Impact parameter dependence of low-x structure functions

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We consider the impact parameter dependence of the polarized and unpolarized structure functions. Unitarity does not allow factorization of the structure functions over the Bjorken x and the impact parameter b variables. On the basis of the particular geometrical model approach we conclude that the spin of constituent quarks may have a significant orbital angular momentum component which can manifest itself through the peripherality of the spin-dependent structure functions.

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I. INTRODUCTION

The behavior and dependence of the structure functions on the Bjorken x is among the most actively discussed subjects in unpolarized and polarized deep-inelastic scattering. The particular role here belongs to the small-x region where the asymptotical properties of the strong interactions can be studied. The characteristic property of the low-x region is an essential contribution of nonperturbative effects [1,2] and one of the possible ways to treat this region is the construction and application of models. Of course, the shortcomings of any model approach to the study of this nonperturbative region are evident. However, one could hope to gain information from these models which cannot be obtained by perturbative methods (cf. [1]). Among the possible extensions of these studies are considerations of the geometrical features of the structure functions, i.e., the dependence of the structure functions on the transverse coordinates or the impact parameter. This subject is not new. The importance of the parton distributions in the transverse plane was stressed in [1] and, e.g., a brief model discussion was recently given in [3]. This work is a revised and extended version of the latter one. As has been demonstrated [4] the b-dependent parton distribution can be related to the Fourier transform of the off-forward matrix elements of parton correlation functions in the limiting case of zero skewedness. Impact parameter dependence would allow one to gain an information on the spatial distribution of the partons inside the parent hadron and the spin properties of the nonperturbative intrinsic hadron structure. The geometrical properties of structure functions play an important role under analysis of the lepton-nuclei deep-inelastic scattering and in the hard production in the heavy-ion collisions.

II. INTERPRETATION OF b-DEPENDENT STRUCTURE FUNCTIONS AT SMALL x

We suppose that the deep-inelastic scattering is determined by the aligned-jet mechanism [1] and consider the b dependence of the structure functions along the lines used in [5]. The aligned-jet mechanism is an essentially nonperturbative and allows one to relate structure functions with the discontinuities of the amplitudes of quark-hadron elastic scattering. These relations are the following [6,7]:

$$q(x)=\frac{1}{2}\mathrm{Im}[F_1(s,t)+F_3(s,t)]|_{t=0},$$

$$\Delta q(x)=\frac{1}{2}\mathrm{Im}[F_3(s,t)-F_1(s,t)]|_{t=0},$$

$$\delta q(x)=\frac{1}{2}\mathrm{Im}F_2(s,t)|_{t=0}. \quad (1)$$

The functions F_i are helicity amplitudes for the elastic quark-hadron scattering in the standard notation for nucleon-nucleon scattering. We consider the high-energy limit or the region of small x.

The structure functions obtained according to the above formulas should be multiplied by the factor $\sim 1/Q^2$—the probability that such an aligned-jet configuration occurs [1].

The amplitudes $F_i(s,t)$ are the corresponding Fourier-Bessel transforms of the functions $F_i(s,b)$.

Relations (1) will be used as a starting point under the definition of the structure functions which depend on the impact parameter. According to these relations it is natural to give the following operational definition:

$$q(x,b)\equiv\frac{1}{2}\mathrm{Im}[F_1(x,b)+F_3(x,b)],$$

$$\Delta q(x,b)\equiv\frac{1}{2}\mathrm{Im}[F_3(x,b)-F_1(x,b)],$$

$$\delta q(x,b)\equiv\frac{1}{2}\mathrm{Im}\,F_2(x,b), \quad (2)$$

and $q(x)$, $\Delta q(x)$, and $\delta q(x)$ are the integrals over b of the corresponding b-dependent distributions, i.e.,

$$q(x)=\frac{Q^2}{\pi^2 x}\int_0^\infty b\,db\,q(x,b),$$

$$\Delta q(x)=\frac{Q^2}{\pi^2 x}\int_0^\infty b\,db\,\Delta q(x,b),$$

$$\delta q(x) = \frac{Q^2}{\pi^2 x}\int_0^\infty b\,db\,\delta q(x,b). \tag{3}$$

The functions $q(x,b)$, $\Delta q(x,b)$, and $\delta q(x,b)$ depend also on the variable Q^2 and have simple interpretations; e.g., the function $q(x,b,Q^2)$ represent the probability to find in the hadron a quark q with fraction of longitudinal momentum x at the transverse distance,

$$b \pm \Delta b, \quad \Delta b \sim 1/Q,$$

from the hadron geometrical center. In the relativistic case a reasonable definition of the hadron geometrical center is the transverse center of momentum [4]

$$\vec{R}_\perp \equiv \sum_i x_i \vec{r}_{i,\perp}.$$

It should be noted that unitarity plays a crucial role in the direct probabilistic interpretation of the function $q(x,b)$. Indeed, because of unitarity, $0 \leqslant q(x,b) \leqslant 1$. The integral $q(x)$ is a quark number density which is not limited by unity and can have an arbitrary non-negative value. Thus, the given definition of the b-dependent structure functions is self-consistent. Of course, the spin distributions $\Delta q(x,b)$ and $\delta q(x,b)$ are not positively defined. The interpretation of the spin distributions follows directly from their definitions: they are the differences of the probabilities to find quarks in the two spin states with longitudinal or transverse directions of the quark and hadron spins.

III. UNITARITY AND *b* DEPENDENCE OF STRUCTURE FUNCTIONS

Unitarity can be fulfilled through the U-matrix representation for the helicity amplitudes of elastic quark-hadron scattering. In the impact parameter representation the expressions for the helicity amplitudes are the following:

$$F_{1,3}(x,b) = U_{1,3}(x,b)/[1 - iU_{1,3}(x,b)],$$

$$F_2(x,b) = U_2(x,b)/[1 - iU_1(x,b)]^2. \tag{4}$$

Unitarity requires $\mathrm{Im}\, U_{1,3}(x,b) \geqslant 0$. The U-matrix form of the unitary representation contrary to the eikonal one does not generate itself essential singularly in the complex x plane at $x \to 0$ and the implementation of unitarity can be performed easily.

The model which provides an explicit form of helicity functions $U_i(x,b)$ has been described elsewhere [5]. A hadron consists of constituent quarks aligned in the longitudinal direction and embedded into the nonperturbative vacuum (condensate). The constituent quark appears as a quasiparticle, i.e., as a current valence quark surrounded by a cloud of quark-antiquark pairs of different flavors. We refer to the effective QCD approach and use the Nambu–Jona-Lasinio (NJL) model [8] as a basis. The Lagrangian in addition to the four-fermion interaction $\mathcal{L}_4$ of the original NJL model includes the six-fermion $U(1)_A$-breaking term $\mathcal{L}_6 \propto K(\bar{u}u)(\bar{d}d)(\bar{s}s)$ [9]. The transition to the partonic picture is described by the introduction of a momentum cutoff $\Lambda = \Lambda_\chi \simeq 1$ GeV, which corresponds to the scale of chiral symmetry spontaneous breaking [10].

This picture for a hadron structure implies that the overlapping and interaction of peripheral condensates in hadron collisions occurs at the first stage. In the overlapping region the condensates interact and as a result virtual massive quark pairs appear. Being released a part of the hadron energy carried by the peripheral condensates goes to the generation of massive quarks. In other words, nonlinear field couplings transform the kinetic energy into the internal energy of dressed quarks. Of course, the number of such quarks fluctuates. The average number of quarks in the considered case is proportional to the convolution of the condensate distributions $D_c^{Q,H}$ of the colliding constituent quark and hadron:

$$N(s,b) \simeq N(s) D_c^Q \otimes D_c^H, \tag{5}$$

where the function $N(s)$ is determined by the transformation thermodynamics of the kinetic energy of interacting condenstates to the internal energy of massive quarks. To estimate the $N(s)$ it is feasible to assume that it is proportional to the maximal possible energy dependence:

$$N(s) \simeq \kappa(1 - \langle x_Q \rangle)\sqrt{s}/\langle m_Q \rangle, \tag{6}$$

where $\langle x_Q \rangle$ is the average fraction of energy carried by the constituent quarks and $\langle m_Q \rangle$ is the mass scale of constituent quarks. In the model each of the constituent valence quarks located in the central part of the hadron is supposed to scatter in a quasi-independent way by the produced virtual quark pairs at a given impact parameter and by the other valence quarks. When smeared over longitudinal momenta the scattering amplitude of the constituent valence quark Q may be represented in the form

$$\langle f_Q(s,b) \rangle = [N(s,b) + N - 1]\langle V_Q(b) \rangle, \tag{7}$$

where $N = N_H + 1$ is the total number of quarks in the system of the colliding constituent quark and hadron and $\langle V_Q(b) \rangle$ is the smeared amplitude of single quark-quark scattering. In this approach the elastic scattering amplitude satisfies unitarity since it is constructed as a solution of the following equation:

$$F = U + iUDF, \tag{8}$$

which is presented here in operator form. The function $U(s,b)$ (generalized reaction matrix)—the basic dynamical quantity of this approach—is then chosen as a product of the averaged quark amplitudes,

$$U(s,b) = \prod_{Q=1}^{N} \langle f_Q(s,b) \rangle, \tag{9}$$

in accordance with the assumed quasi-independent nature of valence quark scattering. The strong interaction radius of the constituent quark Q is determined by its Compton wavelength and the b dependence of the function $\langle f_Q \rangle$ related to the quark form factor $F_Q(q)$ has a simple form $\langle f_Q\rangle \propto \exp(-m_Q b/\xi)$. The helicity flip transition, i.e., $Q_+ \to Q_-$, occurs when the valence quark knocks out a quark with opposite helicity and same flavor [11].

The explicit expressions for the helicity functions $U_i(x,b)$ at small x have been obtained from the functions $U_i(s,b)$ [5] by the substitution $s \simeq Q^2/x$ and at small values of x they are the following:

$$U_{1,3}(x,b) = U_0(x,b)[1+\beta_{1,3}(Q^2)m_Q\sqrt{x}/Q],$$

$$U_2(x,b) = g_f^2(Q^2)\frac{m_Q^2 x}{Q^2}\exp[-2(\alpha-1)m_Q b/\xi]U_0(x,b), \tag{10}$$

where

$$U_0(x,b) = i\tilde{U}_0(x,b) = i\left[\frac{a(Q^2)Q}{m_Q\sqrt{x}}\right]^N \exp[-Mb/\xi]. \tag{11}$$

a, α, β, g_f, and ξ are the model parameters, some of them in this particular case of quark-hadron scattering depending on the virtuality Q^2. The meaning of these parameters is not crucial here; note only that m_Q is the average mass of constituent quarks in the quark-hadron system of $N = N_H + 1$ quarks and M is their total mass, i.e., $M = \Sigma_{i=1}^N m_i$. We consider here for simplicity the pure imaginary case. We need to keep the subleading terms in the expressions for $U_1(x,b)$ and $U_3(x,b)$ since the $\Delta q(x,b)$ is determined by their difference. For $U_2(x,b)$ one can keep only the leading term.

Then using Eqs. (4) we obtain, at small x,

$$q(x,b) = \frac{\tilde{U}_0(x,b)}{1+\tilde{U}_0(x,b)},$$

$$\Delta q(x,b) = \frac{\beta_-(Q^2)m_Q\sqrt{x}}{2Q}\frac{\tilde{U}_0(x,b)}{[1+\tilde{U}_0(x,b)]^2}, \tag{12}$$

$$\delta q(x,b) = \frac{g_f^2(Q^2)m_Q^2 x}{2Q^2}\exp[-2(\alpha-1)m_Q b/\xi] \times \frac{\tilde{U}_0(x,b)}{[1+\tilde{U}_0(x,b)]^2}, \tag{13}$$

where $\beta_-(Q^2) = \beta_3(Q^2) - \beta_1(Q^2)$. From the above expressions it follows that $q(x,b)$ has a central b dependence, while $\Delta q(x,b)$ and $\delta q(x,b)$ have peripheral profiles. Their qualitative dependence on the impact parameter b is depicted in Fig. 1. The function $\Delta q(x,b)$ has a maximum located at

$$b_{max}(x) = \frac{\xi N}{M}\ln\left[\frac{a(Q^2)Q}{m_Q\sqrt{x}}\right].$$

From Eqs. (12),(13) it follows that factorization of x and b dependences is not allowed by unitarity and this provides constraints for the model parametrizations of structure functions which depend on x and b variables. Indeed, it is clear from Eqs. (12),(13) that factorized form of the input "amplitude" $\tilde{U}_0(x,b)$ cannot survive after unitarization due to the presence of the denominators. It is to be noted here that from the relation of the impact parameter distributions with the off-forward parton distributions [4] it follows that the same conclusion on the absence of factorization is also valid for the off-forward parton distributions with zero skewedness.

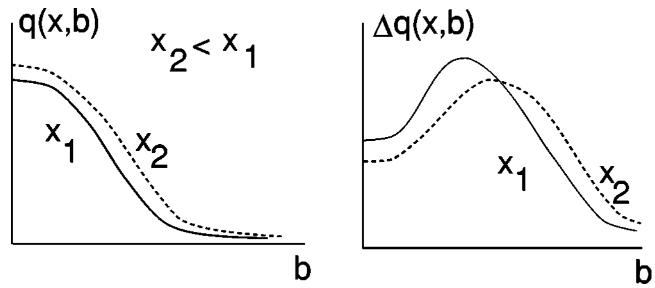

FIG. 1. b dependence of the structure functions $q(x,b)$ and $\Delta q(x,b)$ at low x.

The following relation between the structure functions $\Delta q(x,b)$ and $\delta q(x,b)$ can also be inferred from the above model-based formulas:

$$\delta q(x,b) = c(Q^2)\frac{\sqrt{x}}{Q}\exp(-\gamma b)\Delta q(x,b). \tag{14}$$

Thus, the function $\delta q(x,b)$ which describes the transverse spin distribution is suppressed by the factors $\sqrt{x}$ and $\exp(-\gamma b)$; i.e., it has a more central profile. This suppression also reduces double-spin transverse asymmetries in the central region in the Drell-Yan production compared to the corresponding longitudinal asymmetries.

The strange quark structure functions have also a more central b dependence than in the case of u and d quarks. The radius of the corresponding quark matter distribution follows from Eq. (13) and is

$$R_q(x) \simeq \frac{1}{M}\ln Q^2/x, \tag{15}$$

while the ratio of the strange quark distributions to the light quark distributions radii is given by the corresponding constituent quark masses; i.e., for the nucleon this ratio would be

$$R_s(x)/R_q(x) \simeq \left(1+\frac{\Delta m}{4m_Q}\right)^{-1}, \tag{16}$$

where $\Delta m = m_S - m_Q$.

Time reversal invariance of strong interactions allows one to write down relations similar to Eqs. (1) for the fragmentation functions also and obtain expressions for the fragmentation functions $D_q^h(z,b)$, $\Delta D_q^h(z,b)$, $\delta D_q^h(z,b)$ which have just the same dependence on the impact parameter b as the corresponding structure functions. The fragmentation function $D_q^h(z,b,Q^2)$ is the probability for the fragmentation of a quark q at transverse distance $b \pm \Delta b$ ($\Delta b \sim 1/Q$) into a hadron h which carries the fraction z of the quark momentum. In this case b is a transverse distance between the quark q and the center of the hadron h. It is positively defined and due to unitarity obeys the inequality $0 \leqslant D_q^h(z,b) \leqslant 1$. The physical interpretations of the spin-dependent fragmentation functions $\Delta D_q^h(x,b)$ and $\delta D_q^h(x,b)$ is similar to that of the corresponding spin structure function.

IV. DISCUSSIONS AND CONCLUSION

It is interesting to note that the spin structure functions have a peripheral dependence on the impact parameter contrary to the central profile of the unpolarized structure function. It could be related to the orbital angular momentum of quarks inside the constituent quark. The important point is what the origin of this orbital angular momenta is. It was proposed [12] to use an analogy with an anisotropic extension of the theory of superconductivity which seems to match well with the picture for a constituent quark. Studies [13] of that theory show that the presence of anisotropy leads to axial symmetry of pairing correlations around the anisotropy direction $\hat{\vec{l}}$ and to the particle currents induced by the pairing correlations. In other words, it means that a particle of the condensed fluid is surrounded by a cloud of correlated particles which rotate around it with the axis of rotation $\hat{\vec{l}}$. Calculation of the orbital momentum shows that it is proportional to the density of the correlated particles. Thus, it is clear that there is a direct analogy between this picture and that describing the constituent quark. An axis of anisotropy $\hat{\vec{l}}$ can be associated with the polarization vector of the current valence quark located at the origin of the constituent quark. The orbital angular momentum $\vec{L}$ lies along $\hat{\vec{l}}$.

The spin of the constituent quark, e.g., the U quark, in the model used is given by the sum

$$J_U = 1/2 = S_{u_v} + S_{\{\bar{q}q\}} + L_{\{\bar{q}q\}} = 1/2 + S_{\{\bar{q}q\}} + L_{\{\bar{q}q\}}. \quad (17)$$

In principle, $S_{\{\bar{q}q\}}$ and $L_{\{\bar{q}q\}}$ can include the contribution of gluon angular momentum; however, since we consider the effective Lagrangian approach where gluon degrees of freedom are overintegrated, we are not concerned with problems of the separation and mixing of the quark angular momentum and gluon effects in QCD (cf. [14]). In the NJL model [10] the six-quark fermion operator simulates the effect of the gluon operator $(\alpha_s/2\pi)G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$, where $G_{\mu\nu}$ is the gluon field tensor in QCD. It is worth noting here that in general the large gluon orbital angular momentum is expected to be almost canceled by gluon spin contribution [15].

The value of the orbital momentum contribution into the spin of the constituent quark can be estimated according to the relation between the contributions of current quarks to a proton spin and the corresponding contributions of current quarks to a spin of the constituent quarks and that of the constituent quarks to proton spin [16]:

$$(\Delta\Sigma)_p = (\Delta U + \Delta D)(\Delta\Sigma)_U, \quad (18)$$

where $(\Delta\Sigma)_U = S_{u_v} + S_{\{\bar{q}q\}}$. The value of $(\Delta\Sigma)_p$ was measured in deep-inelastic scattering (DIS). Thus, on the grounds of the experimental data for polarized DIS we arrive at the conclusion that a significant part of the spin of the constituent quark in the model should be associated with the orbital angular momentum of the current quarks inside the constituent one [12].

Then the peripherality of the spin structure functions can be correlated with the large contribution of the orbital angular momentum, i.e., with the quark coherent rotation. Indeed, there is a compensation between the total spin of the quark-antiquark cloud and its orbital angular momenta, i.e., $L_{\{\bar{q}q\}} = -S_{\{\bar{q}q\}}$, and therefore this correlation follows if the above compensation has a local nature and is valid for a fixed impact parameter.

The important role of orbital angular momentum was known long before the EMC discovery [17] and reappeared after as one of the transparent explanations of the polarized deep-inelastic scattering data [18]. Lattice QCD calculations in the quenched approximation also indicate a significant quark orbital angular momentum contribution to the spin of a nucleon [19]. It would be interesting to find out the possible experimental signatures of the peripheral geometrical profiles of the spin structure functions and the significant role of the orbital angular momentum. One of such indications could be an observation of the different spatial distributions of charge and magnetization at Jefferson Lab [20]. It would also be important to have precise data for the strange form factor.

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