Large angle MSW solution in grand unified theories with SU(3)×U(1) horizontal symmetry

Ryuichiro Kitano*

Theory Group, KEK, Oho1-1, Tsukuba, Ibaraki 305-0801, Japan
and Department of Particle and Nuclear Physics, The Graduate University for Advanced Studies, Oho 1-1, Tsukuba,
Ibaraki 305-0801, Japan

Yukihiro Mimura†

Theory Group, KEK, Oho1-1, Tsukuba, Ibaraki 305-0801, Japan

We construct a model with an SU(3)×U(1) horizontal symmetry in the context of grand unified theories. In our models, the bimaximal lepton mixing and suitable neutrino masses for the large angle MSW solution are obtained without any fine tuning due to the spontaneously broken $SU(3)_H$ symmetry. The three generations of quarks and leptons are unified as members of the $SU(3)_H$ fundamental representation, and the $U(1)_H$ charge gives the origin of the fermion mass hierarchy and mixing angles. We present two explicit examples of SU(5) $_{GUT}$ and SO(10)$_{GUT}$ models, in which the Yukawa structures are given successfully.

I. INTRODUCTION

Flavor physics is a recent topic in particle physics. The Cabibbo-Kobayashi-Maskawa (CKM) [1] parameters will be measured precisely in *B*-factory experiments [2]. The recent SuperKamiokande data suggest that the mixing angles of the lepton sector are large in contrast with those in the quark sector [3]. For atmospheric neutrinos, $\nu_\mu \to \nu_\tau$ oscillations are favored and the best fit values are $\sin^2 2\theta = 1.0$ and $\Delta m^2 = 3.2 \times 10^{-3}$ eV^2 [4]. For the solar neutrino problem, the only solution which is not disfavored at 95% confidence level is the large angle Mikheev-Smirnov-Wolfenstein (MSW) solution [5], i.e., $\sin^2 2\theta \sim 1$ and $\Delta m^2 \sim 10^{-5} - 10^{-4}$ eV^2 [6]. If we assume three flavor neutrino mixings, the solar neutrino deficit is explained by $\nu_e \to \nu_\mu$ oscillations and the CHOOZ experiment gives a severe constraint on the mixing angle between the electron neutrino and the heaviest neutrino, $\sin^2 2\theta_{e3} \lesssim 0.1$ [7]. These two nearly maximal mixings and a small mixing suggest that the lepton mixing matrix is the so-called bimaximal type, which is a completely different structure than that of the CKM matrix. Understanding the patterns of the mixing and fermion masses is one of the most challenging puzzles for particle physicists.

One approach to answer this puzzle is to consider a horizontal flavor symmetry [8–10]. The horizontal symmetry provides the Yukawa coupling structure and enables us to predict the structure definitely.

The two generation model successfully explains the magnitude of the Cabibbo angle [11], namely

$$\sin \theta_C \sim \sqrt{\frac{m_d}{m_s}}. \tag{1}$$

We reproduce this relation by using a flavor symmetry. However, the simple extension to three generations does not

succeed. For example, the CKM matrix element V_{cb} is not the same order of $\sqrt{m_s/m_b}$ but is instead of order m_s/m_b [12].

The observation of the large mixing between $\nu_\mu - \nu_\tau$ plays an important role in understanding the 2-3 structure. The structure of the second and third generations are revealed to be nontrivial. The nonparallel family structure has been suggested [13], and this structure is also called lopsided family structure [14] and can be organized by an Abelian symmetry [9].

The lopsided family structure may give us a neutrino mass matrix which leads to large angle MSW solution for solar neutrino problem. However, the lopsided family structure does not always predict the large mixing angle unless the mass (squared) ratio is assumed to take a particular value. There is no reason to obtain large angle MSW solution in the lopsided structure.

We will give a simple lopsided family structure by imposing a $SU(3)_H \times U(1)_H$ horizontal symmetry. In our model, the fermion mass hierarchy is produced by the $U(1)_H$ symmetry. Contrary to the standard approach, we do not suppose that the vacuum expectation values (VEV's) of fundamental (antifundamental) representation of $SU(3)_H$ symmetry are nonhierarchical, and this feature provides large mixing angles in lepton sector.

In this paper, we assume a supersymmetric grand unified theory (SUSY GUT) [15] realized at the high energy scale ($M_G \sim 10^{16}$ GeV). The SUSY GUT is an attractive theory which explains many problems in the standard model. In particular, the tiny neutrino masses are elegantly explained by the seesaw mechanism [16] which requires heavy right-handed neutrinos with masses $M_N \sim 10^{14}$ GeV. This is just below the GUT scale. It is natural that the origin of the scale M_N is GUT physics, so that it is important that we understand the flavor structures in the context of GUT's.

This paper is organized as follows. In Sec. II, we give the Yukawa structures which are realized in the model and show that they reproduce the correct mass ratios and mixing angles for the quarks and leptons. In Sec. III, we give an example of the SU(5) GUT model and explain how we can obtain the Yukawa matrices. In Sec. IV, we construct the SO(10) GUT

---

*Email address: ryuichiro.kitano@kek.jp

†Present address: Department of Physics, Oklahoma State University, Stillwater, OK 74078. Email address: mimura@okstate.edu

model in which the large angle MSW-type mass structure is given. Section V is devoted to our conclusions.

II. STRUCTURE OF THE MODEL

We will construct the model to explain the CKM matrix, fermion masses, and lepton mixings suitable for the large angle MSW solution by means of a horizontal symmetry.

The observed CKM matrix, the quark mass ratios, and lepton mass ratios at GUT scale [17] are approximately expressed by powers of the Cabibbo angle $\lambda \sim 0.22$ as follows:

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 1 \end{pmatrix},$$

$$m_u:m_c:m_t \sim \lambda^7:\lambda^4:1, \quad m_d:m_s:m_b \sim \lambda^4:\lambda^2:1, \quad (2)$$

$$m_e:m_\mu:m_\tau \sim \lambda^5:\lambda^2:1.$$

For the neutrinos, the mass ratio is given by

$$m_{\nu_2}:m_{\nu_3} \sim \lambda^{1-2}:1. \quad (3)$$

We obtain Eq. (3) from the Δm^2 ratio of the atmospheric neutrino and large angle MSW solution, which is

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \sim 10^{-2} \sim \lambda^3. \quad (4)$$

The Maki-Nakagawa-Sakata (MNS) matrix [18] is nearly bi-maximal,

$$V_{MNS} \sim \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & \epsilon \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix}, \quad (5)$$

where the element ϵ is a small parameter constrained by the CHOOZ experiment.

We construct a supersymmetric SU(5) GUT model with a $SU(3)_H \times U(1)_H$ horizontal symmetry whose spontaneous breaking induces the generation mixing and the mass differences in Eqs. (2), (3), and (5). The quarks and leptons are unified as $SU(3)_H$ triplets, which explains why there are three generations.

In our model, the Yukawa couplings may be expressed in the following form:

$$\sum_{i,j=1}^{3} \left(\frac{\Phi}{M_{Pl}} \right)^{x_i+x_j} \xi_i \xi_j^T \frac{1}{M_*^2}. \quad (6)$$

The three[1] ξ's are $SU(3)_H$ antifundamental representations whose VEV's break the horizontal symmetry at the scale

---

[1] In our models, we prepare three pairs of ξ's, though minimal pairs to break $SU(3)_H$ is two. Of course, in absence of ξ_3, the exterior product $\bar{\xi}_1 \times \bar{\xi}_2$ plays a role of ξ_3 effectively. However, such a minimal choice has a difficulty in constructing realistic models.

M_*. The Φ is $SU(3)_H$ singlet field, and has nonvanishing $U(1)_H$ charge. The VEV of Φ is λM_{Pl} and provides a hierarchy in the Yukawa coupling as follows.

With the appropriate $U(1)_H$ charge assignment of ξ fields as shown later in Secs. III and IV, the Yukawa coupling of up-type quark is given by

$$Y_u \sim (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix} \Big/ M_*^2. \quad (7)$$

The VEV's of ξ's are given without loss of generality by

$$\langle \xi_1 \rangle = M_* \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix}, \quad \langle \xi_2 \rangle = M_* \begin{pmatrix} 0 \\ b \\ c \end{pmatrix}, \quad \langle \xi_3 \rangle = M_* \begin{pmatrix} d \\ e \\ f \end{pmatrix}, \quad (8)$$

where the $a \to f$ are parameters of the order unity. In this basis, Y_u is given by

$$Y_u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (9)$$

where we omit the parameters of order unity. This Yukawa matrix gives the mass ratio for the up-type quarks as

$$m_u:m_c:m_t \sim \lambda^6:\lambda^4:1, \quad (10)$$

which is consistent with Eq. (2) except for the up quark but it is maybe within the uncertainty of parameters. The prediction for the magnitude of the top quark Yukawa coupling is also consistent with the experimental value $Y_t \sim 1$.

The Yukawa matrix for the down-type quarks and charged leptons are given by

$$Y_d = Y_e^T \sim (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda & \lambda \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix} \Big/ M_*^2$$

$$\sim \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda & \lambda \end{pmatrix}. \quad (11)$$

This Yukawa structure gives the down-type quark and charged lepton mass ratios as follows:

$$m_d:m_s:m_b = m_e:m_\mu:m_\tau \sim \lambda^4:\lambda^2:1. \quad (12)$$

This is consistent with Eq. (2). The difference between the charged lepton masses and down-type quark masses may be explained by the introduction of a new Higgs field such as the SU(5) **45** representation in the usual way [19].

Inputting the bottom quark mass, this model predicts the VEV ratio of the two Higgs doublets as

$$\tan \beta \equiv \frac{\langle H \rangle}{\langle \bar{H} \rangle} \sim 17. \tag{13}$$

We obtain the correct CKM matrix which comes from Yukawa matrices (9) and (11) as

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 1 \end{pmatrix}. \tag{14}$$

The interesting point is that we can obtain the bimaximal mixing of leptons without any fine tuning in the following mass matrix. The neutrino mass matrix is proportional to

$$(\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^2 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^4 \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix} \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}. \tag{15}$$

We can see in Eq. (15) that the $SU(3)_H$ breaking VEV's give the large mixing for the second and third generations. Moreover, the first and second generation mixing is also large. The easiest way to understand the large first and second generation mixing is by changing the basis in Eq. (8) to

$$\langle \xi_1 \rangle \sim M_* \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \langle \xi_2 \rangle \sim M_* \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \langle \xi_3 \rangle \sim M_* \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{16}$$

This change of basis corresponds to removing the large mixing of second and third generations from the mass matrix (15). In this basis, the neutrino mass matrix (15) is replaced by

$$\begin{pmatrix} \lambda^2 & \lambda^2 & \lambda \\ \lambda^2 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{pmatrix}. \tag{17}$$

It turns out that this matrix gives the large first and second generation mixing.

On the other hand, in this basis, the charged lepton Yukawa matrix is obtained from Eqs. (11) and (16) as

$$Y_e' \sim \begin{pmatrix} \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^4 & \lambda & \lambda \\ \lambda^4 & \lambda & \lambda \end{pmatrix}, \tag{18}$$

which gives the large mixing between the second and third generations. Therefore the lepton mixing matrix is bimaximal. The mass ratios of neutrinos are also suitable for the large angle MSW solution and are as follows:

$$m_{\nu 1} : m_{\nu 2} : m_{\nu 3} \sim \lambda^4 : \lambda^2 : 1. \tag{19}$$

The model predicts that the mixing between the first and the third generations is of order λ and thus small. It is interesting

that the mixing angle is comparable to the CHOOZ bound, and will be observed in future long baseline experiments.

This natural derivation of the bimaximal mixing is due to the $SU(3)_H$ symmetry. The conventional way to obtain the bimaximal mixing by an Abelian symmetry requires an accidental cancellation in the determinant. Consider the case that the neutrino mass matrix is given by

$$\begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix}. \tag{20}$$

This type of mass matrix can be given by the Abelian flavor symmetry models in which the second and the third generations have the same charges. If there is an accidental cancellation in the determinant of the 2-3 submatrix so that the eigenvalues of this submatrix are of order λ^2 and unity, the matrix in which the 2-3 submatrix is diagonalized by the large mixing is given by

$$\begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^2 & 0 \\ \lambda^2 & 0 & 1 \end{pmatrix}. \tag{21}$$

This gives the large first and second generation mixing if there is no cancellation in the (1,2) component while diagonalizing the 2-3 submatrix, and the correct mass ratio for the MSW solution is reproduced. However, without the accidental cancellation, the mixing angle between the first and second generation is naturally of order λ^2 and the mass ratios are unacceptable, scaling as $\lambda^2 : 1 : 1$. As we have seen above, this cancellation can be controlled by the non-Abelian horizontal symmetry.

III. MODEL

In this section, we present a SUSY SU(5) GUT model, in which the Yukawa structure in the previous section is reproduced.

TABLE I. The particle contents of the SU(5) model.

	$SU(5)_{GUT}$	$SU(3)_H$	$U(1)_H$	$U(1)_{PQ}$
10	**10**	**3**	$-1/4$	1
$\bar{5}$	$\bar{\mathbf{5}}$	**3**	$3/4$	1
H	**5**	**1**	-1	0
$\bar{H}$	$\bar{\mathbf{5}}$	**1**	1	0
ξ_1	**1**	$\bar{\mathbf{3}}$	$-1/4$	-1
ξ_2	**1**	$\bar{\mathbf{3}}$	$-9/4$	-1
ξ_3	**1**	$\bar{\mathbf{3}}$	$-13/4$	-1
$\bar{\xi}_1$	**1**	**3**	$1/4$	1
$\bar{\xi}_2$	**1**	**3**	$9/4$	1
$\bar{\xi}_3$	**1**	**3**	$13/4$	1
T	**10**	**1**	$-1/2$	0
$\bar{T}$	$\overline{\mathbf{10}}$	**1**	$1/2$	0
F	**5**	**1**	$3/2$	0
$\bar{F}$	$\bar{\mathbf{5}}$	**1**	$-3/2$	0
G	**5**	**1**	$1/2$	0
$\bar{G}$	$\bar{\mathbf{5}}$	**1**	$-1/2$	0
N	**1**	**1**	$5/2$	0
Φ	**1**	**1**	1	0
$\bar{\Phi}$	**1**	**1**	-1	0

The particle content of this model is listed in Table I. The $U(1)_H$ charge assignment is an example of realistic models. The $U(1)_H$ symmetry is anomaly free with respect to the $SU(5)_{GUT}$, therefore the Nambu-Goldstone boson associated with $U(1)_H$ break down is purely massless and harmless [20].

The fields 10 and $\bar{5}$ are the usual matter fields as follows:

$$10:(q, u^c, e^c), \quad \bar{5}:(d^c, l). \tag{22}$$

The fields H and $\bar{H}$ are the Higgs fields. The $SU(2)_L$ doublet parts of these fields remain as the usual Higgs doublets at the electroweak scale and their VEV's give the masses of the quarks and leptons. The vectorlike fields T, F, G, and N are the Froggatt-Nielsen (FN) fields which generate the usual Yukawa interaction terms for quarks and leptons by being integrated out [8]. The $SU(5)$ and $SU(3)_H$ singlet fields Φ and $\bar{\Phi}$ are the origin of the mass hierarchy and mixing angle of the quarks and leptons through the nonrenormalizable interactions.

The superpotential which creates the Yukawa couplings for the matter fields is constructed as follows:

$$W = W_{matter} + W_{Higgs} + W_{mass}. \tag{23}$$

The matter part W_{matter} contains $SU(3)_H$ nonsinglet fields. It is given by

$$W_{matter} = \left\{ \xi_1 + \left(\frac{\Phi}{M_{Pl}}\right)^2 \xi_2 + \left(\frac{\Phi}{M_{Pl}}\right)^3 \xi_3 \right\} \cdot 10\,\bar{T}$$

$$+ \left\{ \left(\frac{\bar{\Phi}}{M_{Pl}}\right)^2 \xi_1 + \xi_2 + \left(\frac{\Phi}{M_{Pl}}\right) \xi_3 \right\} \cdot \bar{5}\,F$$

$$+ \left\{ \left(\frac{\bar{\Phi}}{M_{Pl}}\right) \xi_1 + \left(\frac{\Phi}{M_{Pl}}\right) \xi_2 + \left(\frac{\Phi}{M_{Pl}}\right)^2 \xi_3 \right\} \cdot \bar{5}\,G, \tag{24}$$

where "$\cdot$" is the inner product of the $SU(3)_H$ indices. We consider the case in which the fields Φ and $\bar{\Phi}$ acquire VEV's as follows:

$$\langle \Phi \rangle = \langle \bar{\Phi} \rangle \sim \lambda M_{Pl}. \tag{25}$$

In this case, the $U(1)_H$ symmetry breaks down at the scale of λM_{Pl} and the factor Φ/M_{Pl} in the superpotential can be replaced by the $\lambda \sim 0.22$ which originates the fermion masses and mixings.

The Higgs part W_{Higgs} contains the Yukawa interaction terms of the FN fields and Higgs fields H and $\bar{H}$ and may be written as

$$W_{Higgs} = THT + \left(\frac{\Phi}{M_{Pl}}\right) T\bar{H}\bar{F} + T\bar{H}\bar{G} + \bar{F}HN + \left(\frac{\bar{\Phi}}{M_{Pl}}\right) \bar{G}HN. \tag{26}$$

The last term W_{mass} in Eq. (23) is the mass term for the FN fields and Higgs fields:

$$W_{mass} = M_* T\bar{T} + M_* F\bar{F} + M_* G\bar{G} + \frac{\bar{\Phi}^5}{M_{Pl}^4} NN$$

$$+ M_* \sum_{i=1}^{3} \xi_i \bar{\xi}_i + M_{GUT} H\bar{H}, \tag{27}$$

where M_* is the FN scale which can be naturally of the order of M_{GUT} and it can arise from the VEV of a singlet field, but we do not specify the scale and its origin. The Majorana mass for the FN field N is given by $\lambda^5 M_{Pl} \sim 10^{14}$ GeV. It is suitable to give the neutrino masses by the seesaw mechanism. The Majorana mass can be given in another way. If M_* is obtained by the singlet VEV and the Planck suppressed mass term in Eq. (27) is forbidden by some symmetry like Z_3, the Majorana mass term can be given by $(\Phi/M_{Pl})^X M_* NN$ and the magnitude can be controlled by M_* and X, which is determined by the $U(1)_H$ charge of N. Therefore, hereafter, we replace the Majorana mass term with $M_N NN$, where $M_N \sim 10^{14}$ GeV.

We can see that $U(1)_{PQ}$ symmetry exists in this superpotential, and the charge assignments are listed in Table I. The nonzero value of the Peccei-Quinn charge of the ξ's indicates that the $SU(3)_H$ breaking simultaneously induces the $U(1)_{PQ}$ breaking, which solves the strong CP problem and creates the axion dark matter if the breaking scale M_* is around 10^{12} GeV [21–23]. In this sense, $M_* \sim M_{GUT}$ may be

unacceptable due to the creation of too many axions. A way out is to dilute out them by the inflation of the universe which is also needed by the GUT monopole dilution [24].

Now we show that the fermion masses and mixings are reproduced by the spontaneous breaking of the horizontal symmetry.

First, let us consider the up-type quarks. The up-type Yukawa couplings are given by the Feynman diagram in Fig. 1. We can extract the Yukawa matrix from this diagram as follows:

$$
Y_u \sim \frac{1}{M_*^2} (\lambda^3 \langle \xi_3 \rangle + \lambda^2 \langle \xi_2 \rangle + \langle \xi_1 \rangle) y_T (\lambda^3 \langle \xi_3 \rangle + \lambda^2 \langle \xi_2 \rangle + \langle \xi_1 \rangle)^T
$$

$$
\sim \frac{1}{M_*^2} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^3 \\ \lambda^2 \\ 1 \end{pmatrix}
$$

$$
\lambda^3 x(\lambda^2) y_T (\lambda^3 \quad \lambda^2 \quad 1) \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix}, \tag{28}
$$

where y_T is the Yukawa coupling constant of THT in Eq. (26). As we can see in Eq. (28), the rank of the Y_u matrix is one, which means that only one of the quarks can acquire the nonvanishing mass while the others remain massless. This situation usually occurs in the FN mechanism. In order to avoid this, more than three pairs of FN fields T and $\bar{T}$ are necessary. Then, the coupling y_T is a matrix and all the components are naturally of order unity in the basis that the mass term $M_* T\bar{T}$ is diagonal. In the case that there are three pairs[2] of T and $\bar{T}$, the Yukawa couplings are given by

$$
Y_u \sim \frac{1}{M_*^2} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}
$$

$$
\times \begin{pmatrix} \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix}
$$

$$
\sim \frac{1}{M_*^2} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix}. \tag{29}
$$

The second and the fourth matrix in the first expression in Eq. (29) is the coupling matrix between 10 and $\bar{T}$ fields and the center matrix is the y_T matrix. This Y_u matrix has the same structure as Eq. (7).

----

[2]We can obtain the same result for the case where there are more than three pairs of FN fields.

$\xi_1 + \lambda^2 \xi_2 + \lambda^3 \xi_3$ $\xi_1 + \lambda^2 \xi_2 + \lambda^3 \xi_3$

$10 \quad \bar{T} \quad \times \quad T \quad | \quad T \quad \times \quad \bar{T} \quad 10$

H
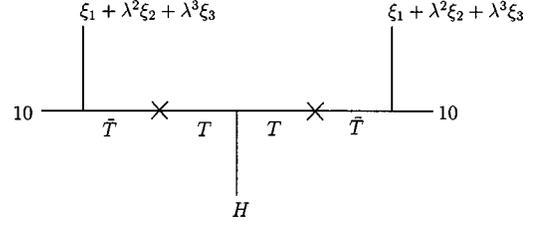
FIG. 1. The Feynman diagram for the up-type Yukawa interactions.

For the down-type quarks and charged leptons, there are two Feynman diagrams because we introduce two kinds of $SU(5)_{GUT}$ **5** representation FN fields F and G, and again more than three pairs of these fields are necessary. The introduction of two kinds of **5** fields is the essential point of this model. The field G gives the down-type quarks and charged leptons mass matrices and F gives the neutrino mass matrix suitably without disturbing each other. The two diagrams are shown in Fig. 2. From the upper diagram, which is G contribution, the Yukawa structure is given as

$$
Y_d^G = (Y_e^G)^T \sim \frac{1}{M_*^2} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix}
$$

$$
\times \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda^2 & \lambda & \lambda \\ \lambda^2 & \lambda & \lambda \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix}
$$

$$
\sim \frac{1}{M_*^2} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda & \lambda \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix}. \tag{30}
$$

The F contribution (the lower diagram) is

$$
Y_d^F = (Y_e^F)^T \sim \frac{1}{M_*^2} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix}
$$

$$
\times \begin{pmatrix} \lambda & \lambda & \lambda \\ \lambda & \lambda & \lambda \\ \lambda & \lambda & \lambda \end{pmatrix} \begin{pmatrix} \lambda & 1 & \lambda^2 \\ \lambda & 1 & \lambda^2 \\ \lambda & 1 & \lambda^2 \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix}
$$

$$
\sim \frac{1}{M_*^2} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^6 \\ \lambda^4 & \lambda^3 & \lambda^5 \\ \lambda^2 & \lambda & \lambda^3 \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix}. \tag{31}
$$

Then, the Yukawa couplings are given as follows:
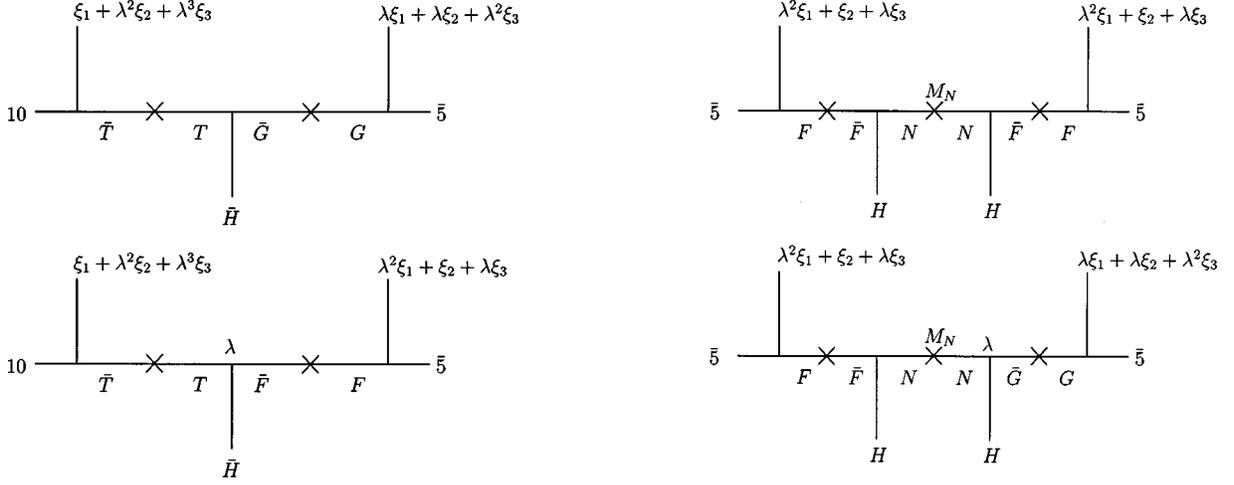
FIG. 2. The Feynman diagrams for the down-type and charged lepton Yukawa interactions.

$$Y_d = Y_e^T \sim Y_d^G + Y_d^F \sim \frac{1}{M_*^2} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle)$$

$$\times \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda & \lambda \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix}. \tag{32}$$

This reproduces the Yukawa structure in Eq. (11).

Now, let us inspect the neutrino mass matrix whose entries originate from the Feynman diagrams shown in Fig. 3. The first diagram gives the main contribution:

$$m_\nu^{FF} \sim \frac{\langle H \rangle^2}{M_N} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^2 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^4 \end{pmatrix}$$

$$\times \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix}. \tag{33}$$

The others are given as follows:

$$m_\nu^{FG} \sim \frac{\langle H \rangle^2}{M_N} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}$$

$$\times \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix}, \tag{34}$$

$$m_\nu^{GF} \sim (m_\nu^{FG})^T, \tag{35}$$
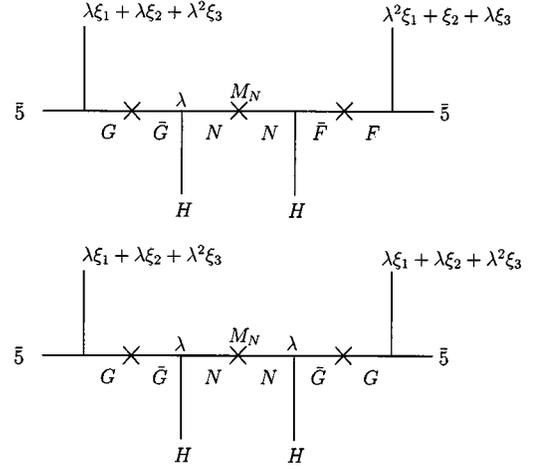
FIG. 3. The Feynman diagrams for the neutrino masses.

$$m_\nu^{GG} \sim \frac{\langle H \rangle^2}{M_N} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^5 & \lambda^4 & \lambda^4 \end{pmatrix}$$

$$\times \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix}. \tag{36}$$

The contributions from Eqs. (34–36) are small compared to m_ν^{FF} and can be safely ignored. The mass matrix $m_\nu \sim m_\nu^{FF}$ gives the bimaximal neutrino mixing as shown in the previous section.

One may think that the G contribution is not necessary if the F contribution gives the neutrino mass matrix suitably. However, the large mixing between the second and third generations does not occur in absence of the G fields. This issue is easily seen in the basis of Eq.(16). In this basis, the charged lepton mass matrix can be read off from Eq. (31) as follows:

$$Y_e'^F \sim \begin{pmatrix} \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^4 & \lambda & \lambda \end{pmatrix}. \tag{37}$$

TABLE II. The particle contents of the SO(10) model.

	SO(10)$_{\rm GUT}$	SU(3)$_H$	U(1)$_H$	U(1)$_{\rm PQ}$
16	**16**	**3**	0	1
H_{10}	**10**	**1**	$-11/2$	0
H_{16}	**16**	**1**	$-13/4$	0
$\bar{H}_{\overline{16}}$	$\overline{\bf 16}$	**1**	13/4	0
ξ_1	**1**	$\bar{\bf 3}$	11/4	-1
ξ_2	**1**	$\bar{\bf 3}$	3/4	-1
ξ_3	**1**	$\bar{\bf 3}$	$-1/4$	-1
$\bar{\xi}_1$	**1**	**3**	$-11/4$	1
$\bar{\xi}_2$	**1**	**3**	$-3/4$	1
$\bar{\xi}_3$	**1**	**3**	1/4	1
S	**16**	**1**	11/4	0
$\bar{S}$	$\overline{\bf 16}$	**1**	$-11/4$	0
J	**10**	**1**	1/2	0
χ	**16**	**1**	$-5/4$	0
$\bar{\chi}$	$\overline{\bf 16}$	**1**	5/4	0
Φ	**1**	**1**	1	0
$\bar{\Phi}$	**1**	**1**	-1	0

This charged lepton mass matrix and the neutrino mass matrix in Eq. (17) do not contain the large second and third generation mixing.

IV. SO(10) EMBEDDING

In this section, we construct the SO(10)$_{\rm GUT}$ model in which the matter fields are unified in a **16** representation. This matter unification is an attractive feature but this is the difficulty of the SO(10)$_{\rm GUT}$ model building simultaneously.

The minimal SO(10)$_{\rm GUT}$ model is not realistic because the Yukawa couplings for the up-type and down-type quarks coincide. To avoid this, we extend the Higgs sector to introduce the **16** representation Higgs field H_{16} whose SU(5) $\bar{\bf 5}$ component mixes with that of usual Higgs field H_{10}, i.e., the down-type quark and charged lepton masses are both given by the VEV of H_{16} and H_{10}. This situation can be realized by the introduction of χ and $\bar{\chi}$ fields which are **16** and $\overline{\bf 16}$ representations of SO(10), respectively, and break the SO(10) symmetry to SU(5) by the SU(5) singlet components acquire the VEV. This idea has been considered in Refs. [25] and [26]. We use this mechanism and we can reproduce the appropriate quark masses, quark mixings, charged lepton masses, neutrino masses and mixings for the large angle MSW solution by imposing the SU(3)×U(1) horizontal symmetry.

The particle content and their horizontal charges are listed in Table II. In this model, the SO(10)$_{\rm GUT}$ and U(1)$_H$ are broken spontaneously by the VEV's of the fields χ, $\bar{\chi}$, Φ, and $\bar{\Phi}$ at the order of $\lambda M_{\rm Pl}$. Below this scale, the SU(5)$_{\rm GUT}$×SU(3)$_H$ symmetry remains.

First, we consider the Higgs sector. The relevant superpotential for the Higgs fields are given by

$$W = M_H H_{16}\bar{H}_{\overline{16}} + \left(\frac{\Phi}{M_{\rm Pl}}\right)H_{10}\bar{H}_{\overline{16}}\bar{\chi}. \tag{38}$$

The breaking of SO(10)$_{\rm GUT}$ and U(1)$_H$ leads to the term $\lambda\langle\bar{\chi}\rangle H_{10}\bar{H}_{\overline{16}}$. If M_H is comparable to $\lambda\langle\bar{\chi}\rangle\sim\lambda^2 M_{\rm Pl}\sim10^{17}$ GeV, the SU(5) $\bar{\bf 5}$ component of H_{10} and that of H_{16} are strongly mixed, and the down-type quark and charged lepton masses are given by both H_{10} and H_{16}. This assumption of the Higgs mass scale is suitable for suppressing proton decay [27], although this mass scale is slightly larger than the ordinary grand unified scale, which is about 10^{16} GeV. We assume such a mass scale $M_H\sim10^{17}$ GeV.

The matter interactions are given as follows:

$$W_{\rm matter} = \left\{\xi_1 + \left(\frac{\Phi}{M_{\rm Pl}}\right)^2\xi_2 + \left(\frac{\Phi}{M_{\rm Pl}}\right)^3\xi_3\right\}\cdot 16\,\bar{S}$$
$$+ \frac{\chi}{M_{\rm Pl}}\left\{\left(\frac{\bar{\Phi}}{M_{\rm Pl}}\right)^2\xi_1 + \xi_2 + \left(\frac{\Phi}{M_{\rm Pl}}\right)\xi_3\right\}\cdot 16\,J. \tag{39}$$

Below the SO(10) breaking scale, $\chi/M_{\rm Pl}$, $\Phi/M_{\rm Pl}$, and $\bar{\Phi}/M_{\rm Pl}$ can be replaced with $\lambda\sim0.22$.

The Yukawa interaction terms of FN fields are given by

$$W_{\rm FN} = SSH_{10} + SJH_{16}. \tag{40}$$

The other nonrenormalizable interaction terms are small and they do not give the leading contribution to fermion masses and mixings. The mass terms of FN fields are given by

$$W_{\rm mass} = \left(\frac{\bar{\Phi}}{M_{\rm Pl}}\right)^2 SJ\chi + \left(\frac{\Phi}{M_{\rm Pl}}\right)\bar{S}J\bar{\chi} + M_*S\bar{S} + \left(\frac{\bar{\Phi}}{M_{\rm Pl}}\right)M_*JJ$$
$$+ \left(\frac{\bar{\Phi}}{M_{\rm Pl}}\right)^8 SS\bar{\chi}\bar{\chi} + \left(\frac{\Phi}{M_{\rm Pl}}\right)^8\bar{S}\bar{S}\chi\chi. \tag{41}$$

By the SO(10) breaking, the first two terms give the mass terms for SU(5) **5** component of $\bar{S}$ ($5_{\bar{S}}$) and J (5_J) and SU(5) $\bar{\bf 5}$ component of S ($\bar{5}_S$) and J ($\bar{5}_J$) as follows:

$$\lambda^3 M_{\rm Pl}\bar{5}_S 5_J + \lambda^2 M_{\rm Pl}5_{\bar{S}}\bar{5}_J. \tag{42}$$

If $M_*\sim M_{\rm GUT}$, the contribution of the next two terms in Eq. (41) is negligible compared to Eq. (42) and the mass eigenstates for the **5** and $\bar{\bf 5}$ FN fields are $(\bar{S}, J)$ and (J, S) pair. It is true for sufficiently small M_* ($M_*\lesssim M_{\rm GUT}$), but as we will show later small M_* leads to small Yukawa couplings for down-type quarks which needs small $\tan\beta$ ($M_*\gtrsim10^{15}$ GeV). The last two terms of Eq. (41) give the Majorana mass terms for SU(5) singlet FN fields.

Now we construct the Yukawa structures. The Yukawa couplings for the up-type quarks are given by the Feynman diagram in Fig. 4. Since this diagram is similar to the SU(5) case (Fig. 1), the desired Yukawa structure is given as in Eq. (29).
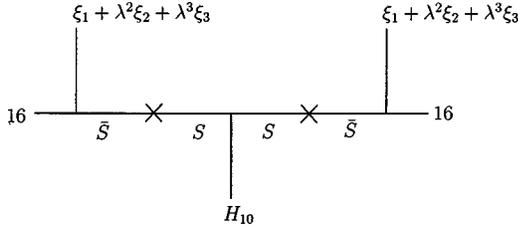
FIG. 4. The Feynman diagram for the up-type Yukawa interactions in the SO(10) model.

The down-type quark and the charged lepton Yukawa structures are given by the sum of the two contributions in Fig. 5. The first diagram gives the Yukawa coupling as follows:

$$Y_d^{(1)} = (Y_e^{(1)})^T \sim \frac{M_*}{\lambda^3 M_{\text{Pl}}} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle)$$

$$\times \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^6 \\ \lambda^4 & \lambda^3 & \lambda^5 \\ \lambda^2 & \lambda & \lambda^3 \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix} \bigg/ M_*^2. \quad (43)$$

Only with this matrix, it is the unwanted type as in the case of F contribution [Eq. (31)] in the $SU(5)_{\text{GUT}}$ model because this type does not give the maximal mixing for atmospheric neutrinos. However, by adding another contribution we can obtain the maximal mixing. It is given by the second diagram in Fig. 5 as follows:

$$Y_d^{(2)} = (Y_e^{(2)})^T \sim \frac{M_*}{\lambda^2 M_{\text{Pl}}} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle)$$

$$\times \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix} \bigg/ M_*^2. \quad (44)$$

Then, the Yukawa couplings for the down-type quarks and the charged leptons are given by

$$Y_d = Y_e^T \sim Y_d^{(1)} + Y_d^{(2)} \sim \frac{M_*}{\lambda^3 M_{\text{Pl}}} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle)$$

$$\times \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda & \lambda \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix} \bigg/ M_*^2. \quad (45)$$

This matrix has the same structure as the $SU(5)_{\text{GUT}}$ case and gives the correct mass ratios and mixings. The difference is the pre-factor $M_*/(\lambda^3 M_{\text{Pl}})$. The prediction for $\tan \beta$ depends on the FN scale M_* as follows:

$$\tan\beta \sim 16 \times \left(\frac{M_*}{M_{\text{GUT}}} \right). \quad (46)$$
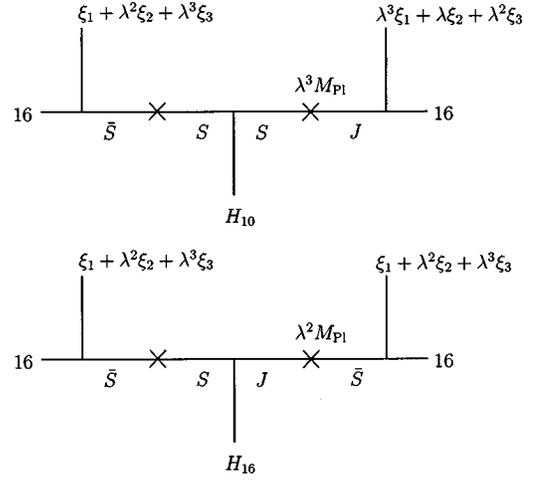
FIG. 5. The Feynman diagrams for the down-type quark and charged lepton Yukawa interactions in the SO(10) model.

The neutrino masses arise from a complicated mechanism. First, the 9×9 Majorana mass matrix for the SU(5) singlet components of S (1_S), $\bar{S}$ ($1_{\bar{S}}$), and 16 (1_{16}) are given by

$$M_{\text{Maj.}} = \begin{array}{c} \\ 1_S \\ 1_{\bar{S}} \\ 1_{16} \end{array} \begin{pmatrix} \overset{1_S}{\lambda^{10} M_{\text{Pl}}} & \overset{1_{\bar{S}}}{M_*} & \overset{1_{16}}{0} \\ M_* & \lambda^{10} M_{\text{Pl}} & M_\xi^T \\ 0 & M_\xi & 0 \end{pmatrix}, \quad (47)$$

where the submatrix M_ξ is the Dirac mass terms from the first line of Eq. (39):

$$M_\xi \sim M_* \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix}. \quad (48)$$

The (1_{16}, 1_{16}) components of Eq. (47) are not exactly zero but are negligible ($\sim \lambda^{10} M_*^2 / M_{\text{Pl}}$). The Majorana masses for the FN fields 1_S and $1_{\bar{S}}$ are of the order of M_* which is too large for the seesaw mechanism. However, the suitable Majorana masses for the 1_{16} fields are given by

$$M_R \sim \lambda^{10} M_{\text{Pl}} \cdot \frac{M_\xi M_\xi^T}{M_*^2} \sim \lambda^{10} M_{\text{Pl}} \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \quad (49)$$

The Dirac mass terms for the neutrinos are given by the Feynman diagram in Fig. 6. From this diagram, the structure of the Dirac mass is as follows:

$$m_D \sim \frac{M_* \langle H \rangle}{\lambda^2 M_{\rm Pl}} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda & \lambda & \lambda \\ 1 & 1 & 1 \\ \lambda^2 & \lambda^2 & \lambda^2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \Big/ M_*. \tag{50}$$

Then the neutrino masses originating from the seesaw mechanism turn out to be

$$m_\nu = m_D M_R^{-1} m_D^T \sim \frac{M_*^2 \langle H \rangle^2}{\lambda^{14} M_{\rm Pl}^3} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda & \lambda & \lambda \\ 1 & 1 & 1 \\ \lambda^2 & \lambda^2 & \lambda^2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda & 1 & \lambda^2 \\ \lambda & 1 & \lambda^2 \\ \lambda & 1 & \lambda^2 \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix} \Big/ M_*^2$$

$$\sim \frac{M_*^2 \langle H \rangle^2}{\lambda^{14} M_{\rm Pl}^3} (\langle \xi_3 \rangle \quad \langle \xi_2 \rangle \quad \langle \xi_1 \rangle) \begin{pmatrix} \lambda^2 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^4 \end{pmatrix} \begin{pmatrix} \langle \xi_3 \rangle^T \\ \langle \xi_2 \rangle^T \\ \langle \xi_1 \rangle^T \end{pmatrix} \Big/ M_*^2, \tag{51}$$

where the matrix whose components are all unity in the first line represents the following calculation as

$$\begin{pmatrix} \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^2 \\ 1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{52}$$

The matrix m_ν in Eq. (51) also has the same structure as the SU(5)$_{\rm GUT}$ model [Eq. (33)] which creates the suitable mass ratio and bimaximal mixing for the large angle MSW solution. The prefactor gives the appropriate order of magnitude for the masses.

The results of the mixing and mass ratio have been confirmed by the numerical analysis in which we diagonalize the whole mass matrix including FN fields.

V. CONCLUSIONS AND DISCUSSIONS

We have considered a horizontal SU(3)$_{\rm H}$ × U(1)$_{\rm H}$ symmetry to construct the appropriate mass matrices for the quarks and leptons. We concentrated on the large mixing angle MSW solution for the solar neutrino problem. The large mixing angle for $\nu_\mu \leftrightarrow \nu_\tau$ can be explained by the lopsided family structure. In such a building block, many models in the context of grand unified model predict that the ratio of mass squared differences $\Delta m_{\rm solar}^2 / \Delta m_{\rm atm}^2$ is similar to $(m_c/m_t)^4$ [26,28]; otherwise, fine tuning is needed to explain the hierarchy.
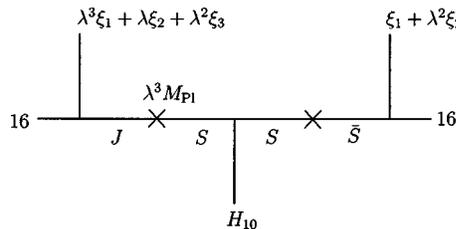
FIG. 6. The Feynman diagram for the neutrino Dirac masses in the SO(10) model.

In this paper, we have constructed realistic models in the context of grand unified models to explain the hierarchy naturally such that

$$\frac{\Delta m_{\rm solar}^2}{\Delta m_{\rm atm}^2} \sim 10^{-2}. \tag{53}$$

We emphasize that the unification of the three flavors enables us to reproduce a neutrino mass matrix consistent with the large angle MSW solution.

Horizontal unification provides interesting features in particle physics. In supersymmetric models, flavor unification is a motivation for the suppression of flavor changing neutral currents (FCNC) [29,30]. However, it is well known that the simple models with a gauged horizontal symmetry do not suppress the FCNC in the gravity-mediated supersymmetry breaking scenario [31].

It is interesting that the strong CP problem can be solved in our models. The VEV's of the ξ fields break the horizontal SU(3)$_{\rm H}$ symmetry. The ξ's have a Peccei-Quinn-like charge and the Peccei-Quinn mechanism will work well. The invisible axion will be created if the breaking scale M_* is of the order of 10^{12} GeV. The assumption of the horizontal scale ($M_* \sim 10^{12}$ GeV) does not lead to gauge coupling divergence below the GUT scale, which is caused by the existence of the vectorlike FN fields at the scale M_*. Another attractive scale for M_* is the GUT scale. In the SO(10) unified

model, the scale M_* is constrained to be 10^{15}–10^{16} GeV. If the scale M_* is at the GUT scale, too many axions will be created and our universe will be overclosed. In such a case, we need the axions to be diluted by the inflation of the universe.

In our formulation, the horizontal unification of $SO(3)_H$ instead of $SU(3)_H$ may be also available. The $SU(3)_H$ unification has the particularly interesting feature that the representation of $SU(3)$ is complex and spontaneous CP violation can occur [32]. The origin of the KM phase might be in the fundamental representation of ξ in our model.

[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1964); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[2] D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[3] SuperKamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998).

[4] SuperKamiokande Collaboration, T. Toshito, talk given at the International Conference on High Energy Physics, Osaka, Japan, 2000.

[5] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); S. P. Mikheev and A. Y. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; Nuovo Cimento C **9**, 17 (1986).

[6] SuperKamiokande Collaboration, Y. Suzuki, a talk given in the 19th International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada (2000).

[7] M. Apollonio *et al.*, Phys. Lett. B **466**, 415 (1999).

[8] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).

[9] M. Leurer, Y. Nir, and N. Seiberg, Nucl. Phys. **B420**, 468 (1994); L. Ibanez and G. G. Ross, Phys. Lett. B **332**, 100 (1994); Y. Grossman and Y. Nir, Nucl. Phys. **B448**, 30 (1995); E. Dudas, S. Pokorski, and C. A. Savoy, Phys. Lett. B **356**, 45 (1995); P. Binetruy, S. Lavignac, and P. Ramond, Nucl. Phys. **B477**, 353 (1996); P. Binetruy, S. Lavignac, S. Petcov, and P. Ramond, *ibid.* **B496**, 3 (1997); A. E. Nelson and D. Wright, Phys. Rev. D **56**, 1598 (1997); N. Irges, S. Lavignac, and P. Ramond, *ibid.* **58**, 035003 (1998); J. K. Elwood, N. Irges, and P. Ramond, Phys. Rev. Lett. **81**, 5064 (1998); Y. Grossman, Y. Nir, and Y. Shadmi, J. High Energy Phys. **10**, 007 (1998); G. Altarelli and F. Feruglio, *ibid.* **11**, 021 (1998); K. Choi, K. Hwang, and E. J. Chun, Phys. Rev. D **60**, 031301 (1999); M. Bando and T. Kugo, Prog. Theor. Phys. **101**, 1313 (1999); S. Lola and G. G. Ross, Nucl. Phys. **B553**, 81 (1999); Y. Nir and Y. Shadmi, J. High Energy Phys. **05**, 023 (1999); S. F. King, Nucl. Phys. **B562**, 57 (1999); S. F. King, *ibid.* **B576**, 85 (2000); J. Ellis, G. K. Leontaris, and J. Rizos, J. High Energy Phys. **05**, 001 (2000); C. H. Albright, K. S. Babu, and S. M. Barr, Phys. Rev. Lett. **81**, 1167 (1998); Q. Shafi and Z. Tavartkiladze, Phys. Lett. B **451**, 129 (1999); Q. Shafi and Z. Tavartkiladze, *ibid.* **487**, 145 (2000).

[10] F. Wilczek and A. Zee, Phys. Rev. Lett. **42**, 421 (1979); Z. G. Berezhiani and D. L. Chkareuli, Yad. Fiz. **37**, 1043 (1983) [Sov. J. Nucl. Phys. **37**, 618 (1983)]; R. Barbieri, L. J. Hall, S. Raby, and A. Romanino, Nucl. Phys. **B493**, 3 (1997); G. Eyal, Phys. Lett. B **441**, 191 (1998); T. Blazek, S. Raby, and K. Tobe, Phys. Rev. D **60**, 113001 (1999); M. Tanimoto, T. Watari, and T. Yanagida, Phys. Lett. B **461**, 345 (1999); R. Der-

misek and S. Raby, Phys. Rev. D **62**, 015007 (2000); T. Blazek, S. Raby, and K. Tobe, *ibid.* **62**, 055001 (2000).

[11] H. Fritzsch, Phys. Lett. **73B**, 317 (1978); Nucl. Phys. **B115**, 189 (1979).

[12] G. C. Branco and J. I. Silva-Macros, Phys. Lett. B **331**, 390 (1994).

[13] J. Sato and T. Yanagida, Phys. Lett. B **430**, 127 (1998); T. Yanagida and J. Sato, Nucl. Phys. B (Proc. Suppl.) **77**, 293 (1999).

[14] C. H. Albright and S. M. Barr, Phys. Lett. B **452**, 287 (1999).

[15] E. Witten, Nucl. Phys. **B188**, 513 (1981); S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981); N. Sakai, Z. Phys. C **11**, 153 (1981).

[16] T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe*, edited by O. Sawada and A. Sugamoto (KEK, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979).

[17] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).

[18] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[19] J. A. Harvey, D. B. Reiss, and P. Ramond, Nucl. Phys. **B199**, 223 (1982).

[20] F. Wilczek, Phys. Rev. Lett. **49**, 1549 (1982); G. B. Gelmini, S. Nussinov, and T. Yanagida, Nucl. Phys. **B219**, 31 (1983); J. L. Feng, T. Moroi, H. Murayama, and E. Schnapka, Phys. Rev. D **57**, 5875 (1998).

[21] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. D **16**, 1791 (1977).

[22] J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979); M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B166**, 493 (1980).

[23] A. R. Zhitnitsky, Yad. Fiz. **31**, 497 (1980) [Sov. J. Nucl. Phys. **31**, 260 (1980)]; M. Dine and W. Fischler, Phys. Lett. **120B**, 137 (1983).

[24] A. H. Guth, Phys. Rev. D **23**, 347 (1981).

[25] C. H. Albright, K. S. Babu, and S. M. Barr, Phys. Rev. Lett. **81**, 1167 (1998); K. S. Babu, J. C. Pati, and F. Wilczek, Nucl. Phys. **B566**, 33 (2000); Y. Nomura and T. Sugimoto, Phys. Rev. D **61**, 093003 (2000).

[26] Y. Nomura and T. Yanagida, Phys. Rev. D **59**, 017303 (1999).

[27] V. Lucas and S. Raby, Phys. Rev. D **55**, 6986 (1997); T. Goto and T. Nihei, *ibid.* **59**, 115009 (1999).

[28] S. A. Bludman, D. C. Kennedy, and P. G. Langacker, Phys.

Rev. D **45**, 1810 (1992); V. Barger, S. Pakvasa, T. J. Weiler, and K. Whisnant, Phys. Lett. B **437**, 107 (1998); C. H. Albright and S. M. Barr, *ibid.* **461**, 218 (1999).

[29] J. Ellis and D. V. Nanopoulos, Phys. Lett. **110B**, 44 (1982).

[30] Y. Nir and N. Seiberg, Phys. Lett. B **309**, 337 (1993); P. Pouliot and N. Seiberg, *ibid.* **318**, 169 (1993); R. Barbieri, G. Dvali, and L. J. Hall, *ibid.* **377**, 76 (1996).

[31] M. Dine, R. Leigh, and A. Kagan, Phys. Rev. D **48**, 4269 (1993); K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **83**, 2522 (1999).

[32] T. D. Lee, Phys. Rev. D **8**, 1226 (1973); S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976); G. C. Branco, Phys. Rev. D **22**, 2901 (1980).