a are normalized such that $\mathrm{Tr}(T_a T_b) = \delta_{ab}/2$.

The parameters of the Lagrangian are the bare mass m, a background matrix field $H = h_0 T_0 + h_8 T_8$, a cubic coupling c and two quartic couplings, λ_1 and λ_2. The various patterns of symmetry breaking and the parametrizations of the coupling constants for this Lagrangian were studied in [22] and will only be briefly reviewed here. For $H=0$, $c=0$ and $m^2 > 0$, the Lagrangian has a global $SU(3)_r \times SU(3)_l \times U(1)_A$ symmetry. The effects of the $U(1)_A$ symmetry breaking by a nonvanishing topological susceptibility (i.e. the presence of instantons in the QCD vacuum) are included by setting $c \neq 0$ which reduces the symmetry to $SU(3)_r \times SU(3)_l$. For nonzero H, chiral symmetry is explicitly broken.

I assume that there are nonzero vacuum expectation values for the σ_0 and σ_8 fields which I denote by $\overline{\sigma_0}$ and $\overline{\sigma_8}$. After shifting these fields by their expectation values and following [23], the Lagrangian can be rewritten as

$$\begin{aligned}\mathcal{L} = {} & \tfrac{1}{2}[\partial_\mu \sigma_a \partial^\mu \sigma_a + \partial_\mu \pi_a \partial^\mu \pi_a - (m_S^2)_{ab} \sigma_a \sigma_b \\ & - (m_P^2)_{ab} \pi_a \pi_b] + (\mathcal{G}_{abc} - \tfrac{4}{3} \mathcal{F}_{abcd} \overline{\sigma_d}) \sigma_a \sigma_b \sigma_c \\ & - 3(\mathcal{G}_{abc} + \tfrac{4}{3} \mathcal{H}_{abcd} \overline{\sigma_d}) \pi_a \pi_b \sigma_c - 2\mathcal{H}_{abcd} \sigma_a \sigma_b \pi_c \pi_d \\ & - \tfrac{1}{3} \mathcal{F}_{abcd} (\sigma_a \sigma_b \sigma_c \sigma_d + \pi_a \pi_b \pi_c \pi_d) - h_a \overline{\sigma_a}, \end{aligned} \tag{2}$$

where

$$\begin{aligned}\mathcal{G}_{abc} = {} & \tfrac{1}{6} c[d_{abc} - \tfrac{3}{2}(\delta_{a0} d_{0bc} + \delta_{b0} d_{a0c} + \delta_{c0} d_{ab0}) \\ & + \tfrac{9}{2} d_{000} \delta_{a0} \delta_{b0} \delta_{c0}], \\ \mathcal{F}_{abcd} = {} & \tfrac{1}{4} \lambda_1 (\delta_{ab} \delta_{cd} + \delta_{ad} \delta_{bc} + \delta_{ac} \delta_{bd}) \\ & + \tfrac{1}{8} \lambda_2 (d_{abn} d_{ncd} + d_{adn} d_{nbc} + d_{acn} d_{nbd}), \\ \mathcal{H}_{abcd} = {} & \tfrac{1}{4} \lambda_1 \delta_{ab} \delta_{cd} + \tfrac{1}{8} \lambda_2 (d_{abn} d_{ncd} + f_{acn} f_{nbd} \\ & + f_{bcn} f_{nad}), \\ (m_S^2)_{ab} = {} & m^2 \delta_{ab} - 6\mathcal{G}_{abc} \overline{\sigma_c} + 4\mathcal{F}_{abcd} \overline{\sigma_c}\, \overline{\sigma_d}, \\ (m_P^2)_{ab} = {} & m^2 \delta_{ab} + 6\mathcal{G}_{abc} \overline{\sigma_c} + 4\mathcal{H}_{abcd} \overline{\sigma_c}\, \overline{\sigma_d}. \end{aligned} \tag{3}$$

Here the summation runs over the index n only and d_{abc} and f_{abc} are the symmetric and antisymmetric structure constants, respectively, of $U(3)$.

The σ_a fields are members of the scalar ($J^\pi = 0^+$) nonet and the π_a fields are members of the pseudoscalar ($J^\pi = 0^-$) nonet. The $\pi_{1,2,3}$ are the pions, the $\pi_{4,5,6,7}$ are the kaons and the π_0 and the π_8 are admixtures of the η and the η' with mixing angle θ_{P}. The situation with the scalar nonet is not as clear and still somewhat controversial [24]. The σ_0 and the σ_8 are admixtures of the σ and the $f_0(1370)$ with mixing angle θ_{S}. The $\sigma_{1,2,3}$ are identified with the $a_0(980)$ and the $\sigma_{4,5,6,7}$ with the κ meson.

The explicit symmetry breaking terms can be determined (see, for instance, Ref. [22]), to be $h_0 = (1/\sqrt{6})(m_\pi^2 f_\pi + 2\, m_K^2 f_K)$, $h_8 = (2/\sqrt{3})(m_\pi^2 f_\pi - m_K^2 f_K)$.

The gap equations (Schwinger–Dyson equations) derived from the CJT effective potential [21] in the tadpole-resummed approximation, or Hartree approximation, are found to be

$$
\begin{aligned}
(\mathcal{S}_{ab}(k))^{-1} = & -k^2+m^2\delta_{ab}-6\mathcal{G}_{abc}\overline{\sigma}_c+4\mathcal{F}_{abcd}\overline{\sigma}_c\overline{\sigma}_d \\
& +4\mathcal{F}_{abcd}\int_k \mathcal{S}_{cd}(k)+4\mathcal{H}_{abcd}\int_k \mathcal{P}_{cd}(k), \\
(\mathcal{P}_{ab}(k))^{-1} = & -k^2+m^2\,\delta_{ab}+6\mathcal{G}_{abc}\,\overline{\sigma}_c+4\mathcal{H}_{abcd}\overline{\sigma}_c\,\overline{\sigma}_d \\
& +4\mathcal{H}_{abcd}\int_k \mathcal{S}_{cd}(k)+4\mathcal{F}_{abcd}\int_k \mathcal{P}_{cd}(k), \\
h_a = & \, m^2\,\overline{\sigma}_a-3\mathcal{G}_{abc}\overline{\sigma}_b\overline{\sigma}_c-3\mathcal{G}_{abc}\int_k \mathcal{S}_{cb}(k) \\
& +3\mathcal{G}_{abc}\int_k \mathcal{P}_{cb}(k)+\tfrac{4}{3}\mathcal{F}_{abcd}\overline{\sigma}_d\overline{\sigma}_b\overline{\sigma}_c \\
& +4\mathcal{F}_{abcd}\overline{\sigma}_d\int_k \mathcal{S}_{cb}(k)+4\mathcal{H}_{bcad}\overline{\sigma}_d\int_k \mathcal{P}_{cb}(k),
\end{aligned}
\tag{4}
$$

where, in the last equation, $a=0{,}8$ and $\mathcal{S}_{ab}(k)$ $[\mathcal{P}_{ab}(k)]$ are the Green's functions for the scalar [pseudoscalar] mesons. $\mathcal{S}_{08}(k)$ and $\mathcal{P}_{08}(k)$, however, are nonzero on account of the mixing between the singlet and the octet states. All other non-diagonal entries are identically zero. As such, it is necessary to rotate these Green's functions into the mass eigenbasis since only physical fluctuations can contribute to the masses:

$$
U^{\mathrm{S}}_{ia}\mathcal{S}_{ab}(k)U^{\mathrm{S}}_{jb}=\tilde{\mathcal{S}}_i(k)\,\delta_{ij},\quad U^{\mathrm{P}}_{ia}\mathcal{P}_{ab}(k)U^{\mathrm{P}}_{jb}=\tilde{\mathcal{P}}_i(k)\,\delta_{ij},
\tag{5}
$$

where $U^{\pi}_{ia}=\delta_{ia}$ for $i,a\neq 0{,}8$ and where U^{π}_{ia} is given by an $O(2)$ rotation by θ_{P} in the 0-8 block. The definition for U^{σ}_{ia} is similarly given by $\theta_{\mathrm{P}}\rightarrow\theta_{\mathrm{S}}$. The thermal integral arising from tadpole diagrams is

$$
\int_k \tilde{\mathcal{S}}_i(k)=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\,\frac{1}{\epsilon_{\mathbf{k}}[(\tilde{M}^2_S)_i]}\left(\exp\left\{\frac{\epsilon_{\mathbf{k}}[(\tilde{M}^2_S)_i]}{T}\right\}-1\right)^{-1}
$$

and similarly for the pseudoscalar tadpole integrals, $\int_k\mathcal{P}_{cd}(k)$. Here, $\epsilon_{\mathbf{k}}[(\tilde{M}^2_S)_i]=[\mathbf{k}^2+(\tilde{M}^2_S)_i]^{1/2}$ is the relativistic energy of the ith scalar quasiparticle with momentum $\mathbf{k}$. I have neglected the vacuum contribution arising from the loop integrals. Implementing a systematic renormalization scheme is difficult but possible in this approximation (see [25]). The results, however, are not significantly altered.

Since in the Hartree approximation the gap equations do not have an explicit momentum dependence, we can assume that $[\mathcal{S}_{ab}(k)]^{-1}=-k^2+M^2_S$ where M_S depends on temperature but not momentum, and similarly for $[\mathcal{P}_{ab}(k)]^{-1}$. Equations (4) and (5) are then fixed point equations and can be numerically solved simultaneously as a function of temperature for M_π, M_K, M_η, $M_{\eta'}$, M_σ, M_κ, M_{f_0}, M_{a_0}, $\overline{\sigma}_0$, $\overline{\sigma}_8$, θ_{P} and θ_{S}. The numerical solutions for a variety of parameters are given in Ref. [22].

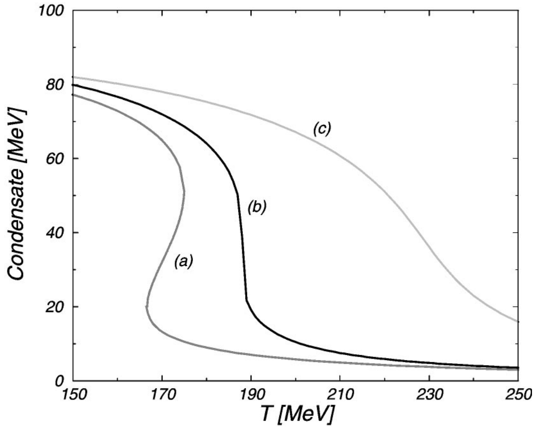

FIG. 2. The $\overline{\sigma}_0$ condensate for various values of the kaon mass with a pion mass of 100 MeV. The kaon mass is 80, 200 and 300 MeV for curves (a), (b) and (c), respectively. Curve (a) is a first order phase transition, curve (b) is close to second order and curve (c) lies in the crossover region.

The condensate and mass gap equations are solved with fixed m, c, λ_1 and λ_2, while varying the background fields, h_0 and h_8. The determination of the coupling constants is detailed in Ref. [22]. For $c\neq 0$, the four couplings in the Lagrangian are fitted to yield the physical tree-level masses of the pion, kaon, σ, η and η', while for $c=0$, the remaining three couplings are determined from the physical tree-level masses of the pion, kaon, σ and η. The background fields are proportional to the current quark masses: $m_{\mathrm{up}}=m_{\mathrm{down}}=a(h_0+h_8/\sqrt{2})$, $m_{\mathrm{strange}}=b(h_0-\sqrt{2}h_8)$. For simplicity, I assume temperature independent proportionality constants, a and b. Requiring that $m_\pi=138$ MeV, $m_K=496$ MeV, $m_{\mathrm{up}}=m_{\mathrm{down}}=10$ MeV and $m_{\mathrm{strange}}=150$ MeV gives $a=4.64\times 10^{-6}$ $[\mathrm{MeV}]^{-2}$ and $b=2.27\times 10^{-6}$ $[\mathrm{MeV}]^{-2}$.

To determine the order of the phase transition, I examined the continuity of the order parameters as a function of temperature. For a first order transition, the condensates are multivalued functions of temperature in the vicinity of the phase transition. For a smooth crossover, the condensates are smooth singlevalued functions of temperature and always nonzero. This behavior is demonstrated in Fig. 2. Only the nonstrange condensate is shown since both condensates exhibit qualitatively the same behavior.

The numerical results are plotted in Fig. 3. For $c\neq 0$, these results agree with those of lattice groups [3,14,15]. For $m_\sigma=1000$ MeV, the authors of Ref. [18] report the ratio of the critical current up-down quark mass to the physical up-down quark mass for $m_{\mathrm{strange}}/m_{\mathrm{up,down}}=32$ to be ~ 0.01. The corresponding value found in the present work is ~ 0.20. The larger value found here is most likely due to the inclusion of thermal fluctuations from the scalar and pseudoscalar nonets. For $m_{\mathrm{up}}=m_{\mathrm{down}}=0$, the transition is first order from zero strange quark mass to some critical strange quark mass.

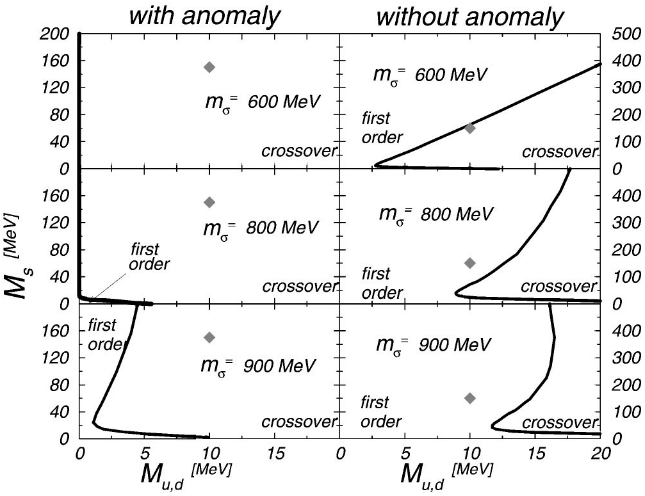

FIG. 3. The lines of second order phase transitions in the plane of the nonstrange and strange current quark masses for $m_\sigma = 600$ MeV, $m_\sigma = 800$ MeV and $m_\sigma = 900$ MeV. The cases where the $U(1)_A$ symmetry is explicitly broken by the axial anomaly, $c \neq 0$, are shown on the left, and the cases where the $U(1)_A$ symmetry is exact, $c = 0$, are shown on the right. The physical mass point ($m_{\mathrm{up,down}} = 10$ MeV and $m_{\mathrm{strange}} = 150$ MeV) is indicated by the diamond.

For $m_\sigma = 800$ MeV, this critical strange quark mass is about 16 MeV, while for $m_\sigma = 900$ MeV, it is 260 MeV.

For $c = 0$, however, the results are dramatically different. In particular, the line of second order transitions does not seem to approach the strange quark mass axis. For $m_\sigma = 600$ MeV, the physical point, $m_{\mathrm{up}} \cong m_{\mathrm{down}} \cong 10$ MeV and $m_{\mathrm{strange}} \cong 150$MeV, is just outside the first order region. For larger values of the σ meson mass, the physical point is well within the first order region. The results also seem to indicate that for $c = 0$ there is a first order phase transition for three flavors provided only that one of the flavors is sufficiently heavier than the other two flavors. The departure of the second order phase transition line from the strange quark mass axis was also predicted using arguments from large-N_c chiral perturbation theory in Ref. [26].

At this point, it should be mentioned that the Hartree approximation sometimes predicts a first order transition when the transition is actually second order. For example, renormalization group arguments predict a second order phase transition for the massless limit of the $O(4)$ linear sigma model [1], while the Hartree approximation predicts a first order transition (see, for instance, Ref. [25]). This is not a problem in the low quark mass region since the transition is expected to be first order. The location of the second order line should not be significantly affected.

Additionally, the cubic and quartic couplings are fixed and temperature independent. The running of the couplings with temperature should be at most logarithmic, while the integrals arising from the tadpole diagrams depend quadratically on the temperature. So it is reasonable that the running of the couplings does not qualitatively alter these results. On the other hand, if the Coleman-Weinberg mechanism is strongly operative, some portion of the crossover region may actually be driven to first order [27].

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