PHYSICAL REVIEW D, VOLUME 63, 017302

Heavy neutrino mixing effects in helicity amplitudes for the process $\mu^+\mu^- \to W^+W^-$

J. K. Singhal, Sardar Singh, Ashok K. Nagawat,* and N. K. Sharma
Department of Physics, University of Rajasthan, Jaipur-302004, India
(Received 20 June 2000; published 30 November 2000)

The helicity amplitudes for the process $\mu^+\mu^- \to W^+W^-$ are reevaluated with the inclusion of heavy neutrino mixing with the standard light neutrino. The effects of mixing are parametrized as a fractional change $\Delta\widetilde{M}^{*\nu}$ in the helicity amplitude for the t-channel ν_μ exchange. The behavior of $\Delta\widetilde{M}^{*\nu}$ is examined. It is found that for a heavy neutrino mass $m_N = 10$ GeV and scattering angle $\theta = 90°$, $\Delta\widetilde{M}^{*\nu} = 0.007|U_{\nu N}|^2$ at $\sqrt{s} = 200$ GeV and $\Delta\widetilde{M}^{*\nu} = 0.00002|U_{\nu N}|^2$ at $\sqrt{s} = 800$ GeV. The heavy neutrino mixing effects tend to vanish in the limit $m_N \ll \sqrt{s}$ even if the mixing element $U_{\nu N}$ is large.

DOI: 10.1103/PhysRevD.63.017302 PACS number(s): 13.15.+g, 14.80.−j

I. INTRODUCTION

Neutral heavy leptons or heavy neutrinos (HNs) are predicted by various extensions of the standard model (SM) [1–3]. These HNs can mix with standard light neutrinos [3]. Although recent searches for HNs reveal no evidence in the mass range 0.25–2.0 GeV, the existence of more heavy neutrinos is not ruled out [4].

The physics potential of high energy muon colliders [5,6] has opened the prospect for a detailed study of W-boson pair production in the process $\mu^+\mu^- \to W^+W^-$. In the SM this occurs at the tree level through the dominant s-channel γ, Z exchange [Fig. 1(a)] and t-channel neutrino (ν_μ) exchange [Fig. 1(b)]. The Higgs-boson exchange [Fig. 1(c)] cancels the bad high energy behavior for the production of longitudinally polarized W bosons in case of massive muons (growth of s-wave scattering amplitude as $\sqrt{s}$). If mixing between light and heavy neutrinos exists, it results in two effects: (i) the $W_{\nu\mu}$ couplings are modified (reduced) and (ii) an additional t-channel heavy neutrino (N_μ) exchange [Fig. 1(d)] contributes. With a view to discern the heavy neutrino mixing effects, we evaluate in this Brief Report the helicity amplitudes for the process $\mu^+\mu^- \to W^+W^-$ with the inclusion of heavy neutrino mixing. We parametrize the mixing effects as a fractional change in the helicity amplitude contribution from the neutrino exchange diagram. We retain the muon mass terms to examine the effects of mixings on the cancellation of bad high energy behavior.

II. HEAVY NEUTRINO MIXING AND MODIFIED HELICITY AMPLITUDES

A. Mixing model

A general discussion of mixing between known neutrino fields and new heavy neutrino fields is available in Ref. [7]. When mixings are allowed, the neutrino mass eigenstates (ν_μ, N_μ) are related to the weak interaction eigenstates (ν_μ^0, N_μ^0) by a unitary transformation [8]

$$\begin{pmatrix} \nu_\mu \\ N_\mu \end{pmatrix} = \begin{pmatrix} U_{\nu\nu} & U_{\nu N} \\ U_{N\nu} & U_{NN} \end{pmatrix} \begin{pmatrix} \nu_\mu^0 \\ N_\mu^0 \end{pmatrix}, \tag{1}$$

where the matrix U diagonalizes the neutrino mass matrix. The elements U_{ij} are constrained by the relations

$$|U_{\nu\nu}|^2 = 1 - |U_{\nu N}|^2, \quad |U_{NN}|^2 = 1 - |U_{N\nu}|^2,$$

$$U_{\nu\nu}U_{N\nu}^* = -U_{\nu N}U_{NN}^*. \tag{2}$$

The heavy neutrino mixing when included in the SM theory does not change the (i) $WW\gamma$, (ii) WWZ, (iii) $\gamma\mu\mu$, and (iv) $Z\mu\mu$ couplings. However, the $W_{\nu\mu}$ coupling is modified. The SM interaction Lagrangian, relevant for ascertaining changes in $W_{\nu\mu}$ coupling in weak eigenstate basis, is

$$L_{\rm int} = \frac{e}{\sqrt{2}\sin\theta_W}[(\bar{\nu}_{\mu L}^0 \gamma^\mu \mu_L^0) W_\mu^+ + \text{H.c.}], \tag{3}$$

where e is the positron charge and θ_W is the weak mixing angle. Using Eq. (1), the $L_{\rm int}$ in the mass eigenstate basis becomes [9]

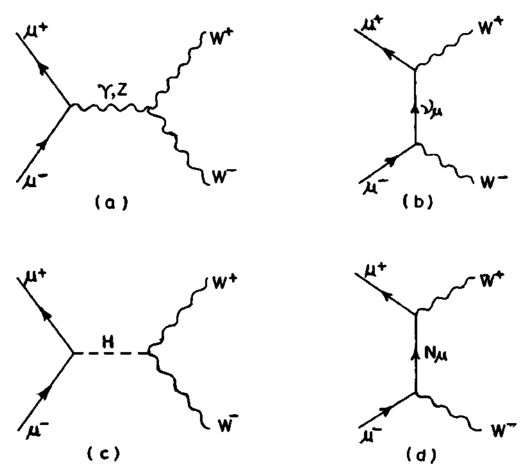

FIG. 1. (a) Dominant s-channel γ- and Z-exchange diagram, (b) t-channel ν_μ-exchange diagram, (c) Higgs-boson exchange diagram, and (d) additional t-channel N_μ-exchange diagram.

*Email address: nagawat@hotmail.com; nagawat@rediff.com

TABLE I. The explicit form of the $A^{\gamma,Z}_{\lambda\bar{\lambda}}$, $A'^{\gamma,Z}_{\lambda\bar{\lambda}}$, $B_{\lambda\bar{\lambda}}$, $B'_{\lambda\bar{\lambda}}$, $C_{\lambda\bar{\lambda}}$, $C'_{\lambda\bar{\lambda}}$, and $H_{\lambda\bar{\lambda}}$ coefficients.

$(\lambda,\bar{\lambda})$	$A^{\gamma,Z}_{\lambda\bar{\lambda}}$	$B_{\lambda\bar{\lambda}}$	$C_{\lambda\bar{\lambda}}$	$A'^{\gamma,Z}_{\lambda\bar{\lambda}}$	$B'_{\lambda\bar{\lambda}}$	$C'_{\lambda\bar{\lambda}}$	$H_{\lambda\bar{\lambda}}$
(+,0) (0,−)	2γ	2γ	$2(1+\beta)/\gamma$	1	1	$1-(\beta+\sigma)\beta^2$ $1-(\beta-\sigma)\beta^2$	-
(0,+) (−,0)	2γ	2γ	$2(1-\beta)/\gamma$	1	1	$1+(\beta+\sigma)\beta^2$ $1+(\beta-\sigma)\beta^2$	-
(+,+) (−,−)	1	1	$1/\gamma^2$	$1/2\gamma$	$\cos\theta/2\gamma$	$\cos\theta/2\gamma^3$	$-1/4\gamma$
(0,0)	$2\gamma^2+1$	$2\gamma^2$	$2/\gamma^2$	$(2\gamma^2+1)/2\gamma$	$\gamma(\beta+\cos\theta)$	$\cos\theta/\gamma^3$	$\gamma(1+\beta^2)/4$

$$L_{\rm int}=\frac{eU^*_{\nu\nu}}{\sqrt{2}\sin\theta_W}[(\bar{\nu}_{\mu L}\gamma^\mu\mu_L)W^+_\mu+\text{H.c.}]+\frac{eU^*_{\nu N}}{\sqrt{2}\sin\theta_W}[(\bar{N}_{\mu L}\gamma^\mu\mu_L)W^+_\mu+\text{H.c.}]. \qquad (4)$$

The heavy neutrino mixing (i) reduces the $W_{\nu\mu}$ coupling by a factor $U^*_{\nu\nu}$ and (ii) induces a $WN\mu$ coupling which allows an additional t-channel heavy neutrino (N_μ) exchange [Fig. 1(d)] contribution.

B. Helicity amplitudes

The helicity amplitudes for the process

$$\mu^-(k,\sigma)+\mu^+(\bar{k},\bar{\sigma})\rightarrow W^-(q,\lambda)+W^+(\bar{q},\bar{\lambda}) \qquad (5)$$

(where the arguments indicate the four-momenta and helicities of the respective particles) are calculated following the technique described by Hagiwara *et al.* for the process $e^+e^-\rightarrow W^+W^-$ [10]. We include heavy neutrino mixing and the Higgs-boson contribution and retain the muon mass terms to see the effects of heavy neutrino mixings on the cancellation of bad high energy behavior [11]. Following the Ref. [10], we separate the contributions to the helicity amplitudes from the various graphs in Fig. 1 as [12]

$$M_{\sigma,\bar{\sigma}\lambda\bar{\lambda}}(\theta)=\sqrt{2}e^2\tilde{M}_{\sigma,\bar{\sigma}\lambda\bar{\lambda}}(\theta)d^{J_0}_{\Delta\sigma,\Delta\lambda}(\theta), \qquad (6)$$

where $\Delta\lambda=\lambda-\bar{\lambda}$, $\Delta\sigma=(\sigma-\bar{\sigma})/2$, $J_0=\max(|\Delta\sigma|,|\Delta\lambda|)$, and θ is the scattering angle of W^- with respect to the μ^- direction in the $\mu^+\mu^-$ c.m. frame. The $d^{J_0}_{\Delta\sigma,\Delta\lambda}$ are the d functions [13]. J_0 is the minimum angular momentum of the system. The amplitude $\tilde{M}_{\sigma\bar{\sigma}\lambda\bar{\lambda}}$ includes contributions from the γ, Z, ν_μ, N_μ, and H exchange diagrams of Fig. 1, i.e.,

$$\tilde{M}=\tilde{M}^\gamma+\tilde{M}^Z+\tilde{M}^{*\nu}+\tilde{M}^{*N}+\tilde{M}^H, \qquad (7)$$

where the asterisk on the ν and N contributions is to remind us of mixing-modified terms. The $\tilde{M}^\gamma$, $\tilde{M}^Z$, and $\tilde{M}^H$ contributions are not affected by the heavy neutrino mixings. The explicit forms of these contributions [with $\beta=\sqrt{1-(4m_W^2/s)}$, $\gamma=\sqrt{s}/2m_W$, $\sqrt{s}=$ total c.m. energy, and m_μ, m_W, m_Z, and m_H the masses of the muon, W, Z, and H bosons, respectively] are

$$\tilde{M}^\gamma=-\beta\delta_{|\Delta\sigma|,I}A^\gamma_{\lambda\tilde{\lambda}}\delta_{J_{0,1}}-\beta\delta_{\Delta\sigma,0}\frac{\sqrt{2}m_\mu}{m_W}\times A'^\gamma_{\lambda\tilde{\lambda}}(\delta_{J_0,1}+\cos\theta_{J_0,0}), \qquad (8)$$

$$\tilde{M}^Z=\beta\left(\delta_{|\Delta\sigma,1|}-\frac{\delta_{\Delta\sigma,-1}}{2\sin^2\theta_W}\right)A^Z_{\lambda\tilde{\lambda}}\frac{s}{s-m_Z^2}\times\delta_{J_0,1}+\beta\delta_{\Delta\sigma,0}\frac{\sqrt{2}m_\mu}{m_W}\left(1-\frac{1}{4\sin^2\theta_W}\right)\times A^{A'Z}_{\lambda\tilde{\lambda}}\frac{s}{s-m_Z^2}(\delta_{J_0,1}+\cos\theta\,\delta_{J_0,0}), \qquad (9)$$

$$\tilde{M}^H=-\delta_{\Delta\sigma,0}\frac{\sqrt{2}m_\mu}{m_W}\frac{1}{2\sin^2\theta_W}H_{\lambda\tilde{\lambda}}\frac{s}{s-m_H^2}\delta_{J_0,0}. \qquad (10)$$

Here the first term in $\tilde{M}^\gamma,\tilde{M}^Z$ is the same as that in Ref. [10]. The second term in $\tilde{M}^\gamma,\tilde{M}^Z$ and the contribution from $\tilde{M}^H$ arise due to the retention of muon mass terms [11]. The coefficients $A_{\lambda\bar{\lambda}}$, $A'_{\lambda\bar{\lambda}}$, and $H_{\lambda\bar{\lambda}}$ are given in Table I. The neutrino-mixing-modified helicity amplitudes are

$$\tilde{M}^{*\nu}=|U_{\nu\nu}|^2\tilde{M}^\nu \qquad (11)$$

and

$$\tilde{M}^{*N}=|U_{\nu N}|^2\frac{1+\beta^2-2\beta\cos\theta}{1+\beta^2-2\beta\cos\theta+(4m_N^2/s)}\tilde{M}^\nu, \qquad (12)$$

with

$$\tilde{M}^{\nu}=\frac{-\sqrt{2}}{\sin^2\theta_W}\frac{\delta_{J_0,2}}{1+\beta^2-2\beta\cos\theta}\left(\delta_{\Delta\sigma,-1}+\delta_{\Delta\sigma,0}\sqrt{\frac{2}{3}\frac{m_\mu}{\sqrt{s}}}\right)$$
$$+\frac{1}{2\sin^2\theta_W\beta}\left[\delta_{\Delta\sigma,-1}\left(B_{\lambda\tilde{\lambda}}-\frac{C_{\lambda\tilde{\lambda}}}{1+\beta^2-2\beta\cos\theta}\right)\right.$$
$$\times\delta_{J_0,1}+\delta_{\Delta\sigma,0}\frac{\sqrt{2}m_\mu}{m_W}\frac{1}{2}\left(B'_{\lambda\tilde{\lambda}}-\frac{C'_{\lambda\tilde{\lambda}}}{1+\beta^2-2\beta\cos\theta}\right)$$
$$\left.\times(\delta_{J_0,1}+\delta_{J_0,0})\right]. \tag{13}$$

Here the $\tilde{M}^{\nu}$ is the contribution of the t-channel neutrino exchange in the absence of neutrino mixing. The terms without m_μ are the same as those in Ref. [10]. The extra terms with $\delta_{\Delta\sigma,0}$ arise due to retention of muon mass. The coefficients $B_{\lambda\bar{\lambda}}$, $B'_{\lambda\bar{\lambda}}$, $C_{\lambda\bar{\lambda}}$, and $C'_{\lambda\bar{\lambda}}$ are given in Table I.

III. HEAVY NEUTRINO MIXING EFFECTS AND HIGH ENERGY BEHAVIOR

We combine the heavy neutrino mixing modified helicity amplitudes $\tilde{M}^{*\nu}$ and $\tilde{M}^{*N}$ in the following form [using Eq. (2)]:

$$\tilde{M}^{*\nu}+\tilde{M}^{*N}=\tilde{M}^{\nu}(1-\Delta\tilde{M}^{*\nu}), \tag{14}$$

where

$$\Delta\tilde{M}^{*\nu}=|U_{\nu N}|^2\left[1-\frac{1+\beta^2-2\beta\cos\theta}{1+\beta^2-2\beta\cos\theta+(4m_N^2/2)}\right]. \tag{15}$$

The parameter $\Delta\tilde{M}^{*\nu}$ aggregates the heavy neutrino mixing effects although heavy neutrino mixing adds one more t-channel heavy neutrino exchange diagram, but the overall effect is to reduce the neutrino exchange contribution in the helicity amplitude [Eq. (14)]. We note the following.

(i) In the presence of heavy neutrino mixing, the helicity amplitudes $\tilde{M}^{\gamma}$, $\tilde{M}^{Z}$, and $\tilde{M}^{H}$ remain unaffected, while the t-channel neutrino exchange helicity amplitude $\tilde{M}^{\nu}$ is modified to $\tilde{M}^{\nu}(1-\Delta\tilde{M}^{*\nu})$.

(ii) The fractional change $\Delta\tilde{M}^{*\nu}$ in the helicity amplitude for t-channel neutrino exchange, occurring due to heavy neutrino mixing, is always less than $|U_{\nu N}|^2$, where $U_{\nu N}$ is the heavy neutrino mixing parameter [see Eq. (15)]. In Fig. 2, the dependence of $\Delta\tilde{M}^{*\nu}/|U_{\nu N}|^2$ on (a) $\sqrt{s}$ for different m_N values, (b) m_N for different $\sqrt{s}$ values, and (c) θ for a fixed $\sqrt{s}$ and different m_N values is shown.

(iii) In the high energy limit, the heavy neutrino mixing effects tend to vanish (even if mixing is large); that is, for $\sqrt{s}\gg m_N$, $4m_N^2/s\to 0$, and as a result $\Delta\tilde{M}^{*\nu}\to 0$.

(iv) Since the muons are massive and may therefore be found in wrong helicity states, the s-wave scattering amplitude which exists in this case (case $J_0=0$) grows as $\sqrt{s}$ for the production of longitudinally polarized W bosons [$(\lambda,\bar{\lambda})=(0,0)$]. In the SM, this is canceled by the H-exchange contribution. To see the effects of heavy neutrino mixing on this cancellation, we consider $\mu^+_{R(L)}\mu^-_{R(L)}\to W^+_L W^-_L$, that is the $J_0=0$, $\Delta\sigma=0$ case. For $\theta=90°$, from Eqs. (8)–(13) we have $\tilde{M}^{\gamma}=0$, $\tilde{M}^{Z}=0$,

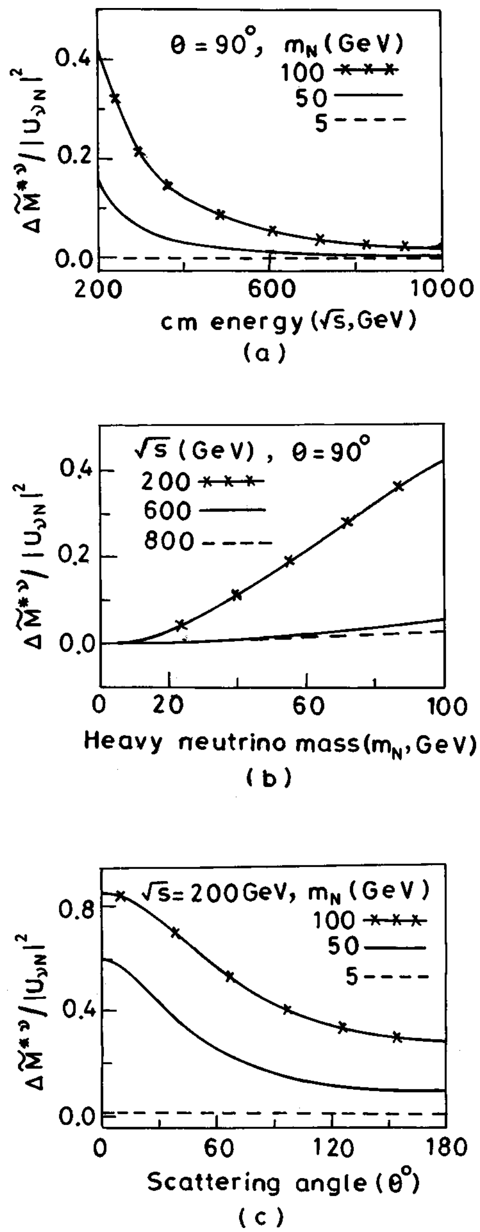

FIG. 2. The variation of $\Delta\tilde{M}^{*\nu}/|U_{\nu N}|^2$ with (a) $\sqrt{s}$, the c.m. energy, (b) m_N, heavy neutrino mass, and (c) scattering angle (θ).

$$\tilde{M}^{H}=-\frac{m_\mu}{m_W^2}\frac{1}{4\sqrt{2}\sin^2\theta_W}\left(1-\frac{2m_W^2}{s}\right)\left(\frac{s}{s-m_H^2}\right)\sqrt{s}$$

and

$$\tilde{M}^{*\nu}+\tilde{M}^{*N}=\frac{m_\mu}{m_W^2}\frac{1}{4\sqrt{2}\sin^2\theta_W}(1-\Delta\tilde{M}^{*\nu})\sqrt{s}.$$

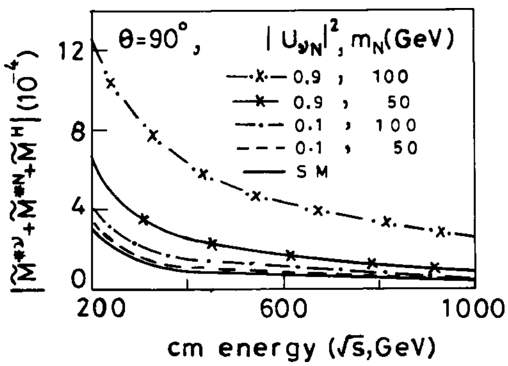

FIG. 3. The variation of $|\widetilde{M}^{*\nu}+\widetilde{M}^{*N}+\widetilde{M}^{H}|$ with $\sqrt{s}$ for various values of the heavy neutrino mass m_N and mixing parameter $|U_{\nu N}|^2$ for the process $\mu^{+}_{R(L)}\mu^{-}_{R(L)}\rightarrow W_L^+W_L^-$ at $\theta=90°$.

We plot in Fig. 3 the helicity amplitude $|\widetilde{M}^{*\nu}+\widetilde{M}^{*N}+\widetilde{M}^{H}|$ as a function of $\sqrt{s}$ for heavy neutrino mass $m_N=50\,\mathrm{GeV}$, 100 GeV and $|U_{\nu N}|^2=0.1$ (small mixing), $|U_{\nu N}|^2=0.9$ (large mixing) [2] along with SM expectations. We take $m_\mu=105.658\,\mathrm{MeV}$, $m_W=80.41\,\mathrm{GeV}$ [2]. For the Higgs-boson mass, we take a representative value $m_H=125\,\mathrm{GeV}$ [14]. Our conclusions are summarized below.

IV. CONCLUSIONS

We considered heavy neutrino mixing effects in the helicity amplitudes for the process $\mu^+\mu^-\rightarrow W^+W^-$. We parametrized the mixing effects of heavy neutrino mixing as a fractional change $\Delta\widetilde{M}^{*\nu}$ in the helicity amplitude contribution from the *t*-channel ν_μ exchange diagram. We find the following.

(i) $\Delta\widetilde{M}^{*\nu}<|U_{\nu N}|^2$, where $U_{\nu N}$ is the heavy neutrino mixing parameter.

(ii) The dependence of $\Delta\widetilde{M}^{*\nu}$ on (a) $\sqrt{s}$, (b) m_N, and (c) θ is as shown in Fig. 2.

(iii) We note from the data for Fig. 2 that for $m_N=10\,\mathrm{GeV}$ at $\sqrt{s}=200\,\mathrm{GeV}$, $\Delta\widetilde{M}^{*\nu}=0.007|U_{\nu N}|^2$ and for $m_N=10\,\mathrm{GeV}$ at $\sqrt{s}=800\,\mathrm{GeV}$, $\Delta\widetilde{M}^{*\nu}=0.00002|U_{\nu N}|^2$. Thus, even if the neutrino mixing parameter $U_{\nu N}$ is large, say, $|U_{\nu N}|^2\sim 1$, then in the high energy limit ($\sqrt{s}=800\,\mathrm{GeV}$) the fractional change $\Delta\widetilde{M}^{*\nu}\sim 0.002\%$ for $m_N=10\,\mathrm{GeV}$ and $\Delta\widetilde{M}^{*\nu}\sim 0.8\%$ for $m_N=50\,\mathrm{GeV}$. At extremely high energies (i.e., $\sqrt{s}\gg m_N$), the heavy neutrino mixing effects tend to vanish ($\Delta\widetilde{M}^{*\nu}\rightarrow 0$).

(iv) From Fig. 3 we note that at low energies the neutrino mixing effectively reduces the neutrino exchange contribution, enhancing the value of $|\widetilde{M}^{*}(\text{neutrino})+\widetilde{M}(\text{Higgs boson})|$ contribution in comparison to the SM value. At high energies, the neutrino mixing effects decrease. In the limit $\sqrt{s}\rightarrow\infty$, as $\Delta\widetilde{M}^{*\nu}\rightarrow 0$, the neutrino mixing effects tend to vanish even if $|U_{\nu N}|^2$ is large.

ACKNOWLEDGMENTS

One of us (J.K.S.) acknowledges financial assistance received from the University Grants Commission, New Delhi, and Director, College Education, Jaipur for granting academic leave.

[1] M. Gronau, L. N. Leung, and J. L. Rosner, Phys. Rev. D **29**, 2539 (1984).

[2] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).

[3] R. E. Shrock, Phys. Rev. D **24**, 1232 (1981); P. Kalyniak and J. N. Ng, *ibid.* **24**, 1874 (1981); A. K. Nagawat, S. Singh, and N. K. Sharma, *ibid.* **42**, 2984 (1990), and references contained therein.

[4] A. Vaitaitis *et al.*, Phys. Rev. Lett. **83**, 4943 (1999).

[5] V. Barger, M. S. Berger, J. F. Gunion, and T. Han, Phys. Rev. D **55**, 142 (1997).

[6] M. S. Berger, ''Muon Collider Physics at Very High Energies,'' hep-ph/0001018; ''SUSY Thresholds at a Muon Collider,'' hep-ph/0003128.

[7] P. Langacker and D. London, Phys. Rev. D **38**, 886 (1988).

[8] In the neutrino mixing model considered here, the left- (right-) handed heavy neutrinos are mixed with left- (right-) handed standard neutrinos of the same family.

[9] We assume that the heavy neutrino N_μ is either in singlet or doublet representation. It may be noted that the couplings are only strictly correct when the heavy neutrinos are in singlet or doublet representations, as is assumed here.

[10] K. Hagiwara, R. D. Peccei, D. Zeppenfeld, and K. Hikasa, Nucl. Phys. **B282**, 253 (1987). The references for earlier important contributions to the subject are given therein.

[11] In Ref. [10], the Higgs exchange contribution is not included and also lepton mass terms are neglected.

[12] We omit a sign factor ϵ which is not required in the Jacob-Wick phase convention; see the footnote on p. 263 of Ref. [10].

[13] M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley, New York, 1957). The *d* functions are also given in Ref. [2], p. 183.

[14] D. Zeppenfeld, R. Kinnunen, A. Nikitenko, and E. Richter-Was, ''Measuring Higgs boson couplings at the LHC,'' Report No. CERN-TH/2000-039.