

Low-energy limits of theories with two supersymmetries

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Given its non-renormalization properties, low-energy supersymmetry provides an attractive framework for extending the standard model and for resolving the hierarchy problem. Models with softly broken $N=1$ supersymmetry have been extensively studied and are phenomenologically successful. However, it could be that an extended $N=2$ supersymmetry survives to low energies, as suggested by various constructions. We examine the phenomenological viability and implications of such a scenario. We show that consistent chiral fermion mass generation emerges in $N=2$ theories, which are vectorial, as a result of supersymmetry breaking at low energies. A rich mirror quark and lepton spectrum near the weak scale with model-dependent decay modes is predicted. A Z_2 mirror parity is shown to play an important role in determining the phenomenology of the models. It leads, if conserved, to a new stable particle, the LMP. Consistency of the $N=2$ framework and its unique spectrum with electroweak precision data is considered, and the discovery potential in the next generation of hadron collider experiments is stressed. Mirror quark pair production provides the most promising discovery channel. Higgs boson searches are also discussed and it is shown that there is no upper bound on the prediction for the Higgs boson mass in the framework of low-energy supersymmetry breaking, in general, and in the $N=2$ framework, in particular. Possible $N=2$ realizations of flavor symmetries and of neutrino masses are also discussed.

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I. INTRODUCTION

Supersymmetry and its boson-fermion symmetry provide an attractive framework for embedding the standard model of electroweak and strong interactions (SM) [1]. The electroweak scale is understood in this framework as roughly the scale of supersymmetry breaking in the global theory and is protected, in general, from destabilization at the quantum level. In particular, softly broken $N=1$ supersymmetry provides a phenomenologically successful extension of the SM [2]. The particle content is the minimal one required by boson-fermion symmetry and, regardless of the exact details of the soft supersymmetry breaking (SSB) spectrum parameters, the corresponding β functions predict gauge coupling unification at a scale of $\mathcal{O}(10^{16})$ GeV [3]. The theory tends to decouple from most electroweak observables [for particles of $\mathcal{O}(300)$ GeV] [4] while the absence of flavor changing neutral currents is a source of information about the high-energy origins of the low-energy effective theory. At high energies, the rigid supersymmetry can be extended to supergravity [1,2]: the first step towards gravity-gauge unification and further embedding of the SM in a theory of quantum gravity of which supergravity is the sub-Planckian limit—for example, superstring theory.

However, there exists a tension between a “bottom-up” approach, which beginning with the SM motivates its $N=1$ supersymmetry extension, and a “top-down” approach, which beginning with a superstring theory often suggests that an extended $N=2$ supersymmetry is broken at some energy directly to $N=0$ [5]. If supersymmetry is to stabilize the weak scale and resolve the hierarchy problem associated with its instability in the SM, then the extended supersymmetry can be broken in this case only near that scale. Indeed, current knowledge of string theory is far from sufficient for understanding its electroweak-scale limit or how the SM would be embedded in such a theory, and therefore it is

premature to draw conclusions from string theory regarding the nature of the weak-scale supersymmetry. Nevertheless, it is an intriguing and highly interesting question to ask whether an $N=2$ supersymmetry extension of the SM at weak-scale energies is phenomenologically viable, what constraints the infrared SM limit imposes on ultraviolet realizations of such a theory, and what would be its signatures. Here, as a first step towards addressing these questions, we will investigate some of their more fundamental aspects, laying the foundation for, and hopefully intriguing, further discussion.

The phenomenology of $N=2$ supersymmetry and its extended spectrum were studied over the years by only a couple of groups [6–8]. Its “ultraviolet” elegance stemming from its constrained structure (for example, there is only one coupling in the theory, the gauge coupling, and the theory is not renormalized beyond one loop) does not translate to an equivalent elegance in the infrared. On the contrary, the $N=2$ intrinsic constraints make it difficult to reconcile the framework with the SM and with observations. Most notably, the theory does not contain chiral Yukawa couplings or any other source of chiral mass generation. Once supersymmetry is broken the fermion mass issue can be resolved. However, one then finds that the $N=2$ theory does not decouple from electroweak observables (nor does it suggest gauge unification). These issues place strong constraints on the properties of the extended spectrum that $N=2$ supersymmetry predicts.

Theories with two supersymmetries contain a rich spectrum: While each SM fermion (boson) is accompanied by a boson (fermion) superpartner to form a chiral or a vector superfield in the $N=1$ extension, each chiral $N=1$ superfield is further accompanied by an anti-chiral superfield to form a vector-like hypermultiplet in the $N=2$ extension. An $N=1$ vector superfield is accompanied in the $N=2$ extension by

an $N=1$ chiral superfield in the appropriate representation, the *mirror* gauge superfield. For example, a SM quark is partnered, in addition to the squark, also with a *mirror* quark and a *mirror* squark. The gauge boson is partnered not only with the gaugino but also with a complex scalar and additional Majorana fermion in the adjoint representation (or singlets in the Abelian case). The number of particles is increased 4 times with respect to the SM.

In describing the extended spectrum we used the $N=1$ superfield language. Indeed, it is possible (and we will do so) to formulate the $N=2$ framework in this language. In order to impose the additional $N=2$ constraints one has to specify a set of global R symmetries. It includes a vectorial $SU(2)_R$ exchange R symmetry which forbids, as mentioned above, any chiral fermion Yukawa or mass terms. Once the vectorial symmetry is broken, all chiral and anti-chiral fermion masses are proportional to the Higgs vacuum expectation values (VEVs) which spontaneously break the SM $SU(2)_L$. One expects that the new particles are entangled with the ordinary SM fields, as the gauge symmetries do not forbid their mixing. This again provides an important set of constraints on the theory and on the dynamics that breaks it. It also provides clear tests of the framework. Most importantly, unlike a $N=1$ theory that must be discovered via its somewhat arbitrary predictions for the spectrum of new bosons and Majorana fermions, an $N=2$ theory would be readily discovered or excluded in the next generation of hadron collider via its strongly constrained predictions of the mirror (anti-chiral) fermion spectrum, which is not expected to be much heavier than the top quark. Henceforth, a study of the $N=2$ framework is timely and well motivated.

The knowledgeable reader may be questioning the validity of any such an extension which contains contributions of three anti-chiral families to the oblique S parameter [9] (which measures quantum corrections from new physics to $Z-\gamma$ mixing). Usually, one assumes that mass-dependent terms are negligible, as is appropriate in the decoupling limit $m_{f_{\text{new}}} \gg m_Z$. In that case, the mass-independent contribution of each chiral or anti-chiral generation to S is positive, contrary to current measurements which imply $S \leq 0$ [10]. Therefore, one may argue that S excludes extra (anti-)chiral families. However, the $N=2$ spectrum is far too rich and complicated to allow for such arguments. Current data do not exclude (though they do not suggest) three anti-chiral families if the spectrum of the $N=2$ mirror fermions breaks the custodial $SU(2)$ symmetry of the electroweak interactions and is (at least partially) ‘‘light’’ [11]. The situation is even more arbitrary if the Majorana fermion spectrum, which also contains custodial $SU(2)$ breaking mass terms, is considered [12]. We therefore proceed and investigate $N=2$ models, further discussing this and other phenomenological issues in a dedicated section. Our main focus, however, is establishing tools for the construction of the chiral fermion spectrum below the $N=2$ breaking scale, while a careful investigation of the oblique S , T , and U parameters will be reported in a future article [13].

The fermion mass problems in these models have many facets. First and foremost, the generation of any chiral spec-

trum must be a result of supersymmetry breaking. Second, the two sectors have to be distinguished with sufficiently heavy mirror fermions and (relatively) light ordinary fermions, with any mixing between the two sectors suppressed, at least in the case of the first two families. In addition there are the issues of the heavy third family and of the very light neutrinos in the ordinary sector and, subsequently, of flavor symmetries and their relation to supersymmetry breaking. In order to address these issues we choose to formulate a global $N=2$ theory as an $N=1$ theory with a second supersymmetry manifest only through global R symmetries which are imposed on the $N=1$ description. [For example, the $SU(2)_R$ mentioned above.] This is a standard procedure [6] that allows, in our case, the use of the $N=1$ spurion formalism [14] in the construction of the fermion spectrum. Specifically, we assume below the following:

(i) The matter content is that of the minimally extended $N=2$ supersymmetric SM (MN2SSM) (and that of a flavor sector, if exists) given in terms of $N=1$ chiral and vector superfields.

(ii) The $N=2$ imposed global symmetries (or a subset thereof) are explicitly broken by non-renormalizable terms in the Kahler potential. These terms are characterized by a scale M . In the limit $M \rightarrow \infty$ the full supersymmetry is restored.

(iii) The only VEVs are ($N=1$ breaking) F -type VEVs which generate all dimensionful and dimensionless couplings in the electroweak theory (aside from the gauge coupling). Electroweak symmetry breaking VEVs (and flavor symmetry breaking VEVs) are then induced in the resulting effective theory.

(iv) To the most part we will also assume that some flavor and ‘‘mirror’’ symmetries, which do not commute with the $N=2$ R symmetries, are conserved in the resulting effective theory.

We will explore the chiral fermion spectrum within this framework and establish phenomenologically viable low-energy limits of the $N=2$ framework which could be probed, given the fermion spectrum, in the near future.

We briefly review $N=2$ supersymmetry in Sec. II. Though we do not focus on the boson spectrum and the dimensionful SSB parameters, they are straightforward to write in the $N=1$ formalism. This will be done as a warm-up exercise in Sec. III (and again, using the spurion formalism, in Sec. VI). The softly broken $N=2$ model will be compared to its $N=1$ equivalent. In Sec. IV we consider the possibility of breaking the chiral symmetries primarily in the SSB scalar potential, generating the fermion spectrum only radiatively [6,15]. While this option is viable in some cases for the SM spectrum, it cannot provide a consistent mirror fermion spectrum. In Sec. V we exploit the $N=1$ spurion formalism to classify the most general supersymmetry breaking framework, and in Sec. VI we use it to discuss a general $N=2$ framework and the possible tree-level origins of the chiral fermion spectrum. We find that certain Kahler operators can generate such a spectrum as long as the supersymmetries are broken at relatively low energies, which we will assume. (Note that large parts of our discussion are applicable to low-energy $N=1$ supersymmetry breaking as well.) The special issue of the heavy SM third family fermions is addressed

in Sec. VII. In Sec. VIII we comment on neutrino physics within the context of $N=2$ supersymmetry. Direct and indirect signatures and other phenomenological issues are discussed in Sec. IX. Unlike $N=1$ supersymmetry, $N=2$ cannot escape detection in the next generation of hadron colliders. We conclude with a summary, an outlook, and a comparison to previous constructions, in Sec. X.

II. THE MN2SSM FRAMEWORK

The $N=2$ supersymmetry algebra has two spinorial generators Q_α^i , $i=1,2$, satisfying

$$\{Q_\alpha^i, \bar{Q}_\alpha^j\} = \sigma_{\alpha\alpha}^\mu P_\mu \delta^{ij}, \quad (1)$$

where σ^μ are, as usual, the Pauli matrices and P_μ is the momentum. The supercharges Q_α^i form a doublet of the (exchange) $SU(2)_R$ R symmetry, which must be imposed when using the $N=1$ formulation to describe a $N=2$ theory. The lowest $N=2$ spin representations, which are the relevant ones for embedding the SM, are the hypermultiplet and vector multiplet. Written in the familiar $N=1$ language, the hypermultiplet is composed of two $N=1$ chiral multiplets $X=(x, \psi_x)$ and $Y=(y, \psi_y)$, with Y occupying representations \mathcal{R} of the gauge groups which are conjugate to that of X , $\mathcal{R}(X)=\mathcal{R}(Y^\dagger)$. Schematically, the hypermultiplet is described by a ‘‘diamond’’ plot

$$\begin{array}{ccc} & \psi_x & +\frac{1}{2} \\ & \nearrow & \\ x & & y^\dagger \quad 0 \\ & \searrow & \\ & \bar{\psi}_y & -\frac{1}{2} \end{array}$$

where the first, second and third rows correspond to helicity $-1/2$, 0 , and $+1/2$ states. The vector multiplet contains a $N=1$ vector multiplet $V=(V^\mu, \lambda)$, where λ is a gaugino and a $N=1$ chiral multiplet $\Phi_V=(\phi_V, \psi_V)$ in the adjoint representation of the gauge group (or a singlet in the Abelian case). Schematically, it is described by

$$\begin{array}{ccc} & V^\mu & 1 \\ & \nearrow & \\ \lambda & & \psi_V \quad \frac{1}{2} \\ & \nearrow & \\ & \phi_V & 0 \end{array}$$

where the first, second and third rows correspond to helicity 0 , $1/2$, and 1 states. The $N=1$ superfields are given by the two 45° sides of each diamond (indicated by arrows), with the gauge field arranging itself in its chiral representation $W_\alpha \sim \lambda_\alpha + \theta_\alpha V$. The particle content doubles in comparison to the $N=1$ supersymmetry case and it is 4 times that of the SM. For each of the usual chiral fermions ψ_x and its complex-scalar partner x , there is a conjugate mirror fermion ψ_y and a complex scalar y (so that the theory is vectorial).

For each gauge boson and gaugino, there is a *mirror* gauge boson ϕ_V and a *mirror* gaugino ψ_V .

The $N=0$ boson and fermion components of the hyper- and vector-multiplet form $SU(2)_R$ representations. States with equal helicity form a $SU(2)_R$ doublet (x, y^\dagger) and an anti-doublet (ψ_V, λ) , while all other states are $SU(2)_R$ singlets. In fact, the full R symmetry is $U(2)_R$ of which the exchange $SU(2)_R$ is a subgroup. There are additional $U(1)_R^{N=2}$, $U(1)_J^{N=2}$ subgroups such that the R symmetry is either $SU(2)_R \times U(1)_R^{N=2}$ or in some cases only a reduced $U(1)_J^{N=2} \times U(1)_R^{N=2}$. The different superfields $X \sim x + \theta\psi_x$, etc., transform under the $U(1)$ symmetries with charges R and J given by

$$R(X) = r = -R(Y), \quad R(\Phi_V) = -2, \quad (2)$$

$$J(X) = -1 = J(Y), \quad J(\Phi_V) = 0, \quad (3)$$

and $R(W_\alpha) = J(W_\alpha) = -1$. The (manifest) supercoordinate θ (which carries a chiral index, denoted explicitly in some cases) has, as usual, charge $R(\theta) = J(\theta) = -1$.

The $SU(2)_R \times U(1)_R^{N=2}$ invariant $N=2$ Lagrangian can be written in the $N=1$ language as [1,6]

$$\begin{aligned} L = & \frac{1}{8g^2} [W^\alpha W_\alpha]_F + [\sqrt{2}igY\Phi_V X]_F + \text{H.c.} \\ & + [2\text{Tr}(\Phi_V^\dagger e^{2gV}\Phi_V e^{-2gV} + X^\dagger e^{2gV}X + Y^\dagger e^{-2gV}Y)]_D, \end{aligned} \quad (4)$$

where $\Phi_V = \Phi_V^a T^a$ and $V = V^a T^a$, T^a being the respective generators. The second F term is the superpotential. The only free coupling is the gauge coupling constant g : The coupling constant of the Yukawa term in the superpotential is fixed by the gauge coupling due to a global $SU(2)_R$. In particular, the $SU(2)_R$ symmetry forbids any chiral Yukawa terms so that fermion mass generation is linked to supersymmetry breaking, as will be discussed in the following sections. Note that the $U(1)_R^{N=2}$ forbids any mass terms $W \sim \mu' XY$ (and the full R symmetry forbids the usual $N=1$ μ term $W \sim \mu H_1 H_2$ to be discussed later). Unlike the $SU(2)_R$, $U(1)_R^{N=2}$ can survive supersymmetry breaking.

The $N=2$ Lagrangian (4) contains several discrete symmetries, which may or may not be broken in the broken supersymmetry regime. There is a trivial extension of the usual $N=1$ R -parity (R_p) Z_2 symmetry which does not distinguish the ordinary fields from their mirror partners:

$$\theta \rightarrow -\theta, \quad X_M \rightarrow -X_M, \quad Y_M \rightarrow -Y_M, \quad (5)$$

where all other supermultiplets are R_p even and where the hypermultiplets have been divided into the odd matter multiplets (X_M, Y_M) and the even Higgs multiplets (X_H, Y_H) . (Note that V is even but W_α is odd.) As in the $N=1$ case, all the ordinary and mirror quarks, leptons and Higgs bosons are R_p even, while the ordinary and mirror gauginos are R_p odd. R_p is conveniently used to define the superpartners (or spar-

TABLE I. Hypermultiplets and vector multiplets in the MN2SSM.

		X, V		Y, Φ_V		
	Notation	Fields	$(SU(3)_C, SU(2)_L, U(1)_Y)$	Notation	Fields	$(SU(3)_C, SU(2)_L, U(1)_Y)$
Matter	Q	(q, \tilde{Q})	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	Q'	(q', \tilde{Q}')	$(\bar{\mathbf{3}}, \bar{\mathbf{2}}, -\frac{1}{6})$
	U	(u, \tilde{U})	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	U'	(u', \tilde{U}')	$(\mathbf{3}, \mathbf{1}, \frac{2}{3})$
	D	(d, \tilde{D})	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$	D'	(d', \tilde{D}')	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$
multiplets	L	(l, \tilde{L})	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	L'	(l', \tilde{L}')	$(\mathbf{1}, \bar{\mathbf{2}}, \frac{1}{2})$
	E	(e, \tilde{E})	$(\mathbf{1}, \mathbf{1}, 1)$	E'	(e', \tilde{E}')	$(\mathbf{1}, \mathbf{1}, -1)$
Higgs	H_1	(H_1, \tilde{H}_1)	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	H'_1	(H'_1, \tilde{H}'_1)	$(\mathbf{1}, \bar{\mathbf{2}}, \frac{1}{2})$
	H_2	(H_2, \tilde{H}_2)	$(\mathbf{1}, \bar{\mathbf{2}}, \frac{1}{2})$	H'_2	(H'_2, \tilde{H}'_2)	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
Vector	g	(g, \tilde{g})	$(\mathbf{8}, \mathbf{1}, 0)$	Φ_g	(ϕ_g, ψ_g)	$(\mathbf{8}, \mathbf{1}, 0)$
	W	(W, \tilde{W})	$(\mathbf{1}, \mathbf{3}, 0)$	Φ_W	(ϕ_W, ψ_W)	$(\mathbf{1}, \mathbf{3}, 0)$
	B	(B, \tilde{B})	$(\mathbf{1}, \mathbf{1}, 0)$	Φ_B	(ϕ_B, ψ_B)	$(\mathbf{1}, \mathbf{1}, 0)$

cles) as the R_P -odd particles [16]. The lightest superparticle (LSP) is stable if R_P remains unbroken.

A second parity, called mirror parity (M_P), distinguishes the mirror particles from their partners:

$$\theta \rightarrow \theta, \quad Y_M \rightarrow -Y_M, \quad Y_H \rightarrow -Y_H, \quad \Phi_V \rightarrow -\Phi_V, \quad (6)$$

and all other superfields (including W_α) are M_P even. It is convenient to use mirror parity to define the mirror particles as the M_P -odd particles. [This definition should not be confused with other definitions of mirror particles used in the literature and which are based on a left-right group $SU(2)_L \times SU(2)_R$ or a mirror world which interacts only gravitationally with the SM world.] The lightest mirror parity odd particle (LMP) is also stable in a theory with unbroken mirror parity. However, if supersymmetry breaking does not preserve mirror parity, mixing between the ordinary matter and the mirror fields is allowed.

There is also a reflection (exchange) symmetry (which must be broken at low energies), the mirror exchange symmetry:

$$X \leftrightarrow Y, \quad \Phi_V \leftrightarrow \Phi_V^T, \quad V \leftrightarrow -V^T. \quad (7)$$

As in the case of this continuous $SU(2)_R$, if the reflection symmetry remains exact after supersymmetry breaking, then for each left-handed fermion there would be a degenerate right handed mirror fermion in the conjugate gauge representation, which is phenomenologically not acceptable.

For easy reference, we list in Table I the minimal particle content of the MN2SSM, where a mirror partner Y (Φ_V) exists for every ordinary superfield X (V) of the minimal $N=1$ supersymmetric extension of the standard model (MSSM). We could eliminate one Higgs hypermultiplet and treat H_1 and H_2 as mirror partners. However, this could lead to the spontaneous breaking of mirror parity when the Higgs bosons acquire VEVs and, as a result, to a more complicated radiative structure than the theory with two Higgs hypermultiplets. (We note, however, that it is possible in these theories that only one Higgs doublet acquires a VEV as the chiral

Yukawa coupling is related to supersymmetry breaking and, unlike in the $N=1$ case with high-energy supersymmetry breaking, is not necessarily constrained by holomorphicity.)

For the above particle content, and imposing the full $U(2)_R$ on the superpotential, the theory is scale invariant and is given by the superpotential (after phase redefinitions)

$$\begin{aligned} W/\sqrt{2} = & g_3(Q' \Phi_g Q + U' \Phi_g U + D' \Phi_g D) \\ & + g_2(Q' \Phi_W Q + L' \Phi_W L + H'_1 \Phi_W H_1 + H'_2 \Phi_W H_2) \\ & + g_1(\frac{1}{6} Q' \Phi_B Q - \frac{2}{3} U' \Phi_B U + \frac{1}{3} D' \Phi_B D - \frac{1}{2} L' \Phi_B L \\ & + \tilde{E}' \Phi_B E - \frac{1}{2} H'_1 \Phi_B H_1 + \frac{1}{2} H'_2 \Phi_B H_2). \end{aligned} \quad (8)$$

After substitution in the Lagrangian (4), the superpotential (8) gives rise in the usual manner [1,2] to gauge-quartic and gauge-Yukawa interactions. All interactions are gauge interactions. Table I and the superpotential (8) define the MN2SSM (in the supersymmetric limit).

III. SOFTLY BROKEN $N=2$

Once the MN2SSM is written as an $N=1$ theory with appropriate spectrum and global symmetries, as explained above, supersymmetry breaking translates to the introduction of (a) SSB dimensionful parameters, which lift the boson-fermion degeneracy and could also break the continuous R symmetries, and (b) dimensionless parameters, which spoil the constrained $N=2$ relations between gauge, Yukawa and quartic couplings. We postpone discussion of the latter to Sec. V, where we also write all parameters as polynomials in the supersymmetry breaking VEVs. The theory studied in this section is the (global) $N=2$ SM, the MN2SSM, with explicitly and softly broken supersymmetries. The breaking is parametrized by the familiar SSB terms (which also parametrize supersymmetry breaking in $N=1$ theories). These terms are soft in the sense that the theory is at most logarithmically divergent even after their introduction. The SSB terms can be chosen to preserve or to break the global sym-

TABLE II. The $N=1$ MSSM (R_P even) soft supersymmetry breaking terms and their M_P even mirrored versions.

	$N=1$ SSB	Mirror term
Scalar masses (Hypermultiplets)	$m_{\tilde{Q}}^2 \tilde{Q}^\dagger \tilde{Q}, m_{\tilde{U}}^2 \tilde{U}^\dagger \tilde{U}, m_{\tilde{D}}^2 \tilde{D}^\dagger \tilde{D},$ $m_{\tilde{L}}^2 \tilde{L}^\dagger \tilde{L}, m_{\tilde{E}}^2 \tilde{E}^\dagger \tilde{E},$ $m_{H_1}^2 H_1^\dagger H_1, m_{H_2}^2 H_2^\dagger H_2$	$m_{\tilde{Q}'}^2 \tilde{Q}'^\dagger \tilde{Q}', m_{\tilde{U}'}^2 \tilde{U}'^\dagger \tilde{U}', m_{\tilde{D}'}^2 \tilde{D}'^\dagger \tilde{D}',$ $m_{\tilde{L}'}^2 \tilde{L}'^\dagger \tilde{L}', m_{\tilde{E}'}^2 \tilde{E}'^\dagger \tilde{E}',$ $m_{H_1'}^2 H_1'^\dagger H_1', m_{H_2'}^2 H_2'^\dagger H_2'$
Gaugino masses	$M_1 \tilde{B} \tilde{B}, M_2 \tilde{W} \tilde{W}, M_3 \tilde{g} \tilde{g}$	$M_1'' \psi_B \psi_B, M_2'' \psi_W \psi_W, M_3'' \psi_g \psi_g$
Trilinear operators	$A_u H_2 \tilde{Q} \tilde{U}, A_d H_1 \tilde{Q} \tilde{D}, A_e H_1 \tilde{L} \tilde{E}$ $A_u H_1^\dagger \tilde{Q} \tilde{U}, A_d H_2^\dagger \tilde{Q} \tilde{D}, A_e H_2^\dagger \tilde{L} \tilde{E}$	$A_u'' H_1 \tilde{Q}' \tilde{U}', A_d'' H_2 \tilde{Q}' \tilde{D}', A_e'' H_2 \tilde{L}' \tilde{E}'$ $A_u'' H_2^\dagger \tilde{Q}' \tilde{U}', A_d'' H_1^\dagger \tilde{Q}' \tilde{D}', A_e'' H_1^\dagger \tilde{L}' \tilde{E}'$
Bilinear scalar operators	$b H_1 H_2$	$b'' H_1' H_2'$
Bilinear fermion operators	$\tilde{\mu} \tilde{H}_1 \tilde{H}_2$	$\tilde{\mu}'' \tilde{H}_1' \tilde{H}_2'$

metries of the $N=2$ theory. However, we leave this issue aside, imposing none of the continuous R symmetries on the SSB terms; i.e., we assume for now maximal breaking. We concentrate instead on (i) those SSB terms that are unique to $N=2$ theories and (ii) those that break the chiral symmetries (in the SSB scalar potential). In order to control radiative mixing between the sectors as well as lepton and baryon number violation we assume that the Z_2 mirror and R parities are conserved, unless otherwise specified.

In accordance with mirror parity conservation, the MN2SSM contains in each sector (i.e., the M_P -even ordinary and M_P -odd mirror sectors) gaugino mass terms (M_λ), scalar mass terms (m_ϕ^2), gauge invariant scalar and fermion bilinear terms (b , often denoted as $B\mu$, and $\tilde{\mu}$, respectively) and trilinear (A) terms. These terms are the the SSB terms familiar from the $N=1$ MSSM, only in two ‘‘copies.’’ In addition, trilinear terms can couple an ordinary particle to two mirror particles. The $A^V y_i \phi_V x_i$ and $\mathcal{A}^V y_i \phi_V x_i^\dagger$ terms in the scalar potential are an example of such inter-sector couplings. In addition, a dimensionful mirror parity conserving effective superpotential $W = -\mu H_1 H_2 - \mu'' H_1' H_2'$ may also arise, providing the usual MSSM μ term and its mirror. The SSBs that may be familiar to the reader from the $N=1$

MSSM case are listed in Table II. Along side are listed their ‘‘mirrored versions,’’ where in accordance with mirror parity conservation two of the fields in the operators are substituted by their mirror partners. If $SU(2)_R$ is conserved, then the different SSB (and μ) parameters are related to each other with a significantly smaller number of free parameters. Note that we included also the non-standard non-holomorphic trilinear \mathcal{A} terms and the SSB Higgsino-mass $\tilde{\mu}$ term. (The SSB Higgsino mass can be absorbed into a redefinition of μ in the superpotential and \mathcal{A} in the SSB scalar potential.) The next group of SSB operators are those M_P -even terms which are new to the $N=2$ models due to its unique spectrum. These are listed listed in Table III.

If mirror parity and $U(1)_{R'}^{N=2}$ are not conserved (see Sec. VII), the effective superpotential could contain $W = \mu' XY$ terms with μ' being an arbitrary mass parameter. In addition, mirror parity violating (MPV) SSB terms can also mix the two sectors. For completeness, we list the MPV SSB operators in Table IV. The mixing terms qq' , uu' for the third family can play an important role in generating the heavy top mass. Similarly, ll' mixing can play a role in the generation of light neutrino masses. This will be discussed in detail in Secs. VII and VIII, respectively. Otherwise, such

TABLE III. SSB operators which are unique to the mirror sector in the MN2SSM.

Mirror gauge scalar masses	$m_{\phi_g}^2 \phi_g^\dagger \phi_g, m_{\phi_W}^2 \phi_W^\dagger \phi_W, m_{\phi_B}^2 \phi_B^\dagger \phi_B$
Trilinear scalar operators	$A_Q^g \tilde{Q}' \phi_g \tilde{Q}, A_U^g \tilde{U}' \phi_g \tilde{U}, A_D^g \tilde{D}' \phi_g \tilde{D}$ $A_Q^W \tilde{Q}' \phi_W \tilde{Q}, A_L^W \tilde{L}' \phi_W \tilde{L}, A_{H_1}^W H_1' \phi_W H_1, A_{H_2}^W H_2' \phi_W H_2$ $A_Q^B \tilde{Q}' \phi_B \tilde{Q}, A_U^B \tilde{U}' \phi_B \tilde{U}, A_D^B \tilde{D}' \phi_B \tilde{D}, A_L^B \tilde{L}' \phi_B \tilde{L}, A_{E'}^B \tilde{E}' \phi_B \tilde{E}'$ $A_{H_1}^B H_1' \phi_B H_1, A_{H_2}^B H_2' \phi_B H_2$ $A_H^W H_1' \phi_W H_2^\dagger, A_H^B H_1' \phi_B H_2^\dagger$

TABLE IV. Mirror parity violating SSB operators in the MN2SSM.

Trilinear operators	$A'_u H'_1 \tilde{Q} \tilde{U}, A'_d H'_2 \tilde{Q} \tilde{D}, A'_e H'_2 \tilde{L} \tilde{E}$ $A'''_u H'_2 \tilde{Q}' \tilde{U}', A'''_d H'_1 \tilde{Q}' \tilde{D}', A'''_e H'_1 \tilde{L}' \tilde{E}'$ $A'_u H_2^\dagger \tilde{Q} \tilde{U}, A'_d H_1^\dagger \tilde{Q} \tilde{D}, A'_e H_1^\dagger \tilde{L} \tilde{E}$ $A'''_u H_1 \tilde{Q}' \tilde{U}', A'''_d H_2^\dagger \tilde{Q}' \tilde{D}', A'''_e H_2^\dagger \tilde{L}' \tilde{E}'$
Scalar mixing	$b'_{H_1} H_1 H'_1, b'_{H_2} H_2 H'_2, b'_Q \tilde{Q} \tilde{Q}', b'_{U'} \tilde{U} \tilde{U}', b'_{D'} \tilde{D} \tilde{D}', b'_L \tilde{L} \tilde{L}', b'_E \tilde{E} \tilde{E}'$
Chiral fermion mixing	$\tilde{\mu}'_{H_1} \tilde{H}_1 \tilde{H}'_1, \tilde{\mu}'_{H_2} \tilde{H}_2 \tilde{H}'_2, \tilde{\mu}'_q q q', \tilde{\mu}'_u u u', \tilde{\mu}'_d d d', \tilde{\mu}'_l l l', \tilde{\mu}'_e e e'$
Gauge fermion mixing	$M'_3 \tilde{g} \psi_g, M'_2 \tilde{W} \psi_W, M'_1 \tilde{B} \psi_B$

terms are assumed to be absent. This completes the listing of (dimensionful) SSB terms in the MN2SSM.

The softly broken MN2SSM resembles, not surprisingly, an extended MSSM. For example, consider electroweak symmetry breaking (EWSB). In the $N=1$ MSSM EWSB is triggered by the SSB terms in the Higgs potential. EWSB in the MN2SSM can be induced, in general, by any combination of the four Higgs doublets and the triplet ϕ_W . However, mirror parity conservation allows only the ordinary MSSM Higgs doublets H_1 and H_2 to acquire VEVs. (Independently of the parity considerations, a triplet VEV is strongly constrained by electroweak data and has to practically vanish [10].) From the discussion of the effective Yukawa couplings it will become evident that in fact it is sufficient that only one Higgs doublet acquires a VEV, which can then be truly identified with the SM Higgs boson. However, here we assume, for simplicity, that the MSSM realization of EWSB with two Higgs doublets is reproduced with the usual Higgs doublets H_1 and H_2 receiving non-zero VEVs v_1 and v_2 , respectively. This is achieved by adjusting the soft parameters which enter the Higgs potential such that $(m_{H_1}^2 + \mu^2)(m_{H_2}^2 + \mu^2) < |b|^2$. Though introduced here by hand, this relation could be satisfied via a generalization of the MSSM radiative symmetry breaking mechanism [2]. However, since we do not discuss any specific pattern of the boundary conditions to the SSB parameters, we postpone discussion of their renormalization for future works. Defining, as usual, $\tan \beta = v_2/v_1$, the Z boson mass m_Z is then given by

$$\frac{1 + \Delta_{\text{hard}}}{2} m_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2, \quad (9)$$

where Δ_{hard} contains the effects of hard-supersymmetry corrections to the quartic terms in Higgs potential, which are discussed in Sec. V. In the softly broken MN2SSM $\Delta_{\text{hard}} \equiv 0$.

There is, however, an important difference between the MSSM and the MN2SSM. While the gauge bosons get

masses via the usual Higgs mechanism and Eq. (9) reduces to its MSSM form, the softly broken MN2SSM contains no mass terms for the usual and mirror fermions. This is due to the absence of tree-level Higgs Yukawa couplings. Is this naive MSSM-like softly broken MN2SSM in which supersymmetry is broken only by dimensionful parameters then viable? The key to the answer lies with the trilinear terms, which are the only terms that break the chiral symmetries in the scalar potential and can therefore induce fermion masses at the quantum level, $m_f \propto A m_\lambda$, $A'' m_{\psi_V}$ (where the gaugino and mirror gaugino masses are responsible for the breaking of fermion number), and similarly for the mirror fermions. The viability of this mechanism is discussed in the next section.

IV. RADIATIVE FERMION MASSES

We begin the discussion of fermion mass generation with the discussion of radiative fermion masses. (More general mechanisms will be discussed in Sec. VI, following the generalization of the softly broken MN2SSM in Sec. V.) The observation that chiral symmetries could be primarily broken in the scalar potential and that the fermion spectrum in supersymmetry could arise radiatively was first made in the context of $N=2$ supersymmetry [6], though it was studied most extensively in the case of $N=1$ supersymmetry [15]. It provides an avenue for the generation of fermion masses in the softly broken MN2SSM which was discussed in the previous section. Such a mass generation mechanism has the advantage that it could be accommodated in any scenario of SSB which includes the generation of trilinear terms. On the other hand, it is highly constrained.

At one loop, the extended supersymmetry gauge interactions lead to loops such as in Fig. 1, with external ordinary fermions where the sfermion \tilde{f} and gaugino λ (or mirror sfermion \tilde{f}' and mirror gaugino ψ_V) propagate in the loop. Equivalent loops exist with external mirror fermions. The sfermion left-right mixing terms $m_{\chi_{SB}}^2$ proportional to $A \langle H \rangle$, where A here is a chiral symmetry breaking trilinear param-

eter and $\langle H \rangle$ is a $SU(2)_L$ -breaking Higgs (doublet) VEV, generates a finite contribution to the chiral fermion mass,

$$\begin{aligned}
 -m_f = & \frac{\alpha_s}{2\pi} C_f [m_{\chi_{SB}}^2 M_3 I(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, M_3^2) \\
 & + m_{\chi_{SB}}^2 M_3'' I(m_{\tilde{f}_1'}^2, m_{\tilde{f}_2'}^2, M_3''^2)] \\
 & + \frac{\alpha'}{2\pi} Y_{f_L} Y_{f_R} [m_{\chi_{SB}}^2 M_1 I(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, M_1^2) \\
 & + m_{\chi_{SB}}^2 M_1'' I(m_{\tilde{f}_1'}^2, m_{\tilde{f}_2'}^2, M_1''^2)], \quad (10)
 \end{aligned}$$

$$I(m_1^2, m_2^2, m_\psi^2) = - \frac{m_1^2 m_2^2 \ln(m_1^2/m_2^2) + m_2^2 m_\psi^2 \ln(m_2^2/m_\psi^2) + m_\psi^2 m_1^2 \ln(m_\psi^2/m_1^2)}{(m_1^2 - m_2^2)(m_2^2 - m_\psi^2)(m_\psi^2 - m_1^2)}, \quad (11)$$

which typically behaves as $\sim \mathcal{O}(1/\max(m_1^2, m_2^2, m_\psi^2))$. The dependence on the left–right squark or slepton mixing, $m_{\chi_{SB}}^2 = A \langle H_\alpha \rangle$ and its mirror $m_{\chi_{SB}}^2 = A'' \langle H_\alpha \rangle$, and on the chiral violation arising from the gaugino and mirror-gaugino Majorana masses, M_3, M_3'' , and/or M_1, M_1'' , is explicitly displayed in Eq. (10).

By observation, the natural size of the resulting fermion mass is

$$m_f \sim \begin{cases} \frac{\alpha'}{4\pi} \langle H \rangle \lesssim \mathcal{O}(100 \text{ MeV}) & \text{lepton,} \\ \frac{2\alpha_s}{3\pi} \langle H \rangle \lesssim \mathcal{O}(1 \text{ GeV}) & \text{quark,} \end{cases} \quad (12)$$

and similarly for the mirror fermions. This is the appropriate mass range for most of the ordinary fermions, but not for the mirror fermions (and t quark), whose masses are given by a similar expression. In the approximation (12) it was implicitly assumed that all of the SSB parameters are of the same order of magnitude, e.g., $m_{\chi_{SB}}^2 m_\lambda I(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, m_\lambda^2) \simeq \langle H \rangle \times \mathcal{O}(1)$, and therefore cannot change the order of magnitude of the resulting fermion mass.

More generally, however, the fermion mass can grow as $|A/\max(m_{\tilde{f}}, m_\lambda)|$ or $|A/\max(m_{\tilde{f}'}, m_{\psi_V})|$. Thus, it may appear that the fermion mass could in fact be as large as $\mathcal{O}(\langle H \rangle)$, which is the correct mass range for the mirror fermions, provided that the size of the trilinear coupling roughly equals in size to the inverse of a loop factor. Unfortunately, this cannot be the case. As already noted in Ref. [6], in the relevant limit of no tree-level Yukawa couplings the trilinear parameters destabilize the scalar potential (along the equal field direction), leading to color and charge breaking. In particular, the trilinear couplings cannot be too large.

which generalizes the expressions given in Ref. [15]. ($m_{\tilde{f}_i}^2, m_{\tilde{f}_i'}^2$ are the sfermion and mirror sfermion mass eigenvalues and $m_{\chi_{SB}}^2 = A'' \langle H \rangle$.)

The first and second terms in Eq. (10) correspond to the strong (gluino and mirror gluino) and hypercharge (the B -ino neutralino and its mirror) contributions, respectively, where α_s and α' are the strong and hypercharge couplings, $C_f = 4/3, 0$ for quarks and leptons, respectively, and Y_f is the fermion hypercharge. We assume, for simplicity, no neutralino mixing. The function I is the loop function

It was recently noted [15] that the scalar potential may be stabilized if effective quartic couplings are generated by the decoupling of chiral superfields with masses of the order of the SSB parameters or in the presence of non-holomorphic trilinear terms $\mathcal{A}H_1^\dagger \tilde{Q} \tilde{U}$ etc. (which do not correspond to flat directions of the scalar potential). Even though the potential is stable in this case, requiring color and charge conservation in the global (or meta-stable) minimum still constrains $A/m_{\tilde{f}}$ from above. (See Fig. 7 of Ref. [15].) In the next section we will show that in theories with low-energy supersymmetry breaking there could also appear arbitrary and hard supersymmetry breaking quartic couplings κ , which could further stabilize the potential. Nevertheless, given the stability constraints $\kappa \gtrsim \sqrt{3}A/m_{\tilde{f}}$ [17,15], A cannot be sufficiently large to accommodate the heavy mirror fermions (for perturbative values of the quartic couplings).

We conclude that radiative fermion mass generation leads to a viable scenario only in the case of (most of) the ordinary fermions. In particular, even though the gauge loop can break the ordinary-mirror mass degeneracy, provided that it is already broken by the SSB parameters, it cannot provide the required two or more orders of magnitude separation between the ordinary and mirror fermion spectra. As for the SM fermions, in the limit that the gaugino-sfermion loop

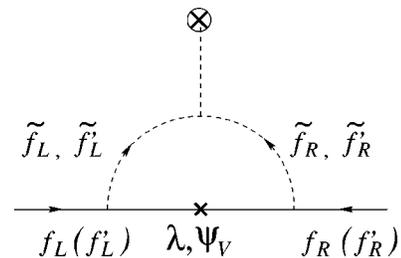


FIG. 1. One-loop contribution to fermion mass from soft chiral symmetry breaking.

dominates (or equivalently only its mirror loop dominates) this reduces to the case studied in Ref. [15], with distinctive phenomenology and signatures. If both loop classes contribute, the flavor structure becomes more complicated, but the model still maintains the same essential features and signatures. We refer the interested reader to Ref. [15] for an extensive discussion.

We note in passing that even if right-handed neutrinos are introduced, the SM neutrinos would remain massless if the fermion spectrum is indeed generated via such (supersymmetric) gauge loops, since the right-handed neutrino is a gauge singlet in the SM. (We return to the neutrinos in Sec. VIII.) The lightness of the neutrinos could be explained in this context by extending the SM by an extremely weakly coupled Abelian factor under which the right-handed neutrinos are not singlets.

Radiative mass generation for the SM fermions must go hand in hand with a mechanism to lift the ordinary-mirror matter mass degeneracy (and with a mechanism to lift the t -quark mass). Such mechanisms will be studied in the following sections, and require one to consider a more general parametrization of supersymmetry breaking. We turn to a general classification of supersymmetry breaking operators in the next section.

V. CLASSIFICATION OF SUPERSYMMETRY-BREAKING OPERATORS

In Sec. IV, the possibility of radiatively induced Yukawa couplings in the effective low-energy theory was shown to lead, regardless of its details, to a typical mass range of $m_q, m_{q'} \lesssim (\alpha_s/\pi)\langle H \rangle$ for the ordinary and mirror quarks, and $m_l, m_{l'} \lesssim (\alpha'/\pi)\langle H \rangle$ for the ordinary and mirror leptons. This is a natural and sufficient solution in the case of most of the ordinary fermions (we postpone that discussion of the third family fermions to Sec. VII) but not in the case their mirrors. The mirror fermion spectrum is constrained by experiment, $m_{f'} \approx \langle H \rangle$, which implies tree-level effective Yukawa couplings. (The constraints, however, are model dependent.) Typically, a Kahler function K describing the effective low-energy theory includes tree-level non-renormalizable Yukawa operators, either supersymmetry conserving, soft, or hard supersymmetry breaking. In order to consider such operators one must step out of the softly broken MN2SSB framework of Sec. III and systematically include all relevant supersymmetry breaking operators. This will be done in this section. Concrete realizations of tree-level Yukawa couplings in the MN2SSM will be considered in the next section.

Our classification applies to any theory which can effectively be described by $N=1$ superfields, and will be adopted to the $N=2$ case only in the next section. In typical $N=1$ (high-energy) supergravity model building with supersymmetry breaking scale $F \approx M_{\text{weak}} M_{\text{Planck}}$, the Yukawa (and quartic) operators listed below are proportional to $(M_{\text{weak}}/M_{\text{Planck}})^n$, $n=1,2$, and hence are often omitted. [Nevertheless, even in that case such terms can shift any boundary conditions for the SSB by $\mathcal{O}(100\%)$ [18].] This proportionality, however, cannot hold in the $N=2$ case if it

is to be phenomenologically viable. Requiring a viable phenomenology constrains the size and symmetries of the effective Yukawa couplings, and hence the scales and symmetries of the $N=2$ theory and of the theory below the supersymmetry breaking scale. Indeed, since the gravitino masses are somewhat arbitrary in the $N=2$ case [7], there is no reason to impose any relation analogous to the above $N=1$ supergravity relation, even when gravity is introduced. After considerations of all operators we will set the supersymmetry breaking scale simply by requiring sizable tree-level Yukawa couplings and stability of the theory against hard operators, and compare it to the requirement that $N=2$ supersymmetry plays a role in the resolution of the hierarchy problem. (We will find the two requirements to be consistent.)

In Secs. III and IV we adopted the description of supersymmetry breaking in terms of an explicitly but softly broken global $N=1$ theory with a second supersymmetry only implicitly manifest in the symmetries of the (super)potential, and its breaking corresponding to explicit breaking of the relevant symmetries in the SSB potential. Though we are about to extend and generalize this description to include dimensionless $N=2$ and $N=1$ breaking couplings, the same modular description of supersymmetry breaking will prove to be a powerful classification tool here as well. Its generalization corresponds to the replacement of explicit soft breaking terms with the spurion formalism [14]. A spurion field $X = \theta^2 F_X$ parameterizes the manifest $N=1$ breaking, and non-renormalizable operators which couple X to the MN2SSM fields parameterize the explicit $N=2$ (exchange symmetry) breaking. The non-renormalizable operators scale as inverse powers of the $N=2$ breaking scale M . The convenient spurion formalism is available only in this $N=1$ formulation. (Note that the spurion X is not to be confused with the generic X superfield component of a hypermultiplet in Sec. II.)

Indeed, one could arbitrarily write down in the infrared theory Yukawa and quartic couplings whose presence leads to quadratically divergent quantum correction to various two-point functions, and which are therefore said to be hard supersymmetry breaking. However, if one requires that in certain $M \rightarrow \infty$ the full (global) $N=2$ supersymmetry be recovered, then these couplings must fall into certain categories of non-renormalizable operators. It is further reasonable to assume that the Kahler potential (which is not protected by non-renormalization theorems) rather than the superpotential accommodates these operators.¹ (As we shall see below, both possibilities of Kahler or superpotential operators lead in practice to couplings of the same size.) The non-renormalizable Kahler potential operators which link the spurion and the MN2SSM fields do not preserve the global symmetries of the full $N=2$ theory, which is equivalent to the symmetry violations by the SSB potential in the previous

¹These operators are induced, in principle, by the dynamics at the scale M . The resulting low-energy effective Kahler potential is not derived, in general, from a holomorphic prepotential function P , $K(\Phi) \neq \text{Im}[\Phi^\dagger(\partial P/\partial\Phi)]$.

TABLE V. The soft supersymmetry breaking terms as operators contained in K_1 and K_2 . $\Phi = \phi + \theta\psi + \theta^2 F$ is a low-energy superfield while X , $\langle F_X \rangle \neq 0$, parametrizes supersymmetry breaking. $F^\dagger = \partial W / \partial \Phi$.

Ultraviolet K operator	Infrared \mathcal{L}_D operator
$\frac{X}{M} \Phi \Phi^\dagger + \text{H.c.}$	$A \phi F^\dagger + \text{H.c.}$
$\frac{XX^\dagger}{M^2} \Phi \Phi^\dagger + \text{H.c.}$	$\frac{m^2}{2} \phi \phi^\dagger + \text{H.c.}$
$\frac{XX^\dagger}{M^2} \Phi \Phi + \text{H.c.}$	$b \phi \phi + \text{H.c.}$
$\frac{X^\dagger}{M^2} \Phi^2 \Phi^\dagger + \text{H.c.}$	$\kappa \phi^\dagger \phi F + \text{H.c.}$
	$y \phi^\dagger \psi \psi + \text{H.c.}$

sections. In addition, non-vanishing values of $X = \theta^2 F_X$ parametrize the breaking of the manifest $N=1$ supersymmetry as well as replace the non-renormalizable operators with explicit ($N=2$ and $N=1$) supersymmetry breaking terms in the low-energy potential. Note that the spurion and its mirror (X, X') transform as a doublet under the $SU(2)_R$ exchange R symmetry, which implies that a non-vanishing VEV $\langle F_X \rangle$ automatically breaks it. [The $SU(2)_R$ of $N=2$ allows one to rotate the supersymmetry breaking VEV such that the mirror $F_{X'} = 0$.] This parametrization has two breaking parameters, F_X and M , corresponding to the spurion VEV and (inverse) couplings, respectively. It corresponds to an one-step breaking scenario, $N=2 \rightarrow N=0$, for $F_X \simeq M^2$, which we will assume.

We now turn to a general classification of K operators. We do not impose any of the global symmetries which parametrize the second supersymmetry, a subset of which can survive its breaking. (This will be done in the next section.) The effective low-energy Kahler potential of a rigid $N=1$ supersymmetry theory is given by

$$\begin{aligned}
 K = & K_0(X, X^\dagger) + K_0(\Phi, \Phi^\dagger) + \frac{1}{M} K_1(X, X^\dagger, \Phi, \Phi^\dagger) \\
 & + \frac{1}{M^2} K_2(X, X^\dagger, \Phi, \Phi^\dagger) + \frac{1}{M^3} K_3(X, X^\dagger, \Phi, \Phi^\dagger, D_\alpha, W_\alpha) \\
 & + \frac{1}{M^4} K_4(X, X^\dagger, \Phi, \Phi^\dagger, D_\alpha, W_\alpha) + \dots \quad (13)
 \end{aligned}$$

where X is the spurion and Φ are the (ordinary and mirror) chiral superfields of the low-energy MN2SSM theory. D_α is the covariant derivative with respect to the (explicit) superspace chiral coordinate θ_α , and W_α is the $N=1$ gauge supermultiplet in its chiral representation, $W_\alpha \sim \lambda_\alpha + \theta_\alpha V$. Once a separation between supersymmetry breaking field X and low-energy Φ fields is imposed, there is no tree-level renormalizable interaction between the two sets of fields, and their mixing can arise only at the non-renormalizable level

TABLE VI. The effective renormalizable $N=1$ superpotential W operators contained in K_1 and K_2 , $\mathcal{L} = \int d^2 \theta W$. Symbols are defined in Table V.

Ultraviolet K operator	Infrared W operator
$\frac{X^\dagger}{M} \Phi^2 + \text{H.c.}$	$\mu \Phi^2$
$\frac{X^\dagger}{M^2} \Phi^3 + \text{H.c.}$	$y \Phi^3$

$K_{l \geq 1}$. (This separation is quite natural in the context of $N=2$ if X and Φ transform under different gauge groups, in particular if X is a gauge singlet field.)

The superspace integration $\mathcal{L}_D = \int d^2 \theta d^2 \bar{\theta} K$ reduces K_1 and K_2 to the usual SSB terms, as well as the superpotential μ parameter $W \sim \mu \Phi^2$, which were discussed in the previous section. It also contains Yukawa operators $W \sim y \Phi^3$ which can appear in the effective low-energy superpotential. These are summarized in Tables V and VI. (We did not include linear terms that may appear in the case of a singlet superfield.) Finally, the last term in Table V contains correlated but unusual quartic and Yukawa couplings. They are soft as they involve at most logarithmic divergences.

Integration over K_3 produces the non-standard soft terms, also discussed in the previous section. These are summarized in Table VII. It also generates contributions to the (“standard”) A and gaugino-mass terms. These terms could arise at lower orders in \sqrt{F}/M from integration over holomorphic functions (and in the case of A , also from K_1). However, this is equivalent to integration over K if $\int d^2 \bar{\theta} (X^\dagger / M^2) \simeq 1$. Note that in the presence of superpotential Yukawa couplings, a

TABLE VII. The non-standard or semi-hard supersymmetry breaking terms as operators contained in K_3 . Here W^α is the $N=1$ chiral representation of the gauge supermultiplet and λ is the respective gaugino. D_α is the covariant derivative with respect to the (explicit) superspace coordinate θ_α . All other symbols are as in Table V.

Ultraviolet K operator	Infrared \mathcal{L}_D operator
$\frac{XX^\dagger}{M^3} \Phi^3 + \text{H.c.}$	$A \phi^3 + \text{H.c.}$
$\frac{XX^\dagger}{M^3} \Phi^2 \Phi^\dagger + \text{H.c.}$	$A \phi^2 \phi^\dagger + \text{H.c.}$
$\frac{XX^\dagger}{M^3} D^\alpha \Phi D_\alpha \Phi + \text{H.c.}$	$\tilde{\mu} \psi \psi + \text{H.c.}$
$\frac{XX^\dagger}{M^3} D^\alpha \Phi W_\alpha + \text{H.c.}$	$M'_\lambda \psi \lambda + \text{H.c.}$
$\frac{XX^\dagger}{M^3} W^\alpha W_\alpha + \text{H.c.}$	$\frac{M_\lambda}{2} \lambda \lambda + \text{H.c.}$

TABLE VIII. The dimensionless hard supersymmetry breaking terms as operators contained in K_4 . Symbols are defined as in Tables V and VII.

Ultraviolet K operator	Infrared \mathcal{L}_D operator
$\frac{XX^\dagger}{M^4} \Phi D^\alpha \Phi D_\alpha \Phi + \text{H.c.}$	$y \phi \psi \psi + \text{H.c.}$
$\frac{XX^\dagger}{M^4} \Phi^\dagger D^\alpha \Phi D_\alpha \Phi + \text{H.c.}$	$y \phi^\dagger \psi \psi + \text{H.c.}$
$\frac{XX^\dagger}{M^4} \Phi D^\alpha \Phi W_\alpha + \text{H.c.}$	$\bar{y} \phi \psi \lambda + \text{H.c.}$
$\frac{XX^\dagger}{M^4} \Phi^\dagger D^\alpha \Phi W_\alpha + \text{H.c.}$	$\bar{y} \phi^\dagger \psi \lambda + \text{H.c.}$
$\frac{XX^\dagger}{M^4} \Phi W^\alpha W_\alpha + \text{H.c.}$	$\bar{y} \phi \lambda \lambda + \text{H.c.}$
$\frac{XX^\dagger}{M^4} \Phi^\dagger W^\alpha W_\alpha + \text{H.c.}$	$\bar{y} \phi^\dagger \lambda \lambda + \text{H.c.}$
$\frac{XX^\dagger}{M^4} \Phi^2 \Phi^{\dagger 2} + \text{H.c.}$	$\kappa (\phi \phi^\dagger)^2 + \text{H.c.}$
$\frac{XX^\dagger}{M^4} \Phi^3 \Phi^\dagger + \text{H.c.}$	$\kappa \phi^3 \phi^\dagger + \text{H.c.}$

Higgsino $\tilde{\mu}$ term can be rotated to a combination of μ and \mathcal{A} terms and vice versa. The two terms, however, are not necessarily equivalent in our case since $N=2$ forbids chiral superpotential Yukawa couplings. In the case of the mirror gaugino ψ_V , our MN2SSM notation replaces $\tilde{\mu}$ with $M''/2$.

Last, superspace integration over K_4 leads to dimensionless hard operators. These are summarized in Table VIII. (Hard operators were also summarized recently in Ref. [19].) Higher orders in $(1/M)$ can be safely neglected as supersymmetry and the superspace integration allow only a finite expansion in $\sqrt{F_X}/M$, that is $\mathcal{L} = f[F_X^n/M^l]$ with $n \leq 2$ and l is

the index K_l in expansion Eq. (13). Hence, terms with $l > 4$ are suppressed by at least $(\langle X \rangle/M)^{l-4}$. We will assume the limit $\langle X \rangle \ll M$ for the $N=1$ supersymmetry preserving VEV $\langle X \rangle$, i.e., $X \sim \theta^2 F_X$, so that all such operators can indeed be neglected and the expansion is rendered finite.

It is useful for our purposes to identify those terms in K which can break the chiral symmetries and generate the desired Yukawa terms in the low-energy effective theory. Clearly, the relevant terms in Tables V and VII can be identified with the chiral symmetry breaking A and \mathcal{A} terms (with any number of primes) which couple the matter sfermions to the Higgs fields of electroweak symmetry breaking and which were discussed in the previous section. Note that since in $N=2$ there are no chiral terms in the superpotential then chiral-symmetry breaking A terms can only arise from K_3 . More importantly, and as advertised above, a generic Kahler potential is also found to contain tree-level chiral Yukawa couplings. These include $\mathcal{O}(F_X/M^2)$ supersymmetry conserving and soft couplings and $\mathcal{O}(F_X^2/M^4)$ hard chiral symmetry breaking couplings. The relative importance and the potentially destabilizing properties of the different operators must be addressed before any symmetry-derived selection rules are applied. Both issues point to the more fundamental questions that one needs to address: What are the scales $\sqrt{F_X}$ and M and what is their relation to the cutoff scale Λ .

We have $\sqrt{F_X} \approx M \approx \mathcal{O}(\text{TeV})$ from the requirement that $N=2$ supersymmetry plays a role in the solution of the SM hierarchy problem. In addition, the cutoff scale for any such calculation is the scale of $N=2$ restoration above which $F_X = 0$, i.e., $\Lambda \approx M$. In this case, all of the dimensionful couplings could be in principle $\sim \mathcal{O}(1)$, regardless of their softness or order in F_X/M^2 . This is desired for the Yukawa couplings of the mirror fermions. It is important to note, however, that quartic couplings are also large. (We mentioned the latter effect in the previous section.) One has to confirm that this choice is not destabilized when the hard operators, which are large, are included. In order to do so, consider the implication of the hardness of the operators contained in K_4 . Yukawa and quartic couplings can destabilize the scalar potential by corrections to the mass terms Δm^2 of the order of

$$\Delta m^2 \sim \begin{cases} \frac{\kappa}{16\pi^2} \Lambda^2 \sim \frac{1}{16\pi^2} \frac{F_X^2}{M^4} \Lambda^2 \sim \frac{1}{16\pi^2} \frac{F_X^2}{M^2} \sim \frac{1}{16\pi^2 c_m} m^2 \\ \frac{y^2}{16\pi^2} \Lambda^2 \sim \frac{1}{16\pi^2} \frac{F_X^4}{M^8} \Lambda^2 \sim \frac{1}{16\pi^2} \frac{F_X^4}{M^6} \sim \frac{1}{16\pi^2 c_m} m^2 \frac{m^2}{M^2}, \end{cases} \quad (14)$$

where we identified $\Lambda \approx M$ and c_m is a dimensionless coefficient omitted in Table V, $m^2/2 = c_m F_X^2/M^2$. The hard operators were substituted by the appropriate powers of F_X/M^2 [and are $\sim \mathcal{O}(1)$]. Once M is identified as the cutoff scale above which the the full supersymmetry is restored, then

these terms are harmless as the contributions are bound from above by the tree-level scalar squared-mass parameters.

This observation is valid for the $N=1$ case whether it is constrained by the $N=2$ symmetries or not (and extends to the case of non-standard soft operators in the presence of a

singlet). In fact, such hard divergent corrections are well known in $N=1$ supergravity with $\Lambda=M=M_{\text{Planck}}$, where they perturb any given set of tree-level boundary conditions for the SSB parameters [18]. In theories with low-energy supersymmetry breaking $\sqrt{F_X}\simeq M\simeq\Lambda\simeq\mathcal{O}(\text{TeV})$, however, it seems particularly difficult to reliably calculate the SSB parameters. Furthermore, if there are no tree-level scalar squared masses, then they may arise from such loops and be given, roughly, by $M^2/16\pi^2$ (avoiding a potential need to introduce a small coefficient c_m in front of the squared mass operators in Table V).

We conclude that, in general, chiral Yukawa couplings appear once supersymmetry is broken, and if it is broken at low energy $\sqrt{F_X}\simeq M\simeq\Lambda\simeq\mathcal{O}(\text{TeV})$ then these couplings could be sizable $y\simeq\mathcal{O}(1)$ yet harmless.

VI. TREE-LEVEL YUKAWA COUPLINGS FROM THE KÄHLER POTENTIAL

In the previous section we classified all supersymmetry breaking operators and set the supersymmetry breaking scale parameters to $\sqrt{F_X}\simeq M\simeq\mathcal{O}(\text{TeV})$. Large tree-level Yukawa couplings (and trilinear mass parameters) appear in that case in the effective theory. Though their parent operators as well as their order in F_X/M^2 may be different, they are all *a priori* of similar magnitude. The issue at hand is therefore not finding possible operators. Rather, one must avoid excessive mixing between quarks (leptons) and their mirrors, which could lead to disastrous contributions to flavor changing neutral currents. For example, one obvious path one could take is to allow tree-level Yukawa couplings of the same origin (i.e., which are derived from the same operator class) for all matter fields. This, however, could exactly lead to such mixing, and furthermore, does not offer any new insight into the ordinary-mirror fermion mass hierarchy. We therefore pursue a more motivated path in which the two sectors are distinguished by the global symmetries of the effective theory, and the symmetries induce selection rules which allow or forbid certain types of Yukawa and soft operators in the different sectors.²

This can be done by either exploiting the global R symmetries which parametrize the hidden supersymmetry or by symmetries which do not commute with the former symmetries and therefore characterize the supersymmetry breaking mechanism. One could also take a linear combination of these choices, both of which correspond to anomalous symmetries. In addition, a specific choice of a symmetry is better motivated if it can provide a hint as for the origin of the ordinary-mirror fermion mass hierarchy. The model and our parametrizations already direct one toward the possible paths:

(i) Recalling that the hard chiral symmetry breaking operators are already distinguished by the presence of covariant superspace derivatives, which transform under any continu-

TABLE IX. The $U(1)_R^{N=2}\times Z_2^{MP}$ assignment for the various $N=1$ superfields. $R(\theta_\alpha)=-1$ and all matter superfields are charged as the quark doublet Q .

Field	Assignment		Assignment for mirror	
	Case I	Case II	Case I	Case II
H_i	0^+	-1^+	0^-	$+1^-$
Q	$+1^+$	$+\frac{1}{2}^+$	-1^-	$-\frac{1}{2}^-$
Φ_V			-2^-	-2^-

ous or discrete R symmetry, suggests choosing an R symmetry (though this choice is not unique).

(ii) While the $SU(2)_R$ symmetry must be broken (or the fermion and mirror fermion remain degenerate in mass), the $U(1)_R$ of $N=2$ may be preserved and provide the desired selection rules. In fact, a $U(1)^2$ subgroup of the complete $U(2)_R$ can survive, where the other $U(1)$ is $U(1)_J$.

(iii) Mirror parity is a useful tool which enables one to distinguish matter from mirror matter, and may provide an alternate set of selection rules.

In order to illustrate the richness of the possible frameworks we use two distinct sets of selection rules, corresponding to the symmetry classes mentioned above: The first group of symmetries is based on the $N=2$ preserving (A) $U(1)_R^{N=2}\times Z_2^{MP}$; the second one is based on an Abelian R -symmetry extension of mirror parity (B) $U(1)_R^{MP}$ which explicitly breaks $N=2$. The latter example could be an ‘‘accidental’’ symmetry related to the supersymmetry breaking mechanism. We note that it can be mapped to a discrete $Z_3\times Z_2$ R symmetry where the Z_2 is the usual mirror parity and the chiral coordinate θ and the mirror matter fields all transform as $(1/3)^-$. (Note that once the transformation properties of one matter field and its mirror are fixed, the $N=2$ superpotential fixes the charge of Φ_V and, as a result, of all other ordinary-mirror bilinears.) The symmetry assignments and the corresponding selection rules appear in Tables IX and X. For illustration, the quark (super)fields u_L and u_R (Q and U) and their mirrors are substituted in the operators. However, we assume identical transformation properties for all quark and lepton fields so that any other (gauge invariant) combination of fields could be substituted instead. [It is possible to choose slightly more complicated examples with (SM) charge and flavor dependent symmetry assignments.]

TABLE X. The $U(1)_R^{MP}$ assignment for the various $N=1$ superfields. $R(\theta_\alpha)=-1$ and all matter superfields are charged as the quark doublet Q .

Field	Assignment	Assignment for mirror
H_i	0	-1
Q	0	-1
Φ_V		-1

²The heaviness of the ordinary third family fermions may seem to challenge some of the resulting frameworks. We postpone this discussion to the next section.

TABLE XI. Low-energy chiral operators, which are holomorphic in the low-energy fields, and their symmetry properties. The first, second, and third classes of operators are soft supersymmetry breaking, hard supersymmetry breaking, and supersymmetry conserving, respectively. Allowed operators (assuming $R_X=0$ or $R_X=2$) are underlined.

Operator class	Operator	Case A1	Case A2	Case B
$XX^\dagger\Phi^3$	$AH_2\tilde{Q}\tilde{U}$	$+2^+$	<u>0^+</u>	<u>0</u>
	$AH_1\tilde{Q}'\tilde{U}'$	-2^+	-2^+	-2
	$AH_1'\tilde{Q}\tilde{U}$	$+2^-$	$+2^-$	-1
	$AH_2'\tilde{Q}'\tilde{U}'$	-2^-	0^-	-3
$XX^\dagger\Phi D^\alpha\Phi D_\alpha\Phi$	$yH_2u_Lu_R$	$+4^+$	$+2^+$	$+2$
	$yH_1u_L'u_R'$	<u>0^+</u>	<u>0^+</u>	<u>0</u>
	$yH_1'u_Lu_R$	$+4^-$	$+4^-$	$+1$
	$yH_2'u_L'u_R'$	0^-	$+2^-$	-1
$X^\dagger\Phi^3$	$yH_2u_Lu_R$	<u>$(+2-R_X)^+$</u>	<u>$(0-R_X)^+$</u>	<u>$0-R_X$</u>
	$yH_1u_L'u_R'$	$(-2-R_X)^+$	$(-2-R_X)^+$	$-2-R_X$
	$yH_1'u_Lu_R$	$(+2-R_X)^-$	$(+2-R_X)^-$	$-1-R_X$
	$yH_2'u_L'u_R'$	$(-2-R_X)^-$	$(0-R_X)^-$	$-3-R_X$

Finally, for completeness we list both operators which are holomorphic (Table XI) or non-holomorphic (Table XII) in the Higgs fields, though the latter do not add any intrinsically new possibilities. Note that it is assumed that only ordinary Higgs doublets, but not their mirrors or any other fields, participate in electroweak symmetry breaking. (In particular, mirror parity or its extensions are not broken spontaneously by electroweak Higgs VEVs.)

A clear tree of possibilities emerges:

(1) Assume that tree-level mirror-fermion masses arise from the hard supersymmetry breaking operators, which occurs naturally in the examples given here.

(2) The chiral symmetries of the ordinary matter fields may then be broken in the scalar potential, leading to radiative (ordinary) fermion masses. Alternatively, an effective $N=1$ Yukawa tree-level superpotential is generated for the ordinary fields.

(3) The symmetry properties of both SSB and supersymmetry conserving operators imply that either both possibilities for the ordinary fermion mass generation are allowed or that both are forbidden, as long as the spurion is not charged under the global R -symmetries.

(a) If both are allowed, a charge assignment for a spurion field could forbid the supersymmetry conserving operators and, as a result, forbid tree-level masses for the ordinary

TABLE XII. As in Table XI but for operators which contain Φ^\dagger .

Operator class	Operator	Case A1	Case A2	Case B	
$XX^\dagger\Phi^\dagger\Phi^2$	$AH_1^\dagger\tilde{Q}\tilde{U}$	$+2^+$	$+2^+$	<u>0</u>	
	$AH_2^\dagger\tilde{Q}'\tilde{U}'$	-2^+	<u>0^+</u>	-2	
	$AH_2'^\dagger\tilde{Q}\tilde{U}$	$+2^-$	0^-	$+1$	
$XX^\dagger\Phi^\dagger\tilde{Q}'\tilde{U}'$	$AH_1'^\dagger\tilde{Q}'\tilde{U}'$	-2^-	-2^-	-1	
	$XX^\dagger\Phi^\dagger D^\alpha\Phi D_\alpha\Phi$	$yH_1^\dagger u_L u_R$	$+4^+$	$+4^+$	$+2$
		$yH_2^\dagger u_L' u_R'$	<u>0^+</u>	2^+	<u>0</u>
$yH_2'^\dagger u_L u_R$		$+2^-$	$+2^-$	$+3$	
$XX^\dagger\Phi^\dagger u_L' u_R'$	$yH_1'^\dagger u_L' u_R'$	0^-	0^-	$+1$	
	$X^\dagger\Phi^\dagger\Phi^2$	$yH_1^\dagger u_L u_R$	<u>$(+2-R_X)^+$</u>	<u>$(+2-R_X)^+$</u>	<u>$0-R_X$</u>
		$yH_2^\dagger u_L' u_R'$	$(-2-R_X)^+$	<u>$(0-R_X)^+$</u>	$-2-R_X$
$yH_2'^\dagger u_L u_R$		$(+2-R_X)^-$	$(0-R_X)^-$	$+1-R_X$	
$X^\dagger\Phi^\dagger u_L' u_R'$	$yH_1'^\dagger u_L' u_R'$	$(-2-R_X)^-$	$(-2-R_X)^-$	$-1-R_X$	

fermions. This provides a simple explanation of the matter-mirror mass hierarchy as a loop factor.

(b) If both are forbidden, a charge assignment for a spurion field could allow the supersymmetry conserving operators and as a result, for tree-level masses for the ordinary fermions. The ordinary-mirror mass hierarchy can now be explained by the hierarchy between the charged and neutral spurion supersymmetry breaking VEVs F_{X_1}/F_{X_2} . (Note that $\langle F_{X_2} \rangle$ itself breaks the R symmetry if X_2 is neutral, while $\langle F_{X_1} \rangle$ may or may not break it.) Alternatively, it could always be that one class of operators (the hard operators, in this case) appears at the tree level while the other class (the superpotential operators) appears only radiatively so that the hierarchy is imprinted in the coefficients of the different operators in K .

Many other examples can be constructed along these lines.

The symmetry principles nicely arrange the different fermion mass operators. They also carry implications to most of the other operators. The scalar squared masses are generically insensitive and may arise from tree-level operator with relatively small coefficients c_m , from quadratically divergent loop corrections, or from gauge(ino) renormalization. On the other hand (and similarly to $N=1$ supergravity) gaugino mass terms break any Abelian R symmetry, so that there must be a spurion combination such that $R(X_1 X_2^\dagger) = +2$, consistent with our proposals above. Our speculation that the $\langle F \rangle$ of the charged spurion corresponds to a lower scale could lead to suppression of gaugino masses. Another group

of operators of phenomenological relevance are the operators corresponding to Higgs mixing at the electroweak scale, $W \sim \mu H_2 H_1$ and $V_{SSB} \sim b H_2 H_1 + \tilde{\mu} \tilde{H}_2 \tilde{H}_1$. Assigning $R(X) = R(H_2 H_1)$ always allows for the superpotential μ term. (See Table VI.) If there is only one spurion, the SSB Higgs (Table V) and Higgsino (Table VII) mixing operators are independent of the (single) spurion charge and cannot be allowed simultaneously. In the case of a multi-spurion scenario, if the spurions carry different R charges, then both could co-exist. (Phenomenologically, both Higgsino mass and Higgs boson mixing in the scalar potential are required in order to avoid very light Higgs and Higgsino particles in the spectrum.)

VII. HEAVY GENERATION

In our discussion so far we distinguished ordinary from mirror matter, but did not distinguish, for example, light and heavy SM (ordinary) fermions. That is, if one of the mechanisms to render ordinary fermions light relative to the mirror fermion is realized, then *all* of the ordinary fermions will be light with masses of roughly the same order of magnitude. However, the SM fermion spectrum contains two special cases: The first case is that of the top quark (or, for that matter, of all of the third family) whose mass is of the order of the mirror fermion masses. The second case is that of the nearly massless neutrinos. We postpone discussion of the neutrinos to the next section and focus here on the case of heavy SM fermions.

While in some cases an internal hierarchy within the SM sector can be put in by hand, it is not always sufficient. For example, if the SM fermion mass is generated radiatively, vacuum stability constraints make it very unlikely that the top (τ) in the quark (lepton) sector receives its mass radiatively (with a large trilinear parameter put in by hand). This would require hard quartic couplings of order $\kappa \geq 4\pi$. An alternative tree-level mechanism may exist, particularly in the latter case. One obvious candidate for such a mechanism is mirror-symmetry breaking in the third family and, consequently, mass mixing between ordinary and mirror third family fermions. As long as such mixing is constrained to only the third family, the implications to flavor changing neutral currents are generically within experimental constraints. Mirror parity breaking in such a scenario is intimately linked to the flavor symmetry structure. We first discuss the phenomenology of such a mechanism, and then speculate on its possible origin from a spontaneously broken Abelian flavor (gauge) symmetry.

If one allows MPV in the third family, then there could be tree-level mixing between the fermions and their mirrors. For explicitness, let us concentrate on the case of the top quark and its mirror with mixing terms: $\tilde{\mu}'_L t_L t'_L$ and $\tilde{\mu}'_R t_R t'_R$. For simplicity, let us further assume that the usual quark mass term $t_L t_R$ is small and can be taken to be zero. The mirror top quark, on the other hand, has a mass term $M t'_L t'_R$, which is assumed to arise at the tree level and $M \simeq M_{\text{weak}}$. (M here is not the supersymmetry breaking scale but simply the large mass parameter in the fermion mass matrix, $M \equiv M_{f'_L f'_R}$.) A

similar structure holds for the bottom sector, with identical $\tilde{\mu}'_L$ [from the SM $SU(2)_L$ symmetry] for the left-handed bottoms but with independent $\tilde{\mu}'_R$ and M parameters.

Defining

$$\psi_j^+ = \begin{pmatrix} t_L \\ t'_R \end{pmatrix}, \quad \psi_j^- = \begin{pmatrix} t'_L \\ t_R \end{pmatrix}, \quad j=1,2, \quad (15)$$

the mass matrix can be written as

$$(\psi^+ \psi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{H.c.}, \quad (16)$$

where

$$X = \begin{pmatrix} \tilde{\mu}'_L & M \\ 0 & \tilde{\mu}'_R \end{pmatrix}, \quad (17)$$

and we neglected a pure SM top mass. (The MPV mixing may be SSB or $N=1$ supersymmetric, though here we use the SSB notation.) The mass eigenstates χ^\pm are readily found:

$$\begin{aligned} \chi_i^+ &= V_{ij} \psi_j^+ = \begin{pmatrix} \cos \phi_+ & \sin \phi_+ \\ -\sin \phi_+ & \cos \phi_+ \end{pmatrix} \begin{pmatrix} t_L \\ t'_R \end{pmatrix}, \\ \chi_i^- &= U_{ij} \psi_j^- = \begin{pmatrix} \cos \phi_- & \sin \phi_- \\ -\sin \phi_- & \cos \phi_- \end{pmatrix} \begin{pmatrix} t'_L \\ t_R \end{pmatrix}. \end{aligned} \quad (18)$$

Here U and V are the unitary matrices chosen to diagonalize the mass matrix:

$$U^* X V^+ = M_{\text{Dirac}}. \quad (19)$$

The mass eigenvalues $M_{\text{Dirac},1,2}^2$ are

$$\begin{aligned} M_{\text{Dirac},1,2}^2 &= \frac{M^2 + \tilde{\mu}'_L{}^2 + \tilde{\mu}'_R{}^2 \mp \sqrt{(M^2 + \tilde{\mu}'_L{}^2 + \tilde{\mu}'_R{}^2)^2 - 4\tilde{\mu}'_L{}^2 \tilde{\mu}'_R{}^2}}{2}, \end{aligned} \quad (20)$$

while the mixing angles ϕ^+ and ϕ^- can be written as

$$\tan \phi_+ = \frac{\tilde{\mu}'_R{}^2 + M^2 - \tilde{\mu}'_L{}^2 - \sqrt{(M^2 + \tilde{\mu}'_L{}^2 + \tilde{\mu}'_R{}^2)^2 - 4\tilde{\mu}'_L{}^2 \tilde{\mu}'_R{}^2}}{2\tilde{\mu}'_L M} \quad (21)$$

$$\begin{aligned} \tan \phi_- &= \frac{\tilde{\mu}'_R{}^2 - M^2 - \tilde{\mu}'_L{}^2 - \sqrt{(M^2 + \tilde{\mu}'_L{}^2 + \tilde{\mu}'_R{}^2)^2 - 4\tilde{\mu}'_L{}^2 \tilde{\mu}'_R{}^2}}{2\tilde{\mu}'_R M}. \end{aligned} \quad (22)$$

The mass splitting between the ordinary and mirror quarks is a function of $\tilde{\mu}'_L$, $\tilde{\mu}'_R$ and M . Two limits are of particular interest:

(i) $\tilde{\mu}'_L$, $\tilde{\mu}'_R$ and M are all of the same order of magnitude.

Assume, as an example, $\tilde{\mu}'_L = \tilde{\mu}'_R = M$. In this limit, $M_{\text{Dirac}_{1,2}}$ are of the same order of magnitude and there is large mixing between the ordinary SM quarks and their mirror partners:

$$M_{\text{Dirac}_{1,2}} = \left(\frac{3 \mp \sqrt{5}}{2} \right)^{1/2} M = \begin{cases} 0.62 M \\ 1.62 M \end{cases}, \quad \tan \phi_{\pm} = \frac{-\sqrt{5} \pm 1}{2}. \quad (23)$$

This case is relevant for the top sector. The top quark can get its large mass while the mirror top is sufficiently heavy to evade current experimental limits that may apply.

(ii) The MPV mixing between the ordinary quarks and the mirror partners is much smaller than M .

Assume, without loss of generality, $\tilde{\mu}'_R \ll M$. One has, to leading order in $\tilde{\mu}'_R/M$,

$$M_{\text{Dirac}_1} = \frac{\tilde{\mu}'_R \tilde{\mu}'_L}{\sqrt{M^2 + \tilde{\mu}'_L{}^2}} \stackrel{\tilde{\mu}'_L \ll M}{=} \frac{\tilde{\mu}'_L \tilde{\mu}'_R}{M}, \quad (24)$$

$$M_{\text{Dirac}_2} = \sqrt{M^2 + \tilde{\mu}'_L{}^2} \left(1 + \frac{\tilde{\mu}'_R{}^2 M^2}{2(M^2 + \tilde{\mu}'_L{}^2)^2} \right) \stackrel{\tilde{\mu}'_L \ll M}{=} \left(1 + \frac{\tilde{\mu}'_L{}^2 + \tilde{\mu}'_R{}^2}{2M^2} \right). \quad (25)$$

The mixing angles in this limit can be similarly obtained and read

$$\tan \phi_+ = -\frac{\tilde{\mu}'_L}{M} \left(1 - \frac{\tilde{\mu}'_R{}^2}{M^2 + \tilde{\mu}'_L{}^2} \right) \stackrel{\tilde{\mu}'_L \ll M}{=} -\frac{\tilde{\mu}'_L}{M} \left(1 - \frac{\tilde{\mu}'_R{}^2}{M^2} \right) \quad (26)$$

$$\tan \phi_- = -\frac{(M^2 + \tilde{\mu}'_L{}^2)}{M \tilde{\mu}'_R} \left(1 - \frac{\tilde{\mu}'_L{}^2 \tilde{\mu}'_R{}^2}{(M^2 + \tilde{\mu}'_L{}^2)^2} \right) \stackrel{\tilde{\mu}'_L \ll M}{=} -\frac{M}{\tilde{\mu}'_R} \left(1 + \frac{\tilde{\mu}'_L{}^2}{M^2} \right). \quad (27)$$

As one expects, one of the eigenstates becomes light when one of the mass mixings is small, while the heavy mass eigenvalues is still $\sim \mathcal{O}(M)$. The fraction of the usual right-handed quark in the light eigenstate, $\sin \phi_-$, is always large since $\tan \phi_-$ is much larger than 1. However, the fraction of the usual left-handed quark, $\cos \phi_+$, depends on the ratio of $\tilde{\mu}'_L/M$. It is large when $\tilde{\mu}'_L$ is much smaller than M . Alternatively, one can have large mixing between the ordinary and mirror quarks when $\tilde{\mu}'_L$ and M are of the same order. A

similar situation happens when $\tilde{\mu}'_R$ is of the same order of magnitude as M while $\tilde{\mu}'_L$ is much smaller.

The latter limit enables one to realize simultaneously a heavy top quark mass and a few GeV bottom quark mass. The parameter $\tilde{\mu}'_L$ is the same for both the top and bottom sectors and should be of the order of $M_{t_{L'}'}$ so that the top quark is sufficiently heavy. However, $\tilde{\mu}'_R$ and M could be different for the two sectors. As long as $(\tilde{\mu}'_R/M)_b$ is ‘‘small,’’ the contribution to the bottom quark mass is ‘‘small.’’ The most attractive choice is to have $M_{b_{L'}' b_{R'}'} \sim M_{t_{L'}' t_{R'}'} \sim \tilde{\mu}'_L \gg (\tilde{\mu}'_R)_b$. Another possibility is to take $M_{b_{L'}' b_{R'}'} \gg M_{t_{L'}' t_{R'}'} \sim \tilde{\mu}'_L \sim (\tilde{\mu}'_R)_b$. This is, however, more difficult to realize since it is difficult to obtain such a large value for $M_{b_{L'}' b_{R'}'}$ which is proportional to the Higgs VEV.

We conclude that once MPV mixing is allowed, it is possible to realize heavy and highly mixed ordinary and mirror top quarks simultaneously with a relatively light (and relatively non-mixed) SM bottom quark. The question we would like to consider next is with regard to the possible mechanisms that give rise to such a mixing. Various possibilities exist, for example, a ‘‘flavored spurion’’ such that $X^\dagger Q_i Q_j$ terms are allowed in the Kahler potential for $i=j=3$. Mirror symmetry could be viewed in this case an accidental symmetry of the first two generations or as a flavor symmetry. (Note that only vector-like mixing terms are allowed by the SM gauge symmetries.) Here, however, we will present a different toy model in which mirror symmetry breaking is a result of a spontaneous breaking of a gauged flavor symmetry.

Assume an additional (horizontal) $U(1)_H$ gauge factor. The superpotential contains, for example, the term $g_H h_Q Q_3 \Phi_H Q'_3$, assuming that Q_3 and Q'_3 are charged under the horizontal gauge symmetry with charge $\pm h_Q$, and g_H is the horizontal gauge coupling. Φ_H is a gauge singlet contained in the $U(1)_H$ $N=2$ vector multiplet. If it develops a VEV $\langle \Phi_H \rangle$, it would create a mixing parameter $\mu'_Q = g_H h_Q \langle \Phi_H \rangle$. In this example, the Kahler potential can still preserve the mirror symmetry, which is broken spontaneously by $\langle \Phi_H \rangle$. (Note that the flavor symmetry itself is not broken by the $\langle \Phi_H \rangle$ VEV.) This proposal provides a simple framework for the generation of the mixing terms employing generation-dependent $U(1)$ symmetries. However, one must overcome certain difficulties before such a proposal can be realized. We outline those difficulties and the possible cures below.

First, the relative size of the mixing parameter is proportional to the hypermultiplet horizontal charge. It may not be straightforward to find an anomaly-free combination that naturally produces the desired hierarchy. Certain fields, however, could be singlets (for example, D_3). Also, a combination of different M parameters could also contribute to the hierarchy.

Second, there is the issue of the mixing of the third generation quarks with the two light generation quarks. If the third family quarks are charged under any symmetry while

the light quarks and the Higgs bosons are neutral, then any inter-family mixing is forbidden. This can be resolved, for example, by the introduction of a SM singlet hypermultiplet S which is also charged under $U(1)_H$ so that an appropriate chiral symmetry breaking term is allowed in Kahler potential, e.g., $X^\dagger S H_1 Q_3 D_2$, which could lead to an A term or a Yukawa coupling proportional to $\langle F_S \rangle$ and $\langle S \rangle$, respectively. In the case that only A terms arise, then the intergenerational mixing is naturally suppressed as the square of the loop factor. The S -VEVs are induced by the dynamics below the supersymmetry breaking scale, e.g., the SSB potential, and are therefore suppressed. An S -VEV breaks the horizontal symmetry spontaneously and its size is constrained by the usual considerations related to the presence of an extra neutral Z' gauge boson [20].

Last, consider the operators $X^\dagger \Phi_H Q_2 Q_2'$, $XX^\dagger \Phi_H D^\alpha Q_2 D_\alpha Q_2'$ etc. While the former operator can be forbidden by the R symmetry, the latter is allowed by the symmetries. If such operators arise, they could lead to ordinary-mirror matter mixing in all three generations. The supersymmetry dynamics must therefore be constrained not to generate such vector-hypermultiplet mixed operators.

The proposed toy model serves to illustrate that the flavor symmetry may be intimately linked to the details of the breaking of supersymmetry and of the global symmetries it induces. In particular, the heaviness of the third family may stem from the heaviness of the mirror fermions, in which case either mirror symmetry plays the role of a flavor symmetry or the flavor symmetry breaks the mirror symmetry.

VIII. NEUTRINO MASSES

Recent results from the atmospheric and solar neutrino oscillation experiments indicate non-zero neutrino masses, although extremely small with respect to the charged leptons [21]. The mass squared difference between two neutrino mass eigenstates is of the order of 10^{-3} eV² from atmospheric neutrino oscillation data (and 10^{-5} eV² for solar neutrinos). The smallness of neutrino masses can be explained most simply by the seesaw mechanism [22], where a right-handed sterile neutrino N_R with a Majorana mass M is introduced. (Again, M here is the large mass parameter in the fermion mass matrix, $M \equiv M_N$.) Assuming a Dirac mass $m_D N_R \nu_L$ and that there is no Majorana mass for the left-handed neutrinos, the light mass eigenvalue is $m \sim m_D^2/M$. For m_D of the order of the electroweak scale, the tiny neutrino masses can be obtained if $M \sim 10^{15}$ GeV is of the order of the unification scale.

The extended neutrino sector in the MN2SSM is strongly constrained by experiment. There are six active neutrinos, the three ordinary neutrinos ν and their mirror partners ν' , all of which couple to the electroweak gauge bosons. Given the constraints from the invisible Z -boson width on the number of active neutrinos in Z decays, $N_\nu = 2.994 \pm 0.012$ [10], the mirror neutrinos cannot be light: Any additional active neutrinos such as ν' must be heavier than $m_Z/2$. (If there are additional sterile neutrinos, there could be more than three light active neutrinos as the coupling $Z \bar{\nu}_i \nu_i$, ν_i being the

mass eigenstate, is suppressed by mixing angles. Nevertheless, we assume only three light active neutrinos.) As an obvious consequence from the last statement, the observed neutrino oscillations cannot be explained by $\nu \rightarrow \nu'$ and must occur among the ordinary neutrinos. In the following, we will only address the question of obtaining the small ordinary neutrino masses while keeping the mirror neutrinos heavy. The mixing between the light neutrino mass eigenstates and an explanation of the oscillation data require a more careful model building (for example, the generational structure of the matrices discussed below needs to be addressed), which is left to future studies. We therefore discuss only one generation of neutrinos.

Clearly, the seesaw mechanism described above does not generalize to $N=2$ supersymmetry: If the small neutrino mass is generated by the usual seesaw mechanism, the sterile neutrino must be heavy, with its mass above the $N=2$ breaking scale. The mirror sterile neutrino must have the same mass because the exchange symmetry is a good symmetry above the $N=2$ breaking scale. The mirror neutrino masses are then also suppressed by the see-saw mechanism, $m_{\nu'} \sim \langle H \rangle^2/M \ll m_Z/2$, which is below experimental bounds. Therefore, the Majorana mass for the sterile neutrino cannot be much larger than the $N=2$ breaking scale, which in the framework of 2×2 see-saw leads to heavy neutrinos (unless the Yukawa couplings are fine-tuned). (We note in passing that if the three right-handed neutrinos remain as light as the left-handed neutrinos, one can explore an explanation of the oscillation data involving also $\nu_{L_i} \rightarrow N_{R_j}$ transitions.)

If mirror parity is conserved, there is no mixing between the usual and mirror neutrinos. The mass matrix is reduced to two diagonal blocks for the usual and mirror sectors. Once sterile neutrinos are introduced, the mirror neutrino mass can be generated via effective tree level Yukawa coupling as for the other mirror fermions. The ordinary neutrino mass, on the other hand, cannot be generated radiatively since the right-handed neutrino is a gauge singlet. Common techniques like the radiative generation of neutrino masses via the introduction of a charged $SU(2)$ singlet and a second Higgs doublet or tree-level neutrino mass by a Higgs triplet [23] may be exploited to give the small neutrino masses, particularly since such fields are available in the spectrum. (One could also introduce by hand tiny tree-level effective Yukawa couplings for the neutrinos or extend the SM group as discussed in Sec. IV.) Here, we will explore a different source of neutrino masses in $N=2$ scenarios, a 3×3 seesaw mechanism which is induced by a small breaking of the mirror parity.

With only the minimal spectrum (no sterile neutrinos) but with MPV mixing between the usual and mirror neutrinos $\tilde{\mu}'_\nu \nu \nu'$, the neutrino mass matrix is given by

$$\begin{pmatrix} 0 & \tilde{\mu}'_\nu \\ \tilde{\mu}'_\nu & 0 \end{pmatrix} \quad (28)$$

in the basis of (ν, ν') . In this simplest framework one has two degenerate mass eigenstates and no mass hierarchy between the mass eigenstates can be generated.

Let us then consider a more general $N=2$ neutrino sector. (For simplicity, we only consider one generation.) Consider the following:

(i) Two sterile neutrino superfields, N and its mirror partner N' , with masses M_N and M'_N , respectively. (We omit hereafter the R index.)

(ii) Dirac masses for the usual and mirror sector $m_D N \nu$, $m'_D N' \nu'$.

(iii) Mirror parity violating terms $\tilde{\mu}'_\nu \nu \nu'$, $\tilde{\mu}'_N N N'$.

(iv) Dirac-type mixing $\tilde{\mu}'_{N\nu} N \nu'$, $\tilde{\mu}'_{N'\nu} N' \nu$.

Under these assumptions the 4×4 neutrino mass matrix reads

$$(\nu, N, \nu', N') \begin{pmatrix} 0 & m_D & \tilde{\mu}'_\nu & \tilde{\mu}'_{N'\nu} \\ m_D & M_N & \tilde{\mu}'_{N\nu} & \tilde{\mu}'_N \\ \tilde{\mu}'_\nu & \tilde{\mu}'_{N\nu} & 0 & m'_D \\ \tilde{\mu}'_{N'\nu} & \tilde{\mu}'_N & m'_D & M'_N \end{pmatrix} \begin{pmatrix} \nu \\ N \\ \nu' \\ N' \end{pmatrix}. \quad (29)$$

However, it is simplified under a well-motivated set of assumptions which we consider for the purpose of illustration:

(i) There is no Dirac mass in the usual neutrino sector, $m_D=0$. This is true, for example, if the ordinary sector fermion masses originate from radiative corrections.

(ii) There is no mixing between the sterile neutrinos N and N' , $\tilde{\mu}'_N=0$. This is the case if the mixing arises from the VEV of some mirror $U(1)$ gauge boson singlet $\langle \Phi_{U(1)\nu} \rangle$, while N and N' are singlets under $U(1)_\nu$.

(iii) There is no Dirac-type mixing, $\tilde{\mu}'_{N\nu} = \tilde{\mu}'_{N'\nu} = 0$. Assuming that Higgs couplings preserve mirror parity, those terms could only arise from Yukawa terms in the Kahler potential involving the superfield combinations $H'_1 L N'$, $H'_2{}^\dagger L N'$ and $H'_2 L' N$, $H'_1{}^\dagger L' N$, once the mirror Higgs bosons acquire VEVs. However, we assume that EWSB is induced only by the ordinary MSSM Higgs doublets, so $\langle H'_1 \rangle = \langle H'_2 \rangle = 0$ and such Yukawa terms do not generate mixing.

The usual sterile neutrino now decouples, and one is left with a 3×3 mass matrix with only three parameters:

$$\begin{pmatrix} 0 & \tilde{\mu}'_\nu & 0 \\ \tilde{\mu}'_\nu & 0 & m'_D \\ 0 & m'_D & M'_N \end{pmatrix}. \quad (30)$$

In the relevant limit one has $\tilde{\mu}'_\nu \ll m'_D \ll M'_N$; i.e., the MPV parameter $\tilde{\mu}'_\nu$ is small and the Dirac mass for the mirror neutrinos, $m'_D \sim \langle H \rangle$, is small with respect to sterile neutrino mass M'_N which is of the order of the $N=2$ breaking scale. The mass eigenvalues in this limit are approximately

$$m_1 \sim \frac{\tilde{\mu}'_\nu{}^2}{m_2}, \quad m_2 \sim \frac{m'_D{}^2}{M'_N}, \quad m_3 \sim M'_N. \quad (31)$$

The smallness of the lightest neutrino masses can be controlled by the small MPV parameter $\tilde{\mu}'_\nu$, while the second lightest neutrino remains heavy as long as m'_D/M'_N is not too small. It is, however, in the mass range implied by electroweak data (see the next section) and a candidate for the LMP. Notice that M'_N cannot be too large or the mirror neutrino mass would be suppressed below the experimental lower limit. As an example, taking $M'_N=1000$ GeV, $m'_D=300$ GeV and $\tilde{\mu}'_\nu \sim 10^{-6}-10^{-4}$ GeV, the neutrino masses read

$$m_1 \sim 10^{-5}-10^{-1} \text{ eV}, \quad m_2 \sim 90 \text{ GeV}, \quad m_3 \sim 1000 \text{ GeV}. \quad (32)$$

This model is a variation of the seesaw mechanism where the small mirror parity violating parameter $\tilde{\mu}'_\nu$ plays the role of the usual Dirac masses.

In conclusion, The neutrino sector in $N=2$ supersymmetry is strongly constrained as one needs to not only generate the small neutrino masses to fit the neutrino oscillation data, but also to maintain a large mass hierarchy between the ordinary and mirror sectors. Here, we presented a simple model for the $N=2$ neutrino sector which relies on small MPV. The model is successful though it is far from unique and other possibilities need to be explored.

IX. PHENOMENOLOGY OF $N=2$ SUPERSYMMETRY

Given its extended spectrum, the phenomenology of the MN2SSM is particularly rich. Its effects are both indirect (electroweak physics) and direct (collider phenomenology and new particle searches). Although many predictions depend on the details of the model, important conclusions can be drawn based only on the general structure of the $N=2$ framework. While some of the MN2SSM characteristics only provide a variation on the phenomenology of the $N=1$ MSSM, many other features are unique to $N=2$, and provide the smoking gun signals for the discovery of $N=2$ supersymmetry. In this section we review some of the more interesting aspects, both indirect and direct, of $N=2$ supersymmetry.

A. Electroweak and Higgs physics

As mentioned in the Introduction, additional chiral quarks and leptons can lead to large positive contribution to the oblique parameter S [9], which is phenomenologically unfavorable, if not excluded [10]. A dedicated electroweak analysis including all mirror particles is well motivated (though technically complicated) and it is currently underway [13]. (It will be reported elsewhere [13].) A preliminary investigation [13] concludes that the correlations between the oblique parameters (rather than S itself) place probably the strongest constraints on the MN2SSM. Consistency requires some relatively light mirror particles, for example, some combination of relatively light mirror neutrinos and mirror Higgsinos. This is not surprising: The well-known result that each new fermion generation leads to a $+2/3\pi$ contribution to S [9] holds only if the extra generation is degenerate in mass with

$m_{f_{\text{new}}} \gg m_Z$. In the case of $N=2$ and the MN2SSM, the masses of the mirror matter fermions are related to the EWSB Higgs VEVs, and so $m_{f'} \approx m_Z$. Furthermore, the origin of its mass is similar to that of the ordinary fermions (supersymmetry breaking operators in the Kahler potential) and there is no reason to assume degeneracy. Indeed, S is found to be negative in certain regions of the parameter space. Therefore, the constraints emerge not from a single oblique parameter but from the correlation between the different parameters [13].

In fact, it was already shown in Ref. [11] that electroweak precision data can accommodate extra generations if there exist heavy (active) neutrinos with masses close to 50 GeV [while their charged $SU(2)_L$ partners are with masses slightly above 100 GeV and the extra quarks are degenerate]. Mass dependent terms are important in this case, and in addition the non-colored spectrum breaks the custodial $SU(2)$ symmetry of the electroweak interactions. This scenario can be naturally fulfilled in the MN2SSM: An example of a mirror neutrino in this mass range was given in the previous section. In addition, Ref. [11] has also found that extra generations may be accommodated if charginos and neutralinos have masses close to 60 GeV. This is an example of negative contributions to S from custodial $SU(2)$ breaking Majorana masses [12]. In $N=2$ such Majorana masses arise naturally; for example, there are mixing terms between the mirror Higgsino doublets and ψ_W . This again can lead to a negative or vanishing value of S .³

Another issue of importance to electroweak physics is the mass of the Higgs boson. The MN2SSM Higgs sector is not as constrained as in the MSSM or other $N=1$ frameworks. The number of Higgs doublets participating in EWSB could vary, in principle, between 1 and 4.⁴ Here we assume two, H_1 and H_2 , as in the MSSM. Even within this MSSM-like framework of two Higgs doublets participating in EWSB, there is no upper bound on the mass of lightest Higgs boson. This is because tree-level Higgs quartic couplings λ arise not only from supersymmetric terms $\lambda \sim g^2$ as in $N=1$, but also from hard supersymmetry breaking operators in the Kahler potential $\lambda \sim g^2 + \kappa$. (See Sec. V.) Consequently, the minimization of the Higgs potential leads to a modified formula for relating m_Z , $m_{H_{1,2}}^2$, $\tan \beta$, and μ , as in Eq. (9). More importantly, because of its dependence on the arbitrary hard couplings κ , the tree-level light Higgs boson mass m_h^2

$\sim \lambda(\nu_1^2 + \nu_2^2)$ is not bound from above by m_Z (or by 130 GeV at loop order) as in the MSSM. This observation is not unique to $N=2$ but rather to theories with low-energy supersymmetry breaking where $\kappa \sim \mathcal{O}(1)$ is possible. It carries important implications for defining theoretically motivated mass range for future Higgs boson searches.

Other indirect implications arise from the fact that the ordinary quark and lepton masses (except the third generation) may arise radiatively, which by itself has interesting consequences [15]. A general feature in theories with radiative fermion masses is that the anomalous magnetic moments are not suppressed by a loop factor relative to the respective fermion mass: $a_f \sim m_f^2/\tilde{m}^2$, where \tilde{m} is the mass of the heavy particles running in the loop. This has particular relevance in the case of the muon whose magnetic moment is well measured and further improvement in its measurement is expected in the near future [24], allowing for such effects to be observed [15].

The light mirror fermions, the possibility of hard Yukawa and quartic couplings as well as of radiative Yukawa couplings, and the large number of new degrees of freedom, all imply that the MN2SSM interacts with the SM and electroweak physics more strongly than the MSSM and leads to quite different predictions for various observables.

B. Collider phenomenology

The experimental limits on the extra heavy quarks and leptons are based on searches for the fourth generation at e^+e^- and $p\bar{p}$ colliders: $m_{\nu'} \geq 40$ GeV, $m_{l'} \geq 80$ GeV, and $m_{b'} \geq 128$ GeV [10]. These lower mass bounds may or may not apply to the $N=2$ mirror quarks and leptons, as they depend on the decay modes of the heavy fermions. Nevertheless, such limits are easily satisfied for $\sqrt{\langle F_X \rangle} \sim M$, corresponding to effective tree level Yukawa couplings for the mirror fermions of the order of unity. On the other hand, the mirror fermion masses are proportional to the EWSB Higgs VEV and therefore cannot be much larger than the electroweak scale $m_{f'} \approx \langle H \rangle = 174$ GeV. (In addition, the oblique S parameter also constrains some of the masses from above.) This upper bound ensures that mirror fermions can be copiously produced at any machine that produces a large number of top quark pairs: the Tevatron, CERN's Large Hadron Collider (LHC) and future lepton colliders. The MN2SSM and the $N=2$ framework can be confirmed or excluded shortly after the next energy frontier is reached.

The experimental signals largely depend on whether the mirror parity M_P and/or the usual R parity R_P are broken below the $N=2$ breaking scale. Given both parities, there are three special particles that play an important role in determining the phenomenology: the lightest R_P -odd (supersymmetric) particle (LSP), the lightest M_P -odd (mirror) particle (LMP), and the lightest mirror supersymmetric particle (LMSP), which is odd under both parities. The LMSP could be the LSP, the LMP both or neither one. If both parities are preserved, the LSP and LMP are stable. In addition, there could also be a third stable particle whose decay into the LSP and LMP is kinematically forbidden. (For example, this could be the LMSP if it is not the LMP or the LSP and it is

³Additional gauged (flavor) $U(1)$, as suggested in Sec. VII, can also contribute negatively to S , depending on the mixing between the extra gauge boson Z' and the ordinary Z [20].

⁴Note that the $N=1$ anomaly cancellation requirement of an even number of Higgs superfields is automatically satisfied in $N=2$ for any number of Higgs hypermultiplets. The $N=1$ requirement of at least two Higgs superfields with opposite hypercharge acquiring VEVs (from fermion mass generation) is relaxed in the framework of low-energy supersymmetry breaking and the MN2SSM due to the appearance of non-holomorphic Yukawa terms. Therefore, there could be, in principle, only one Higgs hypermultiplet with only the Higgs, not its mirror, acquiring a VEV.

not heavy enough to decay into them.) A stable charged (electromagnetic or color) particle is excluded up to $\sim \mathcal{O}(20 \text{ TeV})$ cosmologically from failure of terrestrial searches for anomalously heavy isotopes of various elements [25]. The only possible massive stable particles are therefore the mirror neutrinos; sneutrinos and mirror sneutrinos; the Higgsinos, mirror Higgsinos and mirror Higgs; ϕ_γ and ψ_γ ; and ϕ_Z and ψ_Z (where we rotated the electroweak group to its SM basis). Note that even if the LSP is not stable due to the broken R_p (RPV), there could still exist a stable neutral LMP, which could be the candidate for dark matter.

Particle decays in the MN2SSM can be classified as follows. Define the following:

(i) $(+, +)$ to denote SM particles (quarks q , leptons l , Higgs bosons H , and gauge bosons g, W, B).

(ii) $(-, +)$ to denote ordinary supersymmetric particles (squarks \tilde{q} , sleptons \tilde{l} , Higgsinos \tilde{H} , and gauginos $\tilde{g}, \tilde{W}, \tilde{B}$).

(iii) $(+, -)$ to denote SM mirror particles (mirror squarks q' , mirror sleptons l' , mirror Higgs bosons H' , and mirror gauge bosons ϕ_V).

(iv) $(-, -)$ to denote mirror sparticles (mirror squarks \tilde{q}' , mirror sleptons \tilde{l}' , mirror Higgsinos \tilde{H}' , and mirror gauginos ψ_V).

Here the first sign in the parentheses is the particle's R_p charge, while the second sign is its M_p charge. The allowed two-body decays are then

$$\begin{aligned} (+, +) \rightarrow (+, -)(+, -), (-, +)(-, +), (-, -)(-, -), \\ (+, +)(+, +), \end{aligned} \quad (33)$$

$$(-, +) \rightarrow (-, -)(+, -), (-, +)(+, +), \quad (34)$$

$$(+, -) \rightarrow (-, -)(-, +), (+, -)(+, +), \quad (35)$$

$$(-, -) \rightarrow (-, +)(+, -), (-, -)(+, +). \quad (36)$$

Three-body and many-body decays can be classified by applying the two-body channels to off-shell processes. (Quartic couplings need also to be considered in some cases.)

As a concrete example, consider a case with a top squark \tilde{t} as the lightest ordinary sparticle, which is heavier than the LSP (taken to be the mirror sneutrino which is also the LMSP, as an example) and LMP (taken to be mirror neutrino). A possible decay chain of the top squark is $\tilde{t} \rightarrow b\tilde{W} \rightarrow bl'\tilde{\nu}' \rightarrow bl\phi_Z\tilde{\nu}' \rightarrow bl\nu\nu'\tilde{\nu}'$, if it is kinematically allowed. (Intermediate steps could be on or off shell.) The final state contains in this case a b jet, a charged lepton, and missing energy. Alternatively, it could decay via a trilinear coupling, $\tilde{t} \rightarrow \tilde{b}'H'^+$, leading to a similar final state $bl(3\nu)\nu'\tilde{\nu}'$. The neutrinos and the neutral LMPs and LSPs all lead in this case to a missing energy signal, as in the usual $N=1$ MSSM case.

Since the masses of the mirror (matter) fermions are related to the electroweak symmetry breaking scale and would be at most a couple hundred GeV, they are most likely to be the first mirror particles to be produced at the colliders and are candidates for the LMP. The mirror quarks particularly deserve attention. They can be copiously pair produced at the

LHC and the Tevatron via gluon fusion. If a mirror neutrino is the LMP and all the superparticles are heavier, each mirror quark can either decay through an on-shell electroweak mirror gauge boson, $q'_i \rightarrow q_i\phi_Z, q_i\phi_\gamma, q_j\phi_W$, where $\phi_{Z,\gamma} \rightarrow \nu\nu'$, $\phi_W \rightarrow l\nu'$, if kinematically allowed, or it directly decays into $q_i\nu\nu', q_jl\nu'$ through an off-shell process. A typical event has two energetic jets (two charged leptons, in some cases) and a missing energy. If the superparticles are not too heavy, mirror quarks can alternatively decay through supersymmetric channels. In addition to the jets and leptons, the final state could have in this case at least two LSPs. (The event reconstruction is more difficult in this case.)

If one of the parities is violated, there is only one stable particle, and if both are violated, all the particles will eventually decay into SM jets and leptons. The LSP and LMP lifetimes depend in this case on the extent of the respective parity violation. If the LMP decays outside the detector, it appears in the detector as a stable particle. Consider a mirror fermion as the (meta-stable) LMP. A neutrino (or any other neutral) LMP leads in this case as well to a missing energy signal. Mirror charged lepton LMPs leave a track in the central tracking chamber and hit the muon chamber, with less activity in the calorimeters. A muon and a mirror charged lepton can be distinguished by the ionization rate dE/dx since the mirror particle is much heavier. If a mirror quark is the LMP and stable inside the detector, it will form a quarkonium or combine with the ordinary quarks to form a mirror hadron. Such states will lead to hadron showers in the hadron calorimeters, and can be distinguished from a regular hadron shower by the wider shower opening angle [26]. If a mirror gauge boson is the LMP (and stable in the detector), the signals will be similar to those mentioned above, depending on whether it is neutral, charged and/or carrying color. If the LMP decays inside the detector, the heavy mirror particle can decay into jets, leptons, or lighter supersymmetric particles. A missing energy signal is still possible if the usual R parity is exact. Otherwise, the signal mimics those of the SM heavy fourth family quarks and leptons and of the RPV MSSM.

We conclude that once the center of mass energy of the future colliders is sufficient to produce the mirror fermions, they can hardly escape detection. Since future colliders will effectively provide top quark factories, sufficient energies will be reached, providing an ideal environment for searching for the $N=2$ mirror quarks (which cannot be much heavier than the top quark).

Higgs boson production is also affected by the MN2SSM spectrum. The existence of extra heavy mirror quarks can greatly enhance the single-Higgs-boson production rate in hadron colliders through gluon and quark fusions, since the effective Yukawa coupling is of the order of unity. Neutral Higgs bosons can be produced radiatively via $gg \rightarrow H^0$ through heavy quark loop [27]. Higgs-strahlung associate production rates for both neutral [28] and charged [29] Higgs bosons through $2 \rightarrow 3$ processes $gg, qq \rightarrow q'q'H$ (and in the case of MPV also $2 \rightarrow 2$ processes $qq \rightarrow q'H$) can also increase greatly. For example, Ref. [30] argued that one generation of mirror heavy quarks can increase the Higgs boson production cross section for $gg \rightarrow H^0$ by a factor of 6–9. In

Ref. [28] the contributions of large Yukawa couplings to associated Higgs boson production was shown for the case of large $\tan\beta$ and of RPV. In addition, decay channels of (a sufficiently heavy) Higgs boson to mirror fermions are also expected to be important. Last, if there are radiative Yukawa couplings, they also affect the Higgs phenomenology and create a misalignment between on-shell and mass ($m_f/\langle H \rangle$) Yukawa couplings of the Higgs boson [15].

The sparticle phenomenology is also richer than in the MSSM. For example, consider the (eight Majorana) neutralinos and (four Dirac) charginos. If mirror parity is unbroken, there are two diagonal blocks in the neutralino and chargino mass matrices, each of which is an analogue of the usual $N=1$ case. The M_P -even neutralino block could provide the LSP while that odd block could provide the LMSP (which may or may not be the LMP and/or LSP). Each sector could decay, however, to particles in the other sector via M_P conserving couplings such as $\tilde{H}'_i \phi_Z \tilde{H}_i$. If mirror parity is violated, then there are off-diagonal mixing terms between the ordinary and the mirror Higgsinos and gauginos, which lead to complicated mixing patterns. Similarly, the mixing patterns of squarks and sleptons are also complicated in the presence of MPV terms, while new production and decay channels (and complicated cascades) are open whether M_P is conserved or violated.

X. CONCLUSIONS

In this paper we have formulated a low-energy $N=2$ supersymmetric framework in which $N=2$ supersymmetry is preserved down to TeV energies. The minimal $N=2$ realization of the SM, the MN2SSM, was presented and its properties studied. While from the low-energy point of view the models do not have a clear added benefit in comparison to the $N=1$ MSSM to justify the extended spectrum, it is ultraviolet constructions that suggest the possibility of $N=2 \rightarrow N=0$ supersymmetry breaking [5]. Therefore, it is important to examine the viability of such a scenario, address the issues it raises (particularly its non-chiral nature), and investigate its signatures. The next generation of hadron collider, in which top quark pairs will be produced in abundance, is ideal to test the $N=2$ framework via its mirror quark sector, adding urgency to such an investigation. To conclude, we review the main issues studied in this paper, comment on some other issues such as unification and cosmology, compare our framework to previous proposals, and propose further avenues for investigation of the $N=2$ framework.

A. Framework

$N=2$ supersymmetry was assumed in this work to break to $N=0$ at low energies. We chose, however, to formulate it as an $N=1$ theory, constrained by a set of global R symmetries which preserve the $N=2$ structure. The spectrum is that given by embedding the SM in $N=2$ hyper- and vector multiplets, but it was written in terms of their $N=1$ superfield components. The minimal embedding corresponds to the MN2SSM: Each MSSM superfield is accompanied by a *mirror* superfield in the conjugate gauge representation. The N

$=2$ symmetries, in turn, constrain the superpotential describing these superfield component interactions. In particular, an exchange symmetry between a particle and its mirror and an Abelian R symmetry imply that the superpotential does not contain chiral Yukawa terms and mass terms, respectively. (Note that theory considered is a global $N=2$ described in the $N=1$ language; i.e., gravity was not introduced.)

The $N=1$ language allows one to use the spurion formalism and supersymmetry breaking is parametrized by two independent parameters, the spurion auxiliary F -VEV and a mass parameter M which suppresses explicit breaking of the global symmetries in the Kahler potential, with $F \simeq M^2$. If supersymmetry is to play a role in resolution of the SM hierarchy problem, then $M \simeq \mathcal{O}(\text{TeV})$. Hence, supersymmetry is broken at low energy and one has to consider all operators to order F^2/M^4 in the Kahler potential. The effective theory below the supersymmetry breaking scale contains various quartic and Yukawa terms and the $N=2$ and $N=1$ relations between couplings are broken. Even though such breaking is hard, it does not destabilize the theory but only affects the calculability of dimensionful parameters.

Below the supersymmetry breaking scale, some of the $N=2$ global symmetries may be preserved and could distinguish the SM matter from its mirror. In addition, parity symmetries, which do not commute with supersymmetry, may be admitted by the supersymmetry breaking mechanism. R parity and mirror parity were used to define sparticles (as in the MSSM) and mirror particles, respectively. If preserved, each parity corresponds to a stable neutral particle, the LSP and LMP, respectively.

B. Fermion mass generation

The explicit supersymmetry breaking terms in the Kahler potential must also break the vectorial symmetries imposed by the $N=2$ global R symmetries so that chiral Yukawa couplings can be generated. This can be done by $N=1$ preserving, softly breaking, or hard breaking tree-level operators $\sim (F/M^2)^n$, $n=1, 2$. It can also be achieved by first breaking the symmetries by trilinear terms in the scalar potential and generating the Yukawa terms at one loop. The latter leads to $\mathcal{O}(\text{GeV})$ quark masses while the former, in principle, could lead to dangerous mixing between the SM fermions and their mirrors. However, by invoking the global (R) symmetries, one can distinguish between operators which generate the SM fermion masses from those generating the mirror spectrum so that the former is proportional to suppressed Yukawa couplings or given by loop corrections while the latter is given by $\mathcal{O}(1)$ Yukawa couplings. In particular, a large mass hierarchy between the SM fermions and the mirror fermions can be achieved and the mixing between the sectors can be suppressed or (if mirror parity is exact) even forbidden. The SM fermion spectrum is controlled, as usual, by small Yukawa couplings while the mirror fermion spectrum is constrained from above by the electroweak scale $m_f \sim \langle H \rangle$.

The heavy SM third family (especially the top quark) is an exception as its mass range is closer to that of the mirror fermions than to that of the lighter SM generations. Though such an hierarchy can be imposed by hand, here we consid-

ered the possibility that a mirror symmetry also plays the role of a SM flavor symmetry, allowing for mirror parity violating mixing in the third family. The heavy top quark is then explained by the heaviness of the third generation mirror top. A toy model with an Abelian flavor symmetry was given along these lines.

A very small violation of mirror parity may also play a role in the smallness of neutrino masses. We suggested that the small neutrino masses can arise from a variation on the see-saw mechanism in which the SM neutrino mass is controlled by the small mixing parameter between the ordinary and mirror neutrinos. The mirror neutrinos must be heavier than $m_Z/2$, as required by the invisible Z width, but could explain the smallness of the oblique parameter S if they are not much heavier than that. This in fact occurs in the same framework as the second eigenvalue is given by $\sim \langle H \rangle^2/M$.

C. Signals and constraints

The contributions to the oblique S parameter from the extra three chiral generations provides a strong constraint on the MN2SSM. In particular, it implies that the mirror fermions cannot be degenerate in mass and cannot (all) be too heavy; for example, the mirror neutrino may be relatively light. In addition, it also suggests light neutralinos and mirror neutralinos (with significant custodial-symmetry-breaking mixing). A dedicated fit to electroweak data in the MN2SSM framework is necessary, but is beyond the scope of this work. It is currently underway and will be reported elsewhere [13].

On the other hand, due to the low-energy supersymmetry breaking, the Higgs boson mass is not constrained from above as in the MSSM since its mass could be proportional to an arbitrary quartic couplings. The Higgs sector could contain two or four doublets, any number of which could participate in supersymmetry breaking. Choosing the MSSM limit of two ordinary Higgs doublets acquiring VEVs, we find a Higgs potential which is more similar to that of a generic two-Higgs-doublet model than to the MSSM.

The spectrum of new particles is very rich, and contains three new sectors (sparticles, mirror matter, mirror sparticles), which do not mix with each other, and two (in some cases even three) stable particles. If either R or mirror parity is broken, mixing is introduced and the number of stable neutral particles is reduced respectively. The most obvious candidates for discovery are the relatively light $\mathcal{O}(100\text{--}300)$ GeV mirror quarks, while any stable particles are likely to correspond to missing energy.

The extra generations of mirror quarks and leptons at the electroweak scale provide the smoking gun for testing the $N=2$ framework at the LHC and the Tevatron. Typical events consist of jets + leptons + missing (transverse) energy. The missing energy is generally greater than in the $N=1$ supersymmetry “events,” as the final states could include two or more neutral heavy stable particles. (If mirror parity is slightly violated, there still is a stable particle as in the usual $N=1$ case, as long as R parity is preserved and vice versa.) Detailed study of the collider phenomenology of low

energy $N=2$ theories is also called upon, but clearly, it would be difficult for $N=2$ to escape discovery if realized at TeV energies.

We note that aside from direct searches for mirror particles, extra generations of heavy quarks could greatly increase the Higgs boson production at hadron colliders via gluon fusion; both single boson production and production in association with quarks are, in principle, enhanced. Therefore, an enhanced Higgs boson production cross section in hadron colliders could be a hint for the existence of mirror particles. In addition, if the SM Yukawa couplings are radiative, it carries strong implications to Higgs physics, such as misalignment of mass and on-shell Yukawa couplings, as well as to low-energy observables such as an enhancement by an inverse loop factor of the anomalous muon magnetic moment.

The phenomenological implications of $N=2$ supersymmetry are rich. Here we focused on those which could be studied in a relatively model-independent fashion. However, the details of the model, for example, the extent of mirror parity violation (if any) and the flavor theory, can determine many aspects of the model phenomenology such as the stable particles, the cascade chains, and indirect effects in low-energy and electroweak SM observables.

D. Other issues of interest

The high precision in which the SM gauge couplings are currently measured strengthens the successful gauge coupling unification picture in $N=1$ theories. (See, for example, Ref. [3].) Unfortunately, MSSM-like Planckian unification of the gauge couplings seems inconsistent with the framework of low-energy $N=2$. Since there are no higher-loop corrections in $N=2$, it is sufficient to examine unification at one-loop order. The one-loop beta function coefficients of the SM gauge couplings in the MN2SSM are large and positive:

$$b_1^{N=2} = \frac{66}{5}, \quad b_2^{N=2} = 10, \quad b_3^{N=2} = 6. \quad (37)$$

Taking M as before to be the $N=2$ breaking scale, above which one has the $N=2$ MN2SSM spectrum, while between m_Z and M resides either the $N=1$ MSSM or $N=0$ SM spectrum, we find that M has to be of the order of 10^{14} GeV (for $N=1$ below M [31]) or 10^{11} GeV (for $N=2$ breaking directly to $N=0$) for MSSM-like unification to hold. This is inconsistent with the assumption that the $N=2$ breaking scale is near the electroweak scale. (The situation does not improve if there is only one Higgs doublet hypermultiplet.) Alternatively, the MN2SSM implies unification 2–4 orders of magnitude above its scale M , i.e., at intermediate energies. Indeed, given the non-asymptotically free behavior of the gauge couplings it is hard to imagine that there is a true desert between a low-energy $N=2$ breaking scale and the sub-Planckian MSSM unification scale of 10^{16} GeV. New physics may manifest itself as an extended gauge group, new thresholds, or even an extended $N=4$ supersymmetry (which is finite), and could play a role in resolving the unification issue. Note that the embedding in $N=4$ must involve also

extending the gauge group as all of the SM representations must be embedded in that case in the adjoint representation (of an extended gauge group), which is an interesting possibility.

Another issue of interest that we did not pursue here is the cosmological and astrophysical implications of such scenarios. It is interesting to revisit issues such as electroweak scale inflation [32] and electroweak scale baryogenesis [33] which can be sensitive to the new rich electroweak structure. In particular, we expect baryogenesis constraints to be affected by the presence of large Yukawa couplings and the large number of Majorana fermions and of singlet fields [34]. A detailed study of neutrino mass and mixing patterns is also of great interest. (Here we only addressed the question of the overall scale of neutrino masses.)

We also note in passing that the cosmological constant is zero in the $N=2$ limit. If the leading contribution from supersymmetry breaking is then of the order of M^8/M_{Planck}^4 (in particular, the M^4 terms are canceled), where gravitational corrections assumed to be suppressed by inverse powers of M_{Planck} , it leads to values of the cosmological constant consistent to the currently preferred value of $\sim(10^{-3} \text{ eV})^4$ [35]. The MN2SSM can provide a natural realization of the general arguments for such a framework [36].

E. Previous works

Previous attempts to construct $N=2$ models were based on either quantum correction or $N=2$ gauge-Yukawa couplings [6]. The former was described in detail in Sec. IV. Although the masses for the usual quarks and leptons (except the third generation) can be generated at the right order of magnitude, the mirror quarks and leptons are typically too light.

Realizing that the radiatively generated mass is not sufficient for the mirror fermions, it was proposed [6] that the mirror fermion mass may be generated at the tree level via the only Yukawa term $[Y\Phi_V X]_F$ in the superpotential if the SM $SU(2)_L$ is extended to $SU(4)_{LR}$. The gauge group is $SU(3)_c \times SU(4)_{LR} \times U(1)_Y$ (3-4-1) and the ordinary and mirror matter representations are

$$\begin{aligned} X &= \begin{pmatrix} X_L \\ \bar{Y}_R \end{pmatrix} \sim (\mathbf{1} \quad \text{or} \quad \mathbf{3,4}, Q_Y), \\ Y &= \begin{pmatrix} Y_L \\ \bar{X}_R \end{pmatrix} \sim (\mathbf{1} \quad \text{or} \quad \bar{\mathbf{3}}, \bar{\mathbf{4}}, -Q_Y), \end{aligned} \quad (38)$$

where X_L, Y_L are the ordinary and mirror $SU(2)_L$ doublets, while \bar{X}_R, \bar{Y}_R are $SU(2)_R$ doublets and their mirrors. Here Q_Y and $-Q_Y$ are the $U(1)_Y$ hypercharge of the superfields X_L and Y_L , respectively. Note that conventional ordinary and mirror particles mix in X and Y . By appropriately arranging the parameters in the scalar potential, the $SU(4)$ mirror gauge boson Φ_4 acquires off-diagonal VEVs, which give masses to the mirror quarks and leptons, while the usual matter fields are kept massless:

$$\mu'_{XY} = \sqrt{2}Y \begin{pmatrix} 0 & 0 & \nu & 0 \\ 0 & 0 & 0 & \nu \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} X. \quad (39)$$

One shortcoming of this approach is that the mirror fermion masses are constrained by the EWSB scale,

$$\mu'^2_U + \mu'^2_D = 2M^2_W, \quad (40)$$

which is not consistent with experiment (based on fourth family searches). Another crucial drawback is that all of the SM fermions have only loop masses, which is unacceptable in the case of the top quark (and the τ lepton). A realization of the 3-4-1 scenario was derived more recently from a spontaneously broken $N=2$ supergravity [7].

Our approach relies instead on low-energy supersymmetry breaking and on the tree-level operators it induces. It is more accommodating for embedding the SM than either the pure loop approach or the 3-4-1 approach and it retains a sufficient predictive power.

F. Outlook

In this paper we addressed some of the more fundamental issues such as the scale of supersymmetry breaking, fermion mass generation, constraints and discovery prospects. In each of these areas there is clearly room for further study and exploration, as indicated in the discussions above, in particular, a consistent analysis of all constraints on the one hand, and of collider signals (including the Higgs sector) on the other.

While we focused on mirror symmetries and on distinguishing matter from mirror matter, we only briefly touched upon the issue of flavor symmetries. In fact, flavor symmetries may be entangled with the mirror symmetries, rendering the heavy SM fermions ‘‘more similar to’’ the mirror fermions. This is a new paradigm for flavor symmetries which offers new avenues for construction of theories of flavor which were not yet explored.

Here we subscribed to an effective approach, allowing the most general Kahler potential, which is constrained only by the symmetries. Ultimately, one would like to derive a suitable Kahler potential (and find the necessary terms) from a spontaneously broken $N=2$ supergravity theory, and perhaps from a more fundamental theory. (This was done in Ref. [7] for the 3-4-1 framework of Ref. [6].) The scale of the more fundamental theory needs also to be explored and it is suggestive that some new physics exists only a few orders of magnitude above the supersymmetry breaking scale. The embedding of $N=2$ extensions of the SM in theories with extra large spatial dimensions (where $N=2$ naturally appears) and/or strongly interacting string theories is also worth exploring.

All in all, the viability and richness of $N=2$ extensions of the SM introduce many questions worth pursuing, from mirror quark searches to Kahler potential construction in the case of low-energy supersymmetry breaking. These are inte-

gral parts of the MN2SSM construction, but their implications extend beyond it: Search algorithms in models with two stable particles and limits on the Higgs boson mass in $N = 1$ models with low-energy supersymmetry breaking are examples of issues that are both central to the MN2SSM phenomenology and extend beyond the $N = 2$ framework and should be explored both within and independently of the MN2SSM.

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