

**Short-baseline neutrino oscillations at a neutrino factory**

V. Barger

*Department of Physics, University of Wisconsin, Madison, Wisconsin 53706*

S. Geer and R. Raja

*Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510*

K. Whisnant

*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011*

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Within the framework of three-neutrino and four-neutrino scenarios that can describe the results of the LSND experiment, we consider the capabilities of short-baseline neutrino oscillation experiments at a neutrino factory. We find that, when short-baseline ( $L \leq 100$  km) neutrino factory measurements are used together with other accelerator-based oscillation results, the complete three-neutrino parameter space can best be determined by measuring the rate of  $\nu_e \rightarrow \nu_\tau$  oscillations, and measuring  $CP$  violation with either  $\nu_e \rightarrow \nu_\mu$  or  $\nu_\mu \rightarrow \nu_\tau$  oscillations (including the corresponding antineutrino channels). With measurements of  $CP$  violation in both  $\nu_e \rightarrow \nu_\mu$  and  $\nu_\mu \rightarrow \nu_\tau$  it may be possible to distinguish between the three- and four-neutrino cases.

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**I. INTRODUCTION**

The recent results from the Super-Kamiokande (SuperK) detector confirm the solar [1] and atmospheric [2] neutrino deficits and strongly suggest the existence of neutrino oscillations. The azimuthal angle and energy dependence of the atmospheric data indicates a mass-squared difference scale,  $\delta m_{\text{atm}}^2 \sim 3.5 \times 10^{-3}$  eV<sup>2</sup>. The Liquid Scintillation Neutrino Detector (LSND) measurements [3] indicate neutrino oscillations with a different scale,  $\delta m_{\text{LSND}}^2 \sim 0.3\text{--}2.0$  eV<sup>2</sup>; there is also a small region of acceptable parameter space at 6 eV<sup>2</sup>. The evidence for oscillations in solar neutrino data [1,4], when taken as a whole, prefer yet a third, lower mass-squared difference scale,  $\delta m_{\text{sun}}^2 \leq 10^{-4}$  eV<sup>2</sup>. Once oscillation explanations for some or all of the data are accepted, the next step is to attempt to find a neutrino mass and mixing pattern that can provide a unified description of all the relevant neutrino data.

In this paper we consider two possible neutrino scenarios which have been proposed to account for the LSND results: (i) a three-neutrino model that also describes the atmospheric neutrino data (in which case the solar data would be explained by a phenomenon other than oscillations [5]), and (ii) a four-neutrino model that also describes both the solar and atmospheric data [6,7]. As we show, the three-neutrino model may be considered a sub-case of the four-neutrino model. We examine the ability of short-baseline experiments at a muon storage ring-based neutrino factory [8–13,15–17] to determine the oscillation parameters in each case, and discuss how the three- and four-neutrino scenarios may be distinguished.

We concentrate on experiments in which muon and tau neutrinos are detected via charge-current interactions and there is good sign determination of the detected muons and tau leptons. The sign determination allows one to distinguish between  $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$  and  $\nu_\mu \rightarrow \nu_\mu$  for stored  $\mu^-$  in the ring, and between  $\nu_e \rightarrow \nu_\mu$  and  $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$  for stored  $\mu^+$ . We consider

primarily the appearance channels  $\nu_e \rightarrow \nu_\mu$ ,  $\nu_e \rightarrow \nu_\tau$ , and  $\nu_\mu \rightarrow \nu_\tau$  (and the corresponding antineutrino channels), for which the uncertainty on the beam flux is not critical to the sensitivity of the measurements. We find that the  $\nu_\mu \rightarrow \nu_\tau$  and  $\nu_e \rightarrow \nu_\mu$  channels provide the best sensitivity to the  $CP$ -violating phase; the  $\nu_e \rightarrow \nu_\tau$  channel allows measurement of an independent combination of mixing parameters. Short-baseline experiments at a neutrino factory are sufficient to determine the complete three-neutrino parameter space in these scenarios when their results are combined with other accelerator-based experiments that are currently underway or being planned. Measuring  $CP$ -violation in both  $\nu_\mu \rightarrow \nu_\tau$  and  $\nu_e \rightarrow \nu_\mu$  allows one to possibly distinguish between the three- and four-neutrino cases. Hence all three off-diagonal oscillation channels are useful. In the four-neutrino case it is not possible to have complete mixing and  $\delta m^2$  parameter determinations without additional measurements in future long-baseline experiments.

**II. THREE-NEUTRINO MODELS**

Here we address the ability of short-baseline neutrino oscillation experiments to probe a class of three-neutrino models that can describe the results of the LSND experiment, together with the atmospheric neutrino deficit observed by the SuperK experiment.

**A. Oscillation formalism**

In a three-neutrino model the neutrino flavor eigenstates  $\nu_\alpha$  are related to the mass eigenstates  $\nu_j$  in vacuum by a unitary matrix  $U$ ,

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j} |\nu_j\rangle, \quad (1)$$

with  $\alpha = e, \mu, \tau$  and  $j = 1, 2, 3$ . The mixing matrix can be parametrized by

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}c_{23} \end{pmatrix}, \quad (2)$$

where  $c_{jk} \equiv \cos \theta_{jk}$ ,  $s_{jk} \equiv \sin \theta_{jk}$ , and  $\delta$  is the  $CP$  non-conserving phase. Two additional diagonal phases are present in  $U$  for Majorana neutrinos, but these do not affect oscillation probabilities.

For three-neutrino models in which the results of the atmospheric and LSND experiments are explained by neutrino oscillations there are two independent mass-squared differences:  $\delta m_{\text{LSND}}^2 = 0.3\text{--}2.0 \text{ eV}^2$  and  $\delta m_{\text{atm}}^2 \approx 3.5 \times 10^{-3} \text{ eV}^2$ . We take  $\delta m_{32}^2 = \delta m_{\text{atm}}^2$  and  $\delta m_{21}^2 = \delta m_{\text{LSND}}^2$ ; see Fig. 1(a). These mass-squared differences obey the condition  $\delta m_{\text{LSND}}^2 \gg \delta m_{\text{atm}}^2$ . Then the general off-diagonal vacuum oscillation probability is

$$P(\nu_\alpha \rightarrow \nu_\beta) \approx 4|U_{\alpha 1}|^2|U_{\beta 1}|^2 \sin^2 \Delta_{\text{LSND}} - 4 \operatorname{Re}(U_{\alpha 2}U_{\alpha 3}^*U_{\beta 2}^*U_{\beta 3}) \sin^2 \Delta_{\text{atm}} \pm 2JS, \quad (3)$$

where

$$\Delta_j \equiv 1.27 \delta m_j^2 (\text{eV}^2) L (\text{km}) / E_\nu (\text{GeV}), \quad (4)$$

$$S \equiv [\sin 2\Delta_{\text{atm}} + \sin 2\Delta_{\text{LSND}} - \sin 2(\Delta_{\text{LSND}} + \Delta_{\text{atm}})] \\ = 2(\sin 2\Delta_{\text{atm}} \sin^2 \Delta_{\text{LSND}} + \sin 2\Delta_{\text{LSND}} \sin^2 \Delta_{\text{atm}}), \quad (5)$$

and  $J$  is the  $CP$ -violating invariant [18–20], which can be defined as  $J = \operatorname{Im}\{U_{e2}U_{e3}^*U_{\mu 2}^*U_{\mu 3}\}$ . The plus (minus) sign in Eq. (3) is used when  $\alpha$  and  $\beta$  are in cyclic (anticyclic) order, where cyclic order is defined as  $e\mu\tau$ . For antineutrinos, the sign of the  $CP$ -violating term is reversed. For the mixing matrix in Eq. (2),

$$J = s_{13}c_{13}^2s_{12}c_{12}s_{23}c_{23}\sin \delta. \quad (6)$$

For recent discussions of  $CP$  violation in neutrino oscillations, see Refs. [9,12–15] and [21–23]. For  $L \leq 100 \text{ km}$ , matter effects [24] are very small and the vacuum formulas are a good approximation to the true oscillation probabilities.

The class of scenarios that we are considering is designed to account for the large  $\nu_\mu \rightarrow \nu_\tau$  mixing of atmospheric neu-

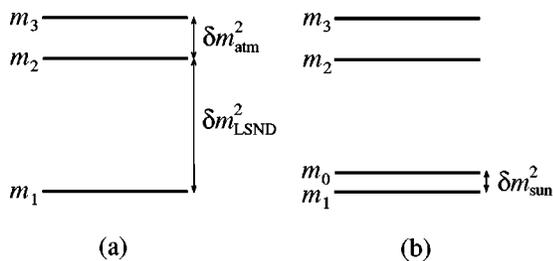


FIG. 1. Ordering and separation of mass eigenvalues in the (a) three-neutrino and (b) four-neutrino scenarios in this paper.

trinos and the small  $\nu_\mu \rightarrow \nu_e$  mixing in the LSND experiment. The  $\nu_e$  survival probability at the leading oscillation scale is given by

$$P(\nu_e \rightarrow \nu_e) = P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 4c_{12}^2c_{13}^2(1 - c_{12}^2c_{13}^2)\sin^2 \Delta_{\text{LSND}}. \quad (7)$$

Results from the BUGEY reactor experiment [25] put an upper bound on  $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  for  $\delta m_{\text{LSND}}^2 > 0.01 \text{ eV}^2$ , which provides the approximate constraint

$$s_{12}^2 + s_{13}^2 \leq 0.01. \quad (8)$$

This leads to the conditions

$$\theta_{12}, \theta_{13} \leq \theta_{23}, \quad (9)$$

with the mixing of atmospheric neutrinos,  $\theta_{23}$ , near maximal ( $\theta_{23} \sim \pi/4$ ). Hence we can take  $s_{12}, s_{13} \ll s_{23}, c_{23}$  and  $c_{12} \approx c_{13} \approx 1$ . The off-diagonal oscillation probabilities are (to leading order in the small mixing angle parameters)

$$P(\nu_e \rightarrow \nu_\mu) \approx 4|s_{12}c_{23} + s_{13}s_{23}e^{i\delta}|^2 \sin^2 \Delta_{\text{LSND}} \\ - 2s_{12}s_{13}c_\delta \sin 2\theta_{23} \sin^2 \Delta_{\text{atm}} + s_{12}s_{13}s_\delta \sin 2\theta_{23}S, \quad (10)$$

$$P(\nu_e \rightarrow \nu_\tau) \approx 4|s_{12}s_{23} - s_{13}c_{23}e^{i\delta}|^2 \sin^2 \Delta_{\text{LSND}} \\ + 2s_{12}s_{13}c_\delta \sin 2\theta_{23} \sin^2 \Delta_{\text{atm}} - s_{12}s_{13}s_\delta \sin 2\theta_{23}S, \quad (11)$$

$$P(\nu_\mu \rightarrow \nu_\tau) \approx 4|s_{12}c_{23} + s_{13}s_{23}e^{i\delta}|^2 |s_{12}s_{23} \\ - s_{13}c_{23}e^{-i\delta}|^2 \sin^2 \Delta_{\text{LSND}} + \sin^2 2\theta_{23} \sin^2 \Delta_{\text{atm}} \\ + s_{12}s_{13}s_\delta \sin 2\theta_{23}S, \quad (12)$$

where  $c_\delta = \cos \delta$  and  $s_\delta = \sin \delta$ . The expressions for the antineutrino channels are obtained by changing the sign of the  $S$ -term in each case. A representative scenario, 1B1 in Ref. [26], that we will use as an example has the parameters

$$\delta m_{32}^2 = 3.5 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{23} = 1.0 \\ \delta m_{21}^2 = 0.3 \text{ eV}^2, \quad \sin^2 2\theta_{12} = \sin^2 2\theta_{13} = 0.015, \quad (13)$$

with  $\delta$  a varied parameter. More generally,  $\delta m_{\text{LSND}}^2$  in the range  $0.3\text{--}2.0 \text{ eV}^2$  is allowed with  $\nu_\mu \rightarrow \nu_e$  oscillation amplitude given by

TABLE I. Leading and next-to-leading contributions to neutrino oscillations in the three-neutrino models under consideration in a short-baseline experiment with  $L/E$  chosen such that  $\Delta_{\text{LSND}} \sim 1$ . The parameters  $s_{12}$ ,  $s_{13}$  and  $\Delta_{\text{atm}} \simeq \Delta_{\text{LSND}}/100$  are all  $\ll 1$ . Also shown is the size of the  $CP$ -violating ( $CPV$ ) term compared to the (dominant)  $CP$ -conserving ( $CPC$ ) term, assuming terms the same order of magnitude cancel in the ratio.

Oscillation	Leading term ( $CP$ conserving)	Next-to-leading term ( $CP$ -violating)	Ratio of $CPV$ to $CPC$ terms
$P(\nu_e \rightarrow \nu_\mu)$	$4 s_{12}c_{23} + s_{13}s_{23}e^{i\delta} ^2 \sin^2 \Delta_{\text{LSND}}$	$4s_{12}s_{13}s_\delta \sin 2\theta_{23} \sin^2 \Delta_{\text{LSND}} \Delta_{\text{atm}}$	$s_\delta \Delta_{\text{atm}}$
$P(\nu_e \rightarrow \nu_\tau)$	$4 s_{12}s_{23} - s_{13}c_{23}e^{i\delta} ^2 \sin^2 \Delta_{\text{LSND}}$	$-4s_{12}s_{13}s_\delta \sin 2\theta_{23} \sin^2 \Delta_{\text{LSND}} \Delta_{\text{atm}}$	$-s_\delta \Delta_{\text{atm}}$
$P(\nu_\mu \rightarrow \nu_\tau)$	$\sin^2 2\theta_{23} \Delta_{\text{atm}}^2$	$4s_{12}s_{13}s_\delta \sin 2\theta_{23} \sin^2 \Delta_{\text{LSND}} \Delta_{\text{atm}}$	$\frac{s_{12}s_{13}s_\delta}{\Delta_{\text{atm}}}$

$$4|s_{12}c_{23} + s_{13}s_{23}e^{i\delta}|^2 \simeq \left( \frac{0.06 \text{ eV}^2}{\delta m_{\text{LSND}}^2} \right)^2. \quad (14)$$

In a short-baseline experiment,  $L/E$  should optimally be chosen such that  $\Delta_{\text{LSND}} \sim 1$ , in which case  $\Delta_{\text{atm}} \simeq \Delta_{\text{LSND}}/100 \ll 1$ ; e.g., for  $\delta m_{\text{LSND}}^2 = 0.3 \text{ eV}^2$  and  $E_\nu = 14 \text{ GeV}$  we might choose  $L = 45 \text{ km}$ , giving

$$\Delta_{\text{atm}} = 0.014 \left( \frac{\delta m_{\text{atm}}^2}{3.5 \times 10^{-3} \text{ eV}^2} \right) \left( \frac{L}{45 \text{ km}} \right) \left( \frac{14 \text{ GeV}}{E_\nu} \right). \quad (15)$$

Note that in the forward direction  $E_\nu = 14 \text{ GeV}$  is the average  $\nu_\mu$  energy for stored unpolarized  $\mu^-$  with  $E_\mu = 20 \text{ GeV}$ . Thus in the probability equations above,  $s_{12}$ ,  $s_{13}$ , and  $\Delta_{\text{atm}}$  are all small parameters, at the few percent level or less. Then to leading order in  $\Delta_{\text{atm}}$

$$S \simeq 4 \Delta_{\text{atm}} \sin^2 \Delta_{\text{LSND}}. \quad (16)$$

Therefore, in these scenarios, the dominant contribution to  $P(\nu_e \rightarrow \nu_\mu)$  and  $P(\nu_e \rightarrow \nu_\tau)$  comes from the leading oscillation (the  $\sin^2 \Delta_{\text{LSND}}$  term), but the dominant contribution to  $P(\nu_\mu \rightarrow \nu_\tau)$  comes from the subleading oscillation (the  $\sin^2 \Delta_{\text{atm}}$  term); these results are summarized in Table I. In each case, the dominant term is  $CP$ -conserving and proportional to the product of two small parameters. The  $CP$ -violating contribution in each case is  $2s_{12}s_{13}s_\delta \sin 2\theta_{23} \sin 2\Delta_{\text{atm}} \sin^2 \Delta_{\text{LSND}}$ , which is a product of three small parameters (assuming  $\delta$  is not small), and hence is smaller than the  $CP$ -conserving contribution (see Table I).

In all, there are six parameters to be determined in the expressions in Eqs. (10)–(12): the three mixing angles, the phase  $\delta$ , and two independent mass-squared differences. However, although there are six off-diagonal measurements possible with  $\mu$  and  $\tau$  detection [the three in Eqs. (10)–(12) plus the corresponding antineutrino channels], the leading and next-to-leading terms in the expressions for these oscillation probabilities are only sensitive to five independent quantities:  $|s_{12}c_{23} + s_{13}s_{23}e^{i\delta}|$ ,  $|s_{12}s_{23} - s_{13}c_{23}e^{i\delta}|$ ,  $s_{12}s_{13}s_\delta$ ,  $\sin^2 \Delta_{\text{LSND}}$ , and  $\sin 2\theta_{23} \Delta_{\text{atm}}$ . For the parameter ranges we are considering,  $\theta_{12}, \theta_{13} > \Delta_{\text{atm}}$ ; then the  $\sin^2 \Delta_{\text{LSND}}$  term in Eq. (12) can be comparable to one of the other terms in  $P(\nu_\mu \rightarrow \nu_\tau)$ , but this term still depends on a subset of these same five independent quantities.

The K2K [27], MINOS [28], ICANOE [29] and OPERA [30] long-baseline experiments will measure the parameters in the leading term of the  $\nu_\mu \rightarrow \nu_\tau$  probability,  $\theta_{23}$  and  $\delta m_{\text{atm}}^2$ , and MiniBooNE [31] will measure the parameters in the leading term of the  $\nu_e \rightarrow \nu_\mu$  probability,  $|s_{12}c_{23} + s_{13}s_{23}e^{i\delta}|$  and  $\delta m_{\text{LSND}}^2$ . Therefore, only two independent quantities will remain to be measured in short baseline experiments: (i) the amplitude of the leading oscillation in the  $\nu_e \rightarrow \nu_\tau$  probability, and (ii) the subleading  $CPV$  term, which has the same magnitude for each off-diagonal channel and can be determined by a comparison of neutrino to antineutrino rates. Hence a combination of short-baseline measurements with the results of the other accelerator-based experiments would allow all of the parameters in this three-neutrino scenario to be determined. Measurements of the other short-baseline off-diagonal oscillation probabilities may then be used to check the consistency of the result and/or improve the accuracy of the parameter determinations.

One can also measure the  $\nu_\mu \rightarrow \nu_\mu$  survival probability, which to leading order in small quantities can be written

$$P(\nu_\mu \rightarrow \nu_\mu) \simeq 1 - 4A(1-A) \sin^2 \Delta_{\text{LSND}} - \sin^2 2\theta_{23} \sin^2 \Delta_{\text{atm}}, \quad (17)$$

where

$$A \simeq s_{23}^2 s_{13}^2 + c_{23}^2 s_{12}^2 + 2s_{23}c_{23}s_{12}s_{13}c_\delta. \quad (18)$$

In short-baseline experiments, both oscillatory terms in Eq. (17) are second order in small quantities. A measurement of  $P(\nu_\mu \rightarrow \nu_\mu)$  could be used in conjunction with other short-baseline measurements to make a complete determination of the oscillation parameters without the use of other data. However, since  $A$  and  $\Delta_{\text{atm}}$  are small, the deviations of  $P(\nu_\mu \rightarrow \nu_\mu)$  from unity are also small, and the normalization of the beam flux would need to be known to high precision for this to be a useful measurement.

An examination of Eqs. (10)–(12) and (16) shows that only the relative sign of  $\delta$  and  $\delta m_{32}^2$  may be determined in short-baseline measurements for the distances and oscillation parameter values that we are considering. Long-baseline experiments at a neutrino factory should be able to determine the sign of  $\delta m_{32}^2$  since there is a significant dependence of the matter effect on the sign of  $\delta m_{32}^2$  for  $L \geq 2000 \text{ km}$  [12–14,16,17].

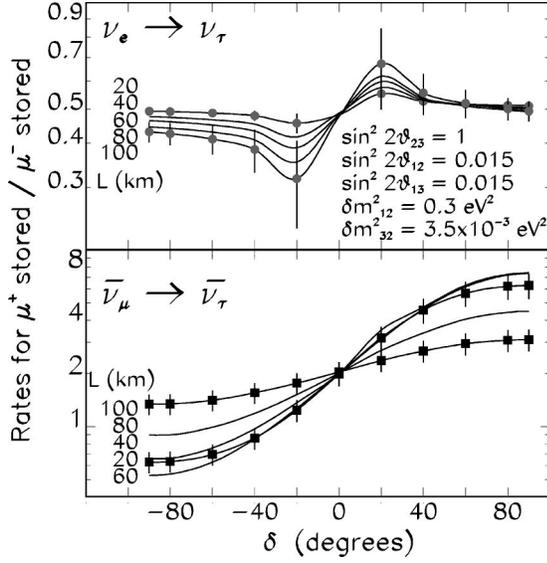


FIG. 2. Ratio of tau production rates for stored  $\mu^+$  to that with stored  $\mu^-$  for  $\nu_e \rightarrow \nu_\tau$  and  $\nu_\mu \rightarrow \nu_\tau$  oscillations, shown versus the  $CP$ -violating phase  $\delta$  for several values of  $L$ . These results assume a stored muon energy  $E_\mu = 20$  GeV and oscillation parameters given by Eq. (13). The representative errors shown are statistical, assuming  $10^{20}$  kt-decays (after accounting for detector efficiency).

### B. Results

Figure 2 shows the ratio of the rate of tau production for stored  $\mu^+$  to that for stored  $\mu^-$  for the  $\nu_e \rightarrow \nu_\tau$  and  $\nu_\mu \rightarrow \nu_\tau$  channels, versus the  $CP$  phase  $\delta$  for several values of baseline length  $L$ , with oscillation parameters given by Eq. (13). Statistical errors are also shown, assuming  $10^{20}$  kt-decays (corresponding, for example, to three years of running with  $10^{20}$  useful muon decays per year and a 1 kt detector having 33% tau detection efficiency). The event rate

calculations are done according to the method outlined in Ref. [11]. For the choice of parameters in Eq. (13),  $s_{12}$  and  $s_{13}$  are larger than  $\Delta_{\text{atm}}$  and the  $\nu_\mu \rightarrow \nu_\tau$  channel shows the largest relative  $CP$ -violating effect (see the last column of Table I). The figure also shows that for  $\nu_\mu \rightarrow \nu_\tau$  oscillations there are some values of  $L$  that lead to a larger  $CP$ -violating effect, as we discuss below.

The last column in Table I shows that the relative size of the  $CP$ -violating effect in the  $\nu_\mu \rightarrow \nu_\tau$  channel decreases with increasing  $\Delta_{\text{atm}}$  (i.e., with increasing  $L/E$ ). Also, for very small  $L/E$ , when  $\Delta_{\text{LSND}} \ll 1$ ,  $S \rightarrow 0$  to leading order in  $\Delta_{\text{atm}}$  and  $\Delta_{\text{LSND}}$ , and the  $CP$  violation becomes negligible. Therefore, for any given set of oscillation parameters, there will be an optimum  $L/E$  that maximizes the  $CP$  violation effects in the  $\nu_\mu \rightarrow \nu_\tau$  channel.

Figure 3a shows the ratio of  $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$  event rates (from  $\mu^+$  decays) to  $\nu_\mu \rightarrow \nu_\tau$  event rates (from  $\mu^-$  decays),  $R_{\mu\tau}$ , versus baseline for 20 GeV muons, for oscillation parameters given by Eq. (13) and three values of  $\delta$  ( $90^\circ$ ,  $0^\circ$ , and  $-90^\circ$ ). Approximate statistical errors are shown for  $10^{20}$  kt-decays. The figure clearly shows that there is one distance that maximizes the size of the  $CP$ -violation effect, which in this case (20 GeV muons) is about  $L = 45$  km. This optimal distance decreases slowly with increasing  $\delta m_{32}^2$ . For  $\delta m_{32}^2$  in the range  $(2.5\text{--}4.5) \times 10^{-3}$  eV $^2$ , we find that the optimal  $L$  is in the range 40–50 km for  $\delta m_{\text{LSND}}^2 = 0.3$  eV $^2$ . The optimal  $L$  scales inversely with  $\delta m_{\text{LSND}}^2$ , and for stored muon energies well above the tau threshold may be approximated by

$$L_{\text{opt}} \approx 45 \text{ km} \left( \frac{0.3 \text{ eV}^2}{\delta m_{\text{LSND}}^2} \right) \left( \frac{E_\mu}{20 \text{ GeV}} \right); \quad (19)$$

e.g., for  $\delta m_{\text{LSND}}^2 = 2$  eV $^2$  the best sensitivity to  $CP$  violation

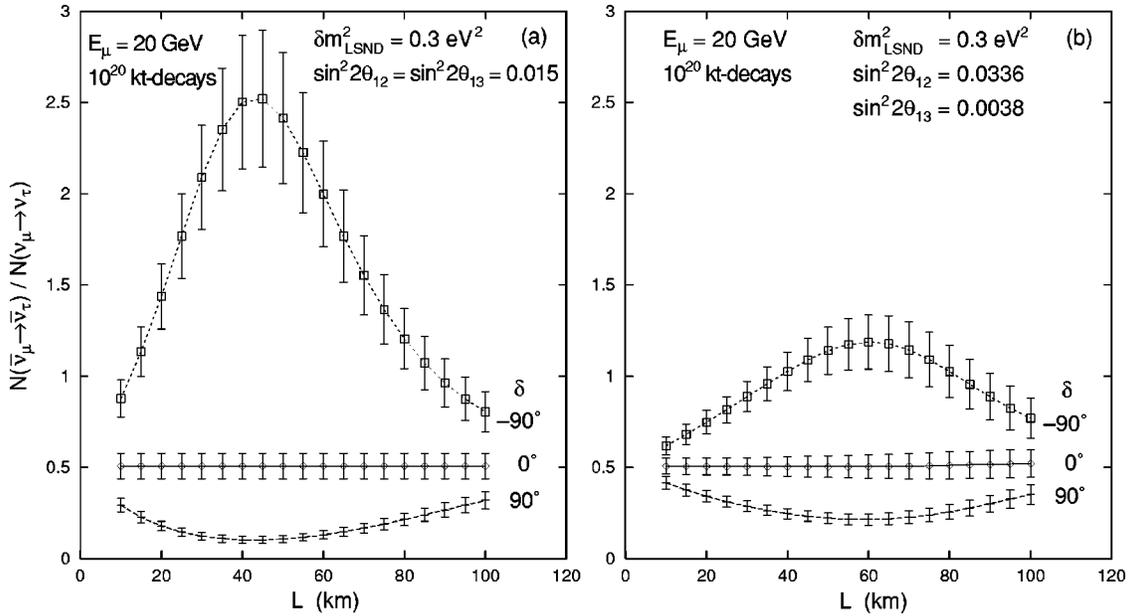


FIG. 3. (a)  $R_{\mu\tau} \equiv N(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau) / N(\nu_\mu \rightarrow \nu_\tau)$  versus  $L$ , for  $E_\mu = 20$  GeV, with oscillation parameters given by Eq. (13), where  $\theta_{12} = \theta_{13}$ . (b) Similar results for  $\sin^2 2\theta_{12} = 0.0336$  and  $\sin^2 2\theta_{13} = 0.0038$ . Statistical errors correspond to  $10^{20}$  kt-decays.

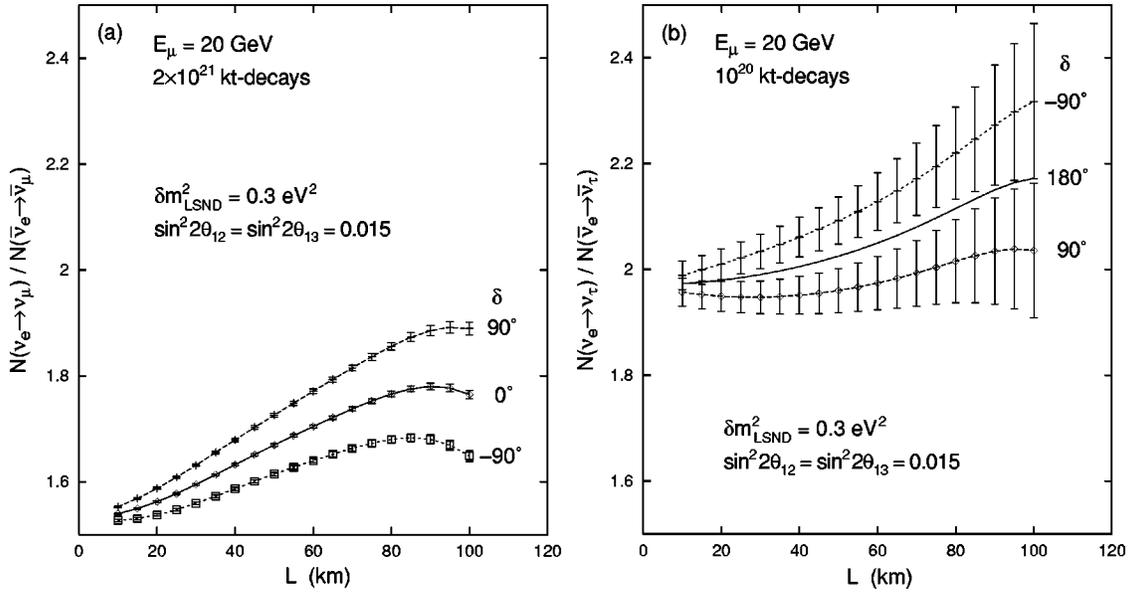


FIG. 4. (a)  $R_{e\mu} \equiv N(\nu_e \rightarrow \nu_\mu) / N(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ , and (b)  $R_{e\tau} \equiv N(\nu_e \rightarrow \nu_\tau) / N(\bar{\nu}_e \rightarrow \bar{\nu}_\tau)$ . A 4 GeV cut has been imposed on the detected muon in (a). Statistical errors correspond to  $2 \times 10^{21}$  kt-decays in (a) and  $10^{20}$  kt-decays in (b).

is obtained with  $L \approx 6$  km. The distance from Fermilab to Argonne is about 30 km, which would be optimal for  $E_\mu = 20$  GeV and  $\delta m_{LSND}^2 = 0.45$  eV<sup>2</sup>. Similar results for an optimal distance for  $CP$  violation in  $\nu_\mu \rightarrow \nu_\tau$  in the context of four-neutrino models have been reported in Ref. [22].

Given Eq. (14) (the LSND constraint on the  $\nu_\mu \rightarrow \nu_e$  oscillation amplitude),  $CP$  violation is maximized when  $\theta_{12} = \theta_{13}$  since  $J$  is proportional to the product of  $s_{12}$  and  $s_{13}$ . Figure 3b shows  $R_{\mu\tau}$  for unequal  $\theta_{12}$  and  $\theta_{13}$ :  $\sin^2 2\theta_{12} = 0.0336$  and  $\sin^2 2\theta_{13} = 0.0038$ , which for  $\delta = 0$  gives the same LSND result as  $\sin^2 2\theta_{12} = \sin^2 2\theta_{13} = 0.015$ . The  $CP$ -violation effects for  $\theta_{12} \neq \theta_{13}$  are not as dramatic as with  $\theta_{12} = \theta_{13}$ , but still may be observable with  $10^{20}$  kt-decays.

An examination of Table I shows that the relative size of the  $CP$  violation in the  $\nu_e \rightarrow \nu_\mu$  or  $\nu_e \rightarrow \nu_\tau$  channels increases with  $L/E$ . However, once  $\Delta_{atm}$  is of order unity or larger the  $\sin 2\Delta_{atm}$  term in  $S$  averages to zero, washing out the  $CP$  violation; this does not happen until  $L$  is much larger than 100 km. Since the flux falls off like  $1/L^2$ , the statistical uncertainty increases roughly like  $L$ , and as long as  $\Delta_{atm} \ll 1$  a wide range of distances have comparable sensitivity to  $CP$  violation in the  $\nu_e \rightarrow \nu_\mu$  or  $\nu_e \rightarrow \nu_\tau$  channels.

Figure 4a shows the ratio  $R_{e\mu}$  of  $\nu_e \rightarrow \nu_\mu$  event rates (from  $\mu^+$  decays) to  $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$  event rates (from  $\mu^-$  decays) versus baseline for oscillation parameters given by Eq. (13), with  $E_\mu = 20$  GeV. The statistical errors correspond to  $2 \times 10^{21}$  kt-decays (which could be obtained, for example, by three years of running with  $10^{20}$  useful muon decays per year, and a 10 kt detector having a 67% muon efficiency). A 4 GeV minimum energy cut has been made on the detected muon. Although  $R_{e\mu}$  is not as sensitive to  $CPV$  effects as  $R_{\mu\tau}$ , the increased statistics (due to a larger overall rate resulting from the use of a larger detector for muons) make  $\nu_e \rightarrow \nu_\mu$  another attractive channel for  $CP$  violation. Figure 4b shows similar results for  $R_{e\tau}$ , the ratio of  $\nu_e \rightarrow \nu_\tau$  event

rates (from  $\mu^+$  decays) to  $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$  event rates (from  $\mu^-$  decays). The statistical errors correspond to  $10^{20}$  kt-decays. It is evident from the figure that the  $\nu_e \rightarrow \nu_\tau$  channel is not as useful for detecting  $CP$  violation (primarily because of the reduced event rate in tau detection), although this channel is the most sensitive to the  $\nu_e \rightarrow \nu_\tau$  oscillation amplitude. The combination of parameters  $|s_{12}s_{23} - s_{13}c_{23}e^{i\delta}|$  in the  $\nu_e \rightarrow \nu_\tau$  amplitude is also present in the  $\sin^2 \Delta_{LSND}$  term of  $P(\nu_\mu \rightarrow \nu_\tau)$  [see Eq. (12)], along with an additional factor involving small mixing angles; hence, it would be more difficult to measure  $|s_{12}s_{23} - s_{13}c_{23}e^{i\delta}|$  in the  $\nu_\mu \rightarrow \nu_\tau$  channel.

Figure 5 shows, for various  $CP$ -violating cases with  $E_\mu = 20$  GeV, the statistical significance (number of standard deviations) that the ratios  $R_{\mu\tau}$ ,  $R_{e\mu}$ , and  $R_{e\tau}$  deviate from their expected values for the  $CP$ -conserving case. While both the  $\nu_e \rightarrow \nu_\mu$  and  $\nu_\mu \rightarrow \nu_\tau$  channels provide good sensitivity near the optimal  $L$ ,  $\nu_e \rightarrow \nu_\mu$  is more sensitive for a wider range of  $L$ , especially for values of  $\delta$  that do not give maximal  $CP$  violation. An additional potential advantage of the  $\nu_e \rightarrow \nu_\mu$  channel is that, in principle, lower energy neutrino factories can be used since there is no need to be above the tau-lepton production threshold. However, as the energy of muons in a neutrino factory decreases, the sensitivity to  $CP$  violation also decreases (see Fig. 6); this is especially true if a lower bound is imposed on the energy of the detected muon.

In principle, measurements can be made with varying  $L/E$ , either by using experiments at more than one baseline, or by using a measure of the neutrino energy (for example, the total observed event energy) at a fixed baseline. However, since there are only five independent quantities in the leading- and subleading-order probabilities in Eqs. (10)–(12), such measurements still cannot completely determine the three-neutrino parameter set; this can only be done at short baselines by a measurement of the subsubleading terms

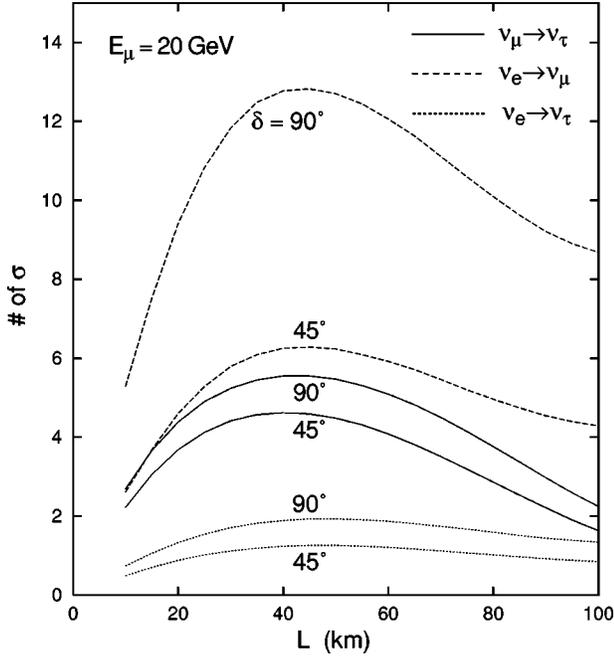


FIG. 5. Statistical significance of  $CP$  violation measurements of  $R_{\mu\tau}$  (solid curves),  $R_{e\mu}$  (dashed), and  $R_{e\tau}$  (dotted) for various positive values of the  $CP$ -violating phase  $\delta$ , when compared to the  $CP$ -conserving case, assuming  $E_\mu = 20$  GeV, and  $10^{20}$  kt-decays for a tau detection and  $2 \times 10^{21}$  kt-decays for muon detection. The other oscillation parameters are given in Eq. (13). Similar results are obtained for negative values of  $\delta$ .

in the off-diagonal probabilities, or of the subleading terms in the diagonal  $\nu_\mu \rightarrow \nu_\mu$  probability [see Eq (17)], both of which would be very challenging experimentally. Similar conclusions apply for additional measurements involving electron detection.

To summarize, when used in conjunction with long-baseline measurements from K2K, MINOS, ICANOE, and OPERA, and with results from MiniBooNE, all six of the three-neutrino oscillation parameters can in principle be determined with short-baseline measurements at a neutrino factory. The short-baseline measurements would determine two parameters not determined by other accelerator-based experiments, and would provide a consistency check by independently measuring three other parameters also measured by other accelerator-based experiments. The  $\nu_e \rightarrow \nu_\tau$  channel is most sensitive to the quantity  $|s_{12}s_{23} - s_{13}c_{23}e^{i\delta}|$ . The  $\nu_e \rightarrow \nu_\mu$  channel provides good sensitivity to  $CP$  violation over a wide range of  $L$  when a large muon detector is used, and the  $\nu_\mu \rightarrow \nu_\tau$  channel is largely useful for detecting  $CP$  violation only near the optimal  $L$ .

### III. FOUR-NEUTRINO MODELS

#### A. Oscillation formalism

Four-neutrino models are required to completely describe the solar, atmospheric, and LSND data in terms of oscillations, since a third independent mass-squared difference is necessary. A fourth neutrino must be sterile, i.e., have negligible interactions, since only three neutrinos are measured in  $Z \rightarrow \nu\bar{\nu}$  decays [32]. Following Ref. [21], we label the fourth mass eigenvalue  $m_0$ . Given the pattern of masses  $m_1$ ,  $m_2$ , and  $m_3$  from the three neutrino case in Sec. II, there is a preferred choice for the scale of  $m_0$  that can fit all of the data, including constraints from accelerator experiments, namely,  $m_0$  must be nearly degenerate with  $m_1$ , so that there are two pairs of nearly degenerate states separated by a mass gap of about 1 eV [6,7]; see Fig. 1b. Then  $\delta m_{10}^2$  governs the oscillation of solar neutrinos,  $\delta m_{32}^2$  governs the oscillation of

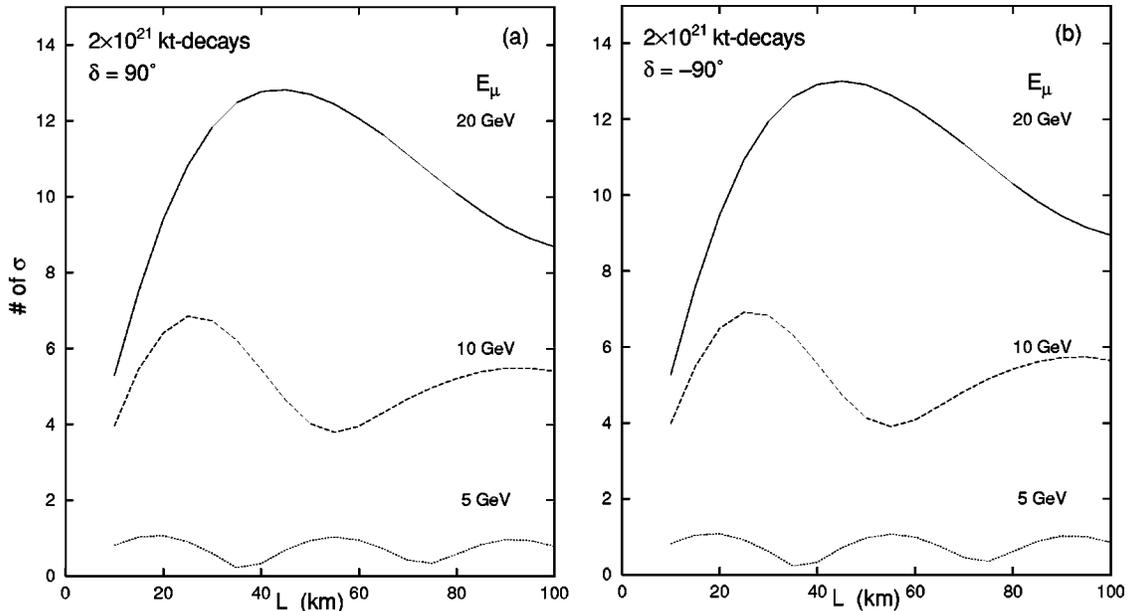


FIG. 6. Statistical significance of  $CP$  violation in  $R_{e\mu}$  for (a)  $\delta = 90^\circ$  and (b)  $\delta = -90^\circ$ , when compared to the  $CP$ -conserving case, assuming  $2 \times 10^{21}$  kt-decays, for  $E_\mu = 20$  GeV (solid curve), 10 GeV (dashed), and 5 GeV (dotted). The other oscillation parameters are given in Eq. (13). A 4 GeV cut has been imposed on the detected muon.

atmospheric neutrinos, and  $\delta m_{21}^2$ ,  $\delta m_{31}^2$ ,  $\delta m_{20}^2$ , and  $\delta m_{30}^2$  all contribute to the LSND oscillations.

Six mixing angles and three (six) phases are needed to parametrize the mixing of four Dirac (Majorana) neutrinos; only three of these phases are measurable in neutrino oscillations. Thus three additional mixing angles, which we label

$\theta_{01}$ ,  $\theta_{02}$ , and  $\theta_{03}$ , and two additional phases are required in extending the three-neutrino phenomenology to the four neutrino case. The simplest situation, which occurs in most explicit four-neutrino models, is that large mixing occurs only between the nearly degenerate pairs; then the four-neutrino mixing matrix can be parametrized as [21]

$$U = \begin{pmatrix} c_{01} & s_{01}^* & s_{02}^* & s_{03}^* \\ -s_{01} & c_{01} & s_{12}^* & s_{13}^* \\ -c_{01}(s_{23}^*s_{03} + c_{23}s_{02}) & -s_{01}^*(s_{23}^*s_{03} + c_{23}s_{02}) & c_{23} & s_{23}^* \\ +s_{01}(s_{23}^*s_{13} + c_{23}s_{12}) & -c_{01}(s_{23}^*s_{13} + c_{23}s_{12}) & & \\ c_{01}(s_{23}s_{02} - c_{23}s_{03}) & s_{01}^*(s_{23}s_{02} - c_{23}s_{03}) & -s_{23} & c_{23} \\ -s_{01}(s_{23}s_{12} - c_{23}s_{13}) & +c_{01}(s_{23}s_{12} - c_{23}s_{13}) & & \end{pmatrix}, \quad (20)$$

where  $s_{jk}$  is here defined as  $\sin \theta_{jk} e^{i\delta_{jk}}$ , and the  $\delta_{jk}$  are the six possible phases for Majorana neutrinos. We set  $\delta_{12} = \delta_{23} = \delta_{02} = 0$  without loss of generality, since only three phases are measurable in neutrino oscillations.

In this four-neutrino scenario, the parameters  $\delta m_{32}^2$ ,  $\delta m_{21}^2$ ,  $\theta_{23}$ ,  $\theta_{12}$  and  $\theta_{13}$  have the same roles as in the three-neutrino scenario in the previous section; the phase  $\delta_{13}$  can be identified with  $\delta$  in the three-neutrino case. The angle  $\theta_{01}$  describes the mixing of the fourth flavor of neutrino with the state it is nearly degenerate with,  $\nu_e$ ; together  $\delta m_{10}^2$  and  $\theta_{01}$  can take on the approximate values appropriate to any of the solar neutrino oscillation solutions [33] [small angle Mikheyev-Smirnov-Wolfenstein (MSW), large angle MSW, low probability MSW (LOW), and vacuum]. The remaining mixing angles  $\theta_{02}$  and  $\theta_{03}$  describe mixing of the fourth neutrino with the two states  $\nu_\mu$ ,  $\nu_\tau$  in the other nearly-degenerate pair. Although models with pure oscillations to sterile neutrinos are disfavored for the solar [34] and atmospheric [35] data, models with mixed oscillations to sterile and active neutrinos [7,36,37], i.e., non-negligible  $\theta_{02}$  and  $\theta_{03}$ , are presumably also possible. Finally,  $\delta_{01}$  and  $\delta_{03}$  are extra phases that may be observable in neutrino oscillations with four neutrinos.

In the limit that terms involving the solar mass-squared difference can be ignored, the expressions for the oscillation probabilities in Eqs. (10) and (11) remain the same. However, the  $\nu_\mu \rightarrow \nu_\tau$  probability becomes

$$P(\nu_\mu \rightarrow \nu_\tau) \approx 4 \left[ (s_{12}c_{23} + s_{13}s_{23}e^{i\delta_{13}})(s_{12}s_{23} - s_{13}c_{23}e^{-i\delta_{13}}) + (s_{02}c_{23} + s_{03}s_{23}e^{i\delta_{03}}) \times (s_{02}s_{23} - s_{03}c_{23}e^{-i\delta_{03}}) \right]^2 \sin^2 \Delta_{\text{LSND}} + \sin^2 2\theta_{23} \sin^2 \Delta_{\text{atm}} + (s_{12}s_{13} \sin \delta_{13} + s_{02}s_{03} \sin \delta_{03}) \sin 2\theta_{23} S. \quad (21)$$

For  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau)$ , the sign of the  $S$ -term is reversed.

## B. Discussion

When combined with measurements from K2K, MINOS, ICANOE, OPERA, and MiniBooNE, the three parameters  $\theta_{12}$ ,  $\theta_{13}$  and  $\delta_{13}$  can in principle be determined by short-baseline measurements of the  $\nu_e \rightarrow \nu_\tau$  amplitude and the  $CPV$  term in the  $\nu_e \rightarrow \nu_\mu$  channel. This is similar to the three-neutrino case in Sec. II. However, measurements of  $\nu_\mu \rightarrow \nu_\tau$  and  $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$  will only give partial information on  $\theta_{02}$ ,  $\theta_{03}$ , and  $\delta_{03}$ . The  $\nu_\mu \rightarrow \nu_\mu$  survival probability can also be measured, which in the limit that the solar mass-squared difference can be ignored is

$$P(\nu_\mu \rightarrow \nu_\mu) \approx 1 - 4A(1-A) \sin^2 \Delta_{\text{LSND}} - \sin^2 2\theta_{23} \sin^2 \Delta_{\text{atm}}, \quad (22)$$

where

$$A \approx 4[s_{23}^2(|s_{03}|^2 + |s_{13}|^2) + c_{23}^2(s_{02}^2 + s_{12}^2) + 2s_{23}c_{23} \text{Re}(s_{02}s_{03} + s_{12}s_{13})]. \quad (23)$$

Both oscillatory terms in Eq. (22) are second order in small quantities in short-baseline experiments. In principle  $P(\nu_\mu \rightarrow \nu_\mu)$  could be used as an additional measurement; however, unless  $\theta_{02}$  and  $\theta_{03}$  are larger than  $\theta_{12}$  and  $\theta_{13}$ , the deviations of  $P(\nu_\mu \rightarrow \nu_\mu)$  from unity are very small, and the normalization of the beam flux would have to be known to high precision for this to be useful.

The parameters  $s_{12}$ ,  $s_{13}$  and  $s_\delta$  determined by short baseline measurements of the  $\nu_e \rightarrow \nu_\tau$  and  $\nu_e \rightarrow \nu_\mu$  channels give predictions for the  $\nu_\mu \rightarrow \nu_\tau$  channel at short baselines that can differ for the three- and four-neutrino cases [Eqs. (12) and (21), respectively]. If the three-neutrino predictions for  $\nu_\mu \rightarrow \nu_\tau$  are found to substantially disagree with the experimental measurements, then the disagreement would provide evidence for the existence of a fourth neutrino. The absence

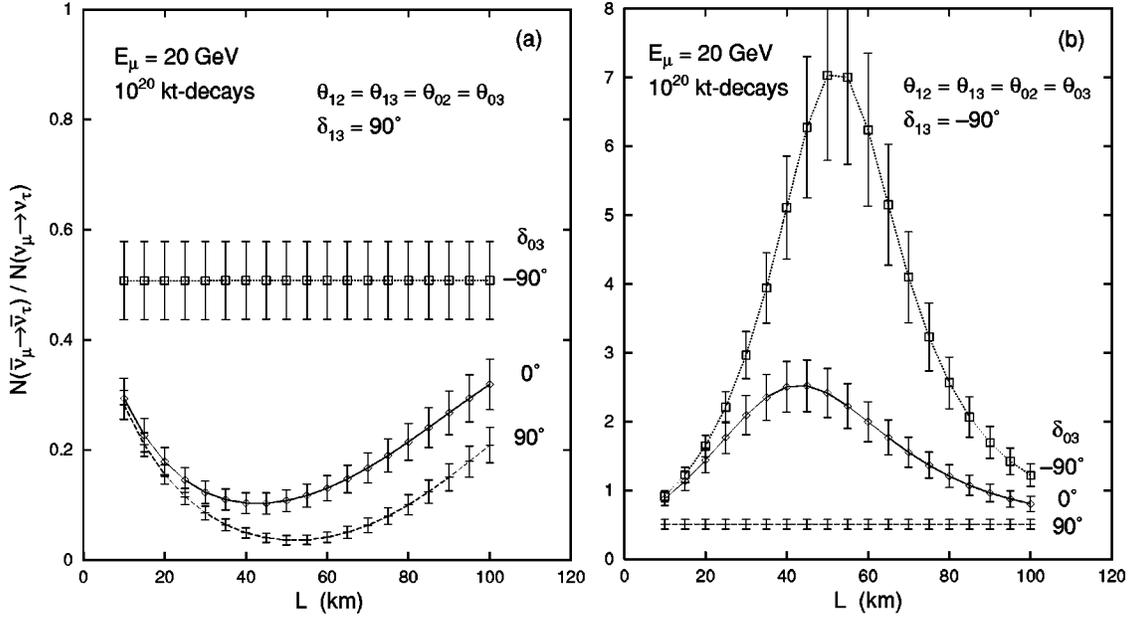


FIG. 7.  $CP$ -violation effects with four neutrinos in  $R_{\mu\tau}$  versus  $L$ , for  $E_\mu = 20$  GeV with oscillation parameters given by Eq. (13),  $\theta_{02} = \theta_{03} = \theta_{12} = \theta_{13}$ ,  $\delta_{03} = 0$  (solid curves),  $90^\circ$  (dashed), and  $-90^\circ$  (dotted) and (a)  $\delta_{13} = 90^\circ$  and (b)  $\delta_{13} = -90^\circ$ . Statistical errors correspond to  $10^{20}$  kt-decays. Corresponding results for three neutrinos are given in Fig. 3.

of such a disagreement would indicate that either there are only three neutrinos or that  $\theta_{02}$  and  $\theta_{03}$  are significantly smaller than  $\theta_{12}$  and  $\theta_{13}$ .

In general, the sensitivity of the measurements of  $\nu_\mu \rightarrow \nu_\tau$  and  $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$  to  $\theta_{02}$ ,  $\theta_{03}$ , and  $\delta_{03}$  in the four-neutrino case are similar to the sensitivities to  $\theta_{12}$ ,  $\theta_{13}$ , and  $\delta$  in the three-neutrino case. For example, given the parameters in Eq. (13) and  $\delta_{13} = 0$  (i.e., no  $CP$  violation in the  $\nu_e \rightarrow \nu_\mu$  and  $\nu_e \rightarrow \nu_\tau$  channels), if  $s_{02} = s_{12}$  and  $s_{03} = s_{13}$ , then the four-neutrino predictions would be given by Figs. 3a and 3b, where  $\delta_{03}$  takes on the values of  $\delta$  in the figures; the corresponding three-neutrino predictions would be given by the  $\delta = 0$  curves. Larger values of  $s_{02}$  and  $s_{03}$  could give a much larger  $CP$ -violating effect, provided that  $\delta_{03}$  was not small. If both  $\delta_{03}$  and  $\delta_{02}$  were nonzero, their  $CP$ -violating effects could add together either constructively or destructively. Hence, larger  $CP$ -violating effects are possible in the  $\nu_\mu \rightarrow \nu_\tau$  channel in the four-neutrino case than with three neutrinos, or  $CP$  violation may be present in  $\nu_\mu \rightarrow \nu_\tau$  when it is not present in the  $\nu_e \rightarrow \nu_\mu$  or  $\nu_e \rightarrow \nu_\tau$  channels (or vice versa), unlike the three-neutrino case, as illustrated in Fig. 7. Even if there is no  $CP$  violation, effects of the angles  $\theta_{02}$  and  $\theta_{03}$  could be seen in the  $\sin^2 \Delta_{\text{LSND}}$  term in Eq. (21), as illustrated in Fig. 8. Here, sensitivities to  $\theta_{02}$  and  $\theta_{03}$  tend to increase with decreasing  $L$ , due to the increased flux at shorter distances.

Proof of the existence of a fourth neutrino does not exclude the possibility that there may be more than four neutrinos. Strictly speaking, an inconsistency between the measurement of  $\nu_\mu \rightarrow \nu_\tau$  oscillations and the three-neutrino predictions would imply only that there are four or more neutrinos. In a model with four or more neutrinos, there are many more mixing angles and phases in the neutrino mixing matrix, and it would not be possible to determine them all

from these oscillation measurements. However, when combined with results at other baselines, most of the four-neutrino parameter set could be determined [21].

#### IV. SUMMARY

In scenarios designed to describe the LSND oscillation results, our results show that short-baseline neutrino factory

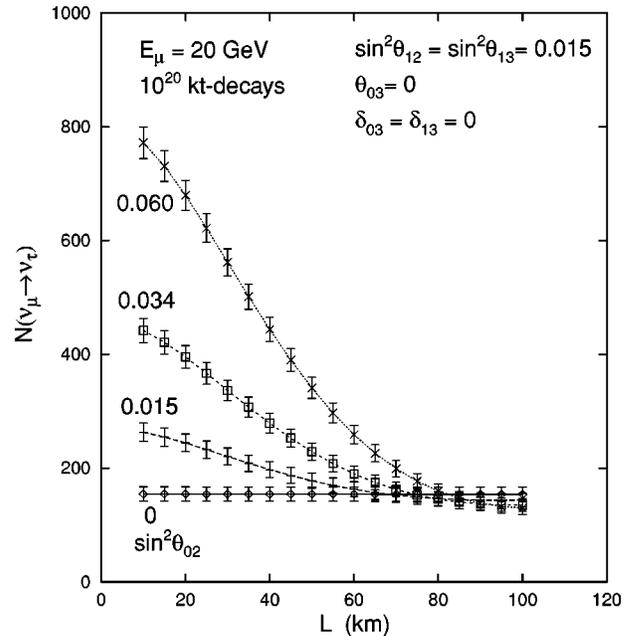


FIG. 8. Effects of four-neutrino mixing on the  $\nu_\mu \rightarrow \nu_\tau$  event rate versus  $L$  for  $E_\mu = 20$  GeV,  $\theta_{03} = \delta_{13} = \delta_{03} = 0$ ,  $\theta_{02} = \theta_{03}$ , and  $\sin^2 2\theta_{02} = 0$  (solid curve), 0.015 (dashed), 0.034 (dotted), and 0.060 (dot-dashed). Statistical errors correspond to  $10^{20}$  kt-decays.

measurements can in principle determine all of the three-neutrino parameters provided the results are used together with future results from other accelerator-based experiments. The  $\nu_e \rightarrow \nu_\tau$  channel is most sensitive to one combination of parameters in the  $CP$ -conserving terms. The  $\nu_e \rightarrow \nu_\mu$  channel provides good sensitivity to  $CP$  violation over a wide range of  $L$  (20–100 km for  $E_\mu = 20$  GeV), assuming that a large muon detector is used, e.g. 10 kt. The  $\nu_\mu \rightarrow \nu_\tau$  channel is also sensitive to  $CP$  violation for a more restricted range of  $L$ , and may be used to explore whether more than three neutrinos exist. For four or more neutrinos, the  $CP$ -violating effect in  $\nu_\mu \rightarrow \nu_\tau$  may be either enhanced or reduced by the additional mixing parameters. If MiniBooNE confirms the LSND oscillation results, then it will be important to measure the rates in all three appearance modes, as well as to search for  $CP$  violation in  $\nu_e \rightarrow \nu_\mu$  and  $\nu_\mu \rightarrow \nu_\tau$ , to obtain indirect evi-

dence for the existence of sterile neutrinos. Although our four-neutrino analysis is based on a specific model with block diagonal mixing matrix in the 2+2 model, we expect the qualitative results to hold more generally. Neutral current measurements would complement the charge current studies of this paper in the search for sterile neutrinos.

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