

## Adiabatic and entropy perturbations from inflation

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We study adiabatic (curvature) and entropy (isocurvature) perturbations produced during a period of cosmological inflation that is driven by multiple scalar fields with an arbitrary interaction potential. A local rotation in field space is performed to separate out the adiabatic and entropy modes. The resulting field equations show explicitly how on large scales entropy perturbations can source adiabatic perturbations if the background solution follows a curved trajectory in field space, and how adiabatic perturbations cannot source entropy perturbations in the long-wavelength limit. It is the effective mass of the entropy field that determines the amplitude of entropy perturbations during inflation. We present two applications of the equations. First, we show why one in general expects the adiabatic and entropy perturbations to be correlated at the end of inflation, and calculate the cross correlation in the context of a double inflation model with two non-interacting fields. Second, we consider two-field preheating after inflation, examining conditions under which entropy perturbations can alter the large-scale curvature perturbation and showing how our new formalism has advantages in numerical stability when the background solution follows a non-trivial trajectory in field space.

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### I. INTRODUCTION

A period of accelerated expansion — inflation — in the early universe has become the standard model for the origin of structure in the universe. Inhomogeneities in the present matter distribution can be traced back to quantum fluctuations in the fields driving inflation which are stretched beyond the Hubble scale during inflation. In the simplest models of inflation driven by a single scalar field, these fluctuations produce a primordial adiabatic spectrum whose amplitude can be characterized by the comoving curvature perturbation  $\mathcal{R}$ , which remains constant on super-Hubble scales until the perturbation comes back within the Hubble scale long after inflation has ended.

As soon as one considers more than one scalar field, one must also consider the role of non-adiabatic fluctuations. This can have important consequences, both in affecting the evolution of the curvature perturbation (often referred to as the “adiabatic perturbation”) and also in the possibility of seeding isocurvature (or “entropy”) perturbations after inflation.

Previous studies have demonstrated that non-adiabatic pressure perturbations can alter the curvature perturbation on super-Hubble scales either during inflation [1,2] or after [3–5]. A general formalism to evaluate the curvature perturbation at the end of inflation in multiple field models was developed in Ref. [6]. In the presence of non-adiabatic fluctuations, one must follow the evolution of perturbed fields on super-Hubble scales, in particular tracking the perturbation in the integrated expansion [7,6,8–10,5], in order to evaluate the large-scale curvature perturbation at late times [11–13,1,6,8,9,14,15].

However no similar formalism has been developed so far to evaluate the isocurvature perturbation in the general case. Instead, isocurvature perturbations have been studied in a

number of particular models of inflation [16,11,12]. These fluctuations typically arise as baryon modes (e.g. [17]) or cold dark matter modes [18], but neutrino isocurvature modes have also been considered [19]. Recently, it has been pointed out [20,21] that it is rather natural to expect the curvature and isocurvature perturbations to be correlated, which yields distinctive observational results [22], in contrast to the isocurvature perturbations usually tested against observations [23].

In this paper we will develop a general formalism to study the evolution of both curvature and isocurvature perturbations in a wide class of multi-field inflation models by decomposing field perturbations into perturbations along the background trajectory in field space (the adiabatic field perturbation), and orthogonal to the background trajectory (the entropy field). We allow an arbitrary interaction potential for the fields, and, although we concentrate upon the case of two scalar fields, the general approach can be easily extended to  $N$  fields, where there will be  $N-1$  entropy fields orthogonal to the background trajectory. This was done for a specific assisted inflation model in Ref. [24]. We will work in the metric based approach of Bardeen [25] in order to define gauge-invariant cosmological perturbations, but our formalism can also be applied to the study of multiple scalar fields in other approaches [26–28,9].

We begin by reviewing the standard results obtained in single field models, emphasizing the suppression of non-adiabatic fluctuations on large-scales. We then extend our analysis to general two-field models, defining an adiabatic field and an entropy field, whose fluctuations, though uncorrelated on small scales, may develop correlations through the subsequent evolution. We present two specific models of two-field inflation, one with non-interacting fields, the other a model of interacting fields which undergo preheating after inflation.

## II. PERTURBATION EQUATIONS FOR MULTIPLE SCALAR FIELDS

We consider  $N$  scalar fields with the Lagrangian density

$$\mathcal{L} = -V(\varphi_1, \dots, \varphi_N) - \frac{1}{2} \sum_{I=1}^N g^{\mu\nu} \varphi_{I,\mu} \varphi_{I,\nu}, \quad (1)$$

and minimal coupling to gravity. In order to study the evolution of linear perturbations in the scalar fields, we make the standard splitting  $\varphi_I(t, \mathbf{x}) \rightarrow \varphi_I(t) + \delta\varphi_I(t, \mathbf{x})$ . The field equations, derived from Eq. (1) for the background homogeneous fields, are

$$\ddot{\varphi}_I + 3H\dot{\varphi}_I + V_{\varphi_I} = 0, \quad (2)$$

where  $V_x = \partial V / \partial x$ , and the Hubble rate,  $H$ , in a spatially flat Friedmann-Robertson-Walker (FRW) universe, is determined by the Friedman equation:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left[ V(\varphi_I) + \frac{1}{2} \sum_I \dot{\varphi}_I^2 \right], \quad (3)$$

with  $a(t)$  the FRW scale factor.

Consistent study of the linear field fluctuations  $\delta\varphi_I$  requires that we also consider linear scalar perturbations of the metric, corresponding to the line element<sup>1</sup>

$$ds^2 = -(1+2A)dt^2 + 2aB_{,i}dx^i dt + a^2 \times [(1-2\psi)\delta_{ij} + 2E_{,ij}]dx^i dx^j, \quad (4)$$

where we have not at this stage specified any particular choice of gauge [29,25,30].

Scalar field perturbations, with comoving wavenumber  $k = 2\pi a/\lambda$  for a mode with physical wavelength  $\lambda$ , then obey the perturbation equations

$$\begin{aligned} \delta\ddot{\varphi}_I + 3H\delta\dot{\varphi}_I + \frac{k^2}{a^2}\delta\varphi_I + \sum_J V_{\varphi_I\varphi_J}\delta\varphi_J \\ = -2V_{\varphi_I}A + \dot{\varphi}_I \left[ \dot{A} + 3\dot{\psi} + \frac{k^2}{a^2}(a^2\dot{E} - aB) \right]. \end{aligned} \quad (5)$$

The metric terms on the right-hand side, induced by the scalar field perturbations, obey the energy and momentum constraints

$$3H(\dot{\psi} + HA) + \frac{k^2}{a^2}[\psi + H(a^2\dot{E} - aB)] = -4\pi G\delta\rho, \quad (6)$$

$$\dot{\psi} + HA = -4\pi G\delta q. \quad (7)$$

<sup>1</sup>We follow the notation of Ref. [29], apart from our use of  $A$  rather than  $\phi$  as the perturbation in the lapse function.

The total energy and momentum perturbations are given in terms of the scalar field perturbations by

$$\delta\rho = \sum_I [\dot{\varphi}_I(\delta\dot{\varphi}_I - \dot{\varphi}_I A) + V_{\varphi_I}\delta\varphi_I] \quad (8)$$

$$\delta q_{,i} = -\sum_I \dot{\varphi}_I \delta\varphi_{I,i}. \quad (9)$$

These two equations can be combined to construct a gauge-invariant quantity, the comoving density perturbation [25]

$$\begin{aligned} \epsilon_m &\equiv \delta\rho - 3H\delta q \\ &= \sum_I [\dot{\varphi}_I(\delta\dot{\varphi}_I - \dot{\varphi}_I A) - \ddot{\varphi}_I\delta\varphi_I], \end{aligned} \quad (10)$$

which is sometimes used to represent the total matter perturbation.

Because the anisotropic stress vanishes to linear order for scalar fields minimally coupled to gravity, we have a further constraint on the metric perturbations:

$$(a^2\dot{E} - aB)' + H(a^2\dot{E} - aB) + \psi - A = 0. \quad (11)$$

The coupled perturbation equations (5)–(9) and (11) are probably most often solved in the zero-shear (or longitudinal or conformal Newtonian) gauge, in which  $a^2\dot{E}_I - aB_I = 0$  [29]. The two remaining metric perturbation variables which appear in the scalar field perturbation equation,  $A_I = \Phi$  and  $\psi_I = \Psi$ , are then equal in the absence of any anisotropic stress by Eq. (11).

Another useful choice is the spatially flat gauge, in which  $\psi_Q = 0$  [30,26]. The scalar field perturbations in this gauge are sometimes referred to as the Sasaki or Mukhanov variables [31], which have the gauge-invariant definition

$$Q_I \equiv \delta\varphi_I + \frac{\dot{\varphi}_I}{H}\psi. \quad (12)$$

The shear perturbation in the spatially flat gauge is simply related to the curvature perturbation,  $\Psi$ , in the zero-shear gauge:

$$a^2\dot{E}_Q - aB_Q = a^2\dot{E} - aB + \frac{1}{H}\psi = \frac{1}{H}\Psi. \quad (13)$$

The energy and momentum constraints, Eqs. (6) and (7), in the spatially flat gauge thus yield

$$\frac{k^2}{a^2}\Psi = -4\pi G\epsilon_m, \quad (14)$$

$$HA_Q = -4\pi G\delta q_Q, \quad (15)$$

where  $\epsilon_m$  is given in Eq. (10), and from Eq. (9) we have  $\delta q_Q = -\sum_I \dot{\varphi}_I Q_I$ .

The equations of motion, Eq. (5), rewritten in terms of the Sasaki-Mukhanov variables, and using Eqs. (14) and (15) to eliminate the metric perturbation terms in the spatially flat gauge, become [32]

$$\ddot{Q}_I + 3H\dot{Q}_I + \frac{k^2}{a^2}Q_I + \sum_J \left[ V_{\varphi_I\varphi_J} - \frac{8\pi G}{a^3} \left( \frac{a^3}{H} \dot{\varphi}_I \dot{\varphi}_J \right) \right] Q_J = 0. \quad (16)$$

### A. Curvature and entropy perturbations

The comoving curvature perturbation [33,34] is given by

$$\begin{aligned} \mathcal{R} &\equiv \psi - \frac{H}{\rho+p} \delta q \\ &= H \sum_I \left( \frac{\dot{\varphi}_I}{\sum_J \dot{\varphi}_J^2} \right) Q_I. \end{aligned} \quad (17)$$

This can also be given in terms of the metric perturbations in the longitudinal gauge as [29]

$$\mathcal{R} = \Psi - \frac{H}{\dot{H}} (\dot{\Psi} + H\Phi). \quad (18)$$

For comparison we give the curvature perturbation on uniform-density hypersurfaces,

$$-\zeta \equiv \psi + H \frac{\delta \rho}{\rho}, \quad (19)$$

first introduced by Bardeen, Steinhardt and Turner [35] as a conserved quantity for adiabatic perturbations on large scales [36,5]. It is related to the comoving curvature perturbation in Eq. (17) by a gauge transformation

$$-\zeta = \mathcal{R} + \frac{2\rho}{3(\rho+p)} \left( \frac{k}{aH} \right)^2 \Psi, \quad (20)$$

where we have used to the constraint equation (14) to eliminate the comoving density perturbation,  $\epsilon_m$ . Note that  $\mathcal{R}$  and  $-\zeta$  thus coincide in the limit  $k \rightarrow 0$ .

Both  $\mathcal{R}$  and  $-\zeta$  are commonly used to characterize the amplitude of adiabatic perturbations as both remain constant for purely adiabatic perturbations on sufficiently large scales as a direct consequence of local energy-momentum conservation [5], allowing one to relate the perturbation spectrum on large scales to quantities at the Hubble scale crossing during inflation in the simplest inflation models [35,37].

A dimensionless definition of the total entropy perturbation (which is automatically gauge-invariant) is given by

$$S = H \left( \frac{\delta p}{\dot{p}} - \frac{\delta \rho}{\dot{\rho}} \right), \quad (21)$$

which can be extended to define a generalized entropy perturbation between any two matter quantities  $x$  and  $y$ :

$$S_{xy} = H \left( \frac{\delta x}{\dot{x}} - \frac{\delta y}{\dot{y}} \right). \quad (22)$$

The total entropy perturbation in Eq. (21) for  $N$  scalar fields is given by

$$S = \frac{2 \left( \dot{V} + 3H \sum_J \dot{\varphi}_J^2 \right) \delta V + 2\dot{V} \sum_I \dot{\varphi}_I (\delta \dot{\varphi}_I - \dot{\varphi}_I A)}{3 \left( 2\dot{V} + 3H \sum_J \dot{\varphi}_J^2 \right) \sum_I \dot{\varphi}_I^2}, \quad (23)$$

where the perturbation in the total potential energy is given by  $\delta V = \sum_I V_{\varphi_I} \delta \varphi_I$ .

The change in  $\mathcal{R}$  on large scales (i.e., neglecting spatial gradient terms) can be directly related to the non-adiabatic part of the pressure perturbation [1,5,38]

$$\dot{\mathcal{R}} \approx -3H \frac{\dot{p}}{\rho} S. \quad (24)$$

We will thus now consider the evolution of the adiabatic and entropy perturbations in both one- and two-field models of inflation.

### B. Single field

Perturbations in a single self-interacting scalar field obey the gauge-dependent equation of motion

$$\begin{aligned} \delta \ddot{\varphi} + 3H \delta \dot{\varphi} + \left( \frac{k^2}{a^2} + V_{\varphi\varphi} \right) \delta \varphi \\ = -2V_{\varphi A} + \dot{\varphi} \left[ \dot{A} + 3\dot{\psi} + \frac{k^2}{a^2} (a^2 \dot{E} - aB) \right], \end{aligned} \quad (25)$$

subject to the energy and momentum constraint equations given in Eqs. (6)–(9).

The scalar field perturbation in the spatially flat gauge has the gauge-invariant definition, Eq. (12),

$$Q_\varphi \equiv \delta \varphi + \frac{\dot{\varphi}}{H} \psi. \quad (26)$$

For a single field this is directly related to the curvature perturbation in the comoving gauge, where the momentum,  $\delta q = -\dot{\varphi} \delta \varphi$ , vanishes

$$\mathcal{R} = \psi + \frac{H}{\dot{\varphi}} \delta \varphi = \frac{H}{\dot{\varphi}} Q_\varphi. \quad (27)$$

It is not obvious that the intrinsic entropy perturbation for a single scalar field, obtained from Eq. (23),

$$S = \frac{2V_\varphi}{3\dot{\varphi}^2(3H\dot{\varphi} + 2V_\varphi)} [\dot{\varphi}(\delta \dot{\varphi} - \dot{\varphi}A) - \ddot{\varphi} \delta \varphi], \quad (28)$$

should vanish on large scales. Because the scalar field obeys a second-order equation of motion, its general solution contains two arbitrary constants of integration, which can describe both adiabatic and entropy perturbations. However  $\mathcal{S}$  for a single scalar field is proportional to the comoving density perturbation given in Eq. (10), and this in turn is related to the metric perturbation,  $\Psi$ , via Eq. (14), so that [39]

$$\mathcal{S} = -\frac{V_\varphi}{6\pi G \dot{\varphi}^2 [3H\dot{\varphi} + 2V_\varphi]} \left( \frac{k^2}{a^2} \Psi \right). \quad (29)$$

In the absence of anisotropic stresses,  $\Psi$  must be of order  $A_Q$ , by Eq. (11), and hence the non-adiabatic pressure becomes small on large scales [6,39,10]. The amplitude of the asymptotic solution for the scalar field at late times (and hence large scales) during inflation thus determines the amplitude of an adiabatic perturbation.

The change in the comoving curvature perturbation is given by

$$\dot{\mathcal{R}} = \frac{H}{\dot{H}} \frac{k^2}{a^2} \Psi, \quad (30)$$

and hence the rate of change of the curvature perturbation, given by  $d \ln \mathcal{R} / d \ln a \sim (k/aH)^2$ , becomes negligible on large scales during single-field inflation.

### C. Two fields

In this section we will consider two interacting scalar fields,  $\phi \equiv \varphi_1$  and  $\chi \equiv \varphi_2$ . The analysis developed here should be straightforward to extend to include additional scalar fields, but we do not expect to see any qualitatively new features in this case, so for clarity we restrict our discussion here to two fields.

In order to clarify the role of adiabatic and entropy perturbations, their evolution and their inter-relation, we define new adiabatic and entropy fields by a rotation in field space. The ‘‘adiabatic field,’’  $\sigma$ , represents the path length along the classical trajectory, such that

$$\dot{\sigma} = (\cos \theta) \dot{\phi} + (\sin \theta) \dot{\chi}, \quad (31)$$

where

$$\cos \theta = \frac{\dot{\phi}}{\sqrt{\dot{\phi}^2 + \dot{\chi}^2}}, \quad \sin \theta = \frac{\dot{\chi}}{\sqrt{\dot{\phi}^2 + \dot{\chi}^2}}. \quad (32)$$

This definition, plus the original equations of motion for  $\phi$  and  $\chi$ , give

$$\ddot{\sigma} + 3H\dot{\sigma} + V_\sigma = 0, \quad (33)$$

where

$$V_\sigma = (\cos \theta) V_\phi + (\sin \theta) V_\chi. \quad (34)$$

As illustrated in Fig. 1,  $\delta\sigma$  is the component of the two-field perturbation vector along the direction of the background

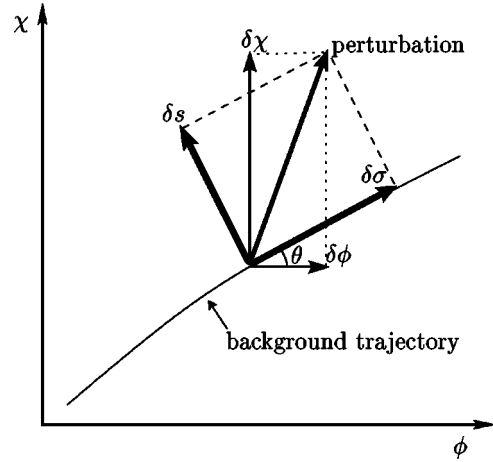


FIG. 1. An illustration of the decomposition of an arbitrary perturbation into an adiabatic ( $\delta\sigma$ ) and entropy ( $\delta s$ ) component. The angle of the tangent to the background trajectory is denoted by  $\theta$ . The usual perturbation decomposition, along the  $\phi$  and  $\chi$  axes, is also shown.

fields’ evolution. Conversely, fluctuations orthogonal to the background classical trajectory represent non-adiabatic perturbations, and we define the ‘‘entropy field,’’  $s$ , such that

$$\delta s = (\cos \theta) \delta\chi - (\sin \theta) \delta\phi. \quad (35)$$

From this definition, it follows that  $s = \text{const}$  along the classical trajectory, and hence entropy perturbations are automatically gauge-invariant [40]. Perturbations in  $\delta\sigma$ , with  $\delta s = 0$ , describe adiabatic field perturbations, and this is why we refer to  $\sigma$  as the ‘‘adiabatic field.’’

The total momentum of the two-field system, given by Eq. (9), is then

$$\delta q_{,i} = -\dot{\phi} \delta\phi_{,i} - \dot{\chi} \delta\chi_{,i} = -\dot{\sigma} \delta\sigma_{,i}, \quad (36)$$

and the comoving curvature perturbation in Eq. (17) is given by

$$\begin{aligned} \mathcal{R} &= \psi + H \left( \frac{\dot{\phi} \delta\phi + \dot{\chi} \delta\chi}{\dot{\phi}^2 + \dot{\chi}^2} \right) \\ &= \psi + \frac{H}{\dot{\sigma}} \delta\sigma. \end{aligned} \quad (37)$$

This expression, written in terms of the adiabatic field,  $\sigma$ , is identical to that given in Eq. (27) for a single field.

We can also write Eq. (37) as

$$\mathcal{R} = (\cos^2 \theta) \mathcal{R}_\phi + (\sin^2 \theta) \mathcal{R}_\chi, \quad (38)$$

where we define the comoving curvature perturbation for each of the original fields as

$$\mathcal{R}_I \equiv \psi + \frac{H}{\dot{\varphi}_I} \delta\varphi_I = \frac{H}{\dot{\varphi}_I} Q_I. \quad (39)$$

However, even fields with no explicit interaction will in general have non-zero intrinsic entropy perturbations on large scales in a multi-field system due to their gravitational interaction, so that  $\mathcal{R}_I$  for each field is not conserved. Although the intrinsic entropy perturbation for each field is still of the form given by Eq. (28), it is no longer constrained by Eq. (14) to vanish as  $k \rightarrow 0$ . This is in contrast to the case of non-interacting perfect fluids, where it is possible to define a constant curvature perturbation for each fluid on large scales [5].

The comoving matter perturbation in Eq. (10) can be written as

$$\epsilon_m = \dot{\sigma}(\delta\dot{\sigma} - \dot{\sigma}A) - \ddot{\sigma}\delta\sigma + 2V_s\delta s, \quad (40)$$

which acquires an additional term, compared with the single-field case, due to the dependence of the potential upon  $s$ , where

$$V_s = (\cos\theta)V_\chi - (\sin\theta)V_\phi. \quad (41)$$

The perturbed kinetic energy of  $s$  has no contribution to first-order as in the background solution  $\dot{s}=0$ , by definition.

The total entropy perturbation, Eq. (23), for the two fields can be written as

$$S = \frac{2}{3\dot{\sigma}^2(3H\dot{\sigma} + 2V_\sigma)} \times \{V_\sigma[\dot{\sigma}(\delta\dot{\sigma} - \dot{\sigma}A) - \ddot{\sigma}\delta\sigma] + 3H\dot{\sigma}^2\dot{\theta}\delta s\}. \quad (42)$$

Combining Eqs. (14), (40) and (42), we can write

$$S = -\frac{V_\sigma}{6\pi G\dot{\sigma}^2[3H\dot{\sigma} + 2V_\sigma]} \left( \frac{k^2}{a^2}\Psi \right) - \frac{2V_s}{3\dot{\sigma}^2}\delta s. \quad (43)$$

Comparing this with the single-field result given in Eq. (29), we see that the entropy perturbation on large scales is due solely to the relative entropy perturbation between the two fields, described by the entropy field  $\delta s$ .

The change in the comoving curvature perturbation given by [1,38]

$$\dot{\mathcal{R}} = \frac{H}{\dot{H}} \frac{k^2}{a^2}\Psi + \frac{1}{2}H \left( \frac{\delta\phi}{\dot{\phi}} - \frac{\delta\chi}{\dot{\chi}} \right) \frac{d}{dt} \left( \frac{\dot{\phi}^2 - \dot{\chi}^2}{\dot{\phi}^2 + \dot{\chi}^2} \right), \quad (44)$$

which can be expressed neatly in terms of the new variables:

$$\dot{\mathcal{R}} = \frac{H}{\dot{H}} \frac{k^2}{a^2}\Psi + \frac{2H}{\dot{\sigma}}\dot{\theta}\delta s, \quad (45)$$

where

$$\dot{\theta} = -\frac{V_s}{\dot{\sigma}}. \quad (46)$$

The new source term on the right-hand-side of this equation, compared with the single-field case, Eq. (30), is proportional

to the relative entropy perturbation between the two fields,  $\delta s$ . Clearly, there can be significant changes to  $\mathcal{R}$  on large scales if the entropy perturbation is not suppressed and if the background solution follows a curved trajectory, i.e.,  $\dot{\theta} \neq 0$ , in field space [10]. This can then produce a change in the comoving curvature on arbitrarily large scales (i.e., even in the limit  $k \rightarrow 0$ ) [1,39].

Equations of motion for the adiabatic and entropy field perturbations can be derived from the perturbed scalar field equations (5), to give

$$\begin{aligned} \delta\ddot{\sigma} + 3H\delta\dot{\sigma} + \left( \frac{k^2}{a^2} + V_{\sigma\sigma} - \dot{\theta}^2 \right) \delta\sigma \\ = -2V_{\sigma A} + \dot{\sigma} \left[ \dot{A} + 3\dot{\psi} + \frac{k^2}{a^2}(a^2\dot{E} - aB) \right] + 2(\dot{\theta}\delta s) \\ - 2\frac{V_\sigma}{\dot{\sigma}}\dot{\theta}\delta s, \end{aligned} \quad (47)$$

and

$$\begin{aligned} \delta\ddot{s} + 3H\delta\dot{s} + \left( \frac{k^2}{a^2} + V_{ss} - \dot{\theta}^2 \right) \delta s \\ = -2\frac{\dot{\theta}}{\dot{\sigma}}[\dot{\sigma}(\delta\dot{\sigma} - \dot{\sigma}A) - \ddot{\sigma}\delta\sigma], \end{aligned} \quad (48)$$

where

$$V_{\sigma\sigma} = (\sin^2\theta)V_{\chi\chi} + (\sin 2\theta)V_{\phi\chi} + (\cos^2\theta)V_{\phi\phi}, \quad (49)$$

$$V_{ss} = (\sin^2\theta)V_{\phi\phi} - (\sin 2\theta)V_{\phi\chi} + (\cos^2\theta)V_{\chi\chi}. \quad (50)$$

When  $\dot{\theta}=0$ , the adiabatic and entropy perturbations decouple.<sup>2</sup> The equation of motion for  $\delta\sigma$  then reduces to that for a single scalar field in a perturbed FRW spacetime, as given in Eq. (25), while the equation for  $\delta s$  is that for a scalar field perturbation in an *unperturbed* FRW spacetime.

The only source term on the right-hand-side in Eq. (48) for the entropy perturbation comes from the intrinsic entropy perturbation in the  $\sigma$ -field. From Eqs. (14) and (40) we have

$$\dot{\sigma}(\delta\dot{\sigma} - \dot{\sigma}A) - \ddot{\sigma}\delta\sigma = 2\dot{\sigma}\dot{\theta}\delta s - \frac{k^2}{4\pi G a^2}\Psi, \quad (51)$$

and hence we can rewrite the evolution equation for the entropy perturbation as

<sup>2</sup>If we employ the slow-roll approximation for the background fields,  $\dot{\phi} \simeq -V_\phi/3H$  and  $\dot{\chi} \simeq -V_\chi/3H$ , we obtain  $\dot{\theta} \simeq 0$ . This reflects the fact that the rate of change of  $\theta$  is slow – instantaneously it moves in an approximately straight line in field space. But the integrated change in  $\theta$  cannot in general be neglected. Even working within the slow-roll approximation, fields do not in general follow a straight line trajectory in field space.



$$\delta\ddot{s} + 3H\delta\dot{s} + \left(\frac{k^2}{a^2} + V_{ss} + 3\dot{\theta}^2\right)\delta s = \frac{\dot{\theta}}{\dot{\sigma}} \frac{k^2}{2\pi G a^2} \Psi. \quad (52)$$

Note that this evolution equation is automatically gauge-invariant and holds in any gauge. On large scales the inhomogeneous source term becomes negligible, and we have a homogeneous second-order equation of motion for the entropy perturbation, decoupled from the adiabatic field and metric perturbations. If the initial entropy perturbation is zero on large scales, it will remain so.

By contrast, we cannot neglect the metric back-reaction for the adiabatic field fluctuations, or the source terms due to the entropy perturbations. Working in the spatially flat gauge, defining

$$Q_\sigma = \delta\sigma_Q = \delta\sigma + \frac{\dot{\sigma}}{H}\psi, \quad (53)$$

and using

$$A_Q = 4\pi G \frac{\dot{\sigma}}{H} Q_\sigma, \quad (54)$$

we can rewrite the equation of motion for the adiabatic field perturbation as

$$\begin{aligned} \ddot{Q}_\sigma + 3H\dot{Q}_\sigma + \left[\frac{k^2}{a^2} + V_{\sigma\sigma} - \dot{\theta}^2 - \frac{8\pi G}{a^3} \left(\frac{a^3 \dot{\sigma}^2}{H}\right)\right] Q_\sigma \\ = 2(\dot{\theta}\delta s) - 2\left(\frac{V_\sigma}{\dot{\sigma}} + \frac{\dot{H}}{H}\right)\dot{\theta}\delta s. \end{aligned} \quad (55)$$

When  $\dot{\theta}=0$ , this reduces to the single-field equation of motion, but for a curved trajectory in field space, the entropy perturbation acts as an additional source term in the equation of motion for the adiabatic field perturbation, even on large scales.

In order for small-scale quantum fluctuations to produce large-scale (super-Hubble) perturbations during inflation, a field must be ‘‘light’’ (i.e., overdamped). The effective mass for the entropy field in Eq. (52) is  $\mu_s^2 = V_{ss} + 3\dot{\theta}^2$ . For  $\mu_s^2 > \frac{3}{2}H^2$ , the fluctuations remain in the vacuum state and fluctuations on large scales are strongly suppressed. The existence of large-scale entropy perturbations therefore requires

$$\mu_s^2 \equiv V_{ss} + 3\dot{\theta}^2 < \frac{3}{2}H^2. \quad (56)$$

### III. APPLICATION TO ENTROPY-ADIABATIC CORRELATIONS FROM INFLATION

Equations (52) and (55) are the key equations which govern the evolution of the adiabatic and entropy perturbations in a two field system. Together with constraint equations (51) and (54) for the metric perturbations, they form a closed set of equations. They allow one to follow the effect on the adiabatic curvature perturbation due to the presence of en-

trophy perturbations, absent in the single field model. This in turn will allow us to study the resulting correlations between the spectra of adiabatic and entropy perturbations produced on large scales due to quantum fluctuations of the fields on small scales during inflation.

A useful approximation commonly made when studying field perturbations during inflation, is to split the evolution of a given mode into a sub-Hubble regime ( $k > aH$ ), in which the Hubble expansion is neglected, and a super-Hubble regime ( $k < aH$ ), in which gradient terms are dropped.

If we assume that both fields  $\phi$  and  $\chi$  are light (i.e., overdamped) during inflation, then we can take the field fluctuations to be in their Minkowski vacuum state on sub-Hubble scales. This gives their amplitudes at Hubble crossing ( $k = aH$ ) as

$$Q_I|_{k=aH} = \frac{H_k}{\sqrt{2k^3}} e_I(k), \quad (57)$$

where  $I = \phi, \chi$ ,  $H_k$  is the Hubble parameter when the mode crosses the Hubble radius (i.e.,  $H_k = k/a$ ), and  $e_\phi$  and  $e_\chi$  are independent Gaussian random variables satisfying

$$\langle e_I(k) \rangle = 0, \quad \langle e_I(k) e_J^*(k') \rangle = \delta_{IJ} \delta(k - k'), \quad (58)$$

with the angled brackets denoting ensemble averages. It follows from our definitions of the entropy and adiabatic perturbations in Eqs. (31) and (35) that their distributions at Hubble crossing have the same form:

$$Q_\sigma|_{k=aH} = \frac{H_k}{\sqrt{2k^3}} e_\sigma(k), \quad \delta s|_{k=aH} = \frac{H_k}{\sqrt{2k^3}} e_s(k), \quad (59)$$

where  $e_\sigma$  and  $e_s$  are Gaussian random variables obeying the same relations given in Eq. (58), with  $I, J = \sigma, s$ .

Super-Hubble modes are assumed to obey the equations of motion given in Eqs. (55) and (52), which we will write schematically as

$$\hat{O}^\sigma(Q_\sigma) = \hat{S}^\sigma(\delta s), \quad (60)$$

$$\hat{O}^s(\delta s) = 0, \quad (61)$$

where  $\hat{O}^\sigma(Q_\sigma)$  and  $\hat{O}^s(\delta s)$  are obtained by setting  $k=0$  on the left-hand side of Eqs. (55) and (52) respectively, and  $\hat{S}^\sigma(\delta s)$  is given by the right-hand side of Eq. (55). As remarked before, there is no source term for  $\delta s$  appearing on the right-hand side of Eq. (52) once we neglect gradient terms. The general super-Hubble solution can thus be written as

$$Q_\sigma = A_+ f_+(t) + A_- f_-(t) + P(t), \quad (62)$$

$$\delta s = B_+ g_+(t) + B_- g_-(t), \quad (63)$$

where the real functions  $f_\pm$  and  $g_\pm$  are the growing or decaying modes of the homogeneous equations,  $\hat{O}^\sigma(f_\pm) = 0$

and  $\hat{O}^s(g_{\pm})=0$ , and  $P(t)$  is a particular integral of the full inhomogeneous equation (60). Note that the growing-mode solution  $f_+ \propto \dot{\sigma}/H$ .

Henceforth we shall consider only slow-roll inflation where the evolution can be approximated by first-order equations [dropping  $\delta\ddot{s}$  and  $\ddot{Q}_\sigma$  in Eqs. (52) and (55)], so that we have<sup>3</sup>

$$Q_\sigma \approx Af(t) + P(t), \quad (64)$$

$$\delta s \approx Bg(t). \quad (65)$$

We can, without loss of generality, take  $f=1=g$  and  $P=0$  when  $k=aH$ , so that the amplitudes of the growing modes at Hubble-crossing are given by Eqs. (59) as

$$A(k) = \frac{H_k}{\sqrt{2k^3}} e_\sigma(k), \quad B(k) = \frac{H_k}{\sqrt{2k^3}} e_s(k). \quad (66)$$

From Eq. (60), we see that the amplitude of the particular integral  $P(t)$  at later times will be correlated with the amplitude of the entropy perturbation,  $B$ , and we can write  $P(t) = B\tilde{P}(t)$ , where  $\tilde{P}(t)$  is a real function independent of the random variables  $e_\sigma, e_s$ .

In order to quantify the correlation, we define

$$\langle x(k)y^*(k') \rangle \equiv \frac{2\pi^2}{k^3} C_{xy} \delta(k-k'). \quad (67)$$

The adiabatic and entropy power spectra are given by

$$\mathcal{P}_{Q_\sigma} \equiv C_{Q_\sigma Q_\sigma} \approx \left( \frac{H_k}{2\pi} \right)^2 [|f^2| + |\tilde{P}^2|], \quad (68)$$

$$\mathcal{P}_{\delta s} \equiv C_{\delta s \delta s} \approx \left( \frac{H_k}{2\pi} \right)^2 |g^2|, \quad (69)$$

while the dimensionless cross correlation is given by

$$\frac{C_{Q_\sigma \delta s}}{\sqrt{\mathcal{P}_{Q_\sigma}} \sqrt{\mathcal{P}_{\delta s}}} \approx \frac{g\tilde{P}}{\sqrt{g^2} \sqrt{|f^2| + |\tilde{P}^2|}}. \quad (70)$$

Note that the adiabatic power spectrum at late times is always enhanced if it is coupled to entropy perturbations [i.e.,  $P(t) \neq 0$ , in Eq. (64)], as the entropy field fluctuations at Hubble-crossing provide an uncorrelated extra source.

As an illustration, we consider the correlations in the adiabatic and entropy perturbations at the start of the radiation era, produced after double inflation, as studied in Ref. [20]. The double-inflation potential for two non-interacting but massive scalar fields is

$$V = \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_\chi^2 \chi^2. \quad (71)$$

Following [11], it is possible to parametrize the background scalar field trajectory in polar coordinates when both fields are slow rolling:

$$\chi \approx \sqrt{\frac{N}{2\pi G}} \sin \alpha, \quad \phi \approx \sqrt{\frac{N}{2\pi G}} \cos \alpha, \quad (72)$$

where  $N = -\ln(a/a_{\text{end}})$  is the number of e-folds until the end of inflation. The background trajectory can then be expressed as

$$N \approx N_0 \frac{(\sin \alpha)^{2/(R^2-1)}}{(\cos \alpha)^{2R^2/(R^2-1)}}, \quad (73)$$

where  $R = m_\chi/m_\phi$ . The scalar field position angle,  $\alpha$ , can be related to the scalar field velocity angle,  $\theta$ , which we used to define the adiabatic and entropy perturbations:

$$\tan \theta \approx -\frac{m_\chi^2}{3H\dot{\sigma}} \sqrt{\frac{N}{2\pi G}} \tan \alpha. \quad (74)$$

The scalar field  $\chi$  is assumed to decay into cold dark matter while the scalar field  $\phi$  decays into radiation. The entropy-isocurvature at the start of the radiation-dominated era is described by

$$S_{\text{rad}} \equiv \frac{\delta\rho_c}{\rho_c} - \frac{3}{4} \frac{\delta\rho_\gamma}{\rho_\gamma}. \quad (75)$$

In Ref. [20], it is shown how the super-Hubble perturbations in the radiation era can be determined in terms of the perturbations during the inflationary era. The fluctuations in both  $\phi$  and  $\chi$  fields can contribute to both the adiabatic and entropy perturbations. The adiabatic component comes directly from the comoving curvature perturbation,  $\mathcal{R}$ , at the end of inflation, and is given by

$$\mathcal{R}_{\text{rad}} \approx -\sqrt{4\pi G} \sqrt{\frac{N_k}{k^3}} H_k [(\sin \alpha_k) e_\chi(k) + (\cos \alpha_k) e_\phi(k)]. \quad (76)$$

The isocurvature perturbation at the start of the radiation-dominated era is related to the entropy perturbation between the two fields at the end of inflation [12]

$$S_{\text{rad}} \approx -\frac{2}{3} m_\chi^2 \frac{1}{H} \left( \frac{\delta\chi}{\dot{\chi}} - \frac{\delta\phi}{\dot{\phi}} \right), \quad (77)$$

which yields

$$S_{\text{rad}} \approx -\sqrt{4\pi G} \sqrt{\frac{N_k}{k^3}} H_k [R^4 \sec \alpha_k + \text{cosec } \alpha_k] e_s(k), \quad (78)$$

<sup>3</sup>We note that in non-slow-roll scenarios the effect of decaying modes may not be negligible on super-Hubble scales, which could affect the correlations between adiabatic and entropy perturbations.

and

$$\mathcal{R}_{\text{rad}} \approx \sqrt{4\pi G} \sqrt{\frac{N_k}{k^3} H_k} \frac{R^2 \tan \alpha_k \sin \alpha_k}{\sqrt{R^2 \tan^2 \alpha_k + 1}} \times \left\{ \left[ \frac{1}{R^2 \tan^2 \alpha_k} + 1 \right] e_\sigma(k) + \left[ \frac{1 - R^2}{R^2 \tan \alpha_k} \right] e_s(k) \right\}. \quad (79)$$

The entropy perturbation during the radiation era only depends on the entropy perturbation at Hubble-crossing during the inflationary era, while the adiabatic perturbation during the radiation era depends on both the adiabatic and entropy perturbations at Hubble-crossing. This is consistent with Eqs. (52) and (47), showing that the entropy perturbation sources the adiabatic perturbation on super-Hubble scales, but not vice versa.

As both Eqs. (78) and (79) depend on the random variable  $e_s$ , the adiabatic and entropy perturbations will be correlated, and we find

$$\frac{C_{\mathcal{R}_{\text{rad}} S_{\text{rad}}}}{\sqrt{\mathcal{P}_{\mathcal{R}_{\text{rad}}}} \sqrt{\mathcal{P}_{S_{\text{rad}}}}} \approx \frac{(R^2 - 1) \sin 2\alpha_k}{2\sqrt{R^4 \sin^2 \alpha_k + \cos^2 \alpha_k}}. \quad (80)$$

This correlation is investigated fully in [20] in terms of the usual scalar field perturbation variables. An interesting point that can easily be seen from Eq. (79) is that  $\mathcal{R}_{\text{rad}}$  will depend only on  $e_\sigma$  if  $R \equiv m_\chi/m_\phi = 1$ . Thus, there will be no correlation if  $R=1$ . As can be seen from Eq. (73),  $\alpha$  will be constant for  $R=1$  and thus so will  $\theta$ ; a straight-line background trajectory will be obtained for  $R=1$ . This is consistent with Eq. (47), where it can be seen that the entropy component only sources the adiabatic component on large scales if  $\dot{\theta} \neq 0$ .

#### IV. APPLICATION TO PREHEATING AFTER INFLATION

In this section we use the entropy-adiabatic decomposition of the perturbation equations to investigate the dynamics of super-Hubble perturbations during a period of preheating at the end of inflation. We consider three models, encompassed by the general effective potential

$$V = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{1}{2} g^2 \phi^2 \chi^2 + \tilde{g}^2 \phi^3 \chi. \quad (81)$$

The essence of preheating lies in the parametric amplification of field perturbations due to the time dependence of their effective mass, e.g.,  $m_\chi^2 \equiv V_{\chi\chi} = g^2 \phi^2$ . In the simplest cases, the inflaton  $\phi$  simply oscillates at the end of inflation.

Preheating typically amplifies long-wavelength modes preferentially. As discussed in [41,42,39], amplification of super-Hubble modes does not lead to a violation of causality, due to the super-Hubble coherence of the inflaton oscillations set up by the prior inflationary phase. If  $\mathcal{R}$  is amplified on super-Hubble scales, this will alter the resulting imprint on the anisotropies of the cosmic microwave background (CMB), and break the simple link between CMB observations and inflationary models.

We consider first the case where the inflaton is massive ( $m \neq 0$ ) and neglect its self-interaction ( $\lambda = 0$ ). The traditional resonance parameter for the strength of preheating at the end of inflation is

$$q = \frac{g^2 \phi_0^2}{4m^2}. \quad (82)$$

In the massive case, where modes move through the resonance bands of the Mathieu chart, and for inflation at high energies where the expansion of the universe is very vigorous,  $q$  needs to be much larger than one if the parametric resonance is to be efficient [43]. It is possible to have large  $q$  even for small coupling,  $g^2 \ll 1$ , as  $m \ll \phi_0 \sim M_{\text{Pl}}$ . We can write the effective mass of the  $\chi$  during inflation as

$$\frac{m_\chi^2}{H^2} \approx \frac{3q}{\pi} \frac{M_{\text{Pl}}^2}{\phi_0^2}, \quad (83)$$

where  $\phi_0$  is the initial value of  $\phi$  at the beginning of preheating. It then follows from Eq. (83) that  $\chi$  must be heavy during inflation for this simple potential if efficient preheating is to be obtained.

Any change in the curvature perturbation  $\mathcal{R}$  on very large scales must be due to the presence of non-adiabatic perturbations. In [44,45], it was shown how, if  $m_\chi^2 \gg m_\phi^2$  during inflation with  $\lambda = 0 = \tilde{g}$ , then the  $\chi$  field and hence any non-adiabatic perturbations on large scales are exponentially suppressed during inflation, and no change to  $\mathcal{R}$  occurs before backreaction ends the resonance.

However, when  $\tilde{g} \neq 0$ , the  $\chi$  field will have a nonzero vacuum expectation value (vev) during inflation *even along the valley of the potential*. In the slow-roll limit for  $\phi$ , this vev is determined by  $V_\chi = 0$ , which gives

$$\chi \approx -\frac{\tilde{g}^2}{g^2} \phi. \quad (84)$$

The  $\tilde{g}$  coupling has the effect of rotating the valley of the potential — which the attractor trajectory approximately follows — from  $\chi=0$ , through an angle

$$\theta \approx -\frac{\tilde{g}^2}{g^2}, \quad (85)$$

where, to ensure that the chaotic inflation scenario is not drastically altered, we assume [46]

$$\frac{\tilde{g}}{g} \ll 1. \quad (86)$$

The effect of  $\tilde{g}$  is to change the attractor for both  $\chi$  and  $\delta\chi$  during inflation, since the  $\chi$  and  $\delta\chi$  equations of motion gain inhomogeneous driving terms proportional to  $\tilde{g}^2 \phi^3$ . This does not necessarily imply that  $\mathcal{R}$  will be amplified by preheating at the end of inflation as purely adiabatic perturbations along the slow-roll attractor now have a component



along  $\chi$  as well as  $\phi$ . In order to determine whether or not the evolution of the comoving curvature perturbation,  $\mathcal{R}$ , on super-Hubble scales is affected, we need to follow the evolution of the entropy field perturbation,<sup>4</sup> defined by Eq. (35), which gives

$$\delta s \approx \delta\chi + \frac{\tilde{g}^2}{g^2} \delta\phi. \quad (87)$$

In the limit  $\tilde{g}/g \rightarrow 0$  we recover  $\delta s \rightarrow \delta\chi$ . Crucially, the evolution equation (52) for the entropy perturbation has *no* inhomogeneous terms in the long-wavelength ( $k \rightarrow 0$ ) limit, even for  $\tilde{g} \neq 0$ , and entropy perturbations will only be non-negligible on super-Hubble scales if the entropy field is light during inflation.

In the slow-roll limit and on large scales, the evolution equation (52) for the entropy perturbation has the approximate solution [47]

$$\delta s \propto a^{-3/2} \left( \frac{k}{aH} \right)^{-\nu}, \quad (88)$$

where

$$\nu^2 = \frac{9}{4} - \frac{\mu_s^2}{H^2}, \quad (89)$$

and the effective mass of the entropy field,  $\mu_s$  is defined in Eq. (56). The power spectrum of entropy perturbations is

$$\mathcal{P}_{\delta s} \propto H^3 \left( \frac{k}{aH} \right)^{3-2\text{Re}(\nu)}. \quad (90)$$

The real part of  $\nu$  vanishes for  $\mu_s^2/H^2 > 9/4$ , leaving a steep  $k^3$  blue spectrum, which is exponentially suppressed with time.

Using Eqs. (50), (81), (85), and (86), one finds that

$$\frac{\mu_s^2}{H^2} \approx \left[ 1 - 4q \left( \frac{\tilde{g}}{g} \right)^4 \left( \frac{\phi}{\phi_0} \right)^2 \right]^{-1} \frac{3qM_{\text{pl}}^2}{\pi\phi_0^2}, \quad (91)$$

$\mu_s^2/H^2$  has a local minimum for  $\tilde{g}=0$ . Thus the additional  $\tilde{g}$  term in Eq. (81) serves to *increase* the entropy mass relative to the Hubble parameter, and so does not avoid the suppression of the entropy perturbation. The  $\tilde{g}$  term therefore does not significantly alter the spectral index of the spectrum of entropy perturbations, which remains steep if  $q \gg 1$ . The strongly blue spectrum implies that non-linear backreaction is dominated by small-scale modes, which go nonlinear long

<sup>4</sup>From Eq. (45) we see that  $\dot{\theta}\delta s$  must be non-zero to change  $\mathcal{R}$  on large scales. Because  $\dot{\theta} \approx 0$ , from Eq. (85), the entropy remains decoupled from the adiabatic perturbation during slow-roll inflation in this model. But at the end of inflation, during preheating,  $\dot{\theta} \neq 0$ .

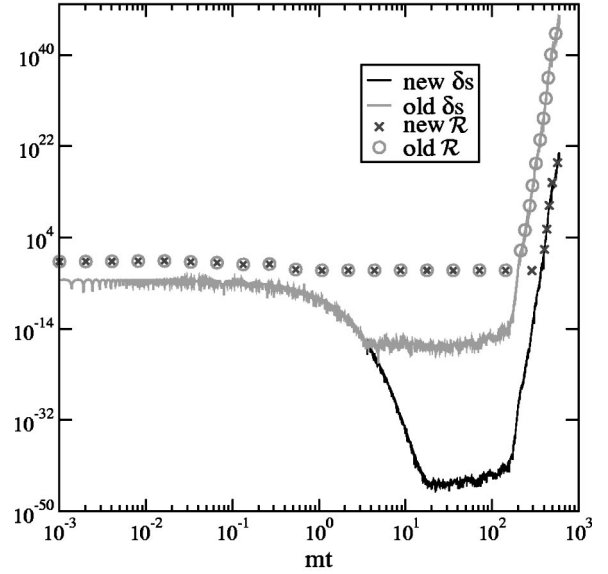


FIG. 2. Numerical simulations of the entropy and comoving curvature perturbations during inflation and preheating, with  $\lambda = 0$ ,  $g = 2 \times 10^{-3}$ ,  $\tilde{g} = 8 \times 10^{-3} g$  and  $m = 10^{-6} M_{\text{pl}}$ . The ‘‘new’’ prefix indicates that the field perturbations were evaluated by numerically integrating Eqs. (52), and (55), while the ‘‘old’’ prefix indicates that the perturbations were evaluated by integrating the original field equations (5). We have not included any higher-order corrections such as back reaction from small-scale perturbations which would shut down the resonant amplification of  $\delta s$  at some point.

before the cosmological modes, implying that resonance ends before  $\mathcal{R}$  changes [43,44].

We have also integrated the field equations numerically to avoid relying on any slow-roll-type approximations. To numerically evaluate the entropy perturbation, one could simulate the original perturbation variables  $\delta\phi$  and  $\delta\chi$ , using Eq. (5), and then work out  $\delta s$  algebraically via Eq. (35). However, this approach is prone to numerical instability when the entropy perturbation is suppressed. To illustrate this, we take  $\tilde{g} = 8 \times 10^{-3} g$  and  $q = 3.8 \times 10^5$ . After about 60 e-folds of inflation, one can see analytically that  $\delta s \sim 10^{-40}$ . Numerically,  $\delta\chi \cos \theta \sim \delta\phi \sin \theta \sim 10^{-8}$  during inflation. So in order to obtain a high enough accuracy to model the suppression of  $\delta s$ , we require that  $\delta\chi \cos \theta$  and  $\delta\phi \sin \theta$  have to be simulated to a relative accuracy of  $\sim 10^{-8}/10^{-40} = 10^{-32}$ . This means approximately 32 significant figures are needed, which is beyond the capability of standard numerical ordinary differential equation integration routines.

If instead we use the new adiabatic and entropy field perturbations and integrate Eqs. (52) and (55), then this numerical instability does *not* occur, since one no longer needs to find the difference between two nearly equal quantities. Simulation results using these equations are compared with the results using the old field perturbation equations (5) in Fig. 2. The simulations show that the growth in  $\mathcal{R}$  is driven by  $\delta s$ , in concordance with Eq. (45). As can be seen, the numerical result using the field perturbation equations fails to track the exponential decay of the entropy during inflation and thus underestimates the delay in the growth of  $\mathcal{R}$ .

In practice, we find a similar instability if we try to construct the gauge-invariant metric perturbation,  $\Psi$ , required in Eq. (52) in terms of the constraint Eq. (51). This includes the intrinsic entropy perturbation in the  $\sigma$  field, which does become small at late times or large scales, but results from the diminishing difference between finite terms. It is more stable numerically to follow the value of  $\Psi$  at late times using the evolution equation

$$\dot{\Psi} + \left( H - \frac{\dot{H}}{H} \right) \Psi = 4 \pi G \dot{\sigma} Q_{\sigma}, \quad (92)$$

which can be obtained from the definition of  $\Psi$  given in Eq. (14) and the metric constraint equations (7) and (11).

Note that the adiabatic-entropy decomposition becomes ill-defined if  $\dot{\sigma} = 0$ , i.e. both fields stop rolling, and this can cause numerical instability during preheating if the trajectory is confined to a narrow valley. This can occur, for instance, when  $\tilde{g} = 0$  and only the  $\phi$  field oscillates. The original field perturbations  $\delta\phi$  and  $\delta\chi$  remain well-defined, although the comoving curvature perturbation  $\mathcal{R}$ , defined in Eq. (37) becomes singular when  $\dot{\sigma} = 0$  [3]. This does not happen for the simulation results shown in Fig. 2 with  $\tilde{g} \neq 0$  where the fields oscillate in a two-dimensional potential well.

The massive inflaton potential ( $m \neq 0$ ) safeguards the conservation of  $\mathcal{R}$  by a bootstrap effect: if preheating is strong,  $q \gg 1$ , then the entropy perturbation is heavy during inflation; on the other hand, if the entropy is light during inflation, then  $q \leq 1$  and preheating is very weak. This is not altered by a rotation of the trajectory in field space ( $\tilde{g} \neq 0$ ) as can be most quickly seen by noting, from Eqs. (49) and (50), that

$$V_{\sigma\sigma} + V_{ss} = V_{\phi\phi} + V_{\chi\chi}. \quad (93)$$

Thus if the  $\chi$  field is very massive ( $V_{\chi\chi} \gg H^2$ ), we must have  $V_{\sigma\sigma} + V_{ss} \gg H^2$ . For slow-roll inflation we require  $V_{\sigma\sigma} \ll H^2$  and hence  $V_{ss} \gg H^2$ .

This situation does not hold if the entropy mass during inflation is not linked to the entropy mass during preheating [46], or in massless ( $m = 0$ ) self-interacting ( $\lambda \neq 0$ ) inflation models [48,38,49]. This latter class of models are almost conformally invariant, allowing analytical results from Floquet theory to be applied. The Floquet index,  $\mu_k$ , which determines the rate of exponential growth, can reach its maximum as  $k/aH \rightarrow 0$ , when  $g^2/\lambda = 2n^2$  for integer  $n$ , thereby implying maximum growth for the longest-wavelength perturbations. Assuming slow-roll inflation driven by  $V \approx \lambda \phi^4/4$ , we see from Eq. (93) that  $V_{\sigma\sigma} + V_{ss} > V_{\chi\chi} = g^2 \phi^2$  and thus that the entropy field is massive ( $V_{ss} > 9H^2/4$ ) whenever

$$\frac{g^2}{\lambda} > 8 \pi \frac{\phi^2}{M_{\text{Pl}}^2}. \quad (94)$$

However, we can have resonance at large scales for  $n = 1$  and  $g^2/\lambda = 2$ , when the entropy field need not be heavy dur-

ing inflation and no exponential suppression takes place, so that the subsequent growth of  $\mathcal{R}$  is explosive [48]. The growth of  $\mathcal{R}$  occurs before backreaction can shut off the resonant growth of the entropy perturbations  $\delta s$  [48,38,50,49]. Although the region of parameter space around  $g^2/\lambda = 2$  is thus ruled out, the same does not hold for  $g^2/\lambda \gg 1$ , since the entropy field is then heavy during inflation and  $\delta s$  is again suppressed.

## V. CONCLUSIONS

We have introduced a new formalism in which to follow the evolution of adiabatic and entropy perturbations during inflation with multiple scalar fields. We decompose arbitrary field perturbations into a component parallel to the background solution in field space, termed the *adiabatic* perturbation, and a component orthogonal to the trajectory, termed the *entropy* perturbation. We have rederived the field equations in terms of these rotated fields in Eqs. (52) and (55). These show that the adiabatic perturbation on large scales can be driven by the entropy perturbation, while the entropy perturbation itself obeys a homogeneous second-order equation on super-Hubble scales. There can only be significant change in the large-scale comoving curvature perturbation if there is a non-negligible entropy perturbation, *and* if the background trajectory in field space is curved.

Our formalism can be applied to evaluate the correlation between the adiabatic and entropy perturbations at the end of inflation. As an example we considered the example of two non-interacting fields in double inflation, calculating the cross correlation between the adiabatic and entropy perturbations.

The effect of preheating on the large-scale curvature perturbation can also be addressed within our formalism. The mass of the entropy field during inflation is a crucial quantity. If the entropy field is heavy, then any fluctuations on large scales are suppressed to negligible values at the beginning of preheating. This squeezing of the entropy perturbation is most accurately modeled numerically using our evolution equation for the entropy perturbation. If it is estimated from the usual field equations, it may contain large numerical errors when there is a non-trivial background trajectory in field space.

*Note added.* After completing this work we became aware of related work by Hwang and Noh [51] who also study entropy perturbations in multiple field inflation. They find that the adiabatic and entropy modes decouple on super-horizon scales when the effect of curvature of the trajectory in field space is neglected, but we have shown that this cannot in general be assumed, even in models of slow-roll inflation.

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